On Some Modifications of the Marchuk's Multigroup Method

Boris D. Abramov
Institute of Physics and Power Engineering,
249020 , Bondarenko Sq.1,Obninsk, Kaluga Region, Russia
E-mail: abbn@ippe.obninsk.ru

ABSTRACT

Approximate methods for solving inhomogeneous boundary-value problems of the nuclear reactor theory are considered. The methods develop the idea of Marchuk's well-known multigroup method. Existence and uniqueness theorems for the corresponding nonlinear equations are proved.

INTRODUCTION

A large amount of work has been done on the development and proof of various models of a multi-group and homogenization methods, and these problems have to a considerable extend been solved [1-3]. However, there are some important mathematical questions concerning the choice of particular formulations of problems of this kind, proof of the existence and uniqueness theorems for the solutions of the corresponding linear and non-linear equations, the investigation of the properties of the approximation and the convergence of appropriate numerical techniques for finding them etc, which still appear to be unanswered.

In this connection, some well-known multigroup methods for solving homogeneous conditionally critical reactor equations have been examined recently, and the corresponding existence theorems have been established [4,5]. In our study, we examine similar aspects concerning existence, uniqueness, and solution techniques for nonlinear equations underlying certain modifications [2,6] of Marchuk's well-known multigroup method (see [1]) for inhomogeneous problems in neutron transport theory.

THE ORIGINAL PROBLEM

1. Consider an inhomogeneous boundary value problem for the neutron transport equation

$$\Omega\nabla y + Cy = Q \qquad (1?)$$

in the domain $G \in R^3$ with the boundary condition

$$y(x,E,\Omega) = f(x,E,\Omega), \quad \Omega n(x) < 0, \quad x \in \Gamma \qquad (1b)$$

on its surface ? of the reactor volume *G*, where $y(x,E,\Omega)$ is the density of neutrons at the point $x \in G \subset R^3$ which have energy $E \in [\underline{E}, \overline{E}]$ and direction of flight $\Omega \in S^1$, S^1 is the unit sphere in R^3, $[\underline{E}, \overline{E}]$ is the interval of admissible values of energy, f, Q are known functions, *n(x)* is the external normal to *G*, *Wn(x)* is the scalar product of vectors in R^3, $? = \Sigma - K$, $K = K_s + K_f$,

$$K_b y = \int dE' \int d\Omega' w_b(x,E,E',\Omega,\Omega') y(x,E',\Omega'), \quad b = s, f \quad . \qquad (2)$$

Note that the regime of exposure of G to neutrons from outside is determined by the function f in(1b) and is fixed. Therefore, the migration of neutrons (in the case of nonconvex domains).from G to

G through spatial regions non belonging to G id, in general, not taken into account. However, this can be done, for example, by adding the corresponding spatial regions to G , and so on [7-9].

Under conventional conditions, a solution to problem (1) is known to exist and can be found , for example, by the method of successive approximations. However, in practice, by virtue of the very complicated (and often unreliably determined) dependence of $\Sigma(x,E)$, $w_b(x,E,E',\Omega,\Omega')$ on the corresponding variables, one usually solves, instead of (1), a simplified equation

$$\Omega\nabla\tilde{y}+\tilde{;;}\,\tilde{y}=Q \qquad (3?)$$

with the same boundary condition

$$\tilde{y}(x,E,\Omega)=f(x,E,\Omega),\quad \Omega n(x)<0,\quad x\in\Gamma, \qquad (3b)$$

which is derived from (1) and (2) by replacing Σ, w_b by some other functions $\tilde{\Sigma},\tilde{w}_b$, that are, as a rule, piecewise-constant with respect to the spatial coordinates and energy. The value of these functions, called the group homogenized constants, are usually chosen on the basis of any proximity conditions of problems (1), (3), such as that the most important integral characteristics of the reactor are preserved in the change from the original model (1) to the approximate model (3), and so on.

2. There is an extensive literature on different ways of choosing the coefficients $\tilde{\Sigma},\tilde{w}_b$ (see[1-5]) Consider some of them. Let be $Y=G\times[\underline{E},\overline{E}]\times S^1$; $\Gamma_\pm=\{x,E,\Omega\in\Gamma\times[\underline{E},\overline{E}]\times S^1:\Omega n(x)\gtrless 0\}$; (,) denote integration in $x,E,\Omega\in Y$; $(\ ,\ )_\pm$ stand for integration over $\Gamma_\pm$ with the weight $|\Omega n(x)|$ and y^* be a solution to the equation

$$-\Omega\nabla y^*+C^*y^*=Q^*,\quad C^*=\Sigma-K^* \qquad (4?)$$

(where C^* is the adjoint operator of C (in the sense of Lagrange)) with the boundary condition

$$y^*(x,E,\Omega)=f^*(x,E,\Omega),\quad \Omega n(x)>0,\quad x\in ?, \qquad (4b)$$

and $\tilde{y}^*$ be a solution to the corresponding multigroup equation

$$-\Omega\nabla\tilde{y}^*+\tilde{C}^*\tilde{y}^*=Q^*,\quad \tilde{C}^*=\tilde{\Sigma}-\tilde{K}^* \qquad (5?)$$

with a similar boundary condition

$$\tilde{y}^*(x,E,\Omega)=f^*(x,E,\Omega),\quad \Omega n(x)>0,\quad x\in\Gamma. \qquad (5b)$$

The problem is to calculate the functional (Q^*,y) or $(f^*,y)_+$ It is well known [1-3] that they can either be found directly (i.e. by solving the original problem (1)); or by the formulas

$$(Q^*,y)=(y^*,Q)+(y^*,f)_-,\quad f^*=0, \qquad (6?)$$

$$(f^*,y)_+=(y^*,Q)+(y^*,f)_-,\quad Q^*=0, \qquad (6b)$$

which follow from Eqs. (1),(4) (when $f^*=0$ or $Q^*=0$, respectively) by virtue of the relation $(1,\Omega\nabla y^*y)=(f^*,y)_+-(y^*,f)_-=(y^*,Q)-(Q^*,y)$; or by the formulas

$$(Q^*,y)=(Q^*,\tilde{y}),\quad f^*=0, \qquad (7?)$$

$$(f^*,y)_+=(f^*,\tilde{y})_+,\quad Q^*=0, \qquad (7b)$$

which follow in a similar manner from Eqs. (3),(4) when the group homogenized constants (the coefficients of the operator $\tilde{C}$) are determined by the condition

$$(y^*, \tilde{C}\tilde{y}) = (\tilde{y}, C^* y^*)\,; \qquad (8)$$

or by the formulas

$$(Q^*, y) = (\tilde{y}^*, Q) + (\tilde{y}^*, f)_-, \quad f^* = 0, \qquad (9?)$$

$$(f^*, y)_+ = (\tilde{y}^*, Q) + (\tilde{y}^*, f)_-, \quad Q^* = 0, \qquad (9b)$$

which follows from Eqs. (1),(5) when $\tilde{C}^*$ is determined by the condition

$$(\tilde{y}^*, Cy) = (y, \tilde{C}^* \tilde{y}^*)\,. \qquad (10)$$

Moreover, in the framework of the last two approaches (7)-(8) and (9)-(10), the desired functionals $(Q^*, y), (f^*, y)_+$ are calculated from the solutions $\tilde{y}, \tilde{y}^*$ to the model multigroup problems(3),(5).

3. Let us define more exactly the statements of the problem. Suppose that the domain G is divided into N subdomains (homogenized zones) $G_n, n = 1,2,..., N < \infty,$ the range $(\underline{E}, \overline{E})$ is subdivided into I subranges (energy groups) $(E_i, E_{i-1}), i = 1,2,..., I < \infty,$ and the unite sphere S^1 is subdivided into $\bar{l}$ segments $S_l^1, l = 1,2,..., \bar{l} < \infty$. Choosing

$$\tilde{\Sigma}(x, E) = \Sigma_n^i, \quad \tilde{w}_b(x, E, E', \Omega, \Omega') = h(E) w_{bn}^{jm \to il} g(E') / < gh >^i \qquad (11)$$

for $x \in G_n,\ E \in (E_i, E_{i-1}),\ E' \in (E_j, E_{j-1}),\ \Omega \in S_l^1,\ \Omega' \in S_m^1$, we determine the constants $\Sigma_n^i, w_{bn}^{jm \to il}$ from condition.(8), setting

$$\Sigma_n^i = \frac{(y^* q_n^i, \Sigma \tilde{y})}{(y^* q_n^i, \tilde{y})}, \quad w_{bn}^{jm \to il} = \frac{(y^* q_n^{il}, K_b q_n^{jm} \tilde{y})}{(y^* q_n^{il}, K^o q_n^{jm} \tilde{y})}, \qquad (12)$$

or, respectively, from condition (10), setting

$$\Sigma_n^i = \frac{(\tilde{y}^* q_n^i, \Sigma y)}{(\tilde{y}^* q_n^i, y)}, \quad w_{bn}^{jm \to il} = \frac{(\tilde{y}^* q_n^{il}, K_b q_n^{jm} y)}{(\tilde{y}^* q_n^{il}, K^o q_n^{jm} y)}, \qquad (13)$$

where $q_n(x), q_i(E), q_l(\Omega)$ are the characteristic functions of the sets $G_n, (E_i, E_{i-1}), S_l^1,$ $q_n^i(x, E) = q_n(x) q_i(E),\ q_n^{il}(x, E, \Omega) = q_n^i(x, E) q_l(\Omega);$ $g(E) \ge 0$ and $h(E) \ge 0$ are given weight functions chosen arbitrarily; and where for $E \in (E_i, E_{i-1})$

$$K^o y = \int dE' \frac{h(E) g(E')}{< gh >^i} \int d\Omega' y(x, E', \Omega'), \quad < gh >^i = \int_{E_i}^{E_{i-1}} dE g(E) h(E) > 0\,. \qquad (14)$$

In this approach, the calculation of the functionals (Q^*, y) and $(f^*, y)_+$ is either reduces to the solution of nonlinear problem (3) with coefficients (11),(12), which depend on a priory unknown functions $\tilde{y}$, y^*, or to the solution of nonlinear problem (5) with coefficients (11), (13), which depend in a similar way on a priory unknown function $\tilde{y}^*$ and y .

4. Note that the multigroup approach formulated extends the well-known approaches described in [1,2] by introducing the weight functions $h(E), g(E)$, which model the spectra of the density y and of the importance (danger) y^* for neutrons within the group intervals. Moreover, the above approach differs from those in [1,2] by the method of approximating the scattering indicatrix in segments (directions). Unlike the usually employed expansion in the Legendre polynomials, this method ensures that $\tilde{w}_b(x,E,E',\Omega,\Omega')$ is positive. In the special case of $\bar{l}=1$, the scheme given by (3),(11),(12) is an analogue of scheme (2.50),(2.60) with anisotropic scattering in [2]. We restrict ourselves primarily to the analysis of its dual scheme (5),(11),(13) under the additional assumptions

$$u(x,E,\Omega) = g(E)\sum_i q_i(E)\,\breve{u}_i(x,\Omega)/<gh>^i, \quad u = Q^*, f^*, \tag{15}$$

Note that equation (5) can be transformed to the system of multigroup equations

$$(-\Omega\nabla + \Sigma^i)\breve{y}_i^* = \sum_{mj} w^{il\to jm} \frac{<gh>^i}{<gh>^j} \int_{S_m^1} d\Omega' \breve{y}_j^*(x,\Omega') + \breve{Q}_i^* \tag{16}$$

with conditions $\breve{y}_i^*(x,\Omega) = \breve{f}_i^*(x,\Omega),\ \Omega n(x) > 0,\ x\in\Gamma$, where the zone index is omitted, $w = w_s + w_f$, $\Omega\in S_l^1$, $\breve{y}_i^* = <h\tilde{y}^*>^i$, $\breve{f}_i^* = <hf^*>^i$, $\breve{Q}_i^* = <hQ^*>^i$ and (in the case (15))

$$\tilde{y}^*(x,E,\Omega) = g(E)\sum_i q_i(E)\breve{y}_i^*(x,\Omega)/<gh>^i. \tag{17}$$

ADDITIONAL INFORMATION

5. Let us refine the statement of the problem. First, we formulate the requirements to the coefficients of the original equation (1). Following [8,9], we set

$$w_{b'}(x,E,E',\Omega,\Omega') = \sum_l (E'/E)^{1/2} n_{b'l}(E')\Sigma_{b'l}(x,E')W_{b'l}(E',E,\Omega',\Omega)$$

where $n_{b'l}(E),\ W_{b'l}(E',E,\Omega',\Omega)$ is the number of secondary neutrons formed in one reaction of type b' of a neutron with the nucleus of the nuclide l, and the probability density of their distribution with respect to energies E and flight directions Ω; $\Sigma_{b'l} = N_l(x)s_{b'l}(E')$ is the macroscopic cross-section of that reaction, $N_l(x)$ is the density of nuclei of the nuclide l; $s_{b'l}(E')$ is the microscopic cross-section,

$$w_s = \sum_{b'\neq c,f} w_{b'}, \quad \int dE\int W_{b'l}(E',E,\Omega',\Omega)d\Omega = 1, \quad \Sigma(x,E) = \sum_{b'}\sum_l \Sigma_{b'l}(x,E).$$

The summation is over all nuclides l and all processes b' of interaction between neutrons and nuclei: radiation capture $(b'=c)$, elastic scattering $(b'=e)$, fission $b'=f$, and so on.

It is assumed that $n_{b'l}$, $s_{b'l}$, $W_{b'l}$, N_l satisfy conditions 3.1 in [8], which refer a number of the most general regularities of the interaction between neutrons and nuclei in thermal motion, such as

$$0 < a \le E^{1/2}\Sigma(x,E) \le g < \infty, \quad W_{b'l}(E',E,\Omega',\Omega) \le C_1(1-m_0^2)^{-1/2}, \tag{18?}$$

$$W_{b'l}(E',E,\Omega',\Omega)/\sqrt{E} \ge c > 0, \quad 0 < a \le E^{1/2}s_{b'l}(E) \le c_1 < \infty, \tag{18b}$$

$$0 < an_0 < E^{1/2} n_{b'l}(E) s_{b'l}(E) \le n^{(0)} c_1 < \infty, \qquad (18c)$$

where the multiplier $1/\sqrt{1-m_0^2}$ in (18a) is omitted if $b' \ne e$; the bounds (18b,c) extend only to those nuclides on which reactions of the given type actually occur (otherwise, it is assumed that $s_{b'l}(E) \equiv 0$); $s_{el}(E) > 0$ for all nuclides without exception; $m_o = \Omega\Omega'$; $n_{el}(E) = 1$, and so on.

6. Following [4-9], the subdomain G_n (or, omitting the index n, the domain G itself) is any open connected measurable subset $G_n \subseteq G$ with a piecewise smooth boundary Γ_n of the class $C^{(1)}$ such that almost all (with respect to the measure in $S^1 \times R^2$) lines $x_o + \Omega t$, $x_o \in p_\Omega$, $t \in (-\infty, \infty)$ having a common point with G_n intersect G_n in a finite number $\tilde{N}_n$ of intervals (t_{nk}^-, t_{nk}^+), where $p_{\Omega n}, x_o$ are the orthogonal projection of G_n and a vector $x \in G$ onto the plane perpendicular to the vector $\Omega \in S^1$, and the dependence of $\tilde{N}_n$ and $t_{nk}^\pm$ on Ω, x_o is not indicated, $k = 1,2,\dots,\tilde{N}_n$, $n = 1,2,\dots N$. It is assumed that any set G_n contains a convex subset of a nonzero measure $m > 0$ in R^3.

In the same way, we define the segments S_1^l as open connected measurable subsets $S_l^1 \in S^1$ with a boundary of zero measure m in S^1. Assume that

$$m(G_{n'} \cap G_n) = m(G \setminus \bigcup_{n=1}^{N} G_n) = 0, \quad n, n' = 1,2,\dots,N, \quad n \ne n',$$

$$m(S_l^1 \cap S_{l'}^1) = m(S^1 \setminus \bigcup_{l=1}^{\bar{l}} S_l^1) = 0, \quad l, l' = 1,2,\dots,\bar{l}, \; l \ne l',$$

$$0 \le \underline{E} = E_I < \dots < E_i < \dots < E_o = \overline{E} < \infty$$

7. Let us introduce the Banach spaces $L_p(Y), L_p(\Gamma_\pm)$ of real function $y(x, E, \Omega)$, summable, with the pth power, on the sets $Y, \Gamma^\pm$ with norms

$$\|y\|_p = \left\{ \int_{\underline{E}}^{\overline{E}} dE \int_{S^1} d\Omega \int_{p_\Omega} \Big(\sum_{k=1}^{\tilde{N}} \int_{t_k^-}^{t_k^+} dt \, |y(x_0 + \Omega t, E, \Omega)|^p \Big) dx_o \right\}^{1/p},$$

$$\|y\|_p^\pm = \left\{ \int_{\underline{E}}^{\overline{E}} dE \int_{S^1} d\Omega \int_{p_\Omega} \Big(\sum_{k=1}^{\tilde{N}} |y(x_0 + \Omega t_k^\pm, E, \Omega)|^p \Big) dx_o \right\}^{1/p},$$

$$\|y\|_\infty = \underset{x,E,\Omega \in Y}{vrai\max} |y(x, E, \Omega)|, \qquad \|y\|_\infty^\pm = \underset{x,E,\Omega \in \Gamma_\pm}{vrai\max} |y(x, E, \Omega)|,$$

and define the operators L, L^{-1}, B for almost all $E,\Omega,x_o \in [\underline{E},\overline{E}]\times S^1 \times p_\Omega$ on the intervals [t_k^-,t_k^+] ($k=1,2,...,\tilde{N}$) by the formulae

$$L\, y = \left[\frac{d}{dt} + \Sigma(x_0+\Omega t,E)\right] y\,(x_0+\Omega t,E,\Omega),\ \ y\in D_o^p, \tag{19?}$$

$$L^{-1}\Phi = \int_{t_k^-}^{t} dt'\Phi(x_o+\Omega t',E,\Omega)\exp[-\int_{t'}^{t} dt''\Sigma(x_o+\Omega t'',E)],,\ \ \Phi\in L_p(Y\ ), \tag{19b}$$

$$B\ (f) = f\ (x_o+\Omega t_k^-,E,\Omega)\exp\left[-\int_{t_k^-}^{t} dt'\Sigma(x_o+\Omega t',E)\right],\ \ f\in L_p(\Gamma_-), \tag{19c}$$

where $D_o^p \subset L_p(Y\ )$ is a linear set of functions $y = L^{-1}\Phi$, $\Phi\in L_p(Y\ )$ of the form (19b) and L^{-1} is the inverse of the linear unbounded operator L , generated by the linear differential expression $(\Omega\nabla+\Sigma)y$ on functions $y\in D_o^p$ according to formula (19a).

Then, choosing $Q\in L_p(Y)$, $f\in L_p(\Gamma_-)$ for a certain $p\in[1,\infty]$, it is possible to formulate the original boundary value problem (1) in the form of the equation

$$y = L^{-1}Ky + L^{-1}Q + B(f),\quad y\in L_p(Y), \tag{20}$$

where $L^{-1}K$ is a linear and continuous in $L_p(Y)$, completely continuous for $1<p<\infty$ (and its square is compact for all $p\in[1,\infty]$), and u_o- positive operator [8-10]. Under the condition

$$r(L^{-1}K)<1, \tag{21}$$

(where $r(L^{-1}K)$ is the spectral radius of the operator $L^{-1}K$ **),** there exist a unique solution to Eq. (20), and that solution can be found by the method of successive approximations

$$y^{(m+1)} = L^{-1}Ky^{(m)} + L^{-1}Q + B(f),\ \ m=0,1,\dots, \tag{22}$$

which converges in $L_p(Y)\ \forall y^{(o)}\in L_p(Y)$ at the rate of geometric progression with the multiplier $r(L^{-1}K)$. Moreover, if $f,Q\geq 0$, $\|f\|_p^- + \|Q\|_p > 0$, then $y>0$ almost everywhere in Y [5-9]..

8. The adjoints $L^*,(L^*)^{-1},B^*$ of the operator L $,L^{-1},B$ are define for almost all $E,\Omega,x_o \in [\underline{E},\overline{E}]\times S^1\times p_\Omega$ on corresponding intervals [t_k^-,t_k^+] by the formula

$$L^*y^* = [-\frac{d}{dt} + \sum(x_o+\Omega t,E)]y^*(x_o+\Omega t,E,\Omega), \tag{23a}$$

$$(L^*)^{-1}\Phi = \int_t^{t_k^+} dt'\Phi(x_o + \Omega t', E, \Omega)\exp\left[-\int_t^{t'} dt'' \sum (x_o + \Omega t'', E)\right], \qquad (23b)$$

$$B^*(f) = f\ (x_o + \Omega t_k^+, E, \Omega)\exp\left[-\int_t^{t_k^+} dt' \Sigma(x_o + \Omega t', E)\right], \qquad (23c)$$

with the respective domains $D_o^{*p}, L_p(Y), L_p(\Gamma_+)$, where D_o^{*p} is a linear set of functions of the form (23b) with $\Phi \in L_p(Y)$, which is dense in $L_p(Y)$ for $1 \le p < \infty$. When $p = \infty$, D_o^p, D_o^{*p} are not dense in $L_p(Y)$ and the adjoint operators cannot be defined uniquely. In this case, the operator $L^*, (L^*)^{-1}, B^*$ are understood as the restrictions of the respective operators by (23) to $D_o^{*1}, L_1(Y), L_1(\Gamma_+)$ (see [7,8]) The same is also true for the adjoint operators K^* of K with a transposed kernel.

Then, choosing $Q^* \in L_q(Y), f^* \in L_q(\Gamma_+)$ for a $q = p/(p-1)$, it is possible to formulate the problem (4) in the form of the equation

,

$$y^* = (L^*)^{-1}K^*y^* + (L^*)^{-1}Q^* + B^*(f^*), \quad y^* \in L_q(Y), \qquad (24)$$

with properties similar to those of Eq. (20).

THE PROBLEM ANDER EXAMINATION

9. Let us justify the scheme (5),(11),(13),(15). It is possible to formulate it in the form of nonlinear equation (24),

$$\tilde{y}^* = A_3(\tilde{y}^*)\ , \qquad (25)$$

where

$$A_3(\tilde{y}^*) = A_2(\tilde{y}^*)\tilde{y}^* + A_1(\tilde{y}^*),\ A_2(\tilde{y}^*) = (\tilde{L}^*)^{-1}\tilde{K}^*,\ A_1(\tilde{y}^*) = (\tilde{L}^*)^{-1}Q^* + \tilde{B}^*(f^*),$$

and $(\tilde{L}^*)^{-1}\tilde{K}^*$, $(\tilde{L}^*)^{-1}$, $\tilde{B}^*$ are operators $(L^*)^{-1}K^*$, $(L^*)^{-1}$, B^* with coefficients (11),(13), which are depend on $\tilde{y}^*$. The following assertion is valid [6].

Theorem 1. *Suppose that the above assumptions on the coefficients of Eq. (1); on the partition of the set Y into zones, groups, and sectors; and on the choice of the functions $0 \le g, h \in L_\infty(E, \overline{E})$, $0 \le f, Q$, $\|f\|_\infty^- + \|Q\|_\infty > 0$ hold. Let conditions (15) and (21) be satisfied. Than Eq. (5) with coefficients (11),(13) and functions $0 \le f^*, Q^*$ such that $0 < \|f^*\|_p^+ + \|Q^*\|_p < \infty$ has a solution $q < \tilde{y}^* \in L_p(Y)$, $1 \le p \le \infty$, and $\tilde{y}^* > 0$ almost everywhere in Y.*

According to this theorem, the existence of solution is independent of the number of zones, groups, or sectors. In this sense, the simplest choice $N = I = \bar{l} = 1$ proves to be sufficient. A different situation occurs with the uniqueness of a solution. Consider this aspect

Definition 1. A partition of the set Y into zones, groups, and sectors is called *regular* if, for an arbitrary $e > 0$, *there exist* $N(e), I(e), \bar{l}(e) < \infty$ *such that* $N > N(e), I > I(e), \bar{l} > \bar{l}(e)$ *imply that* $a_n^i, b_{bn}^{ijlm} < e$ *for all* $b, i, j, l, m, n,$ *where*

$$a_n^i = \frac{\left\| q_n^i (\Sigma - \tilde{\Sigma}) y \right\|_q}{(\tilde{y}^*, q_n^i y)}, \quad b_{bn}^{ijlm} = \frac{\left\| q_n^{il} (K_b - \tilde{K}_b) q_n^{jm} y \right\|_q}{(\tilde{y}^* q_n^{il}, K^o q_n^{jm} y)} .$$

The following assertion is true for such partitions [6].

Theorem 2. *Let a partition be regular and the number of zones, groups, and sectors be sufficiently large. Then, Eq. (25) has a unique solution, which can be found by the method* $\tilde{y}_k^* = A_3(\tilde{y}_{k-1}^*)$ *(k=1,2,…) converging to* $\tilde{y}^*$ *at the rate of a geometric progression with a multiplier that is arbitrary close to* $r[\tilde{A}_2(\tilde{y}^*)]$ *for any first guess* $\tilde{y}_0^*$ *sufficiently close to* $\tilde{y}^*$.

Note that under the additional assumptions

$$0 < \underline{E}, \qquad 0 \le \underline{w}_b \frac{g(E')h(E)}{< gh >^i} \le w_b(x, E, E', \Omega, \Omega') \le \frac{g(E')h(E)}{< gh >^i} \overline{w}_b < \infty \ , \qquad (26)$$

(where $E \in (E_i, E_{i-1}),\ \underline{w}_s > 0$), which concern the separation of the admissible values of energy from zero and the boundedness of the indicatrix, theorem 1 can be strengthened by removing the conditions (15).

Theorem 3. *Let the additional conditions (26) be valid. Than the problem (5),(11),(13) has a positive solution* $q < \tilde{y}^* \in L_q(Y),\ q = (p-1)/p$ *for all* $f, Q \ge 0,\ f \in L_p(\Gamma_-),\ Q \in L_p(Y),$ $\|f\|_p^- + \|Q\|_p > 0;\ f^*, Q^* \ge 0,\ f^* \in L_q(\Gamma_+),\ Q^* \in L_q(Y),\ \left\|f^*\right\|_q^+ + \left\|Q^*\right\|_q > 0,\ 1 \le p \le \infty.$

10. In conclusion, we note that problem (3),(11),(12), which is dual to problem (5),(11),(13), has similar properties [6].

Theorem 4. Let $0 < g, h \in L_\infty(\underline{E}, \overline{E}),\ f^*, Q^* \ge 0,\ \infty > \left\|f^*\right\|_\infty^+ + \left\|Q^*\right\|_\infty > 0$. Suppose that the assumptions of Sections 5,6 hold and $u(x, E, \Omega) = h(E) \sum_i q_i(E)\, \breve{u}_i(x, \Omega) / < gh >^i$, where $\breve{u}_i = < gu >^i,\ u = Q, f$. Than, under condition *(21), Eq. (3) with coefficients (11),(12) and with* $f, Q \ge 0,\ f \in L_p(\Gamma_-), Q \in L_p(Y),\ \|f\|_p^- + \|Q\|_p > 0$, *has a solution* $\tilde{y} \in L_p(Y)$ *that is positive almost everywhere in* Y. *When the partition is regular and the number of zones, groups, and segments is sufficiently large, the solution is unique and can be obtained by the method of successive approximations* $\tilde{y}_{m+1} = \tilde{L}_{(m)}^{-1} \tilde{K}_{(m)} \tilde{y}_m + \tilde{L}_{(m)}^{-1} Q + \tilde{B}_{(m)}(f),\ m = 0,1,\ldots$, *where this method con-*

verges to $\tilde{y}$ *at the rate of a geometric progression with a multiplier arbitrarily close to* $r(\tilde{L}^{-1}\tilde{K})$ *for any first guess* $\tilde{y}_o \in L_p(Y)$ *sufficiently close to* $\tilde{y}$ *. Here* $\tilde{L}^{-1}_{(m)}$, $\tilde{K}_{(m)}$, $\tilde{B}_{(m)}$ *are the operators* L^{-1}, K, B *with coefficients (11),(12) calculated with the weight* $\tilde{y}_m$*; and* $\tilde{L}^{-1}$, $\tilde{K}$ *with the weight* $\tilde{y}$ *. Under conditions (26), these properties are extended to all* $f \in L_p(\Gamma_-)$, $Q \in L_p(Y)$,

$f, Q \geq 0$, $\|f\|^-_p + \|Q\|_p > 0$; $f^*, Q^* \geq 0$, $f^* \in L_q(\Gamma_+)$, $Q^* \in L_q(Y)$, $\|f^*\|^+_q + \|Q^*\|_q > 0$,

$q = (p-1)/p$, $1 \leq p \leq \infty$.

REFERENCES

1. G. I. Marchuk, V. I. Lebedev. Numerical Methods in the Theory of Neutron Transport, Harwood Academic Publishers, Switzerland (1986).
2. B.R.Bergel'son, A.P.Suvorov, B.Z. Torlin. Multigroup Methods for Neutron Protection Calculation. Moscow, Energoatumisdat,1970.
1. M.N.Nikolaev, B.G.Ryasanov, M.M.Savos'kin, A,M.Tsibulya. Multigroup Approximation in Neutron Transport Theory. Moscow, Energoatomisdat, 1984.
2. B.D.Abramov. Some Methods of Multigroup Approximation in Nuclear Reactor Theory, Comp, Maths Math.Phys.,**34**, 2, 175-191, (1994).
3. B.D.Abramov. Some Models of Multigroup Approximation and Reactor Cell Homogenization Theory. Proc. of the Joint Int. Conf. on Math. Methods and Supercomputing in Nucl. App., "Saratoga Springs", New York, 1977, **2**, p. 1527-1536.
4. B.D.Abramov. Some Modifications of Marchuk's Multigroup Method for Inhomogeneous Boundary Value Problems in Neutron Transport Theory, Comp, Maths Math.Phys.,**40,** 11, 1739-1752, (2000).
5. V.S.Vladimirov. Mathematical Problems of One-Speed Particle Transport Theory. Tr. Mat. Inst. im. V.A.Steklova Akad. Nauk SSSR, 1961, v. 61.
6. S.B.Shikhov. Problems of Mathematical Reactor Theory, Moscow, Atomisdat, 1973.
7. B.D.Abramov, S.B.Shikhov. Methods of Splitting by Subdomain for the Neutron Transport Equation, Comp. Maths Math.Phys**.,30**, 6, 74-86, (1990).
8. M.A.Krasnosel'skii, V.A.Lifshits, A.V.Sobolev. Positive linear Systems. Moscow, Nauka, 1985.