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# Some Remarks on Time Series Analysis for BWR Stability Studies

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## Abstract

The results of an experimental or analytical BWR stability analysis are local or global reactor parameter time series, in particular power time histories that are measured or calculated at the locations of the in-core detectors. The time series reflect a stable or unstable oscillatory power evolution at any operational point of the BWR core after a specific or non-specific (noisy) parameter perturbation. In particular, the analysis of measured noisy time series concerning the stability information is a crucial task, because it is this procedure which mainly determines the experimental uncertainty of the BWR stability analysis. In the framework of an international benchmark exercise (NEA time series analysis benchmark, 1999), it was revealed that, at present, in the field of signal analysis, many different methodologies are used and the uncertainties of the various approaches are in some cases very different. Hence, a code containing seven time series analysis approaches for the calculation of the linear stability characteristic "Decay Ratio" (DR) (the so-called asymptotic DR at the natural reactor frequency NF) has been developed at the Paul Scherrer Institut (PSI). This paper is an overview of our experiences and results obtained on the basis of the application of this method, and a preliminary assessment of the uncertainties.

## 1. Introduction

Parameter oscillations are typical for nonlinear feedback systems. Stable or unstable oscillatory solutions are part of the solution manifold of differential equation systems describing nonlinear dynamic systems, such as a boiling water reactor (BWR). Due to the feedback loops, all coupled parameters start to oscillate when one of the reactor core parameters is perturbed in a specific (parameter perturbation) or non-specific (noisy) way. At a stable operational point, the oscillation is decaying, at an (linear) unstable one the oscillation is self-sustained (limit cycle) or diverging. A limit-cycle oscillation can bifurcate under variation of a particular system parameter and the analysis of this nonlinear system behavior is the objective of nonlinear stability analysis. We should note that BWR system analysis, in principle, is equivalent to the analysis of a nonlinear

dynamic system. However, according to the famous theorem of Hartman and Grobman (Guggenheimer and Holmes, 1984), a nonlinear system behaves in the sufficiently close neighbourhood of a hyperbolic fixed point (a steady state) like a linear one. This is the justification for using linear stability indicators like the decay ratio (DR) of the least damped power oscillation or the Nyquist diagram in the framework of BWR stability analysis. From the practical point of view, it is usual to use the DR as the stability indicator. This means that the practical task is focused on the definition of a region in the BWR reactor characteristics, the power-flow map, and outside this region one never expects unstable operational points. This paper is devoted to the analysis of the time series measured by the BWR in-core power detectors, the "Local Power Range Monitors" (LPRM) (Hennig, 1999) and the Average Power Range Monitors (APRM), consisting of several connected LPRM's whose measured signals are averaged. Since the measured time series consists of complex noisy oscillation patterns, statistical methods are necessary to extract the stability indicator, in this case the so-called asymptotic DR, which is the DR of the least damped oscillation with the so-called natural frequency NF.

In general, the statistical methods used in the time series analysis framework are well known, and the main task is to identify the uncertainty of the estimated stability characteristics. These were the objectives of the NEA benchmark task "Time series analysis methods for oscillations during BWR operation" (Verdu, Ginestar et.al., 2000). As a result of the findings of this benchmark exercise, a new computer code (HPTSAC2000) was developed at PSI which includes seven different methodologies (Askari and Hennig, 2000) for the DR estimation. In this way, we are able to calculate the DR on the basis of different and independent methodologies. The statistical methods were divided into parametric and non-parametric methods. The non-parametric methods, in principle, are based on the "classical" autocorrelation function technique and the spectral analysis of stochastic time signals (Priestley, 1981), (MATLAB, 1992, 1993). The stability indicator will be determined on the basis of the autocorrelation function and the power spectrum of the time signals. These methods have been developed at PSI and successfully used in the noise analysis field (Behringer, 1988). The parametric methods are based on the well-known system identification technique: Our system output is, for example. a measured power time series and we want to reconstruct the linear system behavior by assuming that the dynamic system can be described by a linear difference equation of an unknown order:

$$y(kT) = a_1 y((k-1)T) - a_2 y((k-2)T) - \ldots - a_{n_a} y((k-n_a)T) + e(kT) + c_1 e((k-1)T) + c_2 e((k-2)T) + \ldots + c_{n_c} e((k-n_c)T) \quad (1),$$

where y(kT) is the measured time series, the system output (or an output vector) at discrete times kT, and T is the sampling interval. In equation (1) we have assumed that the time series is generated by the stochastic (so-called colored noise) process $C(q)e(kT) = \sum_{n=0}^{n_c} c_n q^{-n} e(kT)$ (with q the time shift operator $q^{-1}y(kT)=y((k-1)T)$), and e(kT) is a white noise Gaussian-process (with E(e)=0, $Var(e)=s^2$). Equation (1) characterizes an autoregressive moving average (ARMA) model and our task is to determine the coefficients of this equation by a suitable approximation algorithm (whereby the ARMA model order estimation, meaning the estimation of $n_a$ and $n_c$ in (1), is a particular optimization problem (Soderstrom and Stoica, 1989)).

In what follows, we present the methods applied at PSI for analyzing stationary linear time series. The paper is organized in the following way. In Chapter 2, the methodologies are described shortly and in Chapter 3 some typical results are demonstrated and discussed.

# 2.Methodology

## 2.1 Non-parametric methods

In what follows we shall assume that the measured time series are stationary and ergodic[1] random processes (Priestley, 1981). Analyzing a random process means to estimate typical statistical quantities such as the autocorrelation (and the cross-correlation) function and auto-power (and cross-power) spectral density. The correlation functions and the power spectral densities reveal deterministic physical effects hidden in the stochastic process recorded by the LPRM detectors. Pure random data are uncorrelated, a correlation of the data at a fixed time delay corresponds to an interdependence of the data in a non-random way. This is also reflected in the frequency dependence of the spectral density function. If the dynamic system does not produce a random process, but the typical time constants of the system generate complex physical information (output includes, for example. a periodic function, like an oscillation), the power spectral density shows the dominance of typical frequencies or, more generally, the spectral density as a function of the frequency is not a constant but is a function with peaks at certain frequencies (see Figs.2 and 3).The autocorrelation function and the spectral density are related (for continuous signals) by a Fourier transform

$$R_{xx}(t) = \int_{-\infty}^{+\infty} S_{xx}(f)\exp(i2\pi ft)df \qquad S_{xx}(f) = \int_{-\infty}^{+\infty} R_{xx}(t)\exp(-i2\pi ft)dt \qquad (2)$$

where $R_{xx}$ is the autocorrelation function (ACF), $S_{xx}$ is the power-spectral density, f is the frequency and $t$ the time delay or displacement, a typical correlation time unit.

In practice, we have to work with discrete signals. If we wish to obtain power spectral density (PSD) in a time interval $0 \le t \le T_0$, we segment this interval into subintervals (which can overlap) (Behringer, 1988) (ks, ks+T), k=0,…,$N_s$-1 where T is segment length (length of the subinterval) and s is the shift which each successive segment undergoes. The direct estimate of the PSD is given by $S_{xx}(f) = \frac{1}{N_s T}\sum_{p=1}^{N_s} |X_p(f)|^2$ where $X_p$(f) is the discrete Fourier transform of the signal record {x(t)}={x(n,t)}, n=0,1,…,N-1.

The idea of the methodology developed by K. Behringer (Behringer and Hennig, 2001) is based on the application of the PSD and the ACF for the estimation of the DR and the NF. In the framework of this approach, a modified ACF was chosen for determining the NF and the DR. The classical autocorrelation function method is based on the linear second-order damped oscillator model. However, this approach failed in many cases because of a noise background problem of the measured time series. The signal data must be band-pass filtered to separate the frequencies around the interesting peak in the PSD (in general, around 0.4…0.6 Hz). Furthermore, the background under the peak in the PSD must be taken into account. Experimental PSD's show that the assumption of an exponentially decaying background under the peak is a good approximation in the frequency domain. Signal filtering and the inclusion of this background term require a correction of the model ACF. The corrected model ACF is then least-squares fitted to the ACF estimated by the previously filtered signal data (we use a 3-parametric fit, in table 2 indicated as IBGR=0, or a 5-parametric fit, IBGR=2, dependent on whether or not the background is taken into account, see (Behringer and Hennig, 2001)). The fitting procedure is relatively complicate and contains a variable lag-time range parameter. Special techniques have been applied to the filtering procedure and the ACF estimation. They are both based on signal segmentation and the use of the Fast Fourier Transform (FFT). An FFT filter has

[1] For ergodic random processes, the time averaged mean value and autocorrelation function are equal to the corresponding ensemble averaged values.

been developed which approximates the shape of an ideal rectangular band-pass filter. The ACF estimation is based on the indirect method with zero-padding and FFT techniques (Bendat and Piersol, 1971). This method is especially advantageous if one is interested only in the initial part of the ACF, which is given by the segment length $N_s$ (number of signal points in a segment) and the sampling frequency $f_s$. The data used were $N_s$=256 and $f_s$=12.5 Hz, which give a lag-time range of about 20 sec. A modification of this method allows an ACF estimation to be made over an arbitrary number of segments $N_{av}$. If $N_{av}$ is a small number (about 5), then "short-time" ACF's are obtained. Furthermore, a gliding segment analysis was introduced. The first ACF estimate is obtained from the (filtered) signal data in the first succeeding $N_{av}$ segments. Each following ACF estimate is based on taking the signal data in the next following segment and keeping the data in the $N_{av}$-1 previous segments. One obtains a set of (strongly correlated) ACF's which move over the signal length. The signal contains $N_{seg}$ segments, the number $N_{acf}$ of ACF estimates is given by $N_{acf}=N_{seg}-N_{av}+1$. The fit code is then repetitively applied to each ACF estimate and gives $N_{acf}$ estimated values for the searched oscillation frequency ($?_c$) and the Decay Ratio (DR). If $N_{acf}$ is sufficiently large (about 50), one can recognize slowly-varying non-stationary signal conditions from plots of the values of $?_c$ and DR. Under stationary conditions, average values of $?_c$ and DR and their standard deviations can be calculated. Hence, the gliding segment analysis allows an uncertainty estimation.

For fitting, two different system transfer functions (TF) are chosen. The difference between these two transfer functions is that their (random) driving force is different. In the first case, a white noise process has been used, and in the other a colored noise (for details see (Behringer and Hennig, 2001)). In other words, the difference is that the first TF corresponds to an AR(2) model, while the second corresponds to an APMA(2,1) model (for definitions of AR and ARMA models, see next section). The DR is calculated from the ACF $R_{xx}(t) = C_0 e^{-l|t|}\left[\cos(w_c t) + \frac{l}{w_c} sin(w_c|t|)\right]$ as $DR = \exp(-2p\frac{l}{w_c})$, where $w_c^2 = w_0^2 - l^2$, $w_c$ is the oscillation frequency, $w_0$ the undamped resonance frequency, and ? is a damping factor. The estimation of the DR should be made in such a way that we obtain information on the calculation uncertainty. In Fig.1 we show the results of an example (case c4_lprm6, see Table 1 at the end of this paper).

## 2.2 Parametric methods (Ljung, 1994)

A dynamic system may be described by a system of partial or ordinary differential equations. In the framework of the signal analysis, we have to construct a dynamic system from the analysis of the discontinuous time series {y(kT),y((k-1)T),…,y((k-n)T)}, which contains nth data time history, where T is sampling time (Hennig, 1997), (Askari, Hennig, 2000).

In the discrete time case, the dynamic system is described by a difference equation or a system of difference equations (a so-called digital filter).

$$A(q)\,y(kT) = B(q)u(kT) + C(q)e(kT)) \qquad (3)$$

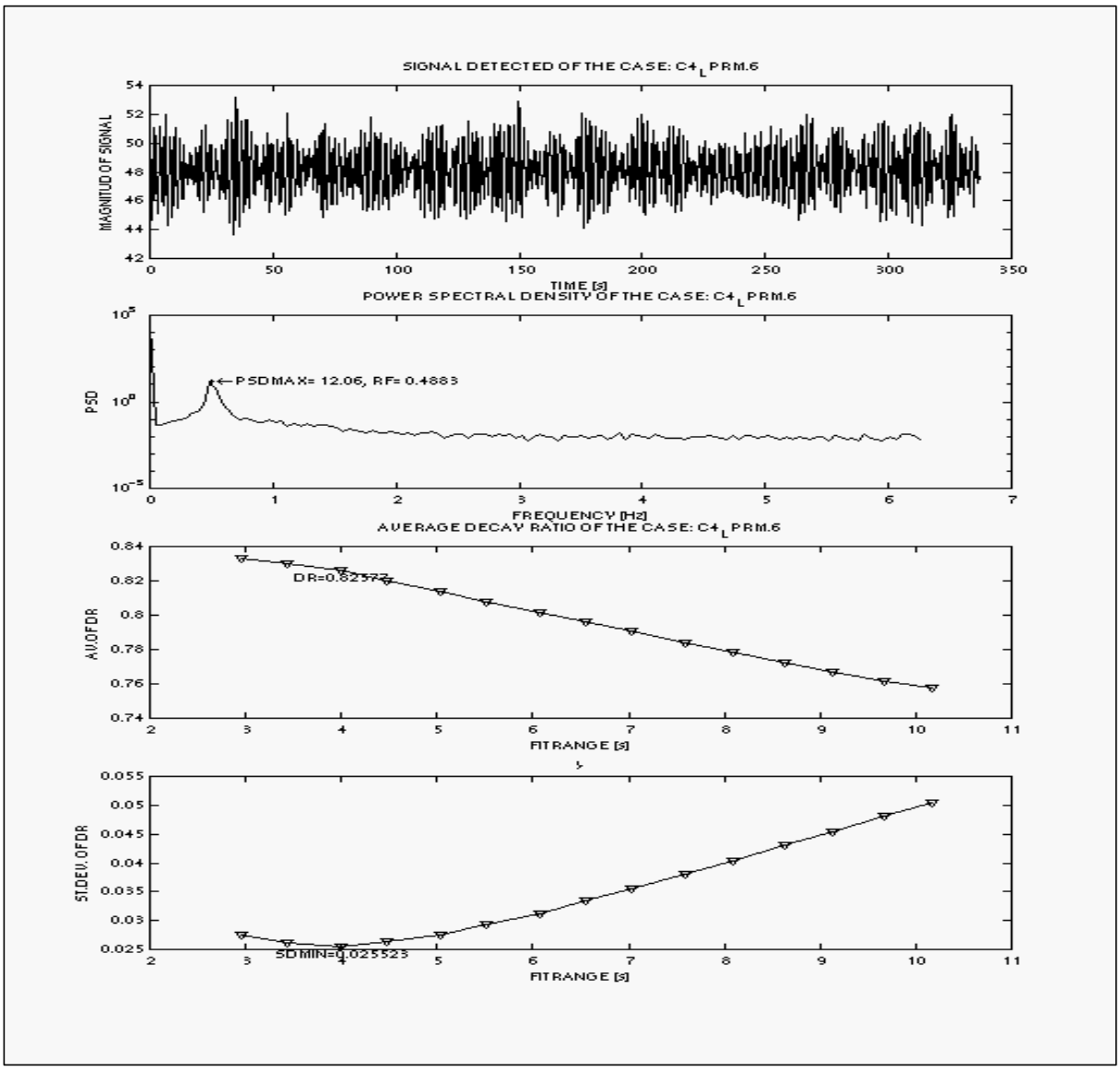


**Figure 1: Non-parametric method (c4_lprm6, 3-parameter fit). The first subplot shows the time series, the second the PSD, the third shows the average DR vs the fit range and the forth the std vs fit range (the fit range was running from 15% to 50% of the full ACF lag-time range of 20s, in steps of 2.5%, see p.4). To find the optimal fit range, we use the empirical finding that the std of the DR moves through a minimum as a function of the fit range. Here we used this criterion for selecting th e best fit and define this std as the uncertainty of the DR.**

In equation (3), q is the time-shift operator, u(kT) is system input, e(kT) a pure stochastic noise process, y(kT) is system out put, and the other coefficients (characteristic system constants) are given by

$$A(q)=\sum_{m=0}^{n_a} a_m q^{-m}\ ;\ \ B(q)=\sum_{m=0}^{n_b} b_m q^{-m};\quad C(q)=\sum_{m=0}^{n_b} c_m q^{-m} \qquad \text{where } q^{-1}y(kT)=y((k-1)*t) \tag{4}$$

where $a_0$,$b_0$ and $c_0$ are equal to zero.

***Parametric model classification***

A dynamic system represented by equation (3), may be used for the classification of a parametric model.

Auto Regressive process (AR): B(q)=0, C(q)=1

Moving Average process (MA): A(q)=1, B(q)=0

Auto Regressive Moving Average (ARMA): B(q)=0

Auto Regressive Moving Average with an eXtra input (ARMAX):

$A(q) \neq 0$, $B(q) \neq 0$, $C(q) \neq 0$

It is important to note that we assumed that a stochastic process generates the time series. This assumption implies that the AR process is produced by white noise e(kT) and the ARMA by colored noise.

***Transfer Function (TF)***

A linear time-invariant system is characterized by a linear input / output relationship, the TF. Because our time series is discontinues, it is useful to use the z-transform technique (Ljung, 1994), (Hennig, 1997) instead of a Fourier or Laplace transform. The z-transform for such data y(kT) is given by

$$Z[y(k)] = Y(z) = \sum_{n=0}^{\infty} y(n) z^{-n} . \qquad (5)$$

Using a z-transform (symbol Z), the system transfer function for an ARMA model is

$\boldsymbol{Z\left[A(q)y(k)\right] = Z\left[C(q)e(k)\right]}$, where e(kT)=e(k) is a stochastic white noise process with mean value m[e(k)]=0 and variance V[e(k)]=$s^2$.

$$\boldsymbol{Y(z)\left(1 + a_1 z^{-1} + .... + a_{n_a} z^{-n_a}\right) = E(z)\left(1 + c_1 z^{-1} + ..... + c_{n_c} z^{-n_c}\right)} \qquad (6)$$

$$\boldsymbol{H(z) = \frac{Y(z)}{E(z)} = \frac{\left(1 + c_1 z^{-1} + ..... + c_{n_c} z^{-n_c}\right)}{\left(1 + a_1 z^{-1} + .... + a_{n_a} z^{-n_a}\right)} = \frac{A \prod_{l=1}^{n_{nc}} (1 - e_l z^{-1})}{\prod_{m=1}^{n_{na}} (1 - m_m z^{-1})}} . \qquad (7)$$

The denominator in equation (7) contributes the poles at z= $m_m$ and the numerator provides the zeros $e_l$ of H(z). If the dynamic system is causal (y(kT)=0 for k<0) and stable, then all poles of the TF lie inside a unit circle (Ljung, 1994), and the DR and the natural (reactor resonance) frequency NF is to be calculated from the complex poles of the TF (Hennig, 1997).

***Model order estimation***

The process of fitting an ARMA (or AR) model to a measured (or predicted) time series involves two separate tasks: (1) the estimation of the model parameters $\{a_i\}$ and $\{c_i\}$; and (2) the determination of the model order $n_a$ and $n_c$.

In contrast to the simple AR model estimation, the parameter estimation for ARMA models is a complex task (Ljung, 1994.). The principle of the ARMA parameter estimation is the minimization of the prediction errors or statistical residuals. For an ARMA process, there is a nonlinear relationship between the prediction error e and y(k) [$e(k) = C(q)^{-1} A(q) y(k)$ ]. Therefore, we need a numerical recursive method to estimate the coefficient vector T=($a_1$, $a_2$,….,$a_k$, $c_1$, $c_2$, ….,$c_k$) (Ljung,1994.),(Soderstrom and Stoica, 1989), (MATLAB, 1992, 1993).

The model order estimation is the crucial point of the system identification methodology. The model order can be determined, for instance, on the basis of the so-called Akaike Information Criterion (AIC). This in turn is based on the minimizing of the prediction error, which is proportional to the sum of the quadratic deviations of the original time series and the predicted one. Therefore, the model order depends on the noise level (increases with increase in the noise level). A too-high model order (overparametrized model) is recognizable, for example, in the pole-zero map of the TF. The different confidential intervals of the poles overlap in such cases.

First, we can say that the expected prediction error variance (EPEV) $E[W_N]=E[E[e^2(k,\theta)]]$ increases with p/N (p=dim T and T is the ARMA coefficient vector). Hence, an over-parametrized model has a large expectation value for the PE variance (PEV) (Ljung, 1994), (Soderstrom and Stoica, 1989):

$$W_N = E\left[e^2(k,q_N)\right] \approx E\left[e(k,q_0) + \left\langle \frac{\partial e(k,q)}{\partial q} \right\rangle_{q=q_0} (q_N - q_0)\right] \tag{8}$$

For $\theta_N=\theta_0$ , the $W_N$ is the minimum variance $\sigma^2$. A Taylor series expansion around $\theta_0$ shows how much the PEV is increased due to the deviation of the $\theta_N$ from the true value $\theta_0$. This means that

$$W_N \approx s^2 + (q_N - q_0)^T E\left[y(k,q_0) y(k,q_0)^T\right](q_N - q_0) \tag{9}$$

and, due to the statistical character of the $\theta_N$ values and the fact that the model parameters are estimated approximately, we consider $\left( \frac{\partial e(k,q)}{\partial q}\Big|_{q=q_0} = y(k,q_0) \right)$. Thus EPEV is given by:

$$E[W_N] \approx s^2 + tr\left\{E\left[(q_N - q_0)(q_N - q_0)^T\right] E\left[y(k,q_0) y(k,q_0)^T\right]\right\} \tag{10}$$

It is possible to demonstrate (Ljung, 1994) that, for a Gaussian distributed $\theta_N$, the second term can be approximated to $s^2 \frac{p}{N}$

$$E[W_N] \approx s^2 + s^2 \frac{p}{N} \tag{11}$$

Equation (11) shows that the EPEV increases for an over-parametrized model (Ljung, 1994), (Sakomoto, Ishigoru and Ktagawa, 1986). However, we can also write PEV as follows

$$W_N = V_N\left(\hat{q}_N\right)\left[1 + b(N,p)\right] \tag{12}$$

Alternatively, we can also use the expression (where we use the log V function as is usual in statistics)

$$W_N = N \log V_N\left(\hat{q}_N\right) + g(N,p) \tag{13}$$

Equations (12) and (13) are two different expressions of the PEV, the first is the linear least-squares fit, and the second the least-squares fit of the logarithm of the function. It is not difficult to see that equations (12) and (13) are asymptotically equivalent, provided $\gamma(N,p)=N\beta(N,p)$. Indeed, for "large N"

$$\log\left\{V_N\left(\hat{q}_N\right)[1+b(N,p)]\right\} = \log V_N\left(\hat{q}_N\right) + \log[1+b(N,p)] \approx \frac{1}{N}\left[N \log V_N\left(\hat{q}_N\right) + Nb(N,p)\right] \tag{14}$$

However, there are many methods of penalizing models with too-high model order. The choice $\gamma(N,p)=2p$ is equivalent to the use of the Minimum Akaike's Information Criterion (MAIC)

$$MAIC = N \log V_N\left(\hat{q}_N\right) + 2p\,. \tag{15}$$

Another powerful criterion is the Ressanen's Minimum Description Length (RMDL), which is given by (Ljung, 1994) and (Sakomoto, Ishigoru and Ktagawa, 1986):

$$RMDL = V_N\left(\hat{q}_N\right) + \log N \frac{p}{N} \tag{16}$$

The minimized PEV is used to choose the best $q_N^0$ among the chosen $q_N$ (different coefficient vectors). MATLAB delivers the function "selstruc.m" for minimization of PEV with different methods for the AR process. An extended procedure for an ARMA process has been incorporated into the HPTSAC2000.m.

When applying the optimization algorithms, the problem revealed is that the algorithm can find different minima (the "minor" or local minima exist additionally alongside the main or global minima). This is typical for some optimization algorithms and, therefore, we have to vary the initially chosen model order, in particular by application of the ARMA model order optimization.

In Fig.3 and 4, we show this methodology for the same case as in the non-parametric section (c4_lprm6). Fig.3 shows the PSD's estimated for the origin and the optimized AR model time series, and Fig.4 shows the ACF of the residuals as a measure of the model order quality (see legend of the Figure).

***Plateau method*** ***(Askari, Hennig 2000)***

In this model, developed by Askari, we assume that the value of the DR (which is calculated with the ARMA model) is considered to be a real number. Subsequently, a set of DRs, can be considered as a sequence of the real numbers ($\{DR_n\}$, $n \in N$, $DR_n \in \boldsymbol{R}$) for which the following limits are valid

$$\lim_{n\to\infty} DR_n = DR^* \quad \text{or} \tag{17a}$$

$$\lim_{e\to 0} DR_n = DR^* \tag{17b}$$

This means that the sequence of the $\{DR_n\}$ converges to a finite number $DR^*$. The limit of the sequence $\{DR_n\}$ exists if, for each $\varepsilon$ (a positive real number), we can find an n* such that, for any n greater than n*, the distance between $DR_n$ and DR* is less than the fixed value

$$\forall \varepsilon>0,\ \varepsilon\in \boldsymbol{R}\ \ \exists\, n^*\in \boldsymbol{N}\quad \left|\forall\, n>n^* \Rightarrow \left|DR_n-DR^*\right|<\varepsilon \right. \tag{18}$$

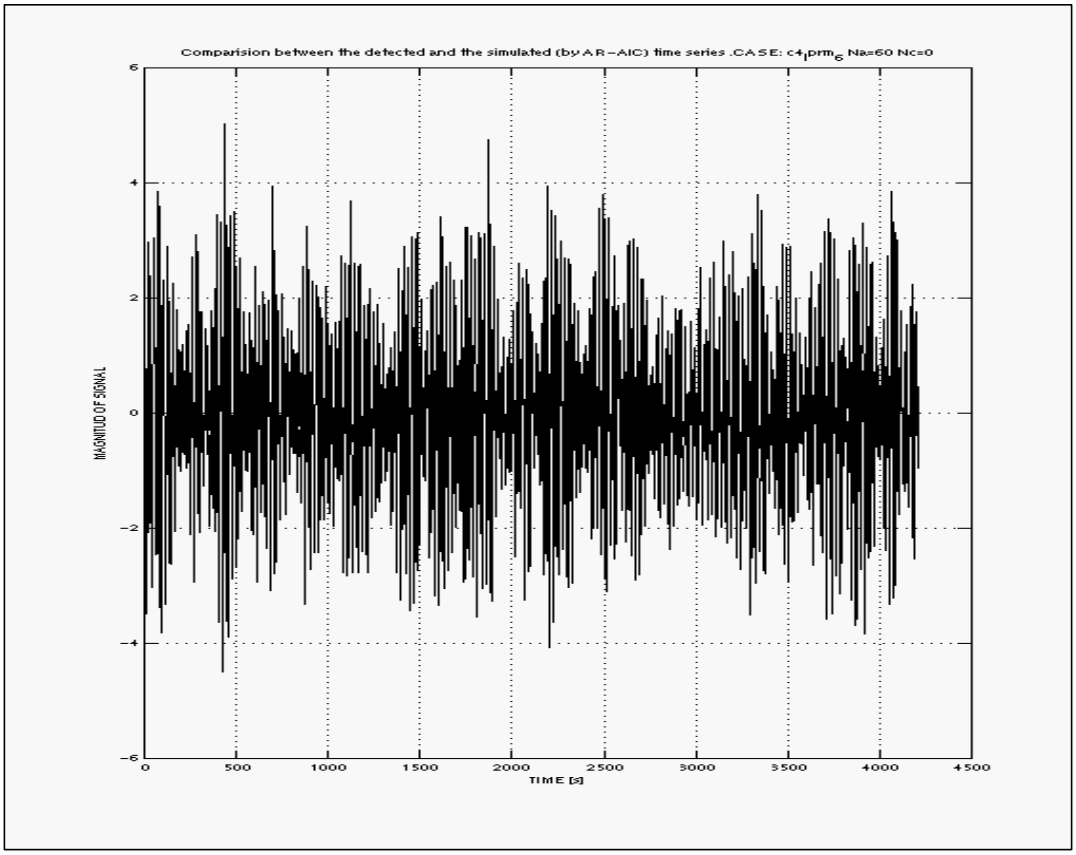


**Figure 2: Parametric analysis. Detected time series (in this Figure the time series simulated by an Akaike optimised AR model is also drawn, but this curve is not resolved on this scale).**

In our model, we are not able to calculate the limit for n (the model order), which tends to infinity, or $\varepsilon$, which tends to zero. This is due to the fact that a high model order introduces a mathematical error in the calculation of DR, and a too-small or zero $\varepsilon$ means that there is no noise and disturbance in our time series. For these reasons, we fixed the number of the model order for which the variation of the DR remains less than the fixed value of the fixed error ($\varepsilon$), and we call this flat model-order region a **plateau**.

We realize our plateau using two definitions of error:

$$e_2^{(n)}=\frac{\left|DR_n-DR_{n-1}\right|}{\max\left(DR_n,DR_{n-1}\right)} \tag{19}$$

$$e^{(n)}=\frac{\left|e_2^{(n)}-e_2^{(n-1)}\right|}{\max\left(e_2^{(n)},e_2^{(n-1)}\right)} \tag{20}$$

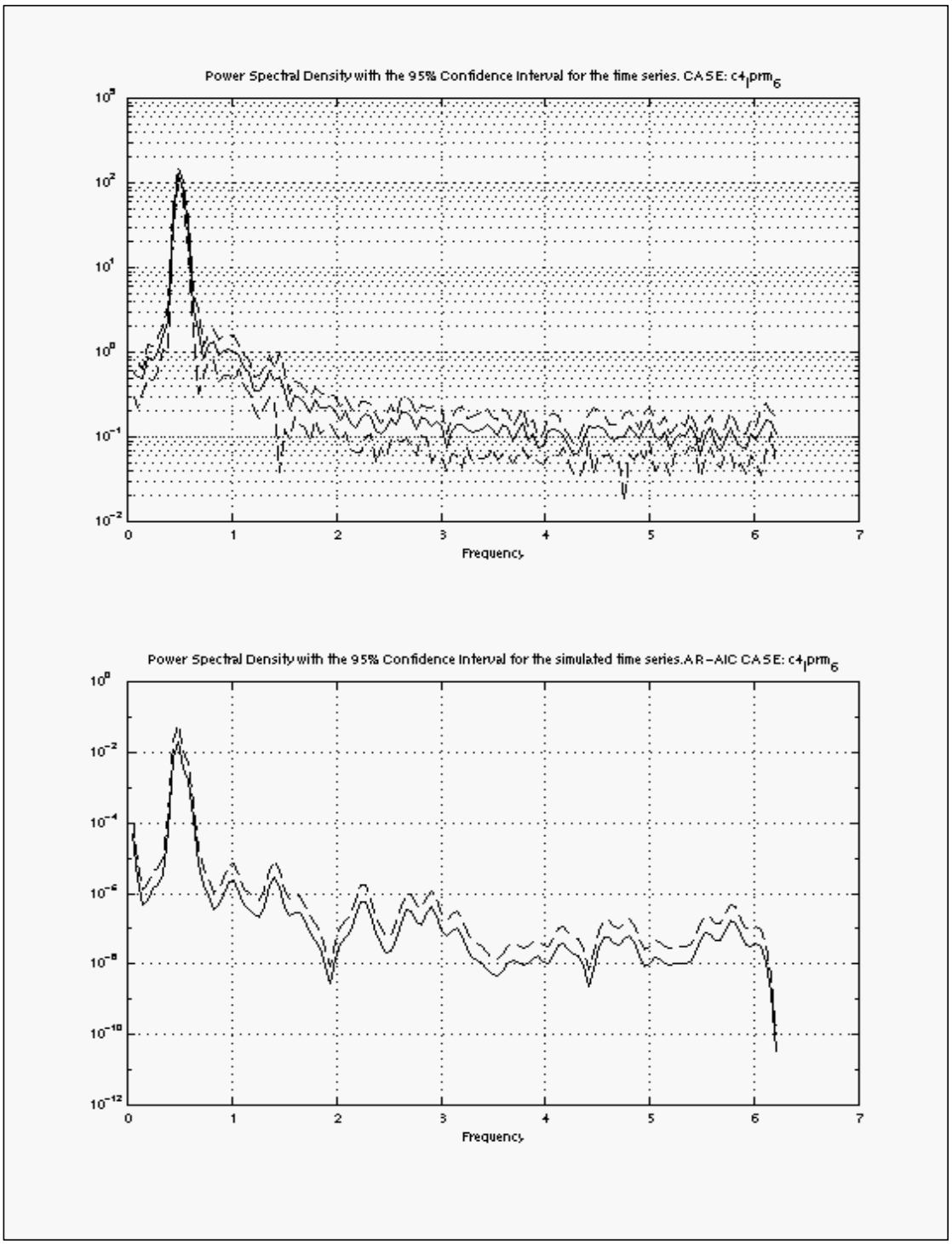


**Figure 3: Parametric analysis. In the upper part is shown the PSD of the detected time series and in the lower part the PSD estimated from the time series simulated by the Akaike optimised AR model.**

The $e_2^n$ calculates the local normalized variation $DR_n$ with respect to $DR_{n-1}$, and $\varepsilon^{(n)}$ relates the variation of three consecutive DR's ($DR_n$,$DR_{n-1}$,$DR_{n-2}$). Once EPSILON, EPSILON2 (two constants) and LPLATEAU (length of the plateau) are fixed, the program looks for the model orders which satisfy the following conditions:

$e^{(n)} \leq$ ***EPSILPON*** (code input)

$e_2^{(n)}$ £ ***EPSILON2*** (code input)

***K*** £ ***LPLATEAU*** (code input)

If the model order n satisfies the condition, the program increases the value of the counter K. The search will continue until K=LPLATEAU. If the conditions are not satisfied for n, the program considers the error equal to the maximum value and increases n.

It is important to note that the existence of the plateau depends strongly on these conditions. The HPTSAC code prints, at the end of a run, the DR values for all possible combination of the $n_a$ and $n_c$ (ARMA characteristic model orders), and the evaluation of the model order will be made at the beginning and end plateau points.

# 3. Results and uncertainties

In Table 2 (see last page of this paper), we summarize typical time series analysis results provided by the 8 methods available at PSI. The chosen time series data are selected from the NEA benchmark data collection (Verdu, Ginestar et al., 1999). In the framework of this analysis we determine the mean values and the standard deviation (std) calculated by the 8 methods, as the systematic uncertainty of the DR/NF estimation (independent of the statistical uncertainty of the individual methods). Further, we have focused our attention mainly on operational points with DR's larger than 0.5 (only a few cases with small DR's) because the uncertainty for small DR's is not significant.

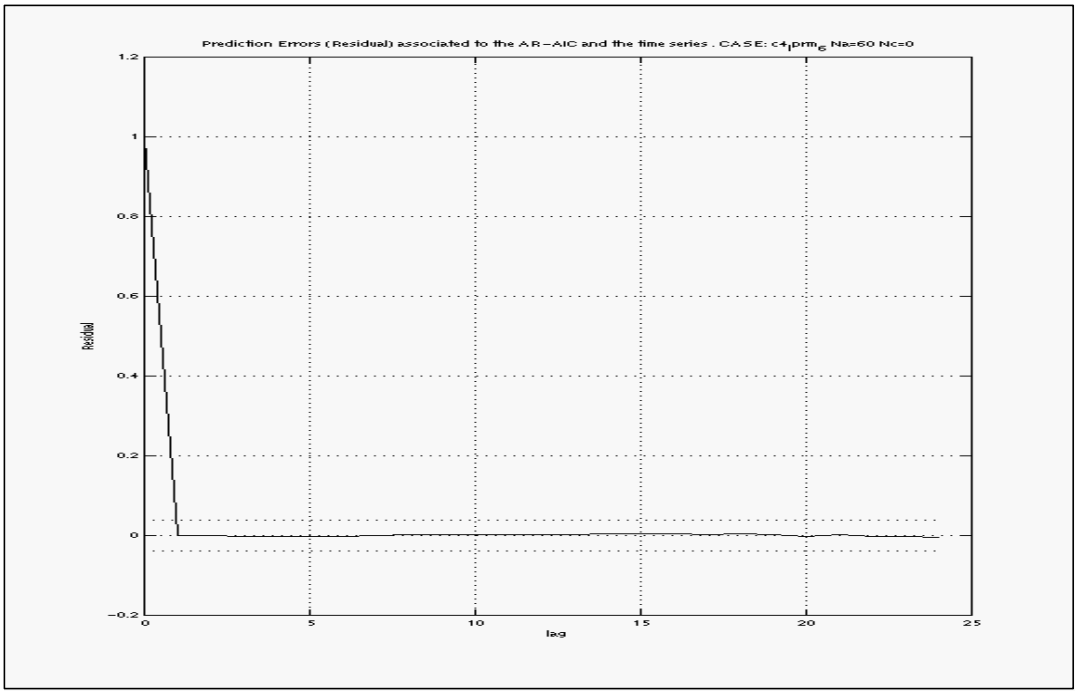


**Figure 4: Parametric analysis. This Figure shows the ACF of the prediction error. The prediction error ACF should be a DELTA-function because the prediction error is uncorrelated.**

If we compare the systematic uncertainty with the statistical uncertainty of the parametric and the plateau methods, we find a correlation. The statistical standard deviation and the standard deviation associated with the systematic uncertainty in many cases are very similar. We shall demonstrate this by some examples (Table1).

**Table 1 Comparison of standard deviations**

| Case | std (3-par. fit) | std (5-param. fit) | std plateau | std systematic |
|---|---|---|---|---|
| C1_aprm1 | 0.02 | 0.06 | 0.07 | 0.06 |
| C1_aprm3 | 0.09 | 0.07 | 0.10 | 0.08 |
| C1_aprm7 | 0.03 | 0.04 | 0.08 | 0.10 |
| C1_aprm14 | 0.08 | 0.03 | 0.09 | 0.05 |
| C4_aprm | 0.03 | 0.02 | 0.04 | 0.1 |
| C4_lprm1 | 0.04 | 0.02 | 0.02 | 0.02 |
| C4_lprm4 | 0.01 | 0.01 | 0.04 | 0.04 |
| C4_lprm5 | 0.03 | 0.02 | 0.04 | 0.02 |
| C4_lprm6 | 0.03 | 0.02 | 0.04 | 0.01 |
| C4_lprm7 | 0.03 | 0.02 | 0.04 | 0.02 |
| C4_lprm8 | 0.04 | 0.03 | 0.05 | 0.02 |
| C4_lprm9 | 0.09 | 0.03 | 0.09 | 0.06 |
| C4_lprm10 | 0.03 | 0.03 | 0.04 | 0.03 |
| C4_lprm11 | 0.04 | 0.01 | 0.04 | 0.03 |
| C4_lprm12 | 0.06 | 0.05 | 0.05 | 0.08 |

The values in Table 1 are not representative in the statistical sense, but it seems that the systematic standard deviation (std) is (for the chosen cases), in general, not much larger than the individual statistical std's. Hence, we can conclude that a DR (or NF) estimated on the basis of one of the methods described above is reliable enough. If we estimate the values with all different methods, and we find a systematic sdt similar to the statistical std, then we can be sure that we are providing reliable stability indicators extracted from the time series.

# 4. Summary and conclusions

Eight different methods for time series analysis used for the DR (NF) estimation are discussed.

A novel non-parametric method was developed, based on time-signal filtering around the eigenfrequency of the reactor and fitting the ACF of the filtered signal by a modified ACF of a second order system. Dependent on whether the PSD background noise is taken into account or not, 3- or 5-parametric-fit procedures are used. A modification of this method allows an ACF estimation over an arbitrary number of time segments, and a gliding segment analysis has been used for estimation of the std of the DR estimation in all segments. The min(std) (minimum std, see Fig.1, referred to there the SDMIN) is defined as the DR uncertainty of this approach.

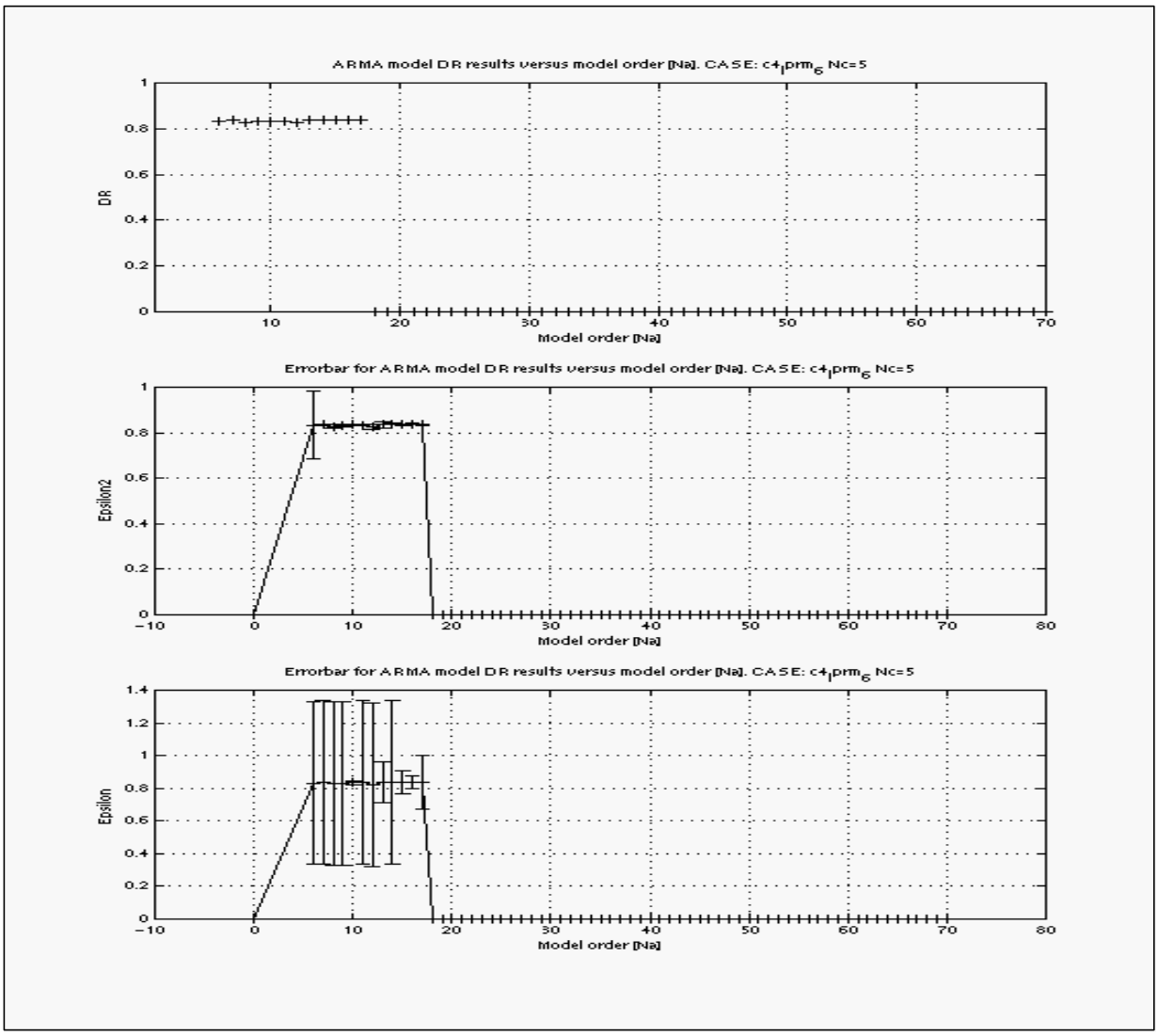


**Figure 5: Parametric methods. Plateau method based on Askari's optimization approach (see text).**

In the framework of parametric methods, two model order optimization algorithms are used. We should note that these algorithms can fail because of the existence of more than one extreme values (minor minima). Hence, we have to be careful to select (and vary) the starting value of the model order estimation.

A comparison of the DR's (NF') estimated by 8 different methods shows (for the chosen cases) that the DR calculation from the detected time series on the basis of non-parametric and parametric methods is relatively reliable for DR larger than 0.7 ($\Delta DR = \pm 0.05$). This statement differs from the statement in (Verdu, Ginestar et al., 1999).

We believe that a comprehensive assessment of the uncertainty of the linear stability characteristics DR (NF) is still pending. However, we have demonstrated that a sufficiently reliable DR estimation can be realized if we fix the DR on the basis of comparison between predictions obtained using different methods.

## Nomenclature (the most important abbreviations and symbols)

ACF Autocorrelation function
AIC Akaike information criterion
RMDL Ressanen's optimization criterion
AR Auto regressive
ARMA Auto regressive moving average
PSD Power spectral density
DR Decay ratio
NF Natural frequency
H Transfer function
Z z-Transform
E Expection value
EPEV Expected prediction error variance
PE Prediction error
$W_N$ Prediction error variance
$\Theta$ ARMA coefficient vector
epsilon, epsilon2, plateau :input values for the HPTSAC2000 code
q Time shift operator
p dim ($\Theta$), dim =dimension
e(kT) White noise process
y(kT) Random process (time series)

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**Table 2 :Decay Ratios/Natural Frequencies calculated by eight different methods**

| Estimated Decay Ratio's (DR) and Natural Frequencies (NF, Hz) DR/NF (model order) | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Non-parametric methods | | Parametric methods | | | | | | Mean | Std |
| Case | 3-parametr. fit IBGR=0 | 5-parametr. fit IBGR=2 | AR (AIC opt) | AR (RES opt) | ARMA (AIC opt) | ARMA (RES opt.) | ARMA (auto plat. | ARMA(plat. sear.) | | |
| c1_aprm1 | 0.42/0.39 | 0.61/0.39 | 0.37/0.47 (15) | 0.37/0.47 (15) | 0.42/0.46 (15,3) | 0.47/0.46 (12,11) | 0.50/0.47 (34,3) | 0.50/0.47 (34,2) | 0.45/0.45 | 0.06/0.04 |
| c1_aprm3 | 0.43/0.49 | 0.52/0.48 | 0.48/0.49 (30) | 0.48/0.48 (30) | 0.42/0.48 (10,5) | 0.42/0.48 (10,5) | 0.66/0.49 (18/11) | 0.50/0.49 (35,2) | 0.49/0.49 | 0.08/0.01 |
| c1_aprm4 | 0.59/0.49 | 0.52/0.49 | 0.52/0.48 (40) | 0.52/0.48 (16) | 0.40/0.48 (22,3) | 0.57/0.50 (10,3) | 0.40/0.48 (22,3) | 0.53/0.49 | 0.51/0.49 | 0.07/0.01 |
| c1_aprm7 | 0.60/0.54 | 0.89/0.54 | 0.65/0.52 (30) | 0.65/0.52 (10) | 0.62/0.54 (6,3) | 0.62/0.54 (6,3) | 0.62/0.54 (6,3) | 0.59/0.52 | 0.66/0.53 | 0.1/0.01 |
| c1_aprm14 | 0.63/0.49 | 0.82/0.49 | 0.68/0.49 (30) | 0.68/0.49 (11) | 0.55/0.48 (5,3) | 0.55/0.48 (5,3) | 0.67/0.49 (27,3) | 0.65/0.48 | 0.65/0.49 | 0.050.01 |
| | | | | | | | | | | |
| c4_aprm | 0.81/0.49 | 0.76/0.49 | 0.79/0.49 (43) | 0.79/0.49 (10) | 0.98/0.48 (18,8) | 0.98/0.48 (16,8) | 0.98/0.48 (29,8) | 0.85/0.50 (15,2) | 0.87/0.49 | 0.1/0.01 |
| c4_lprm.1 | 0.87/0.49 | 0.88/0.49 | 0.91/0.48 (62) | 0.91/0.48 (17) | 0.90/0.49 (9,5) | 0.90/0.49 (6,4) | 0.90/0.49 (14,5) | 0.90/0.49 (16,2) | 0.90/0.49 | 0.02/0.01 |
| c4_lprm.2 | 0.92/0.48 | 0.89/0.49 | 0.93/0.48 (63) | 0.93/0.48 (17) | 0.91/0.49 (16,8) | 0.92/0.49 (9,8) | 0.91/0.49 (20,8) | 0.90/0.49 (12,2) | 0.91/0.49 | 0.01/0.01 |
| c4_lprm.3 | 0.94/0.49 | 0.93/0.49 | 0.95/0.48 (69) | 0.95/0.48 (15) | 0.90/0.49 (14,8) | 0.89/0.49 (11,8) | 0.90/0.49 (14,8) | 0.91/0.49 (10,2) | 0.92/0.49 | 0.02/0.01 |
| c4_lprm.4 | 0.91/0.48 | 0.91/0.49 | 0.94/0.48 (70) | 0.94/0.48 (21) | 0.93/0.48 (10,7) | 0.83/0.49 (7,5) | 0.91/0.48 (16,7) | 0.88/0.49 (20,2) | 0.91/0.48 | 0.04/0.01 |
| c4_lprm.5 | 0.83/0.49 | 0.81/0.49 | 0.86/0.48 (63) | 0.86/0.48 (15) | 0.84/0.50 (9,7) | 0.84/0.49 (7,5) | 0.86/0.48 (20,5) | 0.85/0.50 (16,2) | 0.84/0.49 | 0.02/0.01 |
| c4_lprm.6 | 0.83/0.49 | 0.80/0.49 | 0.84/0.48 (15) | 0.84/0.48 (15) | 0.83/0.50 (8,7) | 0.83/0.50 (8,7) | 0.84/0.50 (12,7) | 0.84/0.50 (14,2) | 0.83/0.49 | 0.01/0.01 |
| c4_lprm.7 | 0.79/0.49 | 0.81/0.49 | 0.77/0.50 (13) | 0.77/0.50 (12) | 0.80/0.52 (8,5) | 0.81/0.50 (6,5) | 0.78/0.50 (11,7) | 0.79/0.50 (10.2) | 0.79/0.50 | 0.02/0.01 |
| c4_lprm.8 | 0.71/0.49 | 0.74/0.49 | 0.70/0.51 (21) | 0.70/0.51 (10) | 0.75/0.50 (12,5) | 0.74/0.52 (7,5) | 0.71/0.51 (23,5) | 0.76/0.51 (18,2) | 0.73/0.51 | 0.02/0.01 |
| c4_lprm.9 | 0.71/0.49 | 0.63/0.49 | 0.77/0.49 (58) | 0.77/0.49 (14) | 0.80/0.51 (8,5) | 0.80/0.51 (8,5) | 0.81/0.49 (15,5) | 0.80/0.51 (18,2) | 0.76/0.53 | 0.06/0.01 |
| c4_lprm.10 | 0.73/0.54 | 0.69/0.54 | 0.74/0.53 (25) | 0.74/0.53 (18) | 0.77/0.52 (7,5) | 0.77,/0.52 (7,5) | 0.78/0.53 (12,5) | 0.75/0.53 (18,2) | 0.75/0.53 | 0.03/0.01 |
| c4_lprm.11 | 0.68/0.54 | 0.74/0.54 | 0.69/0.52 (15) | 0.69/0.52 (15) | 0.72/0.53 (8,5) | 0.72/0.53 (9,7) | 0.77/0.52 (15,7) | 0.77/0.53 (18,2) | 0.71/0.53 | 0.03/0.01 |
| c4_lprm.12 | 0.58/0.54 | 0.82/0.54 | 0.65/0.53 (33) | 0.65/0.53 (10) | 0.70/0.55 (11,7) | 0.75/0.54 (9,7) | 0.77/0.54 (12,7) | 0.67/0.54 (32,2) | 0.70/0.54 | 0.08/0.01 |