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A THREE GRID CORRECTION SCHEME FOR TRANSIENT NONLINEAR NODAL CALCULATIONS

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ABSTRACT

A three grid correction scheme(3GCS) is designed using three-dimensional (3D) one-node-per-assembly(1 N/A) coarse grid equation and its axially integrated two-dimensional (2D) 1 N/A equation to speed up 3D four-node-per-assembly(4 N/A) iterative calculation of nodal multi-group diffusion equation. The computational effectiveness of the 3GCS is examined and compared with that of the bi-conjugate gradient stabilized (BICGSTAB) iteration scheme with and without preconditioners in terms of the CPU time taken to obtain nonlinear nodal solution to OECD/NEACRP PWR transient benchmark problems. It is demonstrated that the 3GCS is more effective than the BICGSTAB iteration scheme with preconditioner and that the combined use of the 3GCS with nodal core reflector boundary condition is very effective in reducing the CPU time of the nodal transient analysis.

1. INTRODUCTION

In our recent paper (Chung, 2000), we demonstrated that the incorporation of nodal core-reflector boundary condition (CRBC) (Lee, 1995) into nonlinear nodal methods (Smith, 1983) and adoption of two grid correction scheme (2GCS) can enhance significantly efficiency of the two most popular transverse integrated nodal methods (TINM), nodal expansion method (NEM) (Finnemann, 1975) and analytic nodal method (ANM) (Smith, 1979), solutions to the static and transient reactor physics problems. In an attempt to enhance further efficiency of the TINM, we herein present a three grid correction scheme (3GCS) which utilizes three-dimensional (3D) one-node-per-assembly (1 N/A) coarse grid diffusion equations in combination with axially integrated two-dimensional (2D) 1 N/A equations (Kim, 1994) to speed up the bi-conjugate gradient stabilized (BICGSTAB) (Van Der Vorst, 1992) iterative solutions of the 3D four-node-per-assembly (4 N/A) TINM equations and examine efficiency of the 3GCS in terms of computational time for the 3D 4 N/A NEM and ANM solutions to the OECD/NEACRP PWR transient benchmark problems (Finnemann, 1992 and Fraikin, 1993).

Multigrid correction method has been known as a very efficient acceleration scheme to solve large sparse linear equations which arise in many physical problems governed by partial differential equation (Briggs, 1987). It uses coarse grid equation to reduce low frequency error terms of fine grid iterative solution which are not easily eliminated in fine grid iterative solution process.

There were several studies (Al-Chalabi, 1994, Alcouffe, 1983, Schiechl, 1999, Zaslavsky, 1995, and Beam, 1999) designed to examine performance of the multigrid method for neutronics diffusion equation, yet a few of them (Al-Chalabi, 1994, Schiechl, 1999, and Beam, 1999) were devoted to its application to the NEM solution to multigroup diffusion equation. The multigrid correction method is characterized by the type of grid cycle and coarse grid equations utilized for speedy error reduction of the fine grid equations. The 3GCS here differs from earlier studies in that it utilizes 3D 1 N/A TINM equation and its axially integrated 2D equation on the modified W cycle.

In the following we present the coarse grid equations of the 3GCS in relation with the fine grid TINM equations to be solved. Then we introduce the modified W cycle employed to accelerate the computational speed of the 3D 4 N/A TINM solutions to OECD/NEACRP PWR transient benchmark problems. Then we compare efficiency of the 3GCS with that of the BICGSTAB iteration scheme with and without pre-conditioner in terms of the CPU time taken for the transient analysis of OECD/NEACRP PWR.

2. COARSE GRID EQUATIONS

The nonlinear TINM is designed to solve the following neutron balance relations for the 3D rectangular node designated by (k,m);

$$\frac{1}{v_g}\frac{d\Phi_g^{k,m}}{dt} = \frac{1}{k_{eff}}\left(\chi_{pg}\sum_{g'=1}^{2}(1-\beta)\Sigma_{fg'}^{k,m}\Phi_{g'}^{k,m} + \chi_{dg}\sum_{d=1}^{D}\lambda_d C_d^{k,m}\right) + \sum_{g'=1}^{2}\Sigma_{g'g}^{k,m}\Phi_{g'}^{k,m} - \sum_{u=x,y,z}\frac{1}{h_u^k}\left(J_{gur}^{k,m} - J_{gul}^{k,m}\right) - \Sigma_{tg}^{k,m}\Phi_g^{k,m} \tag{1}$$

and

$$\frac{dC_d^{k,m}}{dt} = \frac{1}{k_{eff}}\sum_{g'=1}^{2}\nu_{dg'd}\Sigma_{fg'}^{k,m}\Phi_{g'}^{k,m} - \lambda_d C_d^{k,m} \quad . \tag{2}$$

In the nonlinear TINM, $J_{gus}^{k,m}$ are related to $\bar{\Phi}_g^{k,m}$ by

$$J_{gus}^{k,m} = -D_{gus}^{k,m}\left(\Phi_{gus}^{k,m} - \Phi_g^{k,m}\right) - \tilde{D}_{gus}^{k,m}\left(\Phi_{gus}^{k,m} + \Phi_g^{k,m}\right) \tag{3}$$

where $\Phi_{gus}^{k,m}$ is the node average flux of the adjacent node in direct contact with the node (k,m) in u direction at left ($s=l$) or right side ($s=r$). $D_{gus}^{k,m}$ is the coupling

coefficient from the first order finite difference approximation for $J_{gus}^{k,m}$. $\tilde{D}_{gus}^{k,m}$ is the corrective coupling coefficient that is to be updated by updating $J_{gus}^{k,m}$ through two-node solution of transverse integrated 1D diffusion equation. Substitution of Eq. (3) into Eq. (1) results in the nodal coarse mesh finite difference (CMFD) equation.

A fully implicit time integration of the CMFD equation can lead to a linear equation which may be expressed by

$$\mathbf{A}_{\text{3D 4 N/A}}^{(n)}\boldsymbol{\Phi}_{\text{3D 4 N/A}}^{(n)}=\mathbf{b}_{\text{3D 4 N/A}}^{(n)}. \tag{4}$$

The subscript "3D 4 N/A " denotes the 3D 4 N/A CMFD equation. The 3D 1 N/A equation to be utilized in the 3GCS can be similarly represented by

$$\mathbf{A}_{\text{3D 1 N/A}}^{(n)}\boldsymbol{\Phi}_{\text{3D 1 N/A}}^{(n)}=\mathbf{b}_{\text{3D 1 N/A}}^{(n)} \quad . \tag{5}$$

The 2D reactor model is simply axially integrated form of the 3D reactor model (cf. Fig. 1). Summing Eq. (1) over the axial node index m from the bottom node to the top node inside the core with the radial index k fixed, therefore, we obtain

$$\begin{aligned}\frac{1}{v_g}\frac{d\Phi_g^k}{dt}=&\frac{1}{k_{eff}}\left(\chi_{pg}\sum_{g'=1}^{2}(1-\beta)\Sigma_{fg'}^k\Phi_{g'}^k+\chi_{dg}\sum_{d=1}^{D}\lambda_d C_d^k\right)\\&+\sum_{g'=1}^{2}\Sigma_{g'g}^k\Phi_{g'}^k-\sum_{u=x,y}\frac{1}{h_u^k}\left(J_{gur}^k-J_{gul}^k\right)-\left(\Sigma_{tg}^k+D_g^kB_{gk}^2\right)\Phi_g^k\end{aligned} \tag{6}$$

and

$$\frac{dC_d^k}{dt}=\frac{1}{k_{eff}}\sum_{g'=1}^{2}\nu_{dg'd}\Sigma_{fg'}^k\Phi_{g'}^k-\lambda_d C_d^k \tag{7}$$

where

$$\Phi_g^k=\frac{\sum_{m=1}^{M}\Phi_g^{k,m}V_m}{\sum_{m=1}^{M}V_m} \quad , \tag{8}$$

$$\Sigma_{qg}^k=\frac{\sum_{m=1}^{M}\Sigma_{qg}^{k,m}\Phi_g^{k,m}V_m}{\sum_{m=1}^{M}\Phi_g^{k,m}V_m} \quad ,(g=1,2;q=a,f,t) \quad , \tag{9}$$

and

$$C_d^k = \frac{\sum_{m=1}^{M} C_d^{k,m} V_m}{\sum_{m=1}^{M} V_m} \quad . \tag{10}$$

The axial buckling B_{gk}^2 is given by

$$D_g^k B_{gk}^2 = \frac{J_{gz}^{k,Top} - J_{gz}^{k,Bot}}{h_z \bar{\Phi}_g^k} \tag{11}$$

where

$J_{gz}^{k,Top}$ = z-directed net current on the boundary surface of the top node

$J_{gz}^{k,Bot}$ = z-directed net current on the boundary surface of the bottom node

h_z = assembly height.

In the same way to derive Eq. (4) from Eq. (1), we find that Eq. (6) reduces to the 2D CMFD equation of the similar form as Eqs. (4) and (5);

$$\mathbf{A}_{\text{2D 1 N/A}}^{(n)} \mathbf{\Phi}_{\text{2D 1 N/A}}^{(n)} = \mathbf{b}_{\text{2D 1 N/A}}^{(n)} \tag{12}$$

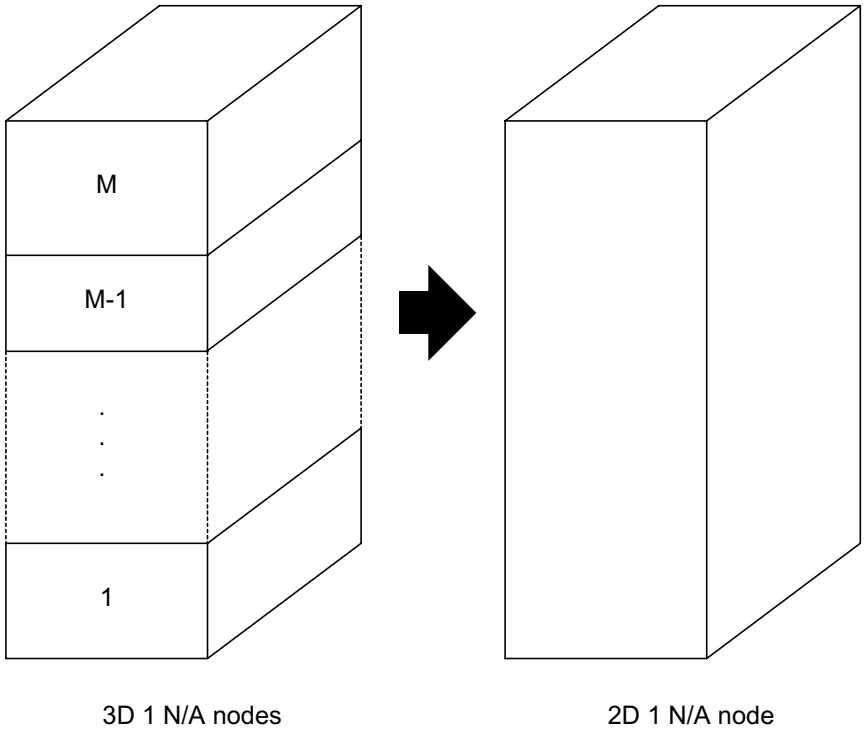

Fig. 1. Construction of 2D 1 N/A node from 3D 1 N/A nodes

3. THREE GRID CORRECTION SCHEME

Table 1 displays the basic idea of coarse grid correction algorithm by which one corrects the fine grid error term by making use of solution to coarse grid residual equation. Suppose that one gets an approximate but intermediate solution, $\mathbf{v}^f$, to fine grid equation $\mathbf{A}^f\mathbf{x}^f = \mathbf{b}^f$. The fine grid error $\mathbf{e}^f$ ($=\mathbf{x}^f - \mathbf{v}^f$) satisfies fine grid residual equation $\mathbf{A}^f\mathbf{e}^f = \mathbf{r}^f$ with fine grid residual $\mathbf{r}^f$ ($= \mathbf{b}^f - \mathbf{A}^f\mathbf{v}^f$). If one knows $\mathbf{e}^f$, the exact solution $\mathbf{x}^f$ can be obtained by simply adding $\mathbf{e}^f$ to $\mathbf{v}^f$. In order to get $\mathbf{e}^f$, one has only to solve the fine grid residual equation. Because this corresponds to solving the original fine grid equation, however, one solves coarse grid residual equation $\mathbf{A}^c\mathbf{e}^c = \mathbf{r}^c$ where coarse grid residual $\mathbf{r}^c$ is obtained from $\mathbf{r}^f$ by applying restriction operator $\mathbf{R}_f^c$. Then, the coarse grid error $\mathbf{e}^c$ is used for approximate estimation of $\mathbf{e}^f$ by using prolongation operator $\mathbf{P}_c^f$ that converts $\mathbf{e}^c$ to $\mathbf{e}^f$. Correction of the intermediate fine grid solution $\mathbf{v}^f$ with the approximate $\mathbf{e}^f$ estimated from $\mathbf{e}^c$ is then expected to eliminate effectively the low frequency errors of the fine grid solution, because they attenuate more effectively in coarse grid equation than in fine grid equation.

Table 1. Coarse grid correction scheme

1. Relax ν_1 times on $\mathbf{A}^f\mathbf{x}^f = \mathbf{b}^f$ on fine grid with initial guess $\mathbf{v}^f$
2. Compute $\mathbf{r}^c = \mathbf{R}_f^c(\mathbf{b}^f - \mathbf{A}^f\mathbf{v}^f)$
3. Solve $\mathbf{A}^c\mathbf{e}^c = \mathbf{r}^c$ on coarse grid
4. Correct fine grid approximation : $\mathbf{v}^f \leftarrow \mathbf{v}^f + \mathbf{P}_c^f\mathbf{e}^e$
5. Relax ν_2 times on $\mathbf{A}^f\mathbf{x}^f = \mathbf{b}^f$ on fine grid with initial guess $\mathbf{v}^f$

To apply coarse grid correction scheme described above for iteration process, one must define the prolongation and restriction operators. Introducing 2GCS in reference 1 in which only the 3D 1 N/A coarse grid equation is used to speed up the solution of 3D 4 N/A fine grid equation, we suggested a restriction operator that was derived from volume average procedure and a prolongation operator that was formed by assuming that fine group flux shape is preserved even after the correction. Because we have 3D 1 N/A coarse grid plus 2D 1 N/A coarse grid in 3GCS, we need to define the restriction and prolongation operators for 2D 1 N/A node. The restriction operator can be formed from volume average procedure from 3D 1 N/A or 3D 4 N/A grid equations to 2D 1 N/A grid

equation. On the other hand, the prolongation operator from 2D 1 N/A to 3D 1 N/A cannot be the same one that was used for 2GCS, because 3D 1 N/A flux is not available explicitly in the correction procedure in the 3GCS. To get around this problem, we devised what we call the modified W cycle for the 3GCS. As shown in Fig. 2, 2D 1 N/A coarse grid equation is not linked to 3D 1 N/A coarse grid equation for error reduction of the latter in this cycle. Instead, the two coarse grid equations are used for correction of the intermediate solutions to the 3D 4 N/A fine grid equation alternately. Note that the prolongation operators in the modified W cycle can be readily defined because approximate 3D 4 N/A flux is available when they need be defined. Note also that the modified W cycle is the modification from the conventional V-cycle or W-cycle (Douglas, 1993) because the prolongation operator for 2D 1 N/A grid is constructed directly from the 4 N/A grid solution by skipping intermediate 1 N/A grid correction.

2D 1 N/A coarse grid equation is also used for determining a trial solution needed at the start of iterative calculation for every time step. Because the trial solution is arbitrary in nature, one may choose the converged 3D 4 N/A fine grid solution of time step n-1, $\mathbf{\Phi}_{\text{3D 4 N/A}}^{(n-1)}$, as the trial solution for the time step n calculation. Observing that the choice of trial solution affects the number of iteration for the converged solution and the CPU time in turn, we tried to determine a trial solution better than $\mathbf{\Phi}_{\text{3D 4 N/A}}^{(n-1)}$ by using 2D 1 N/A coarse grid equation and the converged 3D 4 N/A fine grid solutions at two consecutive time steps n-2 and n-1, namely, $\mathbf{\Phi}_{\text{3D 4 N/A}}^{(n-2)}$ and $\mathbf{\Phi}_{\text{3D 4 N/A}}^{(n-1)}$. At the beginning of time step n, we compute the 2D 1 N/A coarse grid residual vector $\mathbf{r}^c$ using $\mathbf{\Phi}_{\text{3D 4 N/A}}^{(n-1)}$. Then we solve the 2D 1 N/A coarse grid equation to determine the coarse grid error vector $\mathbf{e}^c$. The coarse grid error vector is then converted to the 3D 4 N/A fine grid error vector, $\mathbf{e}^f$, using the prolongation operator which is constructed from the difference of the fine grid solutions at t_{n-1} and t_{n-2}, $\mathbf{\Phi}_{\text{3D 4 N/A}}^{(n-1)} - \mathbf{\Phi}_{\text{3D 4 N/A}}^{(n-2)}$. This $\mathbf{e}^f$ is then added to $\mathbf{\Phi}_{\text{3D 4 N/A}}^{(n-1)}$ and the result is used as the trial solution of the time step n calculation.

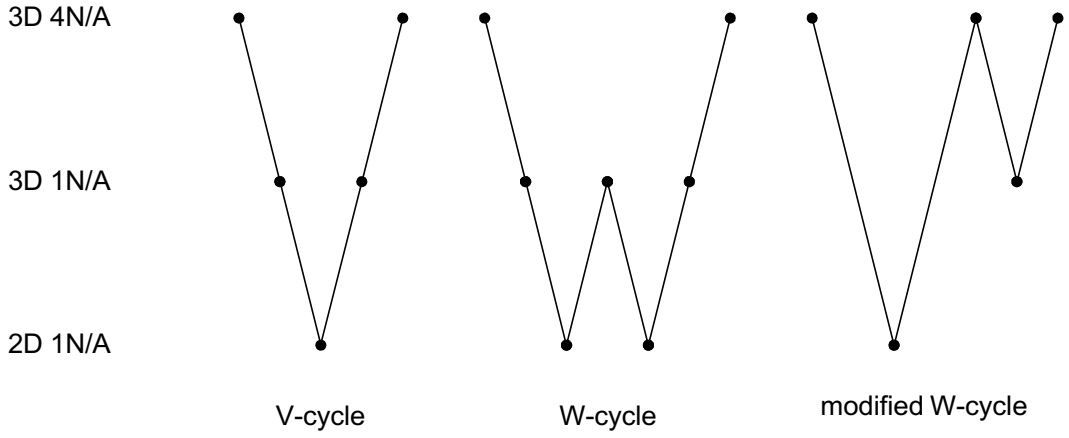

Fig. 2.Three grid cycle

4. NUMERICAL RESULTS AND DISCUSSION

The effectiveness of 3GCS is examined in terms of NEM and ANM analysis for the fast and slow transients in NEACRP 3D PWR. First, we tested the iterative convergence characteristics of 3GCS for fast HZP transient A1 case in comparison with the BICGSTAB iteration scheme with Jacobi and ILU pre-conditioners (Saad, 1996). The transient problem requires solving the linear system of equations with known source term repeatedly every time step. Figure 3 shows reduction trend of normalized residual norm as a function of the iteration number in the iterative solution process of the linear system of equations defining the transient A1 problem at 0.25 sec after the transient is initiated. As shown in Fig. 3, the 3CGS reduces normalized residual norm down to 10^{-5} in less than 10 iterations, while the BICGSTAB iteration scheme without pre-conditioner requires over 80 iterations to achieve the same convergence. The BICGSTAB iteration scheme with pre-conditioners converges with fewer iterations than that without preconditioner. The ILU appears more effective than Jacobi in reducing normalized norm per iteration.

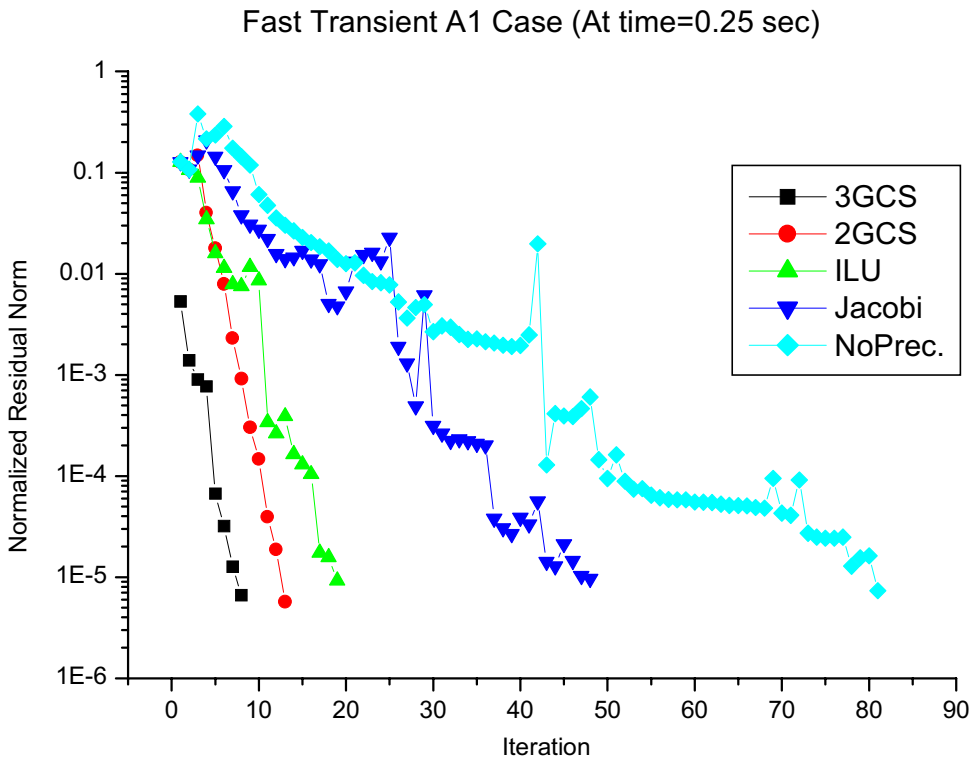

Fig. 3. Convergence of acceleration schemes for A1 case

The iteration number shown in Fig. 3 does not reflect the efficiency of the specific iteration scheme. To compare efficiency of the 3CGS with that of other schemes, we measured the CPU time spent for obtaining the transient solutions to all the NEACRP PWR problems by NEM and ANM with and without the CRBC. Table 2 summarizes

NEM results with or without explicit representation of reflectors by the CRBC. The Jacobi and ILU preconditioners reduce the CPU time by about 30% for all test problems. The 3GCS is about two times faster than these preconditioner schemes for fast transient problems, yet it becomes less effective for slow transient problems- especially for the case D slow transient problem. It is interesting to note that the ILU preconditioner scheme is not so effective as the Jacobi in the comparison of CPU time, contrary to the previous observation (in number of iteration. This may ascribe to the fact that the CPU time per iteration by ILU is longer than that by Jacobi, because ILU involves inversion of incomplete LU matrix per each BICGSTAB iteration.

Table 2. CPU time of accelerated iteration schemes for NEM calculations without CRBC

preconditioner	No Prec.	Jacobi	ILU	3GCS
A1	227.7(1.00)	187.2(0.82)	176.7(0.78)	76.1(0.33)
B1	221.5(1.00)	172.6(0.78)	170.3(0.77)	71.6(0.32)
C1	1912.3(1.00)	1361.8(0.71)	1177.3(0.62)	597.1(0.31)
A2	94.8(1.00)	67.8(0.72)	67.1(0.71)	32.6(0.34)
B2	84.4(1.00)	65.3(0.77)	64.2(0.76)	27.7(0.33)
C2	766.5(1.00)	541.3(0.71)	512.0(0.67)	245.1(0.32)
A	239.2(1.00)	193.1(0.81)	185.5(0.78)	121.4(0.51)
B	328.8(1.00)	263.1(0.80)	238.3(0.72)	181.3(0.55)
D	346.6(1.00)	269.4(0.78)	251.0(0.73)	232.9(0.68)

Table 3 summarizes ANM results without the use of CRBC. The CPU times of the ANM solutions are longer than those of the NEM solutions when the times of two node calculations are included. But comparison of efficiency of one acceleration scheme over another shows similar trend as the NEM case. Jacobi and ILU preconditioners reduce the CPU time of the BICGSTAB without preconditioners by about 20 ~ 30% while the 3GCS 40 ~ 70%. As observed in the NEM calculation, the 3GCS is relatively less efficient for slow transient problems than for fast transient problems. In slow transients, the rod ejection speed is so slow that local flux changes continue to take place over the whole rod ejection period. Because coarse grid equations are more effective to error reduction in problems with global flux changes than in problems with local flux changes, the 3GCS may become less effective in the slow transient problems than in the fast transient ones.

Table 3. CPU time of accelerated iteration schemes for ANM calculation without CRBC

preconditioner	No Prec.	Jacobi	ILU	3GCS
A1	234.4(1.00)	187.5(0.80)	182.1(0.78)	84.8(0.36)
B1	224.7(1.00)	179.0(0.80)	184.0(0.82)	81.8(0.36)
C1	2012.1(1.00)	1427.1(0.71)	1315.3(0.65)	689.6(0.34)
A2	92.6(1.00)	71.6(0.77)	71.7(0.77)	38.1(0.41)
B2	87.6(1.00)	68.3(0.78)	70.1(0.80)	33.5(0.38)
C2	822.7(1.00)	583.3(0.71)	552.4(0.67)	280.7(0.34)
A	241.5(1.00)	199.2(0.82)	200.7(0.83)	134.1(0.56)
B	369.8(1.00)	289.8(0.78)	253.7(0.69)	197.4(0.53)
D	366.4(1.00)	285.7(0.78)	273.8(0.75)	233.7(0.64)

Tables 4 and 5 show NEM and ANM results with the use of CRBC. Use of CRBC reduces the number of unknowns because reflectors are excluded from the computational region. As a result, the CPU time reduces significantly. Note that the CPU time in Tables 4 and 5 is by about 30% less than that in Table 1 and Table 2 results. We observe that the 3GCS is very effective in reducing the CPU time of the two TINM solutions with CRBC.

Table 4. CPU time of accelerated iteration schemes for NEM calculation with CRBC

preconditioner	No Prec.	Jacobi	ILU	3GCS
A1	123.1(1.00)	128.2(1.04)	128.0(1.04)	55.6(0.45)
B1	126.2(1.00)	125.3(0.99)	131.8(1.04)	54.7(0.43)
C1	1076.8(1.00)	1035.5(0.96)	890.6(0.83)	492.3(0.46)
A2	52.7(1.00)	51.0(0.97)	50.0(0.95)	23.4(0.44)
B2	50.3(1.00)	47.8(0.95)	49.6(0.99)	21.4(0.43)
C2	433.6(1.00)	398.0(0.92)	376.8(0.87)	178.5(0.41)
A	166.7(1.00)	165.6(0.99)	142.6(0.86)	82.5(0.49)
B	193.3(1.00)	191.7(0.99)	187.0(0.97)	104.2(0.54)
D	234.7(1.00)	223.4(0.95)	202.1(0.86)	120.1(0.51)

Table 5. CPU time of accelerated iteration schemes for ANM calculation with CRBC

preconditioner	No Prec.	Jacobi	ILU	3GCS
A1	152.0(1.00)	154.1(1.02)	152.1(1.00)	69.8(0.46)
B1	152.2(1.00)	148.9(0.98)	148.6(0.98)	68.5(0.45)
C1	1297.9(1.00)	1167.5(0.90)	1020.9(0.79)	587.0(0.45)
A2	59.0(1.00)	56.7(0.96)	59.4(1.00)	28.3(0.48)
B2	60.6(1.00)	59.1(0.98)	59.1(0.98)	27.4(0.45)
C2	543.5(1.00)	508.1(0.93)	438.0(0.81)	226.9(0.42)
A	171.4(1.00)	166.9(0.97)	156.5(0.91)	89.8(0.52)
B	235.6(1.00)	238.9(1.02)	216.3(0.92)	129.4(0.55)
D	234.4(1.00)	243.5(1.04)	218.6(0.93)	130.2(0.56)

5. CONCLUSION

In this paper we presented a 3GCS which make use of the 3D 1 N/A and 2D 1 N/A coarse grid equations on the modified W cycle to speed up the 4 N/A nonlinear ANM and NEM solution to transient neutronics benchmark problems. Computational effectiveness of the 3GCS is then examined in terms of the NEM and ANM analysis of OECD/NEACRP transient benchmark problems. We demonstrated that the 3GCS can reduce the CPU time of the BICGSTAB iteration scheme without preconditioner by 40 ~70 %, depending on the benchmark problems. We also demonstrated that the 3GCS is more effective in reducing the CPU time than the BICGSTAB with conventional Jacobi and ILU preconditioners. Therefore we safely conclude that the 3GCS must be very useful for enhancing computational efficiency of the iterative solver in the popular nonlinear TINM reactor analysis codes.

NOMENCLATURE

χ	fission spectrum
β	delayed neutron fraction
Σ	homogenized macroscopic cross section
Φ	nodal volume-averaged neutron flux
λ	decay constant of delayed neutron precursor
C	delayed neutron precursor concentration
h	width of numerical node
J	surface averaged neutron current

Subscripts

g	neutron energy group
d	delayed neutron precursor group
u	node direction
s	the side of node
l	left side of node in u direction
r	right side of node in u direction
p	prompt neutron
t	removal reaction
f	fission reaction
gg'	scattering reaction from g' to g

Superscripts

k	radial node index
m	axial node index
n	time step index
f	fine grid
c	coarse grid

REFERENCES

Al-Chalabi, Rifat M. and Turinsky, Paul J., 1994. Application of Multigrid Method to Solving the NEM Form of Multigroup Neutron Diffusion Equation. *Trans. Am. Nucl. Soc*., **71**, p. 259.

Alcouffe, R., 1983. The Multigrid Method for Solving the Two-Dimensional Multigroup Diffusion Equation. Proc. Topl. Mtg. Advances in Reactor Computations, Salt Lake City, UT, p. 340.

Beam, Tara M., Ivanov, Kostadin, Baratta, Anthony J., and Finnemann, H., 1999. Nodal Kinetics model upgrade in the Penn Stste coupled TRAC/NEM codes. *Annals of Nuclear Energy*. **26**, pp.1205-1219.

Briggs, W. L., 1987. *A Multigrid Tutorial*, SIAM, Philadelphia.

Chung, Ku Young and Kim, Chang Hyo, 2000. Application of Multigrid Correction

Scheme Nonlinear Nodal Method Solutions with Use of Core Reflector Boundary Conditions. Proc. of the 2000 Int. Topical Mtg. on Advances in Reactor Physics and Mathematics and Computation into the Next Millenium, PHYSOR 2000, Pittsburgh, USA.

Douglas, C. C. and Douglas, J., 1993. A unified convergence theory for abstract multigrid or multilevel algorithms, serial and parallel. *SIAM J. Numer. Anal.* **30**, pp. 136-158.

Finnemann, H., 1975. A Consistent Nodal Method for the Analysis of Space-Time Effects in Large LWRs. Proc. Joint NEACRP/CSNI Specialists; Mtg. On New Developments in Three-Dimensional Neutron Kinetics, MRR 145, pp.131, January 22-24, Munich.

Finnemann, H., 1992. NEACRP 3D LWR Core Transient Benchmark - Final Specifications. NEACRP-L-335(Revision 1).

Fraikin, R. and Finnemman, H. NEA-NSC 3D/1D PWR Core Transient Benchmark (Uncontrolled Withdrawal of Control Rods at Zero Power) - Final Specifications. OECD/NEA, September 1993.

Kim, Chang Hyo, Cho, Jin Young, and Joo, Han Gyu, 1994. Approximate Three-Dimensional Correction Factors in Two-Dimensional Reactor Calculations. *Nucl. Sci. Eng*. **118**, pp. 108-121.

Lee, Eun Ki, Kim, Chang Hyo, and Joo, Hyung Kook, 1995. New Core-Reflector Boundary Conditions for Transient Nodal Reactor Calculations. *Nucl. Sci. Eng*. **121**, pp. 114-129.

Saad, Yousef, 1996. *Iterative methods for sparse linear system*, PWS Publishing Company, Boston, pp. 266-277.

Scheichl, R. 1999. Parallel Solver for the Transient Multigroup Neutron Diffusion Equations. Mathematics Preprint 98/30, University of Bath, United Kingdom.

Smith, K. S., 1979. An Analytic Nodal Method for Solving the Two-Group, Multidimensional Ststic and Transient Neutron Diffusion Equations. Master Thesis, Massachusetts Institute of Technology.

Smith, K. S., 1983. Nodal Storage Reduction by Nonlinear Iteration. *Trans. Am. Nucl*. **44**, p. 265.

Van Der Vorst, H. A., 1992. BI-CGSTAB: A fast and smoothly converging varient of BI-CG for the solution of nonsymmetric linear systems. *SIAM J. Sci. Stat. Comput*. **13**, pp. 631-644.

Zaslavsky, Leonid Yu., 1995. An Adaptive Algebraic Multigrid For Reactor Criticality Calculations. *SIAM J. Sci. Comput.* **16**, No. 4, pp. 840-847.