SCALING RELATIONS AMONG (p,nx) and (p,px) CONTINUOUS SPECTRA AT INTERMEDIATE ENERGIES

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Abstract- *If one assumes a simple single-scattering model for the emission of low energy loss nucleons from intermediate energy proton beams on nuclear targets, scaling relations can be derived that should relate data for a range of beam energies, angles and nuclear masses. Two such systems are examined for (p,nx) reactions on heavy nuclei, with the goal of providing means to intercompare data, make interpolations and allow modest extrapolations of very difficult data sets.*

I. INTRODUCTION

Collisions of intermediate energy protons with thin samples will yield continuous spectra of outgoing protons or neutrons, with maximum energies at forward angles near the beam energies. These energetic products will have enough energy to induce a wide range of secondary reactions and beyond in thick samples of interest for many applications of accelerator-driven neutron sources. Experimental determination of the cross sections, especially for the neutrons, is quite difficult, and so some reliable means of intercomparing the few reliable measurements is desired. This is accomplished here by means of 'scaling' relations, using fewer kinematic variables than the most general. Two ways to combine measured energy losses ω and three-momentum transfers q are presented here, each forming a single scaling variable, called y or x_B. The many complications arising for strongly interacting particles and nuclei are treated by assuming one and only one collision of the beam particle with a bound nucleon, using a simple eikonal or Glauber method to count the probabilities for this simplest process.. Only if the measured continuum cross sections give the same transformed responses under these conditions can scaling be claimed. There are many examples of such scaling for the much simpler cases of continuum scattering of electrons from nuclei. [1,2] Cases of heavy nuclear targets will be emphasized in the work reported here, since these are of greatest interest for neutron sources.

II. METHODS

One scaling system defines y as the component along the vector of q of the momentum of the struck bound nucleon by y= sqrt(ω(ω+2M)) -q, with lab frame energy losses ω and momentum transfers q, with M as the free nucleon mass. Analyses of all published proton-induced inclusive (p,px) and (p,nx) reactions in this format have been published, together with meson-induced scattering. These have been presented for scattering, without charge exchange (NCX)[3], and for single charge exchange (SCX),[4] where details can be found. Scattering from a free nucleon is found at y=0 in this system, and negative values of y (for head on collisions) lead to lower energy losses or higher ejectile energies. Figure 1 shows an example for Pb(p,nx) at four proton energies and several small angles.[5-8] At the single energy of 800 MeV most of the data agree for y<0, where the approximations are most appropriate. This is called 'y-scaling of the first kind' for electrons[2]. The curve connects the y-responses for Pb(p,px) at 13 deg, [10] showing a close similarity to the neutron data for some range of y. Although successful at a single energy, these examples show that no y-scaling of the neutron spectra is found for a wide range of proton energies.

More recently, electron scattering studies[9] have shown that the low energy loss regions of nuclear continuous spectra show simple behavior with the Bjorken scaling variable, $x_B=(q^2-\omega^2)/2M\omega$, with $x_B=1$ for free scattering (y=0) and $x_B>1$ for lower energy losses than would be possible for free scattering on a nucleon of mass M. This variable measures the fraction of the total momentum of the entire target nucleus held by the one struck nucleon. If high momentum results from hard intranuclear collisions, for instance, significant yields of scattered electrons are found out to $x_B=2.5$. [9] This is kinematically as if the incident projectile had struck a single object with 2.5 times the free nucleon mass.

Measured inclusive spectra are transformed in this Bjorken scaling system as

$$F_B = d^2\sigma/d\Omega d\omega \ \ 2M\omega^2 / A_{eff} \ \ d\sigma/d\Omega(\text{free}) \ (q^2 + \omega^2),$$

using the Jacobian for the change of kinematic variable, with $d\sigma/d\Omega$ the differential cross section for free proton-nucleon elastic scattering, [11] evaluated in the ‘optimum reference frame’ to minimize the off shell effects of not following true free scattering. [12] In practice, Coulomb force effects are included in q, and nuclear binding energies (and Coulomb energies for SCX) are used to adjust ω. The number of nucleons A_{eff} seen once and only once for NCX is computed by the Glauber double integral, [13] using known distributions of nucleons within nuclei [14] and total cross sections for the collision between the proton beam and bound nucleons at 70% of their free space values at each energy. [11] These smaller in-medium cross sections are expected theoretically [15] and found to be required for hadron scattering to match the scaling observations from electron scattering.[16] These same techniques and parameters were used for the extensive hadronic y-scaling work. [3,4] A wide range of hadronic NCX spectra has now been compared in this Bjorken fashion,[17] and here some results for (p,nx) SCX spectra will be presented. In this case, the number of neutrons N_{eff} to be seen once and only once is computed from $N_{eff}=N\ A_{eff}\ /\ A$. Intermediate energy nucleons have large cross sections, and only nucleons in the nuclear surface are available to be sensed once and only once, with $N_{eff}=7.20$ out of the 126 neutrons in lead at an energy of 800 MeV.

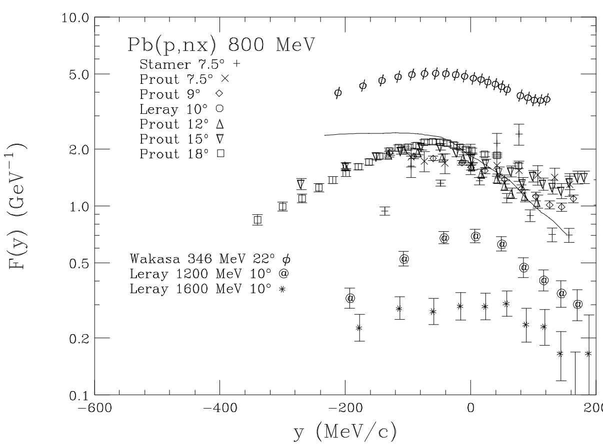

Fig. 1. A wide range of inclusive (p,nx) spectra arising from proton beams on thin lead samples [5,6,7,8] gives the y-scaling responses shown. Momentum transfers range from 190 to 450 MeV/c for these data.

III. EXAMPLES

Figure 2 shows the same Pb(p,nx) spectra as shown in Fig. 1 in the Bjorken format, with a very ambitious scale of x_B. For 800 MeV protons at 10 deg and $x_B=4$, the outgoing neutron energy is 740 MeV, for instance. The solid line again traces the same Bjorken response at 800 MeV and 13 deg for lead for the (p,px) reaction, with a very reliable and accurate measuring system.[10,17] Even at the single 800 MeV energy, x_B-scaling of the first kind is not observed.

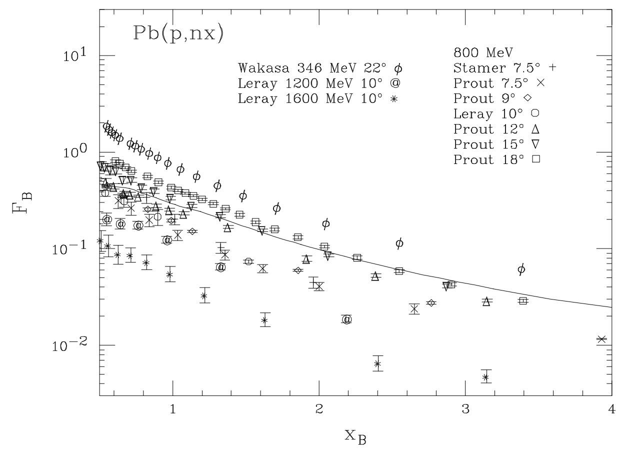

Fig. 2. The same data used in Fig. 1 are shown as Bjorken scaling responses. The agreement among y-scaling responses at 800 MeV is lost in this system. We do note similar shapes for the responses at all energies.

Using the more successful y-scaling system, Fig. 3 extends the comparisons of (p,nx) responses to larger angles, where the kinematics allow a larger coverage in y. Figure 3 shows concurrence among some data sets at negative y, with the striking exception of one data set[18] at 800 MeV, and with an apparent change at the highest energy of 1600 MeV.[8] One of the virtues of a successful scaling comparisons could be an awareness of suspicious data. The solid curve shows the Pb(p,px) response at 25deg.

In electron scattering, the fact that all nuclei give the same y-scaling responses for any given kinematic situation is called scaling of the second kind. [2] For hadrons, this tests the means to compute N_{eff} . Fig. 4 shows 1200 MeV [8] and 597 MeV [19] (p,nx) spectra for a wide range of nuclei, exhibiting good scaling of the second kind also for hadrons. It is also to be noted that these responses are very similar at these two energies.

It is the heaviest nuclei that are of greatest interest for accelerator-driven neutron sources, so Fig. 5 combines spectra for four beam energies and four nuclei at nearly the same angles. Data are from experiments [8,18,19] using both time of flight and proton recoil methods. Quite good agreement is found except for the 800 MeV experiment of Amian et al. [18] This figure illustrates another advantage of a scaling system, in that discrepancies can be made evident, and data sets evaluated for reliability.

Fig. 5 shows that one could interpolate in beam energy or among heavy nuclei to approximate neutron doubly differential cross sections, by inverting the equation above, to an accuracy of about 30%, with the limit being the statistical uncertainty in the measured cross sections for high energy neutrons.

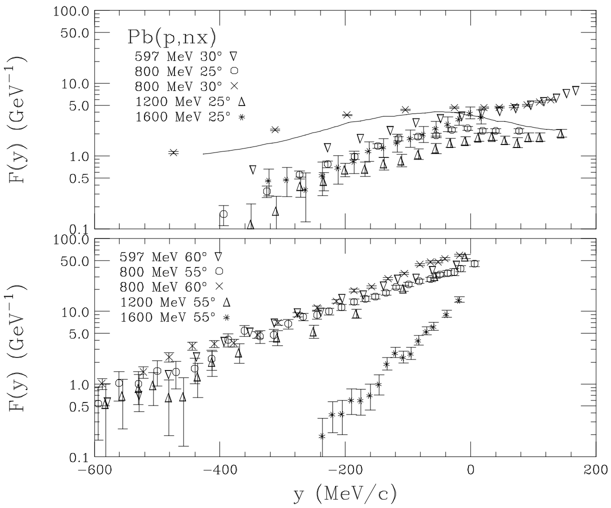

Fig. 3. Y-scaling responses for Pb(p,nx) are shown for larger angles than for Fig. 1, with momentum transfers q from 600 to 1000 MeV/c and from 1000 to 2000 MeV/c.

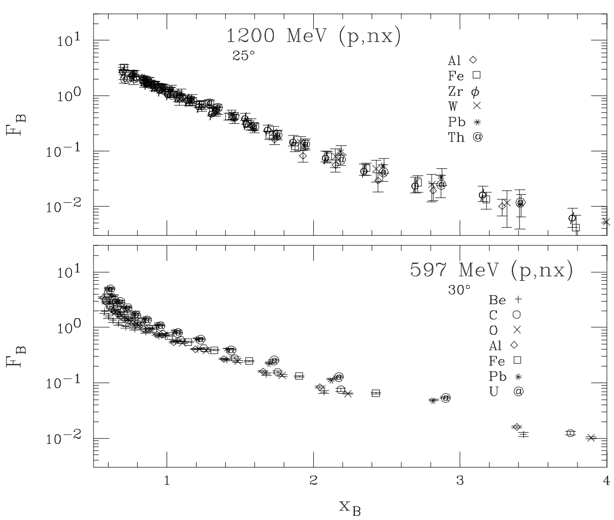

Fig. 4. Scaling of 'the second kind' is that in which any nucleus gives the same response under the same scattering conditions; this tests the N_{eff} approximations for SCX. The two examples shown here, for a wide range of nuclear masses, demonstrate the validity of this scaling for (p,nx) reactions at intermediate energies. Proton-nucleon total cross sections within the nuclear medium are taken at 70% of their free space values to compute N_{eff}.

An overall review of 'scaling of the second kind' is shown in Fig. 6, with interpolated responses at x_B=2 and 3 for a number of beam energies at momentum transfers from 500 to 900 MeV/c. Each data set is quite flat, demonstrating this scaling, save for the Amian 800 MeV experiment. [18] Evidently, the Glauber method with in-medium total cross sections may be relied upon for the mass-dependence of low energy loss hadronic spectra at intermediate energies over a very wide range of nuclear masses.

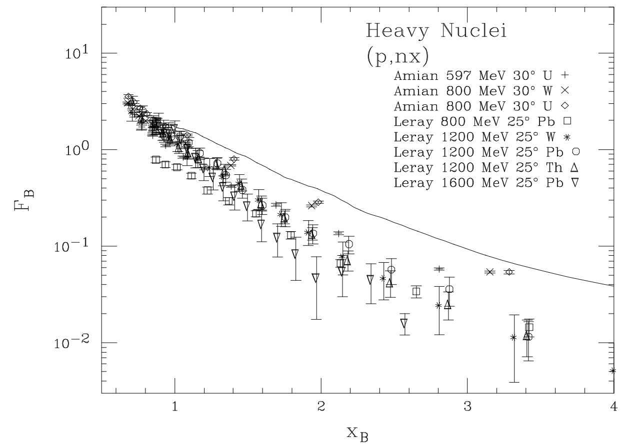

Fig.5. Heavy nuclei are preferred for the production of neutrons by intermediate energy protons, so several examples at four beam energies are shown here,[8,18,19] with good agreement with the Bjorken scaling system, with the exception of one data set.[18] The curve shows the 800 MeV response for Pb(p,px) at 30 deg.

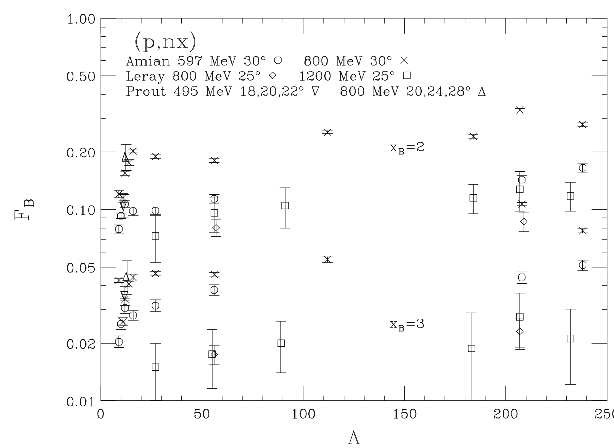

Fig. 6. A summary of scaling of the second kind is made by interpolating Bjorken responses for many SCX reactions. Save for one data set,[18] near scaling is found for x_B=2 and 3, with momentum transfers for the reactions ranging from 500 to 800 MeV/c.

IV. CONCLUSIONS

Of the two scaling systems widely used for electron inclusive scattering, that for y (or ψ') scaling [1] succeeds for 800 MeV Pb(p,nx) spectra at small angles, as shown in Fig. 1, although other beam energies fail to scale with these data. A reasonable interpolation system might be attempted for other proton beam energies. This system finds closer agreement among spectra arising from several beam energies as the angle is increased, shown in Fig. 3.

The Bjorken scaling variable[9] gives another organizing principle for inclusive hadron NCX spectra[17] and for the SCX spectra shown here. This system shows poorer scaling than does the y-system at small angles, noted in comparing Figs. 1 and 2. Scaling of the second kind, using effective nucleon numbers computed in the Glauber model with reduced proton-nucleon total cross sections in the nuclear medium, works very well, for many more cases than shown here in Fig. 4. This enables reliable extensions of scarce data for high outgoing neutron energies to any nuclear target in either y or x_B scaling.

Bjorken scaling of the first kind, at a fixed proton energy, is not found (Fig. 2), but the smooth trends with angle could enable reasonable interpolations. Inclusive (p,nx) responses are found to lie somewhat below those for comparable (p,px) studies, but too few cases of matching data are available to form a complete comparison. The smooth behaviors of Bjorken or y responses enable one to note examples of data sets that seem exceptional, and may thus allow critical assessments of difficult and important experiments. Further, the expectations of simple scaling systems may be of use to compare the outputs of large INC codes, often not in agreement with the highest energy ranges of neutron experiments.

A more complete study of hadronic SCX Bjorken scaling responses, including pion SCX, has been submitted for publication. A detailed exposition of the methods and applications of the scaling systems for accelerator-driven neutron sources is in preparation.

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