DETERMINATION OF THE UNCERTAINTIES OF THE CONSTITUTIVE RELATIONSHIPS OF THE CATHARE 2 CODE

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ABSTRACT

This paper deals with the general problem of uncertainty analysis of best-estimate codes and more precisely with the question of the determination of the uncertainties due to the empiricism of the closure laws. Generally the only proposed approach is the expert judgement. CEA (France) proposes a statistical method of data analysis, called Circé and used for the Cathare code.

The principles of the used algorithm are explained in detail. An example of application is precisely described and the results calculated by Circé are checked. But the bases of the method are especially pointed out. Its purpose is the best possible analysis of existing data, that is to say the Separate Effect Tests experiments used for Cathare qualification, which raises the question of the adequacy of this qualification matrix.

As it stands, Circé is however a powerful tool, which is planned to be systematically used for the determination of the uncertainties of the closure laws of Cathare. The first studies have started and give encouraging results.

1. INTRODUCTION

The general context of this paper is the uncertainty analysis of the best-estimate codes.

In France, the best-estimate Cathare code has been developed by CEA (the research institution), EDF (the utility), Framatome (the vendor) and CEA-IPSN (the safety authority). It is used for safety analysis, accident management, definition of plant operating procedure and research and development (Bestion, 1996). It is also used to quantify the conservatism margins and for licensing by EDF. This last point led

EDF and IPSN to develop rules and methodologies for the use of best-estimate codes. One of the major points is to develop tools for uncertainty analysis.

The aim of uncertainty analysis is the calculation of an uncertainty band for the code response, important from the safety point of view. An example of such a response is the peak cladding temperature (PCT) for a Large Break Loss of Coolant Accident (LB-LOCA), which must not exceed 1204°C, to respect the appendix K of rule 10 CFR 50.46. This uncertainty may have several forms: for example pdf (probability density function) or 95^{th} percentile limit.

The general question of uncertainty analysis has been broadly studied throughout the world. NRC proposes the CSAU methodology (Code scaling, applicability and uncertainty), which identifies clearly the different steps necessary for an uncertainty analysis (Wulff, 1990) and (Lellouche, 1990). The general idea is to determine all the uncertainty sources of the code, also called basic uncertainties, to quantify them and to combine them in order to obtain the final uncertainty for the studied application.

Several methods exist for the propagation (the combination) of the basic uncertainties: establish a calculation matrix and calculate a response surface for CSAU, Wilks formula for GRS (Glaeser, 2000) and IPSN (Chojnacki, 1996) etc. In return, the quantification of the basic uncertainties is a problem still not very tackled. The list of these basic uncertainties is long: choice of the models (number of fluids, of equations, etc.), discretization, uncertainties about the initial and boundary conditions, user effect, etc. A significant uncertainty source comes also from the empiricism of the closure laws (also called correlations or constitutive relationships) used to describe the transfer terms present in the balance equations. The Circé method, developed by CEA, is intended to quantify the uncertainties of the correlations of a code and is applied to Cathare 2 (de Crécy, 1997). It is a statistical method of data analysis which may replace the expert judgement generally used.

2. PRESENTATION OF CIRCE METHOD

2.1 General approach

Circé is a statistical method of data analysis. It means two things:

1) A data analysis

There are sometimes theoretical expressions for the closure laws, but they are generally applicable only in simple configurations, not always encountered in a reactor transient. That is the reason why the physicists use the results of a large number of Separate Effect Tests (SET) experiments, to improve or even establish these closure laws. In CEA, we have at our disposal a large database, which can be used not only for the definition of the correlations but also for the quantification of their uncertainties. This process lies thus on the following hypothesis: the database used for establishing the correlations of Cathare is considered as sufficient to calculate the uncertainty of

these same correlations and to make it possible their use for uncertainty propagation in a reactor transient.

2) A statistical method

With Circé, the uncertainties of the constitutive relationships are defined with statistical tools: pdf (probability density function), mean values and standard deviations. Let us consider, for example, the Cathare correlation describing the heat exchange coefficient between the fluid and the wall, in film boiling, denoted as h_{boi}. It is a function of pressure, liquid Reynolds number, void fraction, etc. To express the idea that h_{boi} is known with a certain uncertainty, an adimensional parameter, ε, is associated with h_{boi}, for example with the following way:

$$h_{boi} = e^{\varepsilon} \times h_{boi}(\text{nominal})$$

where h_{boi}(nominal) is the value of the exchange coefficient calculated by standard Cathare, value which can be shifted by varying ε, the new shifted value being denoted as h_{boi}.

By using an adequate database, Circé expresses the uncertainty of this correlation by calculating the mean value and the standard deviation of the associated ε parameter. An hypothesis of normal law is also made for the distribution of ε. Other means for describing the uncertainty of h_{boi} would have been possible, for example giving only a variation interval, without any statistical consideration. That is not the choice made with Circé.

2.2 Main principles of the Circé statistical tool

Generally the closure laws are not measurable in the experiments devoted to their study. The experimenter knows only physical quantities which are very sensitive to the studied correlation(s): for example, T_{wall} wall temperatures for h_{boi}. These physical quantities are called responses.

By analysing the code-experiment differences on the responses (R_{exp}-R_{code}), and possibly the experimental uncertainties (δR_{exp}), Circé calculates the mean value and the standard deviation of the ε parameters associated with the studied correlations. Circé makes it possible to consider together several closure laws.

The calculation of the statistical features of ε (mean value and standard deviation) from the (R_{exp}-R_{code}) differences is possible without very numerous Cathare sensitivity calculations thanks to the ASM tool implemented into Cathare. The ASM (Adjoint Sensitivity Method) is a method which calculates the exact derivatives of a response with respect to different parameters, with a low CPU cost, even for a large number of parameters (Cacuci, 1990). This powerful tool works as a post-processing module of Cathare (Perret, 1997) and is of special interest for Circé because it provides the derivatives $\frac{\partial R_{code}}{\partial \varepsilon}$ (for example, the derivatives $\frac{\partial T_{wall}}{\partial \varepsilon}$ with ε associated with the film boiling exchange coefficient).

To sum up, for using the Circé statistical tool, the user must:

- perform standard Cathare calculations of the considered experiment, in order to obtain R_{code},

- use the ASM post-processing in order to have the derivatives $\frac{\partial R_{code}}{\partial \varepsilon}$,

- find the experimental values of the responses: R_{exp}, and the experimental uncertainties δR_{exp} in the test reports.

These quantities are the input data of Circé. Generally, only 1, 2 or 3 correlations are considered together (and consequently the same number of parameters is often considered). To be reasonably precise, Circé needs a large number of responses, typically several tens. For one experiment, the user must consider as many tests as possible, and for each test a lot of responses.

The precise algorithm used by Circé is described below.

2.3 Description of the algorithm used by Circé

• *The parameters*

Let us consider d parameters, associated to a set of constitutive relationships.

As explained before, there is generally one parameter ε by correlation, expressed via an exponential formulation e^{ε}, but it is not compulsory: other kinds of parameters may be defined. The exponential formulation is interesting because with this formulation, the closure law cannot change of sign, even for strongly negative values of ε, what is generally indispensable (take the example of a heat exchange coefficient). The nominal value of the closure law corresponds with the 0 value of ε.

• *What is calculated by Circé*

The aim of Circé is the calculation of the statistical features of the $\vec{\varepsilon}$ vector defined by $\vec{\varepsilon} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ ... \\ \varepsilon_d \end{pmatrix}$, where ε_1 is the first parameter, ε_2 the second one, … ε_d the last one. Thus Circé calculates a mean vector $\vec{m} = \begin{pmatrix} m_1 \\ m_2 \\ ... \\ m_d \end{pmatrix}$, also called bias, and a covariance matrix C. By definition, a covariance matrix is symmetric, its diagonal terms are the variances of the parameters, i.e. the square of the σ standard deviations and its extra-diagonal terms are the covariance terms, related to the correlation coefficients of the parameters considered two by two. In the studies performed up to now with Circé, the

covariance terms are considered as equal to 0. It means that, in these studies, the different parameters are assumed independent. This hypothesis is nevertheless not compulsory. However, to sum up and simplify, we will say that the C matrix is defined by: $C = \begin{pmatrix} \sigma_1^2 & 0 & ... & 0 \\ 0 & \sigma_2^2 & ... & 0 \\ ... & ... & ... & 0 \\ 0 & 0 & 0 & \sigma_d^2 \end{pmatrix}$.

The extensive description of the algorithm used by Circé is rather long. To simplify, only the case where the mean vector is assumed equal to $\vec{0}$ (and is consequently not calculated by Circé), is described below. The main principles are quite the same if a bias is calculated.

- $\vec{\varepsilon}_j$ *realizations and estimation of C by the maximum of likelihood*

To calculate the covariance matrix of $\vec{\varepsilon}$, we consider the results of the SET experiments, where the physical phenomena described by the studied correlations are influential. Let n be the number of responses: n >> d.

For each R_j response (j = 1,n), we may define the notion of $\vec{\varepsilon}_j$ realization of the $\vec{\varepsilon}$ vector of the parameters. To simplify, this definition is given in the case where the experimental uncertainties are not taken into account. A $\vec{\varepsilon}_j$ realization associated with R_j is one of the values of the $\vec{\varepsilon}$ vector such as the code response calculated with $\vec{\varepsilon} = \vec{\varepsilon}_j$ i.e. $R_{j,code}(\vec{\varepsilon} = \vec{\varepsilon}_j)$ is equal to the experimental response $R_{j,exp}$. Two items are apparent. Firstly, the $\vec{\varepsilon}_j$ realizations are unknown, unless a lot of Cathare sensitivity calculations are performed. Secondly, if more than one parameter are considered, several $\vec{\varepsilon}_j$ realizations may exist for each R_j response.

The algorithm used by Circé is based on the principle of maximum likelihood, quite usual in statistics. This principle gives an estimation $\widehat{C}$ of the C matrix. It is the equation (1):

$$\widehat{C} = \frac{1}{n} \sum_{j=1}^{n} \vec{\varepsilon}_j \vec{\varepsilon}_j^{\,T} \qquad (1)$$

The problem to use this relationship is that the $\vec{\varepsilon}_j$ realizations are unknown. We need consequently a model between the $\vec{\varepsilon}_j$ realizations of the considered parameters and the code-experiment differences on the responses: $R_{j,exp}$-$R_{j,code}$, which are known. This relationship is given by the linear model.

- *The linear model*

A first order development makes it possible to write:

$$R_{j,exp} - R_{j,code} = \frac{\partial \vec{R}_j}{\partial \varepsilon}^T \vec{\varepsilon}_j \qquad (2)$$

This relationship does not allow to explicitly calculate the $\vec{\varepsilon}_j$ realizations, if more than one parameter are considered, because it is a scalar equation whereas $\vec{\varepsilon}_j$ is a vectorial unknown. But it makes it possible to use Bayes' theorem.

• *Bayes' theorem*

This theorem deals with conditional probabilities and is explained in any statistics manual. Its general idea is to start from an initial estimation of the C covariance matrix, referred to as C "a priori", and denoted as $C^{(i-1)}$. This first estimation is corrected after "observation" of each $\vec{\varepsilon}_j$ realization of $\vec{\varepsilon}$, via $R_{j,exp}$-$R_{j,code}$. A new matrix is obtained, referred to as "a posteriori" matrix, and denoted as $C^{(i)}$. The process is continued until both "a priori" and "a posteriori" matrices are very close. It is consequently an iterative process. The (i) exponent used for the "a posteriori matrix" indicates the iteration performed by Circé. Generally, the iterative process is started with the identity matrix, and around 1000 iterations are necessary to converge.

More precisely, if the $\vec{\varepsilon}$ vectorial random variable is assumed to obey a normal law, then each $\vec{\varepsilon}_j$ is a realization of a random variable which obeys itself a normal law depending on j. Bayes' theorem gives an estimation of the $\bar{\vec{\varepsilon}}_j$ mean vector and of the C_j covariance matrix of each $\vec{\varepsilon}_j$, after observation of $R_{j,exp}$-$R_{j,code}$ and from the "a priori" $C^{(i-1)}$ matrix . It is:

$$\bar{\vec{\varepsilon}}_j = C^{(i-1)} \times \frac{\partial \vec{R}_j}{\partial \varepsilon} \times \frac{R_{j,exp} - R_{j,code}}{\frac{\partial \vec{R}_j}{\partial \varepsilon}^T C^{(i-1)} \frac{\partial \vec{R}_j}{\partial \varepsilon}} \qquad (3)$$

$$C_j = C^{(i-1)} - \frac{C^{(i-1)} \frac{\partial \vec{R}_j}{\partial \varepsilon} \frac{\partial \vec{R}_j}{\partial \varepsilon}^T C^{(i-1)}}{\frac{\partial \vec{R}_j}{\partial \varepsilon}^T C^{(i-1)} \frac{\partial \vec{R}_j}{\partial \varepsilon}} \qquad (4)$$

It is apparent that Bayes' theorem does not calculate explicitly each $\vec{\varepsilon}_j$ but gives only an estimation of its mean vector and its covariance matrix. The $\vec{\varepsilon}_j \vec{\varepsilon}_j^T$ products are needed in (1) to calculate the "a posteriori" $C^{(i)}$ matrix. They are replaced by their expectation value, given by the formula:

$$E(\vec{\varepsilon}_j \vec{\varepsilon}_j^T) = \bar{\vec{\varepsilon}}_j \bar{\vec{\varepsilon}}_j^T + C_j \qquad (5)$$

Finally, the following iterative relationship between the $C^{(i-1)}$ "a priori" matrix and the $C^{(i)}$ "a posteriori" matrix is obtained, by combining (3), (4) and (5) in (1):

$$C^{(i)} = C^{(i-1)} + \frac{1}{n}\sum_{j=1}^{n} \frac{C^{(i-1)}\,\frac{\partial \vec{R}_j}{\partial \varepsilon}\,\frac{\partial \vec{R}_j}{\partial \varepsilon}^{T}\,C^{(i-1)}}{\frac{\partial \vec{R}_j}{\partial \varepsilon}^{T}\,C^{(i-1)}\,\frac{\partial \vec{R}_j}{\partial \varepsilon}} \left(\frac{\left(R_{j,exp} - R_{j,code}\right)^2}{\frac{\partial \vec{R}_j}{\partial \varepsilon}^{T}\,C^{(i-1)}\,\frac{\partial \vec{R}_j}{\partial \varepsilon}} - 1 \right)$$

• *Comments*

1) The iterative process therefore consists of two parts:

- A "maximisation" part where the principle of maximum likelihood is applied. It gives the relationship (1).

- An "expectation" part, where each $\vec{\varepsilon}_j \vec{\varepsilon}_j^{\,T}$ product is replaced by an estimation of its expectation value, thanks to Bayes' theorem.

This algorithm is well-known in statistics and is called E-M algorithm, E for Expectation part, and M for Maximisation part (Dempster, 1981).

2) In the case of calculation of a $\vec{m}$ bias, the principle is unchanged: an iterative process is used. At each iteration (i), the $C^{(i)}$ covariance matrix is always calculated on the basis of two steps (Expectation and Maximisation). The $\vec{m}^{(i)}$ bias is also calculated with the same iterative process, on the basis of maximum likelihood too. But instead to apply it to the $\vec{\varepsilon}_j$ realizations, it is applied to the $R_{j,exp}$-$R_{j,code}$ realizations: If $\vec{\varepsilon}$ obeys a normal law, with a $\vec{m}$ mean vector and a C covariance matrix, then $R_{j,exp}$-$R_{j,code}$ obeys also a normal law, with the mean value equal to $\left(\frac{\partial \vec{R}_j}{\partial \varepsilon}^{T} \times \vec{m} \right)$ and the variance equal to $\left(\frac{\partial \vec{R}_j}{\partial \varepsilon}^{T} C \frac{\partial \vec{R}_j}{\partial \varepsilon} \right)$. The likelihood of the n-sample $R_{j,exp}$-$R_{j,code}$ is written by using these values. The determination of its maximum with respect to the components of $\vec{m}$ leads to a linear d×d system, updated and solved at each (i) iteration.

2.4 Summary of the features and hypotheses of Circé

The algorithm used by Circé is mainly based on the principle of maximum likelihood. It is this principle which, among all the possibilities for $\vec{m}$ and C, i.e. implicitly for the different $\vec{\varepsilon}_j$ realizations, chooses the most "likely" one.

Circé is based on two main hypotheses, reminded here:

1) The parameters are assumed to obey a normal law. If the exponential formulation is chosen, it means that the correlations are assumed to obey a log-normal law. This hypothesis is very plausible, and may be checked when the Circé calculation has been carried out, by observation of residuals.

2) The model chosen for the dependence between the parameters and the responses is a linear one. More precisely, it means that a linear development is performed between $R_{j,exp}$ and $R_{j,code}$. Circé results are valid only if the bias and the standard deviations are low. That is unfortunately not always the case, especially for the bias. This strong hypothesis can be made less restricting with "iterative Circé", described below.

2.5 An improvement of Circé: "Iterative Circé"

A simple improvement of Circé makes it possible to use it even for high biases. It is "Iterative Circé", based on a Gauss-Newton approach. The referred iterations are Gauss-Newton ones and not those of E-M algorithm. The principle is explained below:

1) A first Circé calculation is performed by considering:

$$R_{j,exp} \quad R_{j,code}(\vec{\varepsilon} = \vec{0}) \quad \frac{\partial R_{j,code}(\vec{\varepsilon} = \vec{0})}{\partial \varepsilon}$$

that is to say the responses and the derivatives at the nominal point, with standard Cathare. This corresponds with a standard use of Circé.

A $\vec{m}_0$ bias and a C_0 covariance matrix are obtained.

2) An iterative calculation is carried out, by performing a second Circé calculation by considering this time :

$$R_{j,exp} \quad R_{j,code}(\vec{\varepsilon} = \vec{m}_0) \quad \frac{\partial R_{j,code}(\vec{\varepsilon} = \vec{m}_0)}{\partial \varepsilon}$$

A new $\vec{m}_1$ bias and a new C_1 covariance matrix are obtained. If the process is convergent, we have: $|\vec{m}_1| < |\vec{m}_0|$.

3) The iterative process is continued, by considering:

$$R_{j,exp} \quad R_{j,code}(\vec{\varepsilon} = \vec{m}_0 + \vec{m}_1) \quad \frac{\partial R_{j,code}(\vec{\varepsilon} = \vec{m}_0 + \vec{m}_1)}{\partial \varepsilon}$$

A new $\vec{m}_2$ bias and a new C_2 covariance matrix are obtained.

The iterations are stopped when a bias low in absolute value is obtained. Generally 3 or 4 iterations are sufficient. Let us denote as N the iteration for which the process converges.

It is possible to show that the final bias (or more precisely, its estimation by the maximum likelihood) is the total bias: $\vec{m}_t = \vec{m}_0 + \vec{m}_1 + ...\vec{m}_N$ and that the final covariance matrix (or more precisely, its estimation) is C_N if each standard deviation deduced from C_N is low with respect to the bias.

With this iterative approach, there is still an hypothesis of linearity, but which is less strong than the hypothesis made with standard Circé. Indeed, instead of assuming that the following relationship, coming from (2), is right:

$$R_{j,exp} - R_{j,code} = R_j(\vec{\varepsilon}_j) - R_j(\vec{0}) = \frac{\partial \vec{R}_j(\vec{0})^T}{\partial \varepsilon} \vec{\varepsilon}_j \qquad (2)$$

we write:

$$R_j(\vec{m}_t + \vec{\varepsilon}'_j) - R_j(\vec{m}_t) = \frac{\partial \vec{R}_j(\vec{m}_t)^T}{\partial \varepsilon'} \vec{\varepsilon}'_j$$

It means that the hypothesis of linearity is made around $\vec{m}_t$ instead of being made around $\vec{0}$.

3. DETERMINATION OF THE UNCERTAINTIES OF THE CATHARE CLOSURE LAWS

3.1 General approach and work program

For each type of reactor transient (Large Break or Intermediate Breaks LOCA), the list of the most important physical phenomena is drawn up. For each phenomenon, the SET experiments where the phenomenon is influential and the correlations used for describing it, are listed. This work is carried out from expert judgement and is close to the PIRT step (Process Identification and Ranking Tables) recommended by the CSAU (Wilson, 1990). Circé is then used for each experiment.

All this work has led to define specifications. A about 7 men×years program has been planned for the determination of the uncertainties of the revision 6 closure laws of Cathare2 version v1.5. This determination has started at the end of 1999 and is planned until 2003. The following studies have already been completed:

- interfacial friction in tube geometry: Vertical Canon experiment,
- interfacial friction in rodbundle geometry: Pericles boil-off experiment,
- wall-fluid heat exchanges: Winfrith tube and Pericles boil-off experiment.

In progress:

- critical flowrates: Super-Moby-Dick, Bethsy nozzle and Rebecca experiments.

3.2 Example of results

The following example is given for the Winfrith tube tests (Savage, 1993).

• *Description of the tests and the Cathare results*

These tests are devoted to the study of the post-DNB heat exchanges between the cladding and the fluid. Their conditions are similar to those of the blowdown and filling phases of a LB-LOCA (the pressures are between 5 and 70 bar). They consist in obtaining a steady-state, with stabilization of a quench front at the bottom or under the measurement section. The wall temperatures are measured at 14 different elevations in the dry part, and the void fractions at 2 elevations. The main processes of heat exchanges are film boiling and to a minor extent, the forced convection with vapor.

When the test conditions are examined in terms of mass velocities, pressures and void fractions, 4 out of the 20 tests considered for the qualification are clearly distinct from the other ones. They are the 1281, 1282, 1284 and 1285 tests with mass velocities higher than 400 kg/m^{2}s, low pressures equal to 5 bar and all the void fractions represented ranging from 0.05 to 0.95. The other tests correspond with lower mass velocities, less than 500 kg/m^{2}s, higher pressures, up to 70 bar, and void fractions higher than 0.5.

Cathare has a different behaviour for these 4 tests. Indeed, the wall temperatures are generally underestimated by Cathare, up to -30%. But in the case of the 4 tests, Cathare overestimates the wall temperatures, up to 20%. This over-estimation is clearly shown in figure 1, where the $\frac{T_{code}}{T_{exp}}$ ratios are plot versus the void fractions.

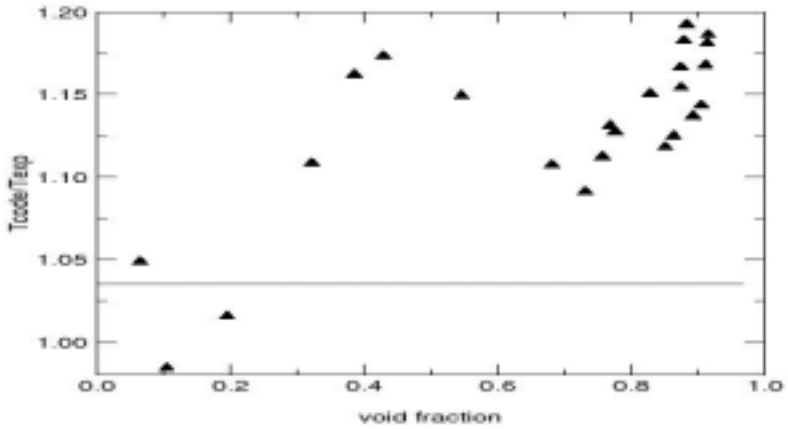

Fig. 1: Winfrith 1281, 1282, 1284 and 1285 tests: ratios of calculated wall temperatures in experimental ones versus void fractions.

- *Circé calculations*

Two parameters are considered by Circé:

– ε_1 associated with forced convection: $h_{conv} = e^{\varepsilon 1} \times h_{conv}(\text{nominal})$

– ε_2 associated with film boiling: $h_{boi} = e^{\varepsilon 2} \times h_{boi}$ (nominal)

In a first attempt, all the tests of the qualification have been considered together, i.e. 20 tests. For each test, the responses considered by Circé are the wall temperatures. But by considering all the tests, the hypothesis of Circé that $\vec{\varepsilon}$ obeys a normal law is not checked. The distribution of residuals is not Gaussian, it is clearly the superposition of two normal distributions. It confirms that two zones must be distinguished. Circé is consequently applied to each zone separately. The results are given for the zone formed by the 1281, 1282, 1284 and 1285 tests.

The results for this zone with "iterative Circé" are, after 5 iterations:

$$\vec{m} = \begin{pmatrix} 1.87 \\ -0.093 \end{pmatrix} \text{and } C' = \begin{pmatrix} 0.003 & 0 \\ 0 & 0.07 \end{pmatrix}$$

The C' matrix is presented instead of the C covariance matrix. C' is the matrix of the σ standard deviations, unlike C which is the matrix of the σ^2 variances.

The meaning of these results is the following: Let us take the example of film boiling (ε_2). With the hypotheses made by Circé, ε_2 obeys a normal law, the mean value and the standard deviation of which have been determined. They are respectively equal to –0.093 and to 0.07. Due to the properties of the normal law pdf, from which the 1.96 factor comes, it can be said that the probability that ε_2 ranges in:

$$[m-1.96\sigma\ ,\ m+1.96\sigma] = [-0.093-1.96 \times 0.07\ ,\ -0.093+1.96 \times 0.07\ ] = [-0.23\ ,\ 0.04]$$

is about 95%. If the correlation itself is considered, and due to the exponential formulation, h_{boi} ranges in:

$$[e^{-0.23} \times h_{boi}(\text{nominal})\ ,\ e^{0.04} \times h_{boi}(\text{nominal})] = [0.80 \times h_{boi}(\text{nominal})\ ,\ 1.04 \times h_{boi}(\text{nominal})].$$

It can be noticed that, due to the negative bias, this variation interval is not centered around h_{boi}(nominal) : h_{boi} must be globally decreased.

The same reasoning may be done for vapor convection, leading to:

ε_1 ranges in [1.865 , 1.875] with a probability equal to 95%,

i.e. for h_{conv}: [6.45×h_{conv} (nominal) , 6.53×h_{conv} (nominal)]

This time, the heat exchanges are globally, and quite strongly, increased.

Another meaning may be pointed out for the biases. They allow to determine how to improve Cathare results so that there is no longer over (or under) estimation. It will be the case if h_{conv} is multiplied by $e^{1.87} = 6.51$, and in the same time, h_{boi} is multiplied by $e^{-0.093} = 0.91$. Globally the exchanges are improved, even if film boiling exchanges are slightly decreased, leading to a decrease of the wall temperatures. This possible improvement of Cathare is only an extra information given by Circé. The variation intervals calculated above do not assume that Cathare has been modified and deal with standard Cathare.

It can be noticed also that the standard deviations are low, when compared to the biases. It indicates that :

1) The use of "iterative Circé" is sufficient. Making an hypothesis of linearity around the bias is correct. It must be noticed that, in return, the use of "iterative Circé" was necessary: the biases are high and the results with standard Circé were significantly different for the biases. We found with standard Circé:

$$\vec{m} = \begin{pmatrix} 1.10 \\ -0.024 \end{pmatrix} \text{ and } C' = \begin{pmatrix} 0.003 & 0 \\ 0 & 0.06 \end{pmatrix}$$

2) The Cathare results are not very dispersed. It can be checked in figure 1.

• *Checking of Circé results*

The biases and standard deviations calculated by Circé may be verified. For the biases, a calculation shifted with the value of the biases is performed and must broadly improve Cathare results: the new results are centered around the experimental ones. An envelop calculation can be performed for the checking of the standard deviations: theoretically 95% of the experimental wall temperatures (i.e. almost all of them) must be lower than those calculated by the envelop calculation. These results are shown for the 4 considered tests, in figure 2.

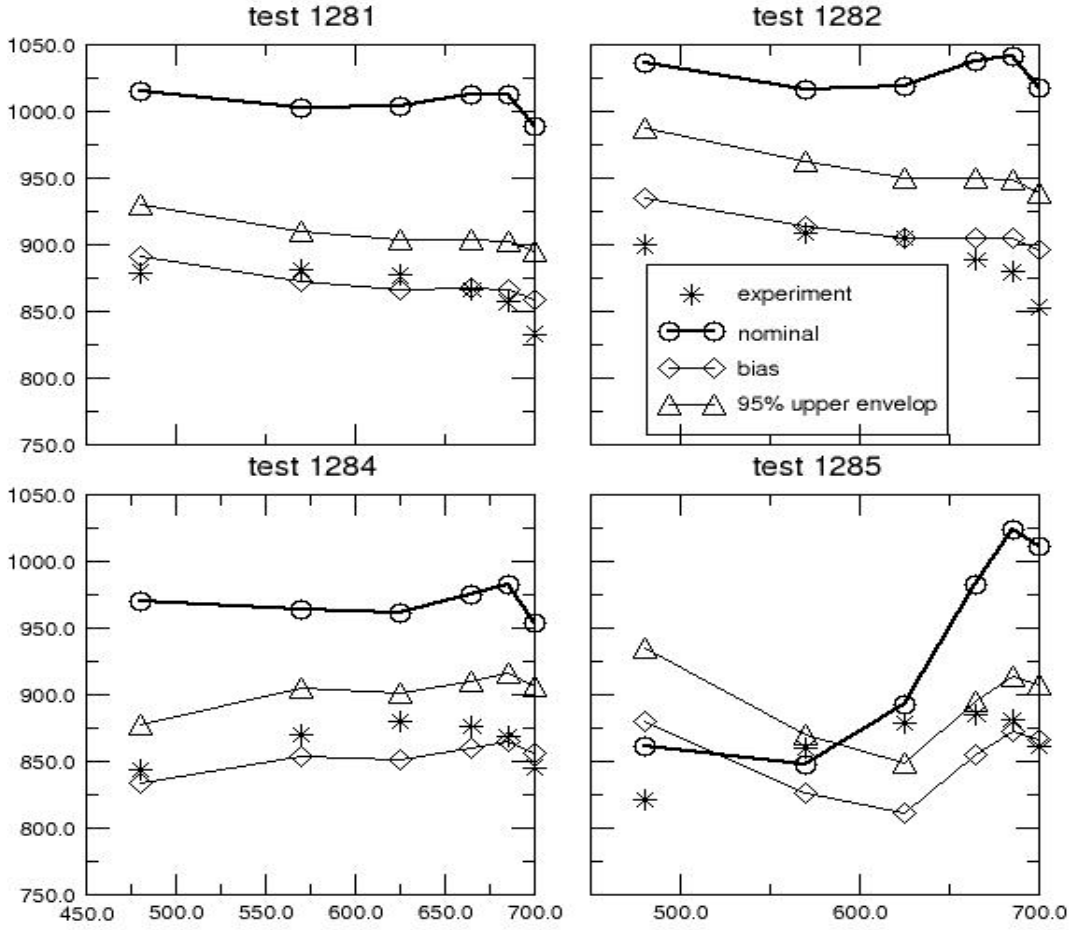

Fig. 2: Winfrith 1281, 1282, 1284 and 1285 tests: checking of Circé results (for each test, wall temperatures versus elevation).

4. CONCLUSIONS

The purpose of this paper is to bring a contribution to the general problem of the determination of the basic uncertainties for uncertainty analysis. The uncertainties due to the empiricism of the closure laws are more especially studied.

Up to now, only the expert judgement was used. Another approach, based on a rigorous statistical method which is usual in other fields than thermal-hydraulics, is proposed. It is the Circé tool, which calculates the mean value (bias) and the standard deviation of the parameters associated with the considered closure laws. The mathematical bases of the method are explained in detail. But the limits of the tool are also clearly given: Circé must not be used like a “black box”. It is the statistical analysis of the database formed by the Separate Effect Tests of the qualification experiments: it means that this database must be adequate. Some hypotheses must also be checked: normality of the considered sample, via the checking of residuals and especially linearity hypothesis around the calculated bias, i.e. low standard deviations. A generalization of Circé without linearity hypothesis is possible and planned.

Even with all these precautions for using it, the contribution made by Circé is considerable. In France, for the Cathare 2 code, revision 6 and version v1.5, an important work program of systematical determination of the uncertainties of the closure laws with Circé has been planned. Cathare will be the only code to be released, in 2003, with the uncertainties of its correlations determined otherwise than with expert judgement.

NOMENCLATURE

C	covariance matrix (of the ε parameters)
C’	matrix of the standard deviations (of the ε parameters)
d	number of considered parameters
ε	parameter (associated with a constitutive relationship)
ε’	parameter considered at the last iteration of “iterative Circé”
h	heat exchange coefficient
m	mean value (bias)
n	number of considered responses
R	response
σ	standard deviation
T	temperature

Subscripts

boi	film boiling
conv	forced convection with vapor
exp	experiment
j	response index
N	total number of iterations for “iterative Circé”
t	total bias for “iterative Circé”
w	wall

Superscripts

(i)	iteration of the E-M algorithm used by Circé
T	transposed

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