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Unified Nodal Method for Solution to the Space-Time Kinetics Problems

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ABSTRACT

The popular transverse integrated nodal method (TINM) like the nodal expansion method (NEM) and analytical nodal method (ANM) and analytic function expansion nodal (AFEN) method are integrated into a single unified nodal formulation for the space-time kinetics calculations in rectangular core geometry. In particular, the nodal coupling equations of the conventional ANM and AFEN method are reformulated on the unified nodal method (UNM) principle for the solution to the transient two-group neutronics benchmark problems. The performance of the UNM formulation is examined in terms of the solution to the NEACRP PWR transient problems. Through comparison of several nodal computational options by the UNM formulation, it is shown that the AFEN method outperforms the nodal expansion method (NEM) and the analytic nodal method (ANM) in prediction accuracy at the sacrifice of the computational time. The advantages of the transient UNM formulation over the conventional TINM and AFEN method formulations are discussed.

1. INTRODUCTION

Modern nodal methods have enhanced greatly the computational effectiveness of neutronics design and analysis of the power reactors in terms of computational speed and accuracy.(Dorning 1979; Sutton and Aviles,1996). In particular, the TINM introduced in 1970's like the NEM (Finnemann, 1977) and the ANM (Smith,1979) have become the popular basis of many production neutronics codes and they are routinely being used by reactor vendors and fuel companies. As a way to improve the computational accuracy of the TINM further, Noh et al.(Noh and Cho, 1994) introduced the AFEN method. They demonstrated that the AFEN method outperforms the TINM in terms of static neutronics benchmark problems. Very recently, Kim et al. (Kim and Cho,2000) introduced an AFEN kinetics calculation method and showed a better computational accuracy of it over the TINM in terms of the two-dimensional (2-D) kinetics benchmark problems. Nevertheless, Three-dimensional (3-D) transient AFEN method calculations has not been reported and the AFEN method has not yet been implemented into production codes.

The formulation of AFEN method is very different from that of the TINM not only because it includes additional unknowns that are absent in the TINM but because it uses the direct solution to the multi-dimensional diffusion equation for the intranodal flux instead of the solution to transverse integrated one-dimensional (1-D) diffusion equations. Consequently, the nodal coupling equations of the AFEN method resemble none of the TINM, which in turn makes it very difficult to incorporate the AFEN method into the existing TINM production codes. However, we recently showed that the AFEN method formulation as well as the popular TINM (NEM and ANM) formulation can be integrated into a single UNM formulation (Lee and Kim,2001). We demonstrated that the UNM formulation can produce the various nodal solutions equivalent to the conventional NEM, ANM, and AFEN method solutions to the static two-group neutronics benchmark problems in rectangular geometry, depending on the nodal computational option of the UNM formulation. The purpose of this paper is to present the transient UNM formulation for the kinetics neutronics calculations, to examine the performance of the UNM kinetics calculations in terms of the 3-D NEACRP PWR kinetics benchmark problems, and to discuss the advantages of the UNM formulation over the conventional ANM and AFEN method formulation. In doing so, we present the transient ANM solution with the analytical transverse leakage approximation (ATL) introduced in reference (Lee and Kim, 2000). Also, we present the first-ever 3-D transient AFEN method calculations and discuss the computational effectiveness of the AFEN kinetics calculation in comparison with that of the TINM kinetics calculation.

2. TRANSIENT UNM FORMULATION

The transient UNM formulation here concerns with solving a set of the time-dependent two-group (2-G) diffusion equations for a given spatial node *m*

$$\frac{1}{v_g}\frac{\partial \phi_g^m(\mathbf{r},t)}{\partial t} = \nabla\cdot D_g^m(t)\nabla\phi_g^m(\mathbf{r},t) - \Sigma_{rg}^m(t)\phi_g^m(\mathbf{r},t) + \frac{\chi_g}{k_{eff}}(1-\beta)\sum_{g'=1}^{2}\nu\Sigma_{fg'}^m(t)\phi_{g'}^m(\mathbf{r},t) + \sum_{\substack{g'=1\\ g'\neq g}}^{2}\Sigma_{sgg'}^m(t)\phi_{g'}^m(\mathbf{r},t) + \chi_g\sum_{d=1}^{6}\lambda_d C_d^m(\mathbf{r},t);\quad g=1,2 \tag{1a}$$

$$\frac{\partial C_d^m(\mathbf{r},t)}{\partial t} = \frac{\beta}{k_{eff}}\sum_{g'=1}^{2}\nu\Sigma_{fg'}^m(t)\phi_{g'}^m(\mathbf{r},t) - \lambda_d C_d^m(\mathbf{r},t);\quad d=1,2,\cdots,6 \tag{1b}$$

$\phi_g^m(\mathbf{r},t)$ and $C_d^m(\mathbf{r},t)$ are the group g flux and the type d delayed neutron precursor density, respectively. The other notations are standard.

A fully implicit temporal integration of Eq. (1) over the time interval between (t_{n-1},t_n) leads to

$$-\nabla\cdot D_g^{(n)}\nabla\phi_g^{(n)}(\mathbf{r})+\Sigma_{rg}^{(n)}\phi_g^{(n)}(\mathbf{r})-\frac{\chi_g}{k_{eff}}\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{(n)}\phi_{g'}^{(n)}(\mathbf{r})-\sum_{\substack{g'=1\\g'\neq g}}^{2}\Sigma_{sgg'}^{(n)}\phi_{g'}^{(n)}(\mathbf{r})$$

$$=\frac{e^{\Omega_g^{(n)}(\mathbf{r})\Delta t}}{v_g\Delta t}\phi_g^{(n-1)}(\mathbf{r})-\frac{\Omega_g^{(n)}(\mathbf{r})\Delta t+1}{v_g\Delta t}\phi_g^{(n)}(\mathbf{r})+\chi_g\sum_{d=1}^{6}\lambda_d e^{-\lambda_d\Delta t}C_d^{(n-1)}(\mathbf{r}) \tag{2a}$$

$$+\frac{\chi_g}{k_{eff}}\sum_{d=1}^{6}\beta_d\left[\frac{\lambda_d}{\lambda_d+\omega^{(n)}(\mathbf{r})}\left(1-e^{-\left(\lambda_d+\omega^{(n)}(\mathbf{r})\right)\Delta t}\right)-1\right]\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{(n)}\phi_{g'}^{(n)}(\mathbf{r})$$

$$C_d^{(n-1)}(\mathbf{r})\approx e^{-\lambda_d\Delta t}C_d^{(n-1)}(\mathbf{r})+\frac{\beta_d}{k_{eff}}\left(\frac{1-e^{-\left(\lambda_d+\omega^{(n)}(\mathbf{r})\right)\Delta t}}{\lambda_d+\omega^{(n)}(\mathbf{r})}\right)\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{(n)}\phi_{g'}^{(n)}(\mathbf{r});\quad d=1,2,\cdots,6 \tag{2b}$$

The superscript m in Eq. (2) is dropped for simplicity of notation. In deriving Eq. (2), $\phi_g^m(\mathbf{r},t)$ is assumed by

$$\phi_g^m(\mathbf{r},t)=e^{\Omega_g^m(\mathbf{r})t}\psi_g^m(\mathbf{r},t) \tag{3a}$$

and the fission source term of Eq. (1b) is approximated by

$$\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{m}(t)\phi_{g'}^{m}(\mathbf{r},t)\approx e^{\omega^{(n)}(\mathbf{r})t}\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{m}(t)\psi_{g'}^{m}(\mathbf{r},t)\ , \tag{3b}$$

where $\omega^{(n)}(\mathbf{r})=\dfrac{\Omega_1^{(n)}(\mathbf{r})+\Omega_2^{(n)}(\mathbf{r})}{2}$.

It is a straightforward matter to obtain a set of the nodal balance equations from the nodal volume integration of Eq. (2a) at each time step;

$$\sum_{u=x,y,z}\frac{1}{a_u}\left(J_{gur}^{(n)}-J_{gul}^{(n)}\right)+\Sigma_{rg}^{(n)}\bar{\phi}_g^{(n)}-\frac{\chi_g}{k_{eff}}\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{(n)}\bar{\phi}_{g'}^{(n)}-\sum_{\substack{g'=1\\g'\neq g}}^{2}\Sigma_{sgg'}^{(n)}\bar{\phi}_{g'}^{(n)}=\bar{S}_g^{(n)} \tag{4}$$

where $\bar{S}_g^{(n)}=\dfrac{1}{V_m}\int_{V_m}S_g^{(n)}(\mathbf{r})dV$,

$$\begin{aligned}S_g^{(n)}(\mathbf{r})=&\frac{e^{\Omega_g^{(n)}(\mathbf{r})\Delta t}}{v_g\Delta t}\phi_g^{(n-1)}(\mathbf{r})-\frac{\Omega_g^{(n)}(\mathbf{r})\Delta t+1}{v_g\Delta t}\phi_g^{(n)}(\mathbf{r})+\chi_g\sum_{d=1}^{6}\lambda_d e^{-\lambda_d\Delta t}C_d^{(n-1)}(\mathbf{r})\\&+\frac{\chi_g}{k_{eff}}\sum_{d=1}^{6}\beta_d\left[\frac{\lambda_d}{\lambda_d+\omega^{(n)}(\mathbf{r})}\left(1-e^{-\left(\lambda_d+\omega^{(n)}(\mathbf{r})\right)\Delta t}\right)-1\right]\sum_{g'=1}^{2}\nu\Sigma_{fg'}^{(n)}\phi_{g'}^{(n)}(\mathbf{r})\end{aligned}\ . \tag{5}$$

Note that $S_g^{(n)}(\mathbf{r})$ is the source term driving the transient and vanishes at the initial steady state. In the UNM the coupling relations between the two nodal unknowns, $\bar{\phi}_g^{(n)}$ and $J_{gus}^{(n)}$, are obtained from the solutions to transverse integrated 1-D equations of Eq. (2a). To show this, let's put Eq. (2a) into

$$\nabla^2 \boldsymbol{\phi}^{(n)}(\mathbf{r}) - \left(\mathbf{D}^{(n)}\right)^{-1} \mathbf{A}^{(n)} \boldsymbol{\phi}^{(n)}(\mathbf{r}) = \left(\mathbf{D}^{(n)}\right)^{-1} \mathbf{S}^{(n)}(\mathbf{r}) \tag{6}$$

and perform the similarity transformation to find

$$\nabla^2 \boldsymbol{\xi}^{(n)}(\mathbf{r}) - \hat{\mathbf{A}}^{(n)} \boldsymbol{\xi}^{(n)}(\mathbf{r}) = \hat{\mathbf{S}}^{(n)}(\mathbf{r}), \tag{7}$$

where

$$\mathbf{D}^{(n)} = \begin{pmatrix} D_1^{(n)} & 0 \\ 0 & D_2^{(n)} \end{pmatrix},$$

$$\mathbf{A}^{(n)} = \begin{pmatrix} \Sigma_{r1}^{(n)} - \dfrac{1}{k_{eff}} \nu\Sigma_{f1}^{(n)} & -\dfrac{1}{k_{eff}} \nu\Sigma_{f2}^{(n)} \\ -\Sigma_{s21}^{(n)} & \Sigma_{r2}^{(n)} \end{pmatrix},$$

$\boldsymbol{\phi}^{(n)}(\mathbf{r})$ = 2-D column vector $\left(\phi_1^{(n)}(\mathbf{r}), \phi_2^{(n)}(\mathbf{r})\right)^T$

$$\boldsymbol{\xi}^{(n)}(\mathbf{r}) = \left(\mathbf{R}^{(n)}\right)^{-1} \boldsymbol{\phi}^{(n)}(\mathbf{r}),$$

$$\hat{\mathbf{A}}^{(n)} = \left(\mathbf{R}^{(n)}\right)^{-1} \left(\mathbf{D}^{(n)}\right)^{-1} \mathbf{A}^{(n)} \mathbf{R}^{(n)} = \begin{pmatrix} \lambda_1^{(n)} & 0 \\ 0 & \lambda_2^{(n)} \end{pmatrix}, \tag{8}$$

$$\hat{\mathbf{S}}^{(n)}(\mathbf{r}) = \left(\mathbf{R}^{(n)}\right)^{-1} \left(\mathbf{D}^{(n)}\right)^{-1} \mathbf{S}^{(n)}(\mathbf{r}),$$

$\mathbf{R}^{(n)}$ = The similarity transformation matrix.

The transverse integration of Eq.(6) then leads to

$$\frac{d^2}{du^2} \boldsymbol{\xi}_u(u) - \hat{\mathbf{A}} \boldsymbol{\xi}_u(u) = \hat{\mathbf{L}}_u(u) + \hat{\mathbf{S}}_u(u), \tag{9}$$

where $\boldsymbol{\xi}_u(u) = \dfrac{1}{a_u a_w} \int_{-\frac{a_w}{2}}^{+\frac{a_w}{2}} \int_{-\frac{a_v}{2}}^{+\frac{a_v}{2}} \boldsymbol{\xi}(u, v, w) dv dw,$ (10a)

$$\hat{\mathbf{L}}_u(u) = -\frac{1}{a_u a_w}\int_{-\frac{a_w}{2}}^{+\frac{a_w}{2}}\int_{-\frac{a_v}{2}}^{+\frac{a_v}{2}}\left(\frac{\partial^2}{\partial v^2}+\frac{\partial^2}{\partial w^2}\right)\boldsymbol{\xi}(u,v,w)dvdw\,. \tag{10b}$$

$$\hat{\mathbf{S}}_u(u) = \frac{1}{a_u a_w}\int_{-\frac{a_w}{2}}^{+\frac{a_w}{2}}\int_{-\frac{a_v}{2}}^{+\frac{a_v}{2}}\mathbf{S}(u,v,w)dvdw \tag{10c}$$

The superscript (n) in Eq. (9) is dropped for simplicity. Equation (9) is equivalent to the transverse integrated 1-D equation given in reference (Engrand et al.,1992) for nonlinear nodal kinetics calculations. It contains two source terms; one representing the transverse leakage and the other from $\hat{\mathbf{S}}^{(n)}(\mathbf{r})$. Because the transverse leakage is a priori unknown, one needs to introduce approximations such as the popular quadratic polynomial approximation (QPA) (Finnemann, 1977) or analytical transverse leakage approximation (ATL) (Lee and Kim, 2000). Without loss of generality, one can represent these approximations by

$$\hat{\mathbf{L}}_u(u) = \sum_{i=0}^{2}\begin{pmatrix} g_{i1u}(u/a_u) & 0 \\ 0 & g_{i2u}(u/a_u)\end{pmatrix}\hat{\mathbf{L}}_{iu}\,, \tag{11}$$

where the shape functions, $g_{ipu}(\tau)$, can be either quadratic polynomials or analytic functions that satisfy the following conditions:

$$g_{ipu}(-\tau) = +g_{ipu}(\tau)\ \ (i=0,2),\ \ g_{1pu}(-\tau) = -g_{1pu}(\tau),$$

$$g_{ipu}(+1/2) = +1\ \ (i=0,1,2),\ \ \int_{-1/2}^{+1/2} g_{0pu}(\tau)d\tau = 1,\ \ \int_{-1/2}^{+1/2} g_{ipu}(\tau)d\tau = 0\ \ (i=1,2).$$

As for $\hat{\mathbf{S}}_u(u)$, we approximate it by using the same shape functions used for the transverse leakage as follows;

$$\hat{\mathbf{S}}_u(u) = \sum_{i=0}^{2}\begin{pmatrix} g_{i1u}(u/a_u) & 0 \\ 0 & g_{i2u}(u/a_u)\end{pmatrix}\hat{\mathbf{S}}_{iu}\,. \tag{12}$$

As will be shown shortly, this approximation makes the transient UNM formulation maintain the same static formulation. With these approximations, Eq. (9) becomes

$$\frac{d^2}{du^2}\boldsymbol{\xi}_u(u) - \hat{\mathbf{A}}\boldsymbol{\xi}_u(u) = \sum_{i=0}^{2}\begin{pmatrix} g_{i1u}(u/a_u) & 0 \\ 0 & g_{i2u}(u/a_u)\end{pmatrix}\left(\hat{\mathbf{L}}_{iu}+\hat{\mathbf{S}}_{iu}\right). \tag{13}$$

We note that, aside from the presence of $\hat{\mathbf{S}}_{iu}$, Eq. (13) is the same as the static 1-D equation. Therefore, one can follow the same mathematical manipulation as the static

UNM case to obtain not only the solution to Eq. (13) but also the coupling relations between $\overline{\phi}_g^{(n)}$ and $J_{gus}^{(n)}$ from it. For example, the solution to Eq. (13) is sought by assuming the following expansion;

$$\xi_{pu}(u) = \sum_{i=0}^{4} \hat{C}_{ipu} f_{ipu}(u/a_u), \tag{14}$$

where the shape functions, $f_{ipu}(\tau)$, satisfy the following conditions:

$$f_{ipu}(-\tau) = +f_{ipu}(\tau) \ (i = 0,2,4), \quad f_{ipu}(-\tau) = -f_{ipu}(\tau) \ (i = 1,3),$$

$$f_{ipu}(+1/2) = +1 \ (i = 0,1,2), \quad f_{ipu}(+1/2) = 0 \ (i = 3,4),$$

$$f'_{0pu}(+1/2) = 0, \quad f'_{1pu}(+1/2) = 2, \quad f'_{2pu}(+1/2) = 6, \quad f'_{ipu}(+1/2) = +1 \ (i = 3,4),$$

$$\int_{-1/2}^{+1/2} f_{0pu}(\tau) d\tau = 1, \quad \int_{-1/2}^{+1/2} f_{ipu}(\tau) d\tau = 0 \ (i = 1,2,3,4).$$

Table 1 shows two sets of basis functions, depending on the QTL and ATL. They are the same basis functions that are used in the static UNM formulation in reference (Lee and Kim,2000) and (Lee and Kim,2001).

Table 1. Functions $g_{ipu}(\tau)$ and $f_{ipu}(\tau)$.

UNM Options	$g_{ipu}(\tau)$	$f_{ipu}(\tau)$
ANM / QTL	$g_{0pu}(\tau) = 1$ $g_{1pu}(\tau) = 2\tau$ $g_{2pu}(\tau) = 6\tau^2 - 1/2$	$f_{0pu}(\tau) = 1$ $f_{1pu}(\tau) = 2\tau$ $f_{2pu}(\tau) = 6\tau^2 - 1/2$ $f_{3pu}(\tau) = a_3 \sinh(2\tilde{\kappa}_{pu}\tau) + b_3\tau$ $f_{4pu}(\tau) = a_4 \cosh(2\tilde{\kappa}_{pu}\tau) + b_4\tau^2 + c_4$
ANM / ATL or AFEN	$g_{0pu}(\tau) = 1$ $g_{1pu}(\tau) = a_1 \sinh(2\tilde{\mu}_{pu}\tau)$ $g_{2pu}(\tau) = a_2 \cosh(2\tilde{\mu}_{pu}\tau) + b_2$	$f_{0pu}(\tau) = 1$ $f_{1pu}(\tau) = a_1 \sinh(2\tilde{\kappa}_{pu}\tau) + b_1 \sinh(2\tilde{\mu}_{pu}\tau)$ $f_{2pu}(\tau) = a_2 \cosh(2\tilde{\kappa}_{pu}\tau) + b_2 \cosh(2\tilde{\mu}_{pu}\tau) + c_2$ $f_{3pu}(\tau) = a_3 \sinh(2\tilde{\kappa}_{pu}\tau) + b_3 \sinh(2\tilde{\mu}_{pu}\tau)$ $f_{4pu}(\tau) = a_4 \cosh(2\tilde{\kappa}_{pu}\tau) + b_4 \cosh(2\tilde{\mu}_{pu}\tau) + c_4$

The five expansion coefficients, $\hat{C}_{ipu}(i=0,1,2,3,4)$, are determined by nodal balance condition, $\bar{\xi}_p = 1/a_u \int_{-a_u/2}^{+a_u/2} \xi_{pu}(u)du$, two conditions at two nodal surfaces, $\xi_{pu}(\pm a_u/2)$, and two weighted residual method (WRM) equations with odd ($w_1(u)$) and even ($w_2(u)$) weighting functions

$$\int_{-a_u/2}^{+a_u/2} w_i(u)[\text{Eq.(13) with } \xi_{pu}(u) \text{ in Eq. (14)}]du \quad (i=1,2), \tag{15}$$

to find

$$\hat{\mathbf{C}}_{0u} = \bar{\boldsymbol{\xi}}_u, \tag{16}$$

$$\hat{\mathbf{C}}_{1u} = \frac{1}{2}\left[\boldsymbol{\xi}_u\left(+\frac{a_u}{2}\right) - \boldsymbol{\xi}_u\left(-\frac{a_u}{2}\right)\right], \tag{17}$$

$$\hat{\mathbf{C}}_{2u} = \frac{1}{2}\left[\boldsymbol{\xi}_u\left(+\frac{a_u}{2}\right) + \boldsymbol{\xi}_u\left(-\frac{a_u}{2}\right)\right] - \bar{\boldsymbol{\xi}}_u, \tag{18}$$

$$\hat{\mathbf{C}}_{3u} = \left(\hat{\mathbf{G}}_{3u}\hat{\mathbf{H}}_{3u}\right)^{-1}\left[\hat{\mathbf{A}}\hat{\mathbf{H}}_{3u}\hat{\mathbf{C}}_{1u} + \hat{\mathbf{L}}_{1u} + \hat{\mathbf{S}}_{1u}\right], \tag{19}$$

$$\hat{\mathbf{C}}_{4u} = \left(\hat{\mathbf{G}}_{4u}\hat{\mathbf{H}}_{4u}\right)^{-1}\left[\hat{\mathbf{A}}\hat{\mathbf{H}}_{4u}\hat{\mathbf{C}}_{2u} + \hat{\mathbf{L}}_{2u} + \hat{\mathbf{S}}_{2u}\right], \tag{20}$$

where

$\hat{\mathbf{C}}_{iu}$ = 2-D column vector $(\hat{C}_{i1u}, \hat{C}_{i2u})(i=0,1,\ldots,4)$,

$\bar{\boldsymbol{\xi}}$ = 2-D column vector $(\bar{\xi}_{1u}, \bar{\xi}_{2u})$,

$$\hat{\mathbf{G}}_{iu} = \begin{pmatrix} \hat{G}_{i1u} & 0 \\ 0 & \hat{G}_{i2u} \end{pmatrix}, \ \hat{\mathbf{H}}_{iu} = \begin{pmatrix} \hat{H}_{i1u} & 0 \\ 0 & \hat{H}_{i2u} \end{pmatrix} (i=3,4). \tag{21}$$

The elements of $\hat{\mathbf{G}}_{iu}$ and $\hat{\mathbf{H}}_{iu}$ are given in reference (Lee and Kim,2001).

The expansion coefficients determined above are related to node average flux and nodal surface average fluxes as follows;

$$\mathbf{C}_{0u} \equiv \mathbf{R}\hat{\mathbf{C}}_{0u} = \mathbf{R}\bar{\boldsymbol{\xi}}_u = \bar{\boldsymbol{\phi}}_u, \tag{22}$$

$$\mathbf{C}_{1u} \equiv \mathbf{R}\hat{\mathbf{C}}_{1u} = \frac{1}{2}\left[\boldsymbol{\phi}_u\left(+\frac{a_u}{2}\right) - \boldsymbol{\phi}_u\left(-\frac{a_u}{2}\right)\right], \tag{23}$$

$$\mathbf{C}_{2u} \equiv \mathbf{R}\hat{\mathbf{C}}_{2u} = \frac{1}{2}\left[\boldsymbol{\phi}_u\left(+\frac{a_u}{2}\right) + \boldsymbol{\phi}_u\left(-\frac{a_u}{2}\right)\right] - \bar{\boldsymbol{\phi}}_u, \tag{24}$$

$$\mathbf{C}_{3u} = \mathbf{R}\hat{\mathbf{C}}_{3u} = \mathbf{M}_{3u}^{-1}(\mathbf{M}_{1u}\mathbf{C}_{1u} + \mathbf{L}_{1u} + \mathbf{S}_{1u}), \tag{25}$$

$$\mathbf{C}_{4u} = \mathbf{R}\hat{\mathbf{C}}_{4u} = \mathbf{M}_{4u}^{-1}(\mathbf{M}_{2u}\mathbf{C}_{2u} + \mathbf{L}_{2u} + \mathbf{S}_{2u}), \tag{26}$$

where

$$\mathbf{M}_{1u} = \mathbf{R}\hat{\mathbf{A}}\hat{\mathbf{H}}_{3u}\mathbf{R}^{-1},\ \mathbf{M}_{3u} = \mathbf{D}\mathbf{R}\hat{\mathbf{G}}_{3u}\hat{\mathbf{H}}_{3u}\mathbf{R}^{-1}, \tag{27}$$

$$\mathbf{M}_{2u} = \mathbf{R}\hat{\mathbf{A}}\hat{\mathbf{H}}_{4u}\mathbf{R}^{-1},\ \mathbf{M}_{4u} = \mathbf{D}\mathbf{R}\hat{\mathbf{G}}_{4u}\hat{\mathbf{H}}_{4u}\mathbf{R}^{-1}, \tag{28}$$

$$\mathbf{L}_{iu} = \mathbf{D}\mathbf{R}\hat{\mathbf{L}}_{iu}\ (i = 1,2) \tag{29}$$

From the definition of the partial currents,

$$\mathbf{j}_{us}^{\pm} = \frac{1}{4}\boldsymbol{\phi}_{us} \mp \frac{1}{2}\mathbf{D}\frac{d\boldsymbol{\phi}_{us}}{du}\ (s = l,r), \tag{30}$$

and noting $\boldsymbol{\phi}_u(u) \equiv \mathbf{R}\boldsymbol{\xi}_u(u)$, one can relate the outgoing partial currents to the incoming partial currents by

$$\begin{aligned}\mathbf{j}_{ur}^{+} &= \mathbf{Q}_{0u}\left(6\bar{\boldsymbol{\phi}} - \mathbf{C}_{4u}\right) - \mathbf{Q}_{1u}\mathbf{C}_{3u} - \mathbf{Q}_{2u}\mathbf{j}_{ul}^{+} + \mathbf{Q}_{3u}\mathbf{j}_{ur}^{-} \\ \mathbf{j}_{ul}^{-} &= \mathbf{Q}_{0u}\left(6\bar{\boldsymbol{\phi}} - \mathbf{C}_{4u}\right) + \mathbf{Q}_{1u}\mathbf{C}_{3u} - \mathbf{Q}_{2u}\mathbf{j}_{ur}^{-} + \mathbf{Q}_{3u}\mathbf{j}_{ul}^{+}\end{aligned}, \tag{31}$$

where $\mathbf{j}_{us}^{\pm}$ = 2-D column vector $\left(j_{1us}^{\pm}, j_{2us}^{\pm}\right)$ and $\boldsymbol{\phi}_{us}$ = 2-D column vector $\left(\phi_{1us}, \phi_{2us}\right)$. They are the *u*-directed partial currents and 1-D fluxes, respectively, at the right surface (*s*=*r*) or at the left surface (*s*=*l*) of the node. The 2x2 diagonal matrices, $\mathbf{Q}_{ku}$ ($k = 0,1,2,3$), are defined in reference (Lee and Kim, 2001). If one substitutes these equations into the nodal balance equation, Eq. (4), one finds that the node average flux is given by

$$\bar{\boldsymbol{\phi}} = \left(\mathbf{A} + 12\mathbf{Q}_0\right)^{-1}\mathbf{S}, \tag{32}$$

where

$$\mathbf{Q}_0 = \sum_{u=x,y,z} \frac{1}{a_u} \mathbf{Q}_{0u} \quad \mathbf{S} = \sum_{u=x,y,z} \frac{2\mathbf{Q}_{0u}\mathbf{C}_{4u} + (\mathbf{I} + \mathbf{Q}_{2u} - \mathbf{Q}_{3u})(\mathbf{j}_{ur}^{-} + \mathbf{j}_{ul}^{+})}{a_u}. \tag{33}$$

In deriving Eqs. (31) and (32), we used $\mathbf{J}_{us} = \mathbf{j}_{us}^{+} - \mathbf{j}_{us}^{-}$ ($s=l,r$). Equations (31) and (32) constitute the basic nodal coupling relations in the UNM formulation which have to be solved for $\overline{\boldsymbol{\phi}}$ and $\mathbf{j}_{us}^{\pm}\ (u = x, y, z; s = l, r)$. The WRM equations (25) and (26) enables one to compute the coefficient vectors, $\mathbf{C}_{3u}$ and $\mathbf{C}_{4u}$, respectively, which are needed in solving the basic UNM coupling equations. The 2x2 matrices $\hat{\mathbf{G}}_{iu}$ and $\hat{\mathbf{H}}_{iu}$ which determines the 2x2 matrices $\mathbf{M}_{iu}$ in the WRM equations are defined differently, depending on the transverse leakage shape functions, $g_{ipu}(\tau)$ (Lee and Kim, 2001).

As demonstrated in the static case (Lee and Kim, 2000 and 2001), it can also be extended to the analytic function expansion nodal (AFEN) method calculations in transient case. In order to show this, let us consider the analytic solution to Eq. (7) for a rectangular prism node. Noting that the approximation on $\hat{\mathbf{S}}_u(u)$ corresponds to the following assumption for $\hat{\mathbf{S}}(\mathbf{r})$,

$$\hat{\mathbf{S}}(\mathbf{r}) = \hat{\mathbf{S}}_0 + \sum_{u=x,y,z} \sum_{i=1}^{2} \begin{pmatrix} g_{i1u}(u/a_u) & 0 \\ 0 & g_{i2u}(u/a_u) \end{pmatrix} \hat{\mathbf{S}}_{iu}\ , \tag{34}$$

where the $g_{ipu}(\tau)$ is given in the second row of table 1.

With Eq. (34) as the source term, one of the approximate analytic solution to Eq. (7) can be given by

$$\begin{aligned} \xi_p(x,y,z) = & \sum_{u=x,y,z} \left[\widetilde{A}_{1pu} \sinh(\kappa_p u) + \widetilde{A}_{2pu} \cosh(\kappa_p u) \right] \\ & + \sum_{u=x,y,z} \begin{bmatrix} \widetilde{B}_{1pu} \sinh(\mu_p v)\sinh(\mu_p w) + \widetilde{B}_{2pu} \sinh(\mu_p v)\cosh(\mu_p w) \\ + \widetilde{B}_{3pu} \cosh(\mu_p v)\sinh(\mu_p w) + \widetilde{B}_{4pu} \cosh(\mu_p v)\cosh(\mu_p w) \end{bmatrix} \\ & + A_o + \sum_{u=x,y,z} \left[A_{1pu} \sinh(\mu_p u) + A_{2pu} \cosh(\mu_p u) \right] \end{aligned} \tag{35}$$

The first two terms corresponds to the homogeneous solution and the remaining terms the particular solution. Rearranging Eq. (35) using the $g_{ipu}(\tau)$ and $f_{ipu}(\tau)$ in the second row in table 1, one can put Eq. (35) into

$$\xi_p(x,y,z) = \hat{C}_{0p} + \sum_{u=x,y,z} \sum_{i=1}^{4} \hat{C}_{ipu} f_{ipu}(u/a_u) + \sum_{u=x,y,z} \sum_{i=1}^{2} \sum_{j=1}^{2} \hat{D}_{ijpu} g_{ipv}(v/a_v) g_{jpw}(w/a_w), \tag{36}$$

The intranodal flux distribution above contains 25 terms with 25 expansion coefficients to be determined. In the spirit of the conventional AFEN formulation, the 25 expansion coefficients are determined by the 19 nodal unknowns per rectangular prism node per neutron group; one node average flux ($\bar{\xi}_p$), six surface average fluxes ($\xi_{pus}; u = x, y, z; s = l, r$), and twelve edge fluxes ($\xi_{pu}^{(i)}; u = x, y, z; i = 1,2,3,4$) and the six WRM conditions designed for obtaining the particular solution of Eq. (7) with Eq. (36) as the general solution, The coupling equations among nodal unknowns are then obtained by using the nodal balance condition, six current continuity conditions, and twelve edge leakage balance equations.

Because the UNM makes use of the transverse integration, the way that the UNM derives the coupling equations among nodal unknowns is different from the AFEN method. In this conjunction let us note that the 1-D flux is given by Eq. (14) when one performs the transverse integration of Eq. (10a) with Eq. (36) for the intranodal flux distribution. Let us also note that the transverse integration of Eq.(10b) with Eq.(36) leads to the ATL equation (11) in which the coefficients are explicitly defined by

$$\begin{aligned}
\hat{L}_{1pu} &= 4\hat{P}_{pv}\hat{D}_{12pw} + 4\hat{P}_{pw}\hat{D}_{21pv} \\
&= \hat{P}_{pv}\left[\xi_{pw}^{(1)} - \xi_{pw}^{(2)} - \xi_{pw}^{(3)} + \xi_{pw}^{(4)} - 2\left(\xi_{ur} - \xi_{ul}\right)\right] \\
&\quad + \hat{P}_{pw}\left[\xi_{pv}^{(1)} + \xi_{pv}^{(2)} - \xi_{pv}^{(3)} - \xi_{pv}^{(4)} - 2\left(\xi_{ur} - \xi_{ul}\right)\right] \\
\hat{L}_{2pu} &= 4\hat{P}_{pv}\hat{D}_{22pw} + 4\hat{P}_{pw}\hat{D}_{22pv} \\
&= \hat{P}_{pv}\left[\xi_{pw}^{(1)} + \xi_{pw}^{(2)} + \xi_{pw}^{(3)} + \xi_{pw}^{(4)} - 2\left(\xi_{ur} + \xi_{ul} + \xi_{vr} + \xi_{vl}\right) + 4\bar{\xi}_p\right] \\
&\quad + \hat{P}_{pw}\left[\xi_{pv}^{(1)} + \xi_{pv}^{(2)} + \xi_{pv}^{(3)} + \xi_{pv}^{(4)} - 2\left(\xi_{ur} + \xi_{ul} + \xi_{wr} + \xi_{wl}\right) + 4\bar{\xi}_p\right]
\end{aligned}, \tag{37}$$

$$\hat{P}_{pu} = \begin{cases} \dfrac{\tilde{\mu}_{pu}^2 K_2\left(\tilde{\mu}_{pu}\right)}{a_u K_3\left(\tilde{\mu}_{pu}\right)} \\ \dfrac{1}{a_u}\left(3 + \dfrac{\tilde{\mu}_{pu}^2}{5} - \dfrac{\tilde{\mu}_{pu}^4}{175} + \dfrac{2\tilde{\mu}_{pu}^6}{7875} - \dfrac{37\tilde{\mu}_{pu}^8}{3031875}\right) \end{cases},$$

$$K_1(x) = \cosh x, \quad K_2(x) = \sinh x, \quad K_3(x) = xK_1(x) - K_2(x).$$

In order to provide the coupling equations equivalent to the original AFEN formulation, the 1-D flux derived from Eq.(36) and expressed by Eq. (14) must satisfy exactly the transverse 1-D equation with the ATL the expansion coefficients of which are defined by Eq.(37). It must also satisfy the current continuity conditions at the two nodal interfaces as well as nodal balance condition. Note that these two conditions are met by Eq. (14) once the coefficients of 1-D flux are determined by Eqs. (16) to (20). Because the nodal coupling equations among nodal unknowns, Eqs. (31) and (32), are derived from the 1-D

flux satisfying these two conditions, they become the nodal coupling equations equivalent to the AFEN counter parts. In other words, the basic nodal coupling equations (31) and (32) are also applicable for the AFEN method calculations. Because of Eq. (37), however, the expansion coefficients in the ATL are given by nodal unknowns;

$$\begin{aligned}
\mathbf{L}_{1u} &= \mathbf{P}_v\left[\boldsymbol{\phi}_w^{(1)} - \boldsymbol{\phi}_w^{(2)} - \boldsymbol{\phi}_w^{(3)} + \boldsymbol{\phi}_w^{(4)} - 2\left(\boldsymbol{\phi}_{ur} - \boldsymbol{\phi}_{ul}\right)\right] \\
&\quad + \mathbf{P}_w\left[\boldsymbol{\phi}_w^{(1)} + \boldsymbol{\phi}_w^{(2)} - \boldsymbol{\phi}_w^{(3)} - \boldsymbol{\phi}_w^{(4)} - 2\left(\boldsymbol{\phi}_{ur} - \boldsymbol{\phi}_{ul}\right)\right] \\
\mathbf{L}_{2u} &= \mathbf{P}_v\left[\boldsymbol{\phi}_w^{(1)} + \boldsymbol{\phi}_w^{(2)} + \boldsymbol{\phi}_w^{(3)} + \boldsymbol{\phi}_w^{(4)} - 2\left(\boldsymbol{\phi}_{ur} + \boldsymbol{\phi}_{ul} + \boldsymbol{\phi}_{vr} + \boldsymbol{\phi}_{vl}\right) + 4\bar{\boldsymbol{\phi}}\right], \\
&\quad + \mathbf{P}_w\left[\boldsymbol{\phi}_w^{(1)} + \boldsymbol{\phi}_w^{(2)} + \boldsymbol{\phi}_w^{(3)} + \boldsymbol{\phi}_w^{(4)} - 2\left(\boldsymbol{\phi}_{ur} + \boldsymbol{\phi}_{ul} + \boldsymbol{\phi}_{wr} + \boldsymbol{\phi}_{wl}\right) + 4\bar{\boldsymbol{\phi}}\right]
\end{aligned} \tag{38}$$

$$\mathbf{P}_u = \mathbf{DR}\begin{pmatrix} \hat{P}_{1u} & 0 \\ 0 & \hat{P}_{2u} \end{pmatrix}\mathbf{R}^{-1}.$$

Because the edge fluxes appear in Eq. (38), the nodal coupling relations are augmented by the so-called CPB equations;

$$\mathbf{T}_{ij}^C\boldsymbol{\phi}_{ij}^C + \mathbf{T}_{ij}^L\boldsymbol{\phi}_{ij}^L + \mathbf{T}_{ij}^R\boldsymbol{\phi}_{ij}^R + \mathbf{T}_{ij}^B\boldsymbol{\phi}_{ij}^B + \mathbf{T}_{ij}^T\boldsymbol{\phi}_{ij}^T = \mathbf{q}_{ij}. \tag{39}$$

The subscript *ij* denotes one of the four corner points in the given node. $\boldsymbol{\phi}_{ij}^s\left(s = C, R, L, B, T\right)$ denotes the flux at the corner point designated by *ij* (*s*=*C*), and corner point fluxes at its right (*s*=*R*), its left (*s*=*L*), its bottom (*s*=*B*) and its top (*s*=*T*). The coefficients $\mathbf{T}_{ij}^s\left(s = C, R, L, B, T\right)$ and $\mathbf{q}_{ij}$ are defined in reference (Noh and Cho, 1994)

3. NUMERICAL RESULTS AND DISCUSSIONS

The transient UNM formulation above covers five different nodal computational options; NEM, ANM/QTL, ANM/ATL, AFEN/MSS, and AFEN/CPB. These options are implemented in the SUNCARDS (Seoul National University Unified Nodal Code for Advanced Reactor Design and Simulation). The two ANM options, ANM/QTL and ANM/ATL, refer to the ANM calculations with the QTL and the ATL approximations, respectively. The two AFEN options, AFEN/MSS and AFEN/CPB, differ from each other in that the former uses the method of succesive smoothing (MSS), while the latter the CPB equations, to determine the edge fluxes. Equations (31) and (32) are the basic nodal coupling relations common in all five options. The sole difference among different nodal options is that the two 2x2 matrices, $\hat{\mathbf{G}}_{iu}$ and $\hat{\mathbf{H}}_{iu}$, which determine the matices $\mathbf{M}_{iu}\left(i = 1,2,3,4\right)$, and the 2-D transverse leakage coefficient vectors, $\mathbf{L}_{iu}\left(i = 1,2\right)$, in Eq. (25) and (26) are differently defined, depending the nodal computational options. Let's take $\mathbf{L}_{iu}\left(i = 1,2\right)$ for example. In the NEM or ANM options, the transverse leakage coefficient vectors are computed by using average transverse leakage vectors of the three adjacent nodes or the averages transverse leakage and two surface transverse leakage of

the given node. While in AFEN options, they are determined by Eqs. (38). This makes it necessary to determine the edge fluxes in addition to the node-average and the surface fluxes by using either the MSS or the CPB equations at the expense of extra computational cost. Unlike the NEM of ANM options, therefore, the AFEN options include the MSS or CPB equations in addition to the coupling equations common in all the nodal options. These differences are discussed in more detail in reference (Lee and Kim, 2001).

In order to examine the performance of several nodal options in terms of computational speed and accuracy, we analized the NEACRP PWR core transient benchmark problems that are designed to compare various 3-D space-time kinetics codes in transient calculations. The NEACRP PWR core is representative of a medium size PWR core consisting of 157 fuel assemblies. The transient problems involves the six cases of rod ejection transients designated A1, A2, B1, B2, C1, and C2. Cases A1, B1, and C1 are initiated from the ejection of a fully inserted control assembly at hot zero power (HZP) (2775W) while cases A2, B2,and C2 are initiated from the ejection of a partially inserted control assembly at the hot full power (HFP) (2775 MW) state. Cases A and B are octant core symmetry problem while C is full core transient. In all cases, the control rod ejection time is 0.1 second.

Table 2 shows the comparison of five different nodal calculations of the UNM formulation by the SUNCARDS code and the reference fine-mesh (4x4x36) ANM/QTL option of the UNM formulation. The UNM calculations are conducted on 1 radial node per FA (N/A) and 4 N/A with the number of the axial nodes fixed at 18. The NEM and ANM/QTL solutions to the NEACRP PWR transients are practically the same as those obtained earlier by many investigators. On the other hand, the ANM/ATL solutions are the new ANM calculations because they are obtained using a new analytical transverse leakage approximation (Lee and Kim,2000). The AFEN solutions are the first-ever 3-D transient calculations, because the 3-D transient calculations by the original AFEN method has not yet been reported

From the results of Table 2, we note that all five nodal options of the UNM calculations on A2, B2, and C2 problems predict very similarly with each other the initial HFP steady state and the transient core parameters of the NEACRP PWR and that the predictions are in good agreement with the reference fine-mesh calculations. In contrast to this, the UNM calculations on the HZP cases, A1, B1, and C1 shows that their agreement with the reference results are dependent on the nodal computational option. The NEM is inferior to ANM and AFEN in prediction of both the steady state and transient HZP core parameters. The 1 N/A AFEN predicts the initial steady state parameters of the NEACPP PWR cores at the HZP better than the 1 N/A ANM, yet the 4 N/A AFEN is no better than 4 N/A ANM in steady state calculations. In particular, the 4 N/A AFEN/CPB calculations predict slightly higher F_q than the other options. As for the transient parameters, the 1 N/A AFEN calculations are closer to the reference results than the 1 N/A ANM calculations in the A1 case and vice versa in B1 and C1 problems.

Table 2. Nodal Transient Calculations of UNM Option for NEACRP PWR Kinetics Benchmark Problems

Case	Node/FA	UNM Option	Soluble Boron (ppm)	F_q	Rod Worth (pcm)	Peak Time (s)	Peak Power (%)	CPU Time (s)
A1	4x4x36	ANM/QTL	561.21	2.8788	827.6	0.5425	131.4	-
	1x1x18	NEM	566.81	2.8306	802.5	0.7075	75.6	174
		ANM/QTL	565.62	2.8482	811.6	0.6375	93.6	169
		ANM/ATL	562.66	2.8595	814.9	0.6175	100.1	154
		AFEN/MSS	558.20	2.8835	827.8	0.5475	130.7	200
		AFEN/CPB	559.57	2.8897	825.2	0.5600	124.1	220
	2x2x18	NEM	562.02	2.8694	822.7	0.5675	119.2	867
		ANM/QTL	561.55	2.8763	826.1	0.5500	128.0	846
		ANM/ATL	560.83	2.8782	827.2	0.5450	130.7	751
		AFEN/MSS	559.41	2.8861	830.1	0.5225	141.4	957
		AFEN/CPB	557.59	2.9371	829.1	0.5300	134.5	1058
B1	4x4x36	ANM/QTL	1247.99	1.9335	829.5	0.5250	240.0	-
	1x1x18	NEM	1262.88	1.9109	845.3	0.4675	330.5	172
		ANM/QTL	1255.46	1.9272	830.5	0.5275	247.2	169
		ANM/ATL	1252.31	1.9263	831.6	0.5225	251.3	163
		AFEN/MSS	1244.69	1.9392	832.8	0.5175	258.1	200
		AFEN/CPB	1247.38	1.9424	834.9	0.5075	268.7	225
	2x2x18	NEM	1251.61	1.9268	835.9	0.5000	274.1	803
		ANM/QTL	1248.46	1.9330	829.7	0.5275	242.1	777
		ANM/ATL	1247.47	1.9333	829.7	0.5275	241.8	781
		AFEN/MSS	1245.98	1.9343	830.3	0.5100	252.3	885
		AFEN/CPB	1244.62	1.9654	830.2	0.5175	244.0	1065
C1	4x4x36	ANM/QTL	1128.29	2.1870	953.5	0.2725	438.5	-
	1x1x18	NEM	1142.89	2.1730	985.8	0.2475	593.6	1332
		ANM/QTL	1135.43	2.1829	960.7	0.2675	478.8	1285
		ANM/ATL	1132.18	2.1829	963.8	0.2650	490.8	1322
		AFEN/MSS	1124.90	2.1924	956.4	0.2725	462.8	1355
		AFEN/CPB	1127.56	2.1991	962.6	0.2675	487.1	1682
	2x2x18	NEM	1131.89	2.1827	963.5	0.2650	483.6	6090
		ANM/QTL	1128.80	2.1867	953.9	0.2725	443.4	5987
		ANM/ATL	1127.89	2.1865	953.9	0.2725	442.4	5910
		AFEN/MSS	1126.14	2.1881	954.3	0.2675	465.4	6569
		AFEN/CPB	1124.88	2.2239	955.0	0.2725	445.8	7874
A2	4x4x36	ANM/QTL	1158.60	2.2117	90.13	0.1000	108.27	-
	1x1x18	NEM	1168.78	2.2216	86.82	0.1000	108.08	47.2
		ANM/QTL	1163.27	2.2194	88.97	0.1000	108.34	38.4
		ANM/ATL	1160.47	2.2301	89.11	0.1000	108.37	38.9
		AFEN/MSS	1155.50	2.2343	91.10	0.1000	108.63	52.1
		AFEN/CPB	1156.87	2.2335	90.51	0.1000	108.59	64.0
B2	4x4x36	ANM/QTL	1185.89	2.1015	100.01	0.1100	106.56	-
	1x1x18	NEM	1197.55	2.1163	96.08	0.1100	106.33	43.8
		ANM/QTL	1191.77	2.1146	94.94	0.1100	106.22	39.7
		ANM/ATL	1189.02	2.1248	95.30	0.1100	106.24	48.2
		AFEN/MSS	1184.15	2.1306	95.15	0.1100	106.26	49.6
		AFEN/CPB	1185.65	2.1279	95.33	0.1200	106.28	59.1
C2	4x4x36	ANM/QTL	1158.60	2.2117	79.91	0.1050	107.53	-
	1x1x18	NEM	1168.78	2.2216	82.20	0.1000	108.08	325.3
		ANM/QTL	1163.25	2.2195	81.32	0.1000	107.90	314.8
		ANM/ATL	1160.46	2.2301	81.48	0.1000	107.92	291.3
		AFEN/MSS	1155.50	2.2344	81.50	0.1000	107.89	356.7
		AFEN/CPB	1156.87	2.2335	81.90	0.1050	108.00	434.5

The core power excursion behavior following the rapid rod ejection from the HZP is strongly affected by the initial steady state FA power at the site of the rod ejection as well as the rod worth, because it is determined mainly by the local power upswing of the FA at the rod ejection site. Figure 1 compares the 1 N/A AFEN method and ANM/QTL calculations for the initial steady state assembly power distribution of the three HZP transient cases. It shows that the 1 N/A AFEN method is more accurate than the 1 N/A ANM/QTL in the core power distribution calculations. In case of A1, the relative power prediction errors of the former are an order less than those of the latter at all FA sites. The same is true in cases B1 and C1 except for the fact that the 1 N/A AFEN method overprdicts the FA power at the rod ejction site(cf. bold-framed FA's in Fig. 1b and 1c) more than the 1 N/A ANM/QTL. On the other hand, Table 2 shows that the 1 N/A AFEN prediction on the reactivity worth of the ejected rod is closer to the reference calculation than the 1N/A ANM/ QTL in case A1 and vice versa in cases B1 and C1. These differences in the initial steady state FA power at the ejected rod site and the ejected rod worth from the reference values are the reasons why the 1 N/A AFEN method prediction is better than the 1 N/A ANM/QTL in the A1 case while vice versa in the B1 and C1 cases.

The seemingly better prediction accuracy of the ANM/QTL over the AFEN method in the B1 and C1 transient analysis is accidental. It is a result of the fact that the rod ejection happens to take place at the FA the initial power of which is better predicted by the ANM/QTL than the AFEN method. We presumed the control rod ejection at the gray-colored FA the assembly power of which is better predicted by the AFEN method than the ANM/QTL. Table 3 shows the transient analysis results for these modified HZP problems designated by MA1, MB1, and MC1. Because the ejected rod worth of the modified problem is different from that of the original problem, the peak time and the power peak are different from those of the the original problem. Yet we observe that the 1 N/A AFEN method predicts the transient parameters better than the 1 N/A ANM/QTL in all the modified transient problems. We also observe that the prediction accuracy of the 1 N/A AFEN method is between those of the 4 N/A NEM and the 4 N/A ANM/QTL.

4. CONCLUSIONS

The saleint feature of the UNM formulation is that the AFEN method formulation as well as the popular NEM and ANM formulations for the solutions to the time-dependent 2-G diffusion equation in rectangular geometry can be represented by the common nodal coupling equations among nodal unknowns. Because they resemble exactly those in the NEM formulation, the UNM formulation can be easily incorporated into the existing NEM production codes. Moreover, the numerical solution schemes proven to be very effective for speedy NEM calculations are straightforwardly applied for the UNM calculations. This is a major practical advantage of the UNM formulation which motivates further efforts to make the most of the outstanding prediction accuracy of the nodal calculations-particularly the AFEN method calculations.

(a) Case A1

0.9948 -2.05 0.13	1.5231 -2.02 0.12	1.0727 -1.73 0.20	1.8084 -1.21 0.25	1.8158 -0.48 -0.35	1.0078 -0.49 -0.58	0.3963 0.48 0.05	0.5391 3.49 -0.96
	1.8476 -2.03 0.07	1.6635 -1.70 -0.05	1.9127 -1.07 -0.03	1.4253 -0.87 -0.07	0.5591 -0.27 0.23	0.5591 2.47 0.05	0.4273 2.48 -0.94
		1.0162 -1.36 -0.05	1.4494 -1.10 -0.09	0.7306 -0.44 0.29	0.7248 1.79 0.26	0.3818 2.44 -0.37	
			1.4354 -0.69 0.17	1.0154 0.92 0.43	0.9428 3.50 0.58	0.5388 3.99 -0.65	
Ref. : ANM/QTL (4x4x36) ANM/QTL (1x1x18) AFEN/CPB(1x1x18)				0.5412 2.75 0.67	0.6682 4.31 -0.33		

(b) Case B1

0.4087 -2.57 0.12	0.7252 -2.52 -0.30	0.9771 -1.74 -0.48	1.0867 -1.79 0.06	1.2616 -0.98 -0.04	1.1270 -0.86 0.02	0.4893 0.43 0.80	0.5579 2.55 -0.36
	0.8255 -2.22 -0.28	0.8858 -2.03 -0.21	1.1218 -1.32 -0.06	1.2505 -1.07 0.17	1.1835 -0.24 0.19	0.9064 0.97 0.52	0.5025 2.01 -0.10
		0.5493 -1.60 0.24	1.0602 -1.09 -0.05	1.2770 -0.34 0.07	1.1981 0.38 0.29	1.0692 1.68 -0.45	
			1.2046 -0.33 0.17	1.2302 0.27 0.46	1.2301 2.00 0.24	0.7910 1.49 -0.92	
Ref. : ANM/QTL (4x4x36) ANM/QTL (1x1x18) AFEN/CPB(1x1x18)				1.1393 2.11 0.34	0.8957 2.14 -0.71		

(c) Case C1

0.2487 -2.25 -0.12	0.4144 -1.93 -0.07	0.3983 -1.66 0.00	0.9831 -1.57 0.00	1.4450 -0.75 -0.12	1.3933 -0.70 0.04	0.6332 0.49 0.81	0.7509 2.64 -0.36
	0.5319 -2.33 -0.19	0.6501 -1.69 -0.22	1.0963 -1.09 0.01	1.4145 -0.96 0.20	1.4446 -0.19 0.19	1.1490 1.04 0.54	0.6654 1.97 -0.12
		0.4998 -1.46 0.14	1.0679 -1.12 -0.01	1.3692 -0.46 -0.02	1.3610 0.22 0.26	1.2786 1.41 -0.42	
			1.1362 -0.71 0.04	1.0639 -0.20 0.17	1.1859 1.55 0.24	0.8613 0.95 -0.88	
Ref. : ANM/QTL (4x4x36) ANM/QTL (1x1x18) AFEN/CPB(1x1x18)				0.494 1.32 0.42	0.644 2.10 -0.82		

Figure 1 Relative Power Errors of ANM/QTL and AFEN Method Calculations

Table 3. Nodal Transient Calculations of UNM Option for Modified NEACRP PWR Kinetics Benchmark Problems

Case	Node/FA	UNM Option	Soluble Boron (ppm)	F_q	Rod Worth (pcm)	Peak Time (s)	Peak Power (%)	CPU Time (s)
MA1	4x4x36	ANM/QTL	561.21	2.8788	848.4	0.4575	342.9	-
	1x1x18	NEM	566.81	2.8306	967.3	0.2650	1607.8	177
		ANM/QTL	565.62	2.8482	914.7	0.3225	921.8	169
		AFEN/CPB	559.57	2.8897	838.9	0.4925	291.1	230
	2x2x18	NEM	562.02	2.8694	870.7	0.3975	502.2	853
		ANM/QTL	561.55	2.8763	853.0	0.4450	374.1	854
		AFEN/CPB	557.59	2.9371	844.4	0.4625	322.3	1060
MB1	4x4x36	ANM/QTL	1247.99	1.9335	876.3	0.3850	540.7	-
	1x1x18	NEM	1262.88	1.9109	823.5	0.5650	214.8	149
		ANM/QTL	1255.46	1.9272	852.6	0.4500	374.2	155
		AFEN/CPB	1247.38	1.9424	859.2	0.3850	554.2	212
	2x2x18	NEM	1251.61	1.9268	862.8	0.4200	441.9	741
		ANM/QTL	1248.46	1.9330	874.5	0.3900	528.4	715
		AFEN/CPB	1244.62	1.9654	878.2	0.3800	555.9	1106
MC1	4x4x36	ANM/QTL	1128.29	2.1870	824.4	0.5550	210.1	-
	1x1x18	NEM	1142.89	2.1730	777.0	0.9800	69.7	167
		ANM/QTL	1135.43	2.1829	803.9	0.6875	133.6	164
		AFEN/CPB	1127.56	2.1991	823.2	0.5675	204.9	221
	2x2x18	NEM	1131.89	2.1827	811.7	0.6300	159.7	773
		ANM/QTL	1128.80	2.1867	822.5	0.5675	202.9	763
		AFEN/CPB	1124.88	2.2239	825.8	0.5450	216.1	1017

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