On Some Modifications of the Point Reactor Kinetics Equations

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ABSTRACT

In the paper some new modifications of the conventional point kinetic method are proposed. The equations are intend for the description of the neutron flux evolution in nuclear reactor with fuel as an arbitrary mixture of the fissile nuclides.

INTRODUCTION

A large amount of works has been done on the development of various models of the neutrons kinetic, and these problems have to a considerable extend been solved. However, there are some important questions concerning the choice of particular formulations of problems of this kind, the solution of the corresponding equations, and so on, which still remain to be unanswered. Some of these questions are discussed in the present paper where, for quite general assumptions concerning the model of neutron interaction with the material, a generalizations of such a well known point kinetics methods as the Ussachoff's method [1], the Henry's method [2], and so on [3-5], are proposed.

THE ORIGINAL PROBLEM

Consider the system of equations

$$\frac{1}{v}\frac{\partial j}{\partial t} + Mj = Fj + \sum_{l}\sum_{m'} \left(l_{(l)}^{(m')} R_{(l)}^{(m')} - F_{(l)}^{(m')} j \right) + Q, \qquad (1?)$$

$$\frac{\partial R_{(l)}^{(m')}}{\partial t} = -l_{(l)}^{(m')} R_{(l)}^{(m')} + F_{(l)}^{(m')} j , \qquad (1?)$$

which are describe the neutron flux $j\,(x, E, \Omega, t)$ evolution in the nuclear reactor, where $Q(x, E, \Omega, t)$ is an independent neutron source; v is the neutron velocity ; $R_{(l)}^{(m')} = c_{(l)}^{(m')} C_{(l)}^{(m')}$; $c_{(l)}^{(m')}(E)$ is the spectrum; $C_{(l)}^{(m')}(x,t)$ - the delayed neutrons precursors concentration with decay constant $l_{(l)}^{(m')}$ and number $m' = m'(l)$, which was generated by mothers nuclide l ; $b_{(l)}^{(m')}$ is its fraction; M, F , $F_{(l)}^{(m')}$ are operators defined by the formulas

$$M = \Omega\nabla + C, \quad C = \Sigma - S, \quad S = K_s, \quad F = K_f \ ,$$

$$K_b j = \int dE' \int d\Omega' w_b(x, E, E', \Omega, \Omega')\, j\,(x, E', \Omega', t), \quad b = s, f \ ,$$

$$F_{(l)}^{(m')} j = c_{(l)}^{(m')}(E) \int dE'\, b_{(l)}^{(m')}(E')\, n_{fl}(E')\, \Sigma_{fl}(x, E') \int d\Omega'\, j\,(x, E', \Omega', t)/4p$$

on functions $j(x,E,\Omega,t)$, satisfying the usual smoothness conditions inside, and the vacuum boundary condition on the surface Γ of the reactor volume G. Here

$$w_{b'l}(x,E,E',\Omega,\Omega')=n_{b'l}(E')\Sigma_{b'l}(x,E')W_{b'l}(E',E,\Omega',\Omega),\ \ w_s=\sum_{b'\neq c,f}\sum_l w_{b'l},$$

$$w_f=\sum_l w_{fl},\ \ \int dE\int W_{b'l}(E',E,\Omega',\Omega)d\Omega=1,\quad \Sigma(x,E)=\sum_{b'}\sum_l \Sigma_{b'l}(x,E),$$

where $n_{b'l}(E)$, $W_{b'l}(E',E,\Omega',\Omega)$ is the number of secondary neutrons formed in one reaction of type b' of a neutron with the nucleus of the nuclide l, and the probability density of their distribution with respect to energies E and flight directions Ω; $\Sigma_{b'l}=N_l(x)s_{b'l}(E')$ is the macroscopic cross-section of that reaction, $N_l(x)$ is the density of nuclei of the nuclide l; $s_{b'l}(E')$ is the microscopic cross-section; the summation is over all nuclides l and all processes b' of interaction between neutrons and nuclei: radiation capture $(b'=c)$, elastic scattering $(b'=e)$, fission $b'=f$, and so on [4].

It should be noted that in these equations $l_{(l)}^{(m')}$ is depend on number l of mothers nuclide.

THE NEW MODIFICATIONS OF THE POINT KINETICS EQUATIONS

Let $y, y^*\geq 0$ be the positive solutions of equations

$$My=Fy/k_{eff}, \tag{2}$$

$$M^*y^*=F^*y^*/k_{eff}, \tag{3}$$

M^*, F^* - the adjoint of operators M, F; k_{eff} - the multiplication factor. With the definition

$$j(x,E,\Omega,t)=P(t)\tilde{y}(x,E,\Omega,t)/(p,\tilde{y}), \tag{4}$$

the new modification of the point reactor kinetics equations became [6]

$$[\frac{d}{dt}+(\bar{a}-\frac{r-\bar{b}}{\Lambda})]P=[\sum_{m=1}^{\bar{m}}\bar{l}^{(m)}\bar{C}^{(m)}+\bar{Q}]/k_p\Lambda, \qquad (5?)$$

$$[\frac{d}{dt}+(\bar{l}^{(m)}-a^{(m)})]\bar{C}^{(m)}=\bar{b}^{(m)}k_pP, \qquad (5?)$$

where

$$P(t)=(p,j),\qquad \bar{C}^{(m)}(t)=\sum_{l,m'\in m}(y^*,R_{(l)}^{(m')}) \tag{6}$$

$$\bar{a}=\frac{(y^*,v^{-1}\partial x/\partial t)}{(y^*,v^{-1}x)},\quad \Lambda=\frac{(y^*,v^{-1}\tilde{y})}{(y^*,F\tilde{y})},\quad \bar{l}^{(m)}=\frac{\sum_{l,m'\in m}(y^*,l_{(l)}^{(m')}R_{(l)}^{(m')})}{\sum_{l,m'\in m}(y^*,R_{(l)}^{(m')})}, \qquad (7?)$$

$$a^{(m)}=\frac{\sum_{l,m'\in m}(\partial y^*/\partial t,R_{(l)}^{(m')})}{\sum_{l,m'\in m}(y^*,R_{(l)}^{(m')})},\ k_p=\frac{(y^*,F\tilde{y})}{(p,\tilde{y})},\ \bar{b}^{(m)}=\frac{\sum_{l,m'\in m}(y^*,F_{(l)}^{(m')}\tilde{y})}{(y^*,F\tilde{y})}, \qquad (7?)$$

$$\overline{b} \;= b_{??} = \sum_{m=1}^{\overline{m}} \overline{b}^{(m)}, \quad \overline{Q} \;= (y^{*}, Q), \;\; x = \tilde{y}/(p,\tilde{y}), \;\; r = 1 - 1/k_{??}\,, \qquad (7?)$$

m - index of the effective delayed neutrons group; $\overline{m}$ - full number of such group; $m' \in m$ - the set of labels $m' = m'(l)$ of the delayed neutron precursors belonging to the group m; $(\,,\,)$ is the symbol of integration in $x \in G$ and E, Ω; $p(x,E,\Omega,t) \geq 0$ - an arbitrary function; $\tilde{y}(x,E,\Omega,t) \geq 0$ - some shape-function, $\tilde{y} = y$ and so on.

The integral balance equations (5)-(7) are followed directly from adjoint weighting and subsequent integrations of Eqs.(1).and are explicit. They differ from conventional point kinetics equations both the coefficient k_p, which describes the distinction between functions p and $F^{*}y^{*}$, and the coefficients $\overline{a}$, $a^{(m)}$, which describe the deformations of the form-functions $\tilde{y}, y^{*}$.

Eqs. (5) provide an arbitrary functional $P = (p, j)$; the dependence of the shape-functions $y^{*}, \tilde{y}$ on t; and the dependence of the decay constants $l_{(l)}^{(m')}$ on l. In particular, in the case

$$\partial y^{*}/\partial t = 0, \quad p = y^{*}/v, \quad l_{(l)}^{(m')} = l^{(m)} d_{mm'}, \;\; b_{(l)}^{(m')} = b^{(m)} d_{mm'} \qquad (8)$$

(where $d_{mm'}$ - the Kronecker's symbol), we have $\overline{a} = a^{(m)} = 0$, $\Lambda k_p = 1$, $P = (y^{*}, j/v)$, and Eqs. (5) are transformed into some Henry's equations [2-5]. If p is the absorptions cross-section of the detector, then P is the detectors response. If

$$w_f(x,E,E',\Omega,\Omega') = \sum_{l} c_{fl}(E) n_{fl}(E') \Sigma_{fl}(x,E')/4p\,, \quad p = \sum_{l} n_{fl} \Sigma_{fl}\,, \qquad (9)$$

then $P = (1, Fj)$ is the rate of the fission neutrons generation, and so on.

In the adiabatic approximation, where it is assumed that at every instant during the transient, the flux shape can be obtained by static calculations, $\tilde{y} = y$ and

$$R_{(l)}^{(m')} \approx F_{(l)}^{(m')} j \,/\, l_{(l)}^{(m')}, \qquad (10)$$

from Eqs.(7) are followed some extension of the conventional definitions [1-5] like the formula

$$\frac{1}{\overline{l}^{(m)}} = \frac{1}{\overline{b}^{(m)}} \sum_{l,m' \in m} \frac{\overline{b}_{(l)}^{(m')}}{l_{(l)}^{(m')}}\,. \qquad (11)$$

In the general case the solution of the original problem (1) can be split up into the solution of the equations (5) for the amplitude P and the solution of the equation

$$\frac{1}{v}\left(\frac{\partial x}{\partial t} + x\frac{dP}{Pdt}\right) = (F_o - M)x + \sum_{l,m'} l_{(l)}^{(m')} \int_{-\infty}^{t} dt' \frac{P(t')}{P(t)} ?^{-l_{(l)}^{(m')}(t-t')} F_{(l)}^{(m')} x(t') + \frac{Q}{P} \qquad (12)$$

for the flux shape x, where

$$F_o = F - F_d\,, \quad F_d = \sum_{l,m'} F_{(l)}^{(m')}. \qquad (13)$$

These splitting are generated some approximations like the conventional quasistatic approximation (where $\partial x/\partial t \approx 0$) and so on. Let us consider some of them

Let us assume that at instant $t = t_{i-1}$ there are some non perturbed operators $M, F, F_o, F_{(l)}^{(m')}$ and functions $y^*_{i-1} = y^*(x,E,\Omega,t_{i-1})$, $\tilde{y}_{i-1} = \tilde{y}(x,E,\Omega,t_{i-1})$, and at any $t \in (t_{i-1}, t_i)$ there are some perturbations

$$F' = F + dF, \quad F'^{(m')}_{(l)} = F^{(m')}_{(l)} + dF^{(m')}_{(l)}, \quad M' = M + dC. \tag{14}$$

Then it is possible to determine the coefficients (7) of the equations (5) with perturbed

$$r' = r + dr \ , \quad \bar{b}' = \bar{b} + d\bar{b} \ , \quad \bar{b}'^{(m)} = \bar{b}^{(m)} + d\bar{b}^{(m)}, \tag{15}$$

to calculate its numerical solution $P(t)$ at $t \in (t_{i-1}, t_i)$, after that to determine $\tilde{y}_i$ from Eq. (12) and so on.. Here the finite difference are used to calculate the derivative in (7),(12),

$$r' - \bar{b}' = r - \bar{b} + dr\,, \qquad dr = (y^*, (dF_o - dC)\tilde{y})/(y^*, F\tilde{y}), \tag{16}$$

the elements $r, \bar{b}, \bar{b}^{(m)}, \Lambda, \bar{l}^{(m)}, k_p, a^{(m)}, y, {}^*\tilde{y}$ are relate to $t = t_{i-1}$ and do not depend on $t \in (t_{i-1}, t_i)$, i =1,2,…It is possible some modifications oh this scheme

It should be noted that it is possible (and some tames more preferable) to use instead of y^* the positive solutions of the uniform equations of type

$$M^* y^* = F_o^* y^* / k_o; \quad M^* y^* = F_d^* y^* / k_d; \quad M^* y^* = F_o^* y^* - a y^* / v; \tag{17}$$

and so on. In this case it is necessary to use instead of $r - \bar{b}$, F the terms of type

$$(1 - 1/k_o), F_o; \quad (y^*, F_o j)/(y^*, F_d j) - 1/k_d, \ F_d; \quad a_o, \ 1/v; \tag{18}$$

and so on. If one use instead of y^* the positive solutions of the non uniform equations of type

$$M^* y^* = q; \quad (-\Omega\nabla + C^*) y^* = 0, \quad y^* = f^* \ ?? \ \Gamma_+; \tag{19}$$

and so on, than he most to use instead of r the terms of type

$$1 - 1/k_q, \quad k_q = (y^*, Fj)/(q, j); \quad 1 - 1/k_f, \quad k_f = (y^*, Fj)/(1, \Omega\nabla y^* j), \tag{20}$$

and so on. Here $\Gamma_\pm = \{x \in \Gamma, E, \Omega : \Omega n(x) \gtrless 0\}$, $q(x.E.\Omega,t) \geq 0$ is some arbitrary function, for example, $q = p$, and so on, and functional

$$(1, \Omega\nabla y^* j) = \int_{\Gamma_+} \Omega n(x) f^*(x,E,\Omega) j(x,E,\Omega,t)\, dgdEd\Omega \tag{21}$$

describes the neutrons leakage from reactors volume (it is equal to the one-sided neutrons current through Γ if $f^* = 1$, and so on (see [6[)).

THE ANALITICAL EXAMPLE

To estimate the correction terms $\bar{a}$? $a^{(m)}$ let us consider the simplified Cauchy problem for the monoenergetic diffusions equation [3-5]

$$\frac{1}{v}\frac{\partial j}{\partial t} + (-D\nabla^2 + \Sigma_a) j = (1 - b) n \Sigma_f j \tag{22}$$

in homogeneous volume G with the routine boundary conditions. It is well known, the solution of this pro blem (for the subcritical reactor) can be written in the form .

$$j\,(x,t)=\sum_{k=0}^{\infty} e^{-a_k t}(j_k, j^{(o)})j_k(x)\,, \qquad (23)$$

where $j_n(x)$ are orthonormal eigenfunctions of the Laplace operator

$$\nabla^2 j_k = -B_k^2 j_k\,,\quad (j_k, j_n)=d_{kn},\quad k,n=0.1...,\qquad (24)$$

and $j^{(0)}=j\,(x,0)$ is an initial condition. In these case $y^*=j_o$, $dy^*/dt=0$, $r=r_o$,

$$a_k=-\frac{r_k-b}{\Lambda},\quad \Lambda=\frac{1}{vn\Sigma_f},\quad r_k=\frac{vn\Sigma_f-DB_k^2-\Sigma_a}{vn\Sigma_f},\quad \overline{a}=\frac{d}{dt}\ln\frac{(j_0,j)}{(p,j)}, \qquad (25)$$

and the solution of the conventional point kinetic equation (Eq. (5?) (without term $\overline{a}$),

$$dP/dt=[(r-b)/\Lambda]P=-a_oP\,, \qquad (26)$$

have the form

$$P(t)=P(0)e^{-a_o t},\quad P(0)=(p,j^{(o)})=\sum_{k=o}^{\infty}(p,j_k)(j_k,j^{(o)}) \qquad (27)$$

and is differ (in case $p\neq j_o$) from the true representations

$$P(t)=\sum_{k=0}^{\infty} e^{-a_k t}(p,j_k)(j_k,j^{(o)})\,. \qquad (28)$$

On the other hand the solution of the modified equation (5?) (with term $\overline{a}$),

$$dP/dt=-(a_o+\overline{a})P\,, \qquad (29)$$

is the same as (28), because in this case

$$P(t)=P(0)\exp[-a_o t-\int_0^t dt'\overline{a}(t')]=P(0)e^{-a_o t}\frac{(p,j)}{(j_o,j)}\frac{(j_o,j^{(o)})}{(p,j^{(o)})}=(p,j)\,. \qquad (30)$$

Thus, taking account of $\overline{a}$ is really provided the multiple precision.

LITERATURE

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