M&C 2001 Salt Lake City, Utah, USA, September 2001.

BWR FUEL ASSEMBLY AXIAL DESIGN OPTIMIZATION USING TABU SEARCH

C. Martín-del-Campo and J. L. François
Universidad Nacional Autónoma de México - Facultad de Ingeniería
Paseo Cuauhnáhuac 8532. 62550 Jiutepec, Mor. México
ceciliam@premia.imta.mx francois@premia.imta.mx

Luis B. Morales
IIMAS, Universidad Nacional Autónoma de México
Apdo. Postal 70-221, México, DF, 04510, México
lbm@servidor.unam.mx

Keywords: BWR Fuel Design, In-core Fuel Management, Tabu Search Method, Optimization.

ABSTRACT

In this paper the implementation of the tabu search (TS) optimization method to the boiling water reactor's fuel assembly axial design is described. The objective of this implementation is to test the TS method for the search of optimal fuel assembly axial designs. This implementation has been linked to the reactor core simulator CM-PRESTO in order to evaluate each design proposed in a reactor cycle operation. The evaluation of the proposed fuel designs takes into account the most important safety limits included in a boiling water reactor in-core analysis based on the Haling principle. Results obtained show that TS is a promising method for solving the axial design problem. However it merits further study in order to find better adaptation of the TS method for the specific problem.

1. INTRODUCTION

Fuel management in a nuclear reactor has great importance for the nuclear industry in terms of safety and economics. For a boiling water reactor (BWR), fuel management is a set of different combinatorial problems that have to be optimized in a coupled form. Radial and axial fuel assembly designs, in-core loading pattern design and control rod pattern design are the principal tasks in reactor fuel management. At present, development of an automated optimization process for each activity in the fuel management area is being performed. In order to select the better method to solve each different combinatorial problem, testing of different optimization methods developed in artificial intelligence is taking place.

In the literature, genetic algorithms (GA) and TS have shown very good results solving combinatorial problems (Poon, 1993; François, 1999; Ben Hmaida, 1999). Specifically the fuel assembly (FA) design problem genetic algorithms have been tested obtaining good results (Martín del Campo, 2001). Now the tabu search method has been

implemented to look for optimal FA designs and the results obtained are comparable to the GA method.

2. FUEL ASSEMBLY AXIAL DESIGN PROBLEM

The fuel assemblies in BWR reactors have several axial zones with different physical characteristics such as the U^{235} enrichment and the gadolinia content in the uranium dioxide used in the fuel, and different water regions. The designer is forced to search an optimal utilization of the fuel under the safety limits related to the power and the reactivity during reactor operation.

The optimal FA design will be the axial distribution of some fuel types (4-8), with the minimum FA mean enrichment, which permits a proposed cycle length (energy) under the safety limits during a reactor operation cycle. In the optimization process the FA mean enrichment is the value to minimize, the cycle length and the safety limits are the constraints to satisfy. For a typical BWR the safety limits are the power parameters such as the maximum linear heat generation rate (MLHGR), the fraction of the limiting average planar heat generation rate (XMPGR), the power peaking factor (PPF), the minimum critical power ratio (MCPR) and the maximum relative nodal power (MRNP), measured during the full power cycle operation, and the reactivity parameters are the hot excess reactivity (HEX) and the shutdown margin (SDM) during the cycle.

In this paper the TS method is used to find a fresh FA axial design for cycle number five of the Laguna Verde Reactor 1 (LVR1). To evaluate the performance of the FA design, the 3D core simulator CM-PRESTO (Scandpower,1993) is utilized, introducing the fresh FA design in the loading pattern for this cycle. The hot excess reactivity and the shutdown margin are calculated at the beginning of cycle. Thermal limits are evaluated at the end of cycle using the Haling calculation (Haling, 1963). The CM-PRESTO simulator has been previously validated for LVR1 applications (François, 2001).

The axial FA is described by an array of 25 integer numbers $<n_1,n_2,....,n_{25}>$ in which one of them indicates the lattice composition type located at the axial position i. The possible values for each n_i are obtained from a list of fuel lattice compositions $(C_1,C_2,....,C_N)$ included on the nuclear data bank used by CM-PRESTO. The nuclear engineer must previously define these fuel lattice compositions.

Two natural and four enriched fuel lattice compositions are used in the FA design utilized as an example in this paper. Table 1 shows the mean enrichment and the gadolinia content of each enriched fuel lattice available for the FA design.

Table 1 Fuel enriched lattice characteristics.

Lattice Type	Enrichment in U-235 (%)	Gadolinia
1	3.81	9G3.0
2	3.96	2G4.0 / 9G3.0
3	3.96	9G3.0
4	3.81	9G4.0

The objective function used in this TS implementation has been previously tested in a GA implementation for BWR axial FA design (Martín del Campo 2001). The objective function to be solved can be written in the following way:

$$\max f(\mathbf{e}) = \Delta \text{Enrichment}(\mathbf{e}) \cdot w_1 - |\Delta \text{Energy}(\mathbf{e})| \cdot w_2 + \Delta \text{MLHGR}_k(\mathbf{e}) \cdot w_3 + \Delta \text{XMPGR}_k(\mathbf{e}) \cdot w_4 + \Delta \text{MRNP}_k(\mathbf{e}) \cdot w_5 + \Delta \text{PPF}(\mathbf{e}) \cdot w_6 + \Delta \text{MCPR}_k(\mathbf{e}) \cdot w_7 + \text{HEX}(\mathbf{e}) \cdot w_8 + \Delta \text{SDM}(\mathbf{e}) \cdot w_9$$

subject to the additional constraint at EOC in accordance with the Haling principle: $\lambda(\mathbf{e}) = \lambda_{trg}$

Where:

$\Delta\text{Enrichment}(\mathbf{e}) = \text{Enrich}_{max} - \text{Enrich}(\mathbf{e})$

$\Delta\text{Energy}(\mathbf{e}) = \text{Energy}(\mathbf{e}) - \text{Energytrg}$

$\Delta\text{MLHGR}(\mathbf{e}) = \text{MLHGR}_{max} - \text{MLHGR}_k(\mathbf{e})$, $k = 1, ..., 25$

$\Delta\text{XMPGR}(\mathbf{e}) = \text{XMPGR}_{max} - \text{XMPGR}_k(\mathbf{e})$, $k = 1, ..., 25$

$\Delta\text{MRNP}(\mathbf{e}) = \text{MRNP}_{max} - \text{MRNP}_k(\mathbf{e})$, $k = 1, ..., 25$

$\Delta\text{PPF}(\mathbf{e}) = \text{PPF}_{max} - \text{PPF}(\mathbf{e})$

$\Delta\text{MCPR}(\mathbf{e}) = \text{MCPR}_k(\mathbf{e}) - \text{MCPR}_{min}$, $k = 1, ..., 25$

$\text{HEX(e)} = \text{KEFF(e)} - \text{KEFF}_{crit}$

$\Delta\text{SDM(e)} = \text{SDM(e)} - \text{SDM}_{min}$

Enrich_{max}	Maximum possible enrichment value
$\text{Enrich}(\mathbf{e})$, w_1	FA mean enrichment obtained by the system and its weighting factor
$\text{Energy}(\mathbf{e})$, w_2	Calculated cycle energy (cycle burn-up) and its weighting factor
Energytrg	Energy target value
MLHGR, w_3	Maximum linear heat generation rate and its weighting factor
XMPGR, w_4	Fraction of the limiting average planar heat generation rate (APLHGR) and its weighting factor
MRNP, w_5	Maximum relative nodal power and its weighting factor
PPF, w_6	Power peaking factor (radial) and its weighting factor
MCPR, w_7	Minimum critical power ratio and its weighting factor
HEX, w_8	Hot excess reactivity at BOC and its weighting factor
SDM, w_9	Shutdown margin at BOC and its weighting factor
KEFF, KEFF_{crit}	Calculated and target k-effective for all control rods out and hot full

	power conditions at BOC
λ, λ_{trg}	Calculated eigenvalue and target eigenvalue at Haling calculation
E	Vector of enrichments (e_1 , e_2 , ..., e_{25})
e_{min} , e_{max}	Enrichment limits in each node
K	Axial node index in the 3D simulator code (1,2,.....,25)

All the weighting factors are fixed values during the optimization process. The enrichment weighting factor (w_1) has a positive value in order to give a high qualification to those designs with low enrichment. The energy weighting factor (w_2) has a positive value in order to penalize those designs that move away from the energy target. The weighting factors w_3 to w_9 are set to zero when their associated safety limit is satisfied, and they take positive values when their associated safety limit is not satisfied. This action penalizes a design qualification only when one or several safety limits are violated.

The proposed cycle length (energy target) for the example case has a value of 9281 MWD/MT. The safety limits are satisfied when:

$$
\begin{array}{lcl}
PPF(\mathbf{e}) & < & PPF_{max} \\
MLHGR(\mathbf{e}) & < & MLHGR_{max} \\
XMPGR(\mathbf{e}) & < & XMPGR_{max} \\
MRNP(\mathbf{e}) & < & MRNP_{max}
\end{array}
$$

$$
HEX_{min} > HEX(\mathbf{e}) < HEX_{max}
$$

$$
\begin{array}{lcl}
SDM(\mathbf{e}) & > & SDM_{min} \\
MCPR(\mathbf{e}) & > & MCPR_{min}
\end{array}
$$

The safety limit values used in this design analysis using Haling calculations are shown in Table 2. These are conservative values that take into account the safety limits underestimation of the Haling approach related to the cycle's control rod pattern depletion. The maximum decrease in SDM from BOC for this cycle is 0.42% delta K (Alonso, 1995), therefore a 1.5% delta K at BOC was chosen as a conservative value for this parameter.

Table 2 Constraints and limit values.

Constraint	Limit value
PPF_{max}	1.5
$MLHGR_{max}$	374 (W/cm)
$XMPGR_{max}$	0.85
$MRNP_{max}$	1.83
$MCPR_{min}$	1.5
HEX_{min}	1.5 (% delta K)
HEX_{max}	2.0 (% delta K)
SDM_{min}	1.5 (% delta K)

3. TABU SEARCH METHOD

The TS method, developed by Glover (1989), is an iterative heuristic procedure for solving complex combinatorial optimization problems. TS is a constrained search procedure designed to overcome local optimality (Glover, 1988) and has been applied successfully in several problems of combinatorial nature (Morales, 1998). In nuclear applications TS has been used for in-core loading optimization (Ben Hmaida, 1999).

Briefly, the TS method minimizes $f(x)$, subject *to* x in X, where f is a cost function, and X is a set of candidate solutions. It starts from an *initial candidate solution* and tries to reach a global minimum by moving from one candidate solution to another. To accomplish this, a set M of simple modifications must be defined. These modifications are called *moves*, which can be applied to a given candidate solution to move to another. The notation $x' = m(x)$, m in M, indicates that m transforms x into x'. This leads to the definition of a *neighborhood*. For each candidate solution x, the neighborhood $N(x)$ is the set of all candidate solutions directly reachable from x by a single move m in M. When $N(x)$ is large, at each step of the iteration process, a subset V^* of $N(x)$ is generated and the move is made from x to the best solution x^* in V^*, whether or not $f(x^*)$ is better than $f(x)$.

Up to this point, the algorithm is close to a local improvement technique, except that the move from x may be to a worse solution x^*, and thus may escape from local minimum of f. To prevent cycling, any move that returns to any local optimum recently visited is *tabu* (forbidden). This is accomplished in a *short-term memory function* by storing the tabu move in a set T, called the *tabu recency list*, of length t. A move will remain tabu during t iterations, so the tabu list can be represented by a queue: at each iteration the opposite move from x to x' is added at the end of T, while the oldest is removed from T. Another mechanism that can improve performance is diversification, that is, to encourage the method to search unexplored regions. This mechanism can be achieved by implementing a *long-term memory function* via frequency counts, a *tabu frequency list*. A move is forbidden if its frequency exceeds a *maximum frequency value*, and this tabu frequency list and its limit value must be updated dynamically when iterations increase. These tabu lists could forbid certain interesting moves, such as moves that lead to a better solution than the best one found so far. Considering this, the *aspiration criterion* is introduced to cancel the tabu status of a move when it is judged to be a useful move.

Stopping rules must also be defined, it can be a fixed lower value f*, or it can be a fixed number of possible solutions investigated, or it can be a fixed number of iterations.

4. APPLICATION OF TABU SEARCH TO THE FA AXIAL DESIGN

In this application, a candidate solution would be the array $<n_1,n_2,....,n_{25}>$ indicating the lattice cell type located at each position. One bottom and two top positions correspond to fixed natural fuel cells. We propose to find the optimal location of four different enriched lattice types (compositions) in the twenty-two internal positions (Fig. 1). This means that there are 4^{22} possible combinations. It is necessary to find the optimal

number of lattices of each type and their location in the 22 positions. A candidate solution can be represented graphically by two lines and twenty-two columns. The position indexes are in the first line, and the lattice types are in the second line and the columns represent the positions.

The first candidate solution is randomly constructed and it is considered the current solution in the iteration number 1. The TS method is designed to select, at each iteration, what is considered the best available *move*, given the current solution. In our application it is necessary to have a *move* type, which allows a change in the quantity of each lattice composition in the fuel, in order to modify the mean FA enrichment and the performance of the FA. It is also necessary to have another *move* type, that allows to modify the lattices' distribution in the FA, in order to search the best location of a certain lattice in the FA. Consequently two types of *moves* are defined, which are described in Fig. 1 using an example.

A move type 1 is entirely defined by the random selection of one position, named pos1, and the random selection of one lattice composition, named lat2, from the list of available lattices. The move type 1, will place lat2 in position pos1. Obviously lat1 (the lattice type in pos1 before the move) and lat2 must be different lattice types to build a real move. For each current solution x, there are 22 x 3 candidate solutions directly reachable from x by a single move type 1.

Move type 2 is a pairwise exchange (or swap), it exchanges the location of two different lattices in the FA. It is defined by the random selection of two different positions, pos1 and pos2, having lattices lat1 and lat2 of different type. For each current solution x, the number of all solutions directly reachable from x by a single move type 2 is equal to [(21 x 21) − I] where I is the number of swaps having the same lattice type in positions pos1 and pos2.

The set N(x) of all neighbors is considerably large, at each step of the iteration process, only a subset V* of N(x) is generated, and the move is made from x to the best solution x* in V*, whether or not f(x*) is better than f(x). The number of moves can be fixed or variable. In this application a maximum number of moves in an iteration step was fixed, but the process will stop the moves when a solution x* is better than solution x.

Associated with each move is the move value, which represents the change in the objective function value. To prevent cycling, any move that returns to any local optimum recently visited is tabu. The tabu recency list is represented by a queue (of length *t*) of an array of four values (pos1, lat1, pos2, lat2). For moves type 1 the position pos2 is always equal to 0. In the tabu recency list a 0 in pos2 does not represent a tabu. The tabu frequency list is represented by an array of 22 values (moves[1], moves[2], ..., moves[22]) containing the position's move frequency. The aspiration criteria cancel the tabu status of a move when it finds a solution with a better function value than the best solution in the past.

The basic aspects of this implementation can be illustrated using the example of Fig 1. In this case, when the move type 1 is done, pos1 and lat2 are randomly selected. In the example of Fig.1, pos1 is equal to 10 and lat2 is equal to C1, the new neighbor

Current solution:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
C_2	C_3	C_2	C_3	C_1	C_2	C_3	C_2	C_0	C_3	C_0	C_1	C_3	C_1	C_2	C_0	C_0	C_1	C_3	C_3	C_1	C_1

Move type 1. Example of random selection: pos1=10, lat2=C_1

Selected position

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

Selected lattice fuel

C_0
C_1
C_2
C_3

Neighborhood solution after the move type 1:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
C_2	C_3	C_2	C_3	C_1	C_2	C_3	C_2	C_0	C_1	C_0	C_1	C_3	C_1	C_2	C_0	C_0	C_1	C_3	C_3	C_1	C_1

Move type 2. Example of random selection: pos1=6, pos2=12

Selected position 1 Selected position 2

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

Neighborhood solution after the move type 2:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
C2	C3	C2	C3	C1	C1	C3	C2	C0	C3	C0	C2	C3	C1	C2	C0	C0	C1	C3	C3	C1	C1

Fig. 1. Moves type 1 and type 2.

solution will have the lattice type C1 at the position number 10. This new candidate solution is evaluated and if its move value shows that it is the best solution, at this search state, the move is accepted, and in the tabu recency list, the array (10,C1,0,C1) is incorporated. The selection of the pair (10,C1) as (pos1,lat1) or as (pos2,lat2) will be tabu in the next *t* iterations. Additionally, in the tabu frequency list, moves[10] is incremented by one.

On the other hand for the move type 2, pos1 and pos2 are randomly selected. In the example of Fig. 1, pos1=6, lat1=C1, pos2=12 and lat2=C3. It is important to clarify that if (pos1,lat1)=(6,1) or (pos2,lat2)=(12,C3) are present in the tabu recency list, this move will be accepted only if the aspiration criterion is satisfied. That means that the new solution improves the best solution reached in the past. In this case the move is accepted and (6,C3,12,C1) is included in the actualized tabu recency list, the removal of lattice C3 from position 6 and/or lattice C1 from position 12 is forbidden.

5. RESULTS AND DISCUSSION

The main results obtained with this tabu search based FA axial design optimization system are presented in this section for cycle five of LVR1. In Fig. 2 the graph of energy and mean enrichment FA evolution is depicted for the best solution in function of the number of candidate solutions evaluated. A good fuel utilization is observed, since the TS method can minimize the mean enrichment in the FA, to obtain the target energy under all the safety limits. At evaluation number 1380, a near optimal solution is found using only a simple TS method. Fig. 3 shows the qualification (function objective value) for the best and the current solutions as a function of the number of candidate solutions evaluated in the process. It is observed that the current solution has oscillations near the best solution. Fig. 4 shows the qualification value of each candidate evaluated, a high diversity of qualifications can be obtained for neighbor solutions. Therefore the search must be well conducted in order to prevent cycling.

The results obtained with TS, in this first implementation, are comparable to those obtained with GA (Martín del Campo, 2001). In Table 3 the results for the best FA design obtained with two different TS runs and a well tested GA solution are shown. Cases TS-1 and TS-2 differ only in the random selection defined by the computer time. Fig 5 shows the lattices' axial distribution for the LVR1 cycle 5 "reference" case and for the TS-1 and GA cases.

Finally a full cycle control rod depletion was performed for the optimal solution of the TS-1 case in order to verify that the safety limits were satisfied during the whole cycle. Table 4 shows the main results.

6. CONCLUSION

The tabu search method has been implemented to the fuel axial design problem in a simple form, and we have obtained results comparable to a well-tested GA implementation. We have demonstrated that the TS method can be used to optimize

axially a realistic BWR fuel assembly, considering the most significant safety constraints used in design analysis for this type of reactor. The objective function can be improved adding other parameters such as the fuel assembly discharge burnup. We conclude that TS is a promising method for solving fuel axial design and other problems related to fuel management optimization in nuclear reactors. It is clear that it will be useful to continue investigating the characteristics of the TS implementation in order to find the best adaptation to the problem of FA axial design optimization.

ACKNOWLEDGMENTS

This work was performed under the auspices of the Mexican Science and Technology National Council (CONACyT) under agreement No. 34657-U, and was partly supported by the National University of Mexico (PAPIIT-UNAM) under project No. IN109400.

Table 3 TS and GA comparison.

Parameter	LVR1-Cycle 5	Case TS-1	Case TS-2	Case GA
Enrich (%)	3.498	3.486176	3.486172	3.486172
Energy (MWD/T)	9280.8	9280.99	9280.85	9281.0
MLHGR (W/cm)	363.3	365.5	365.8	365.6
XMPGR	0.793	0.809	0.802	0.806
PPF	1.532	1.529	1.527	1.529
MRNP	1.816	1.823	1.827	1.824
HEX (%)	1.557	1.584	1.62	1.718
SDM (%)	2.44	1.81	1.68	1.84
MCPR	1.579	1.585	1.587	1.584
Better evaluation	-	1379	1762	431
Total evaluations	-	2000	2200	1365
QUALIFICATION	332	404	398	405

Table 4 Optimal solution TS-1 case control rod depletion thermal limits.

Step (MWd/T)	MLHGR (W/cm)	XMPGR	MCPR
0	435.3	0.909	1.77
1000	425.2	0.893	1.76
2000	422.4	0.89	1.75
3000	432.9	0.924	1.52
4000	445.6	0.926	1.53
5000	438.3	0.908	1.49
6000	429.6	0.901	1.46
7000	410.8	0.864	1.46
8000	376.8	0.806	1.54
9000	381.5	0.836	1.51
9375	383.2	0.848	1.52

REFERENCES

Alonso C. Laguna Verde 1 Cycle – 5 Cycle Management Report. GE Nuclear Energy J11-3010, April 1995.

Ben Hmaida I.A., Carter J.N., de Olivera C.R.E., Goddard A.J.H., Parks G.T., 1999. Nuclear In-Core Fuel Management Optimization Using Tabu Search Method. Mathematics and Computation Reactor Physics and Environmental Analysis in Nuclear Applications, Madrid, Spain, pp. 1658-1666.

Francois J. L., Lopez Z. H., 1999. SOPRAG: A System for Boiling Water Reactor Reload Pattern Optimization Using Genetic Algorithms. *Annals of Nucl. Energy*. **26**, 1053-1063.

François J.L., Esquivel J.L., Cortés C., Esquivias J., Martín del Campo. C., 2001. Validation of a methodology for fuel management analysis of Laguna Verde nuclear power plant. *Annals of Nucl. Energy*. **28**, 489-501.

Glover F., Greeberg H.J., 1988. New approaches for heuristic search: A bilateral linkage with artificial intelligence. *European Journal of Operational Research 00,* EOR00065 North-Holland.

Glover F., 1989. Tabu search part I, *ORSA journal on Computing 1*, 190-206.

Haling R.K., 1963. Operating strategy for maintaining an optimum power distribution throughout life. *ANS Topical Meeting on Nuclear Performance of Power Reactor Cores*, TID 7672.

Martín del Campo C., François J.L., López H.A., 2001. AXIAL: a system for boiling water reactor fuel assembly axial optimization using genetic algorithms. *Annals of Nucl. Energy*, to appear.

Morales L.B., Maldonado F., 1998. Constructing Optimal Schedules for certain Tournaments using Tabu Search. *Investigacion Operativa.* **6**, 127-135.

Poon, P.W., Parks, G.T., 1993, Application of Genetic Algorithms to In-Core Fuel Management Optimization, Proc. Topical Mtg Math Methods and Supercomputing in Nuclear Applications, Karlsruhe, Germany, Vol. 2, p. 777.

Scandpower, 1993. User Manual CM-PRESTO-91.

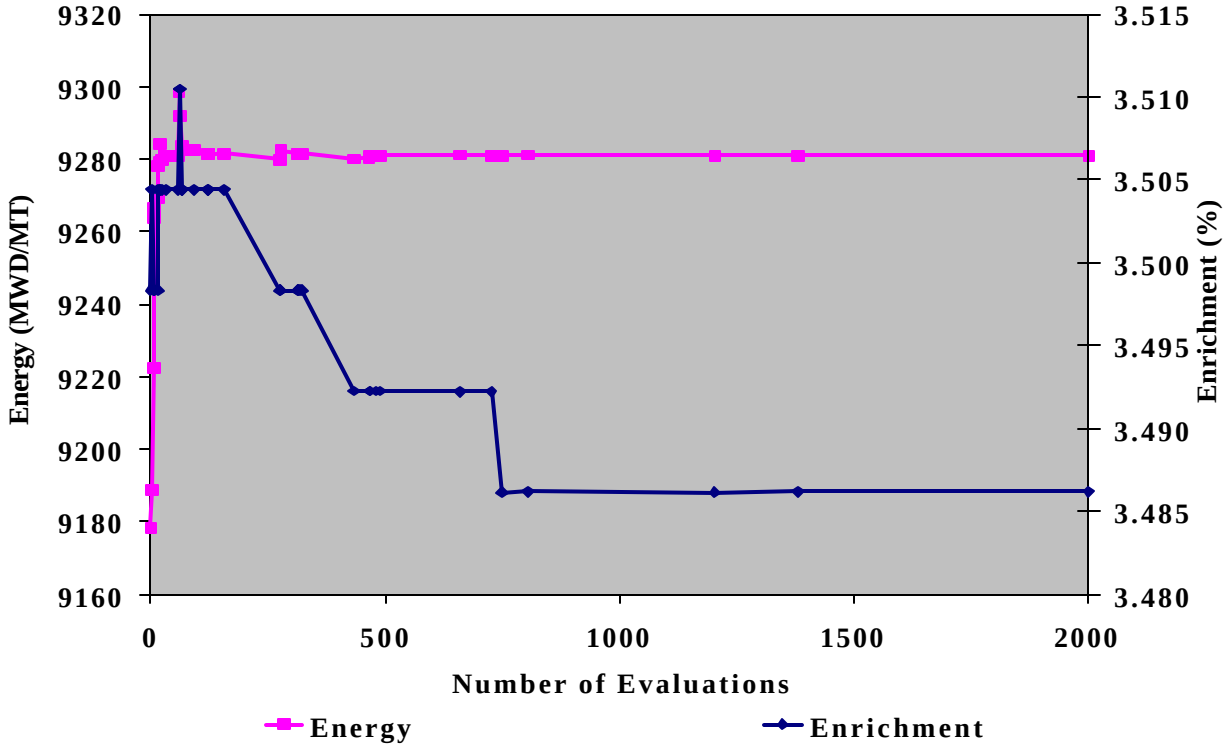

Fig. 2. Energy and enrichment for the best solution vs the number of evaluations.

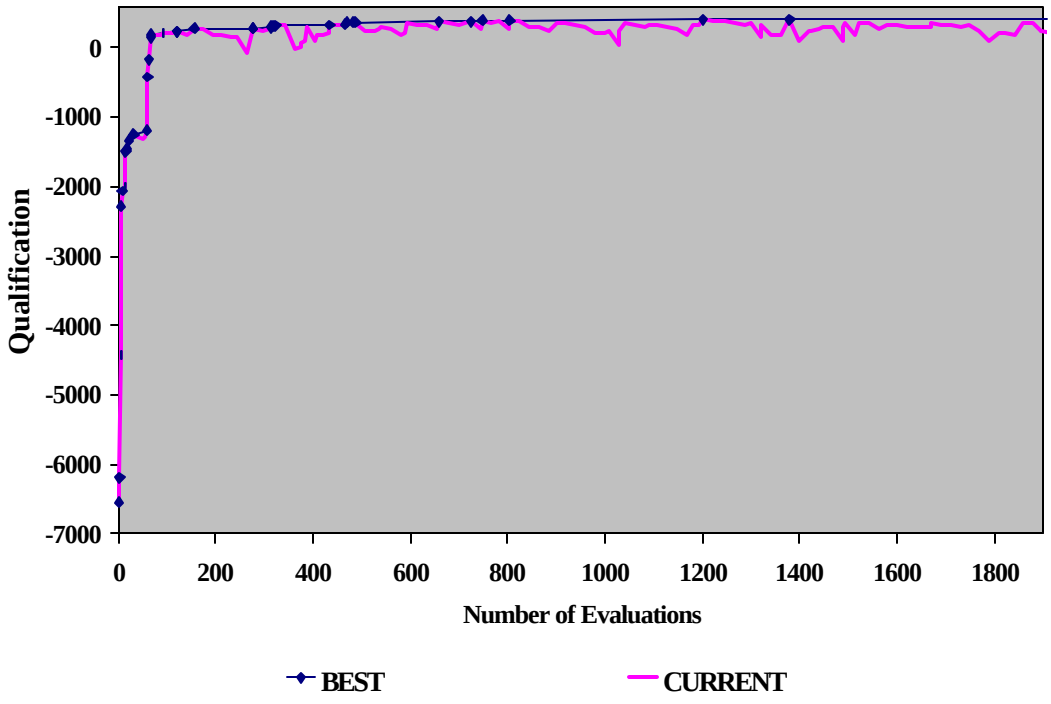

Fig. 3. Qualification of the best and the current solution vs the number of evaluations.

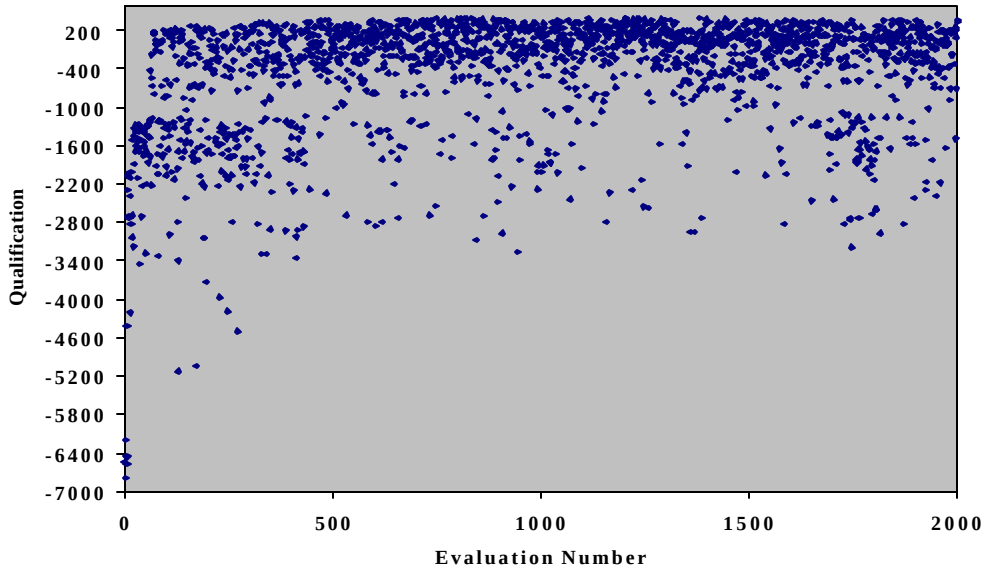

Fig. 4 Qualification value of each candidate evaluated.

LVR1 Cycle 5	TS-1 case	GA case
Nat	Nat	Nat
Nat	Nat	Nat
1	1	1
1	3	3
1	3	2
2	3	3
2	1	3
2	2	1
3	4	3
3	1	1
3	1	1
3	1	3
3	3	3
3	1	4
3	3	1
4	4	1
4	2	1
4	3	3
4	1	1
4	4	1
4	1	1
4	1	1
4	1	1
4	1	1
Nat	Nat	Nat

Fig. 5. FA axial description for the optimal solution.