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TECHNIQUE OF UNCERTAINTIES ESTIMATION IN MODELING OF SLOW TRANSIENT PROCESSES IN NUCLEAR REACTORS.

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ABSTRACT

Presentation is devoted to problem of accuracy in prediction of nuclear system's characteristics during lifetime performance. Here they are analyzed sources of uncertainties including influence of quality of production of nuclear reactor components (accuracy of knowing of real sizes and densities of fuel elements, control rods etc) and influence of dynamical changes of parameters such as uncertainties in density of boiled coolant and oscillation of fuel lattice simultaneously with control bodies. It's considered one way based on perturbation theory for analysis of sensitivity of resulting vector of nuclide's concentration and criticality parameters (such as bias of reactivity and control bodies positions). For example in presentation they are considered two different models of nuclear reactor, one of them is related to concepts with "breeding of reactivity" and the second – to units for long autonomic operation without human control.

INTRODUCTION

The overall world trend in research in the nuclear engineering field is to be seen as an increase in the proportion of detailed-theoretical modeling of the characteristics of nuclear systems. In order to satisfy the increasing requirements for accuracy in the theoretical prediction of the characteristics of nuclear reactors and nuclear systems of different types, mathematical methods, algorithms and programs are being developed which make it possible to model comprehensively a particular system as a complex object, to take into account the effect of uncertainty of the initial data and approximations of theoretical models, and to perform a detailed analysis of various physical processes with allowance for their actual interaction, technical conditions and limitations.

The packages of programs created to solve a problem in this field are universal at the algorithm level and their structure enables them to be adjusted to the type of system in question through selection of corresponding program units modeling a specific physical process (preparation of initial data, calculation of fields of radiation, temperatures, nuclides, etc.). One of the problems of organizing a complex calculation is that of coordinating the information flows (initial data for some modules and results of calculations for others). Here it is important to evaluate the quality of the results obtained by particular program modules from the point of view of using

them in joint modeling. This calls for estimation of the error introduced by the initial data in the program modules involved in the complex calculation and estimating the accuracy requirements for such modules.

New conceptions of the application of nuclear engineering including different ways of managing radioactive waste, transmutation of long-lived radioactive fission products and minor actinides call for prediction of the long-term consequences with different scenarios for the development of power engineering. Systems analysis in this area involves estimating the effect of uncertainty of nuclear data and the models employed on the results of calculating the status of the whole system. This is particularly relevant for estimating the prospects of using accelerator-controlled nuclear systems for eliminating long-lived toxic waste from nuclear power plants (*E.O. Adamov, 1999.*). For this it is necessary to determine the requirements with respect to initial nuclear data and theoretical models on the basis of the systems analysis requirements.

This paper is devoted to developing a mathematical model, methods, algorithm and programs for calculating the evolution of fields of nuclide concentrations and estimating the sensitivity of the result to deviations of the nuclear data, including those bearing a random character.

The evolution of the field of nuclide concentrations in materials of the system in question is described by a dynamic model in lumped parameters. The coefficients of the matrix of the nuclide transmutations are determined from calculation of the steady-state neutron and photon fields and are normalized taking into account the complete capacity of the plant. Here it is assumed that variations in the parameters of the system are small and that a small enough time interval can be selected for the spectral characteristics within it to be considered constant. The stability of the solution of the system of equations is achieved through the use of an analytical algorithm.

Problem of breeding of reactivity (minimization of fall of them during campaign) is connected to search of the most effective ways of use of a nuclear energy. It must be noted that to increase the level of safety (of defense) of any nuclear system it is needed to minimize the requirements to control (active control) system. Therefore it is needed to make small total uncertainty of reactivity bias though lifetime performance. But sources of uncertainties will be very different. And they can include not only errors of modeling but value of sizes and masses changes which are given by manufacturing procedure. Later they are considered two examples of nuclear systems with analyzing of sources of uncertainties.

1. MATHEMATICAL MODEL AND APPROXIMATIONS EMPLOYED

The evolution of the field of nuclide concentrations in the materials of a nuclear reactor or system is described by a dynamic model in lumped parameters (*Evgenij Ivanov, 1998*). Time is considered here as an independent variable.

$$\frac{d\vec{\rho}}{dt} = \hat{B} \cdot \vec{\rho} + \hat{A} \cdot \Phi \cdot \vec{\rho}\,, \tag{1}$$

where $\vec{\rho}(t) = \left[\rho_1(t), \rho_2(t), \ldots \rho_N(t)\right]^T$ is the column vector of the nuclide concentrations; $\hat{B}$ is the square matrix of the nuclide transmutation as a result of radioactive decay; $\hat{A}$ is the square matrix of transmutations as a result of neutron reactions; Φ is the integrated neutron flux.

The system of equations may be written in the form of a system of equations for the segregated chains of transmutations which are represented by the usual differential equations

$$\begin{cases} \dfrac{dN_0(t)}{dt} = -\alpha_0 \cdot N_0(t) + q_0 \\ \dfrac{dN_1(t)}{dt} = -\alpha_1 \cdot N_1(t) + \lambda_0 \cdot N_0(t) \\ \cdots \\ \dfrac{dN_k(t)}{dt} = -\alpha_k \cdot N_k(t) + \lambda_{k-1} \cdot N_{k-1}(t) \end{cases} \quad (2)$$

where q_0 is the source of the initial nuclide of the chain; N_0 is the concentration of the initial isotope; N_k is the isotope which is determined in the chain; α_i is the probability of disappearance of the i-th nuclide; λ_l is the coefficient of transmutation of the i-th nuclide to the $(i+l)^{th}$.

The complete solution is presented in the form of the sum.

$$N_m(t) = \sum_{ic} n_{ic}(t), \quad (3)$$

where $N_m(t)$ is the concentration of the m-th nuclide; $n_{ic}(t)$ is the solution along the ic^{th} chain.

The solution for the segregated chain is represented as:

$$n_k(t) = n_0(0) \cdot \prod_{i=0}^{k-1} \lambda_i \cdot \sum_{j=0}^{k} \frac{\exp(-\alpha_j \cdot t)}{\prod_{i \neq j} (\alpha_i - \alpha_j)} + q_o \prod_{i=0}^{k-1} \lambda_i \left[\frac{1}{\prod_{j=0}^{k} \alpha_j} - \sum_{j=0}^{k} \frac{\exp(-\alpha_j \cdot t)}{\alpha_j \cdot \prod_{i \neq j} (\alpha_i - \alpha_j)} \right], \quad (4)$$

Not all coefficients of nuclide transmutations are estimated with the same accuracy. By analyzing the sensitivity of the results to variations in the initial data, it is possible to estimate the permissible error with which the rates of processes on individual nuclides ought to be estimated. It is convenient to introduce into the consideration a pseudo-nuclide, the concentration of which is proportional to the energy released in the course of irradiation or to some other functional that can be calculated in similar fashion. This nuclide is determined for those material zones where the power conservation condition has to be fulfilled. It is not difficult to generalize such an approach to any model of the conserved value, i.e. to introduce

certain weighting coefficients for the material zones, which determine the contribution to the general conserved functional. The proposed algorithms are applicable for the most general statement of the problem without significant limitations as to the type of nuclear system modeled /4/.

The deviation in concentrations is presented in the form

$$\delta N_m(t) = \sum_{ic} \delta n_{ic}(t), \tag{5}$$

If the calculations are performed over finite time intervals, the solution for each of them is the result of the action of the linear operator on the nuclide vector:

$$\rho_j(t) = u_{ij}(t) \cdot \rho_i(0), \tag{6}$$

The variation for each isotope

$$\delta\rho_l(T) = \sum_i \rho_i(0) \cdot \frac{\partial u_{il}(T)}{\partial \alpha_{km}} \cdot \delta\alpha_{km}, \tag{7}$$

where $\frac{\partial u_{il}(T)}{\partial \alpha_{km}}$ is the derivative of $u_{il}(t)$ with respect to the parameter α_{km};, $\delta\alpha_{km}$ is the variation of the parameter of k-th nuclide incineration for the m-th process ($\delta\alpha_{km} = \delta\sigma_{km} \cdot \Phi$).

In the event of small deviations we have:

$$\frac{\delta\rho_l(T)}{\delta\sigma_{km}} = \sum_i \rho_i(0) \cdot \frac{\partial u_{il}(T)}{\partial \sigma_{km}}, \tag{8}$$

where σ_{km} is the cross-section of the m-th process on the k-th nuclide.

It is necessary to estimate the contributions for each chain:

$$\frac{\delta_{i,ic}\rho_l(T)}{\delta\sigma_{km}} = \rho_i(0) \cdot \frac{\partial u_{il}^{ic}(T)}{\partial \sigma_{km}}, \tag{9}$$

All variations in the linear model may be represented as:

$$\delta n_{ik}(t) = \sum_k \frac{\partial n_{ic}(t)}{\partial \alpha_k} \cdot \frac{\partial \alpha_k}{\partial \sigma_c^l} + \sum_k \frac{\partial n_{ic}(t)}{\partial \lambda_k} \cdot \frac{\partial \lambda_k}{\partial \sigma_c^l}, \tag{10}$$

The formula for estimating the derivative with respect to the nuclide elimination parameter:

$$\frac{\partial n_k(t)}{\partial \alpha_m} = n_0(0)\cdot\prod_{i=0}^{k-1}\lambda_i\cdot\left[\sum_{j\neq m}\left(\frac{\exp(-\alpha_m\cdot t)}{(a_j-\alpha_m)\cdot\prod\limits_{i\neq m}(\alpha_i-\alpha_m)}+\frac{\exp(-\alpha_j\cdot t)}{(a_j-\alpha_m)\cdot\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}\right)\right]$$

$$-n_0(0)\cdot\prod_{i=0}^{k-1}\lambda_i\cdot\left[\frac{t\cdot\exp(-\alpha_j\cdot t)}{\prod\limits_{j\neq m}(\alpha_j-\alpha_m)}\right]-q_o\cdot\prod_{i=0}^{k-1}\lambda_i\cdot\left[\frac{1}{\alpha_m\prod\limits_{j=0}^{k}\alpha_j}+\left(\frac{t}{\alpha_m}-\frac{1}{\alpha_m^2}\right)\cdot\frac{\exp(-\alpha_m\cdot t)}{\prod\limits_{j\neq m}(\alpha_j-\alpha_m)}\right]$$

$$+q_o\cdot\prod_{i=0}^{k-1}\lambda_i\cdot\left[\sum_{j\neq m}\left(\frac{\exp(-\alpha_m\cdot t)}{\alpha_m\cdot(a_j-\alpha_m)\cdot\prod\limits_{i\neq m}(\alpha_i-\alpha_m)}+\frac{\exp(-\alpha_j\cdot t)}{\alpha_j\cdot(a_j-\alpha_m)\cdot\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}\right)\right].$$

, (11)

The derivative with respect to the parameter for transmutation of nuclide to nuclide

$$\frac{\partial n_k(t)}{\partial \lambda_l} = n_0(0)\cdot\prod_{i\neq m}^{k-1}\lambda_i\cdot\sum_{j=0}^{k}\frac{\exp(-\alpha_j\cdot t)}{\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}+q_o\cdot\prod_{i\neq m}^{k-1}\lambda_i\cdot\left[\frac{1}{\prod\limits_{j=0}^{k}\alpha_j}-\sum_{j=0}^{k}\frac{\exp(-\alpha_j\cdot t)}{\alpha_j\cdot\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}\right]\cdot, \tag{12}$$

The derivative with respect to the initial nuclide concentration

$$\frac{\partial n_k(t)}{\partial n_0(0)} = \prod_{i=0}^{k-1}\lambda_i\cdot\sum_{j=0}^{k}\frac{\exp(-\alpha_j\cdot t)}{\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}+q_o\cdot, \tag{13}$$

The derivative with respect to the magnitude of the nuclide source

$$\frac{\partial n_k(t)}{\partial q_0} = \prod_{i=0}^{k-1}\lambda_i\cdot\left[\frac{1}{\prod\limits_{j=0}^{k}\alpha_j}-\sum_{j=0}^{k}\frac{\exp(-\alpha_j\cdot t)}{\alpha_j\cdot\prod\limits_{i\neq j}(\alpha_i-\alpha_j)}\right], \tag{14}$$

For the linear chains considered earlier, we have:

$$\rho_l(t) = \sum_{ic}\rho_{ic}(t), \tag{15}$$

where l is the isotope index; ic is the number of the chain. Variations can be represented as:

$$\delta_\varepsilon\left(\rho_l(t)\right) = \sum_{ic} \delta_\varepsilon\left(\rho_{ic}(t)\right), \tag{16}$$

where $\delta_\varepsilon\left(\rho_\nu(t)\right)$ is the variation in the contribution of each chain from the parameter ε.

In the general case the solution is presented in operator form:

$$\vec{N}(T) = \hat{P}(T) \cdot \vec{N}(0), \tag{17}$$

where $\vec{N}(0)$ is the vector of the initial concentrations; $\vec{N}(T)$ is the concentration vector; $\hat{P}(T)$ is the linear operator of transmutation.

Then, over the time intervals:

$$\vec{N}^{\langle k\rangle} = \hat{P}^{\langle k-1\rangle} \cdot \hat{P}^{\langle k-2\rangle} \cdots \hat{P}^{\langle 0\rangle} \cdot \vec{N}^{\langle 0\rangle}, \tag{18}$$

Since the operators are piecewise constant, we have for the relative variation with respect to the m-th parameter:

$$\frac{\partial \vec{N}^{\langle l\rangle}}{\partial \varepsilon_m} = \hat{P}^{\langle l-1\rangle} \cdot \frac{\partial \vec{N}^{\langle l-1\rangle}}{\partial \varepsilon_m} + \frac{\partial \hat{P}^{\langle l-1\rangle}}{\partial \varepsilon_m} \cdot \vec{N}^{\langle l-1\rangle}, \tag{19}$$

where $\frac{\partial \vec{N}^{\langle l\rangle}}{\partial \varepsilon_m}$ is the coefficient of sensitivity to the m-th parameter in the l-th interval; $\frac{\partial \hat{P}^{\langle i\rangle}}{\partial \varepsilon_m}$ is the functional derivative $\hat{P}^{\langle i\rangle}$.

The general solution is:

$$\begin{aligned} \frac{\partial N^{(k)}}{\partial \varepsilon_m} &= \frac{\partial \hat{P}^{(k)}}{\partial \varepsilon_m} \times P^{(k-2)} \cdots P^{(1)} \cdot N^{(0)} \\ &+ P^{(k)} \times \frac{\partial P^{(k-1)}}{\partial \varepsilon_m} \times P^{(k-2)} \cdots P^{(1)} \cdot N^{(0)} + P^{(k)} \times P^{(k-1)} \cdots P^{1} \cdot \frac{\partial N^{(0)}}{\partial \varepsilon_m} \end{aligned}, \tag{20}$$

The variation may be determined as

$$\delta_m(\vec{N}(t)) = \frac{\partial \vec{N}(t)}{\partial \varepsilon_m} \cdot \delta\varepsilon_m, \tag{21}$$

where $\delta\varepsilon_m$ is the mean square deviation of the m-th parameter; $\frac{\partial \vec{N}(t)}{\partial \varepsilon_m}$ is the coefficient of sensitivity; $\delta_m(\vec{N}(t))$ is the absolute value of the mean square deviation.

Thus, the coefficients in the model of nuclide transmutations are calculated as integrals of the corresponding values obtained in the steady-state particle transport model.

2. SOME NUMERICAL RESULTS

A package of programs called PATRICK (Perturbed Analytical Treatment of the Reactor Inventory Modeling in the Condensed Parameters of Kinetics) has been developed (*Evgenij Ivanov, 1998*). It is constructed on the basis of a mathematical model assuming conditional segregation of different physical processes. The calculation of an operating cycle of the nuclear system is performed by means of successive recalculation of the steady-state neutron and photon fields and the corresponding rates of nuclear processes. The proposed algorithms are applicable for the most general statement of the problem without substantial restrictions on the type of modeled nuclear system, except that the basic transmutations of nuclides are determined by neutron and photon nuclear reactions in the middle and low energy range (up to ~20 MeV). Despite the fact that the above processes in the nuclear system relate to the type of so-called neutron controlled processes, the problem can also be easily generalized to a consideration of other types of particles, provided that the sense of the spectral calculation and the algorithm for introducing the controlling parameters is changed. The changing of the reactor control algorithm and the spectral calculation does not change the form of the calculation scheme and the formal construction of the kinetic model. The methods and algorithms of the program package provide a means of performing theoretical investigations associated with the modeling of the operating cycle of a nuclear system in real geometry with the aid of precision programs for calculating neutron and photon distributions.

The package of programs is intended for solving various problems associated with variations in the nuclide composition of materials in nuclear systems. In particular, it is possible to single out four principal tasks for which the proposed methods and programs may be employed.

- For precise calculation of the nuclide composition for a given regime of the nuclear system. For calculating the functionals of nuclide concentrations such as breeding capacity, activity and power density in irradiated materials, potential danger coefficients, etc. For calculating the nuclide composition with the aid of the Monte Carlo method taking the statistical error of the result into account.

- For analyzing the errors in calculations of nuclide composition and its functionals associated with errors in the initial data and the calculation models by the small perturbation method. For solving inverse problems, i.e. questions of planning the supply of nuclear data.

- For solving optimization problems in the design of special-purpose equipment intended for producing or eliminating specific nuclides, etc. This also includes problems of optimizing the operating conditions of equipment with a view to reducing the toxicity of unloaded fuel or certain constructional elements.

- For analyzing nuclear power fuel cycles, and modeling and optimizing material flows in a power system.

With the aid of modules of the PATRICK package of programs one can use nuclide transmutation rates obtained in the calculation of steady-state neutron and photon fields for modeling changes in the composition of materials of the power plant. For generating libraries of nuclide transmutation chains, estimating the accuracy of calculation and preliminary selection of the spatial calculator nodes and the time interval, one uses the perturbation theory. Apart from analysis of the errors, the method of small perturbations is employed for calculating the effective neutron flux and modeling the cyclic chains of transmutation of nuclides under irradiation. The PATRICK programs may be used for analyzing the stability of equilibrium points in investigations of nuclear power fuel cycles. The programs and methodology on which the PATRICK package is based have been developed with an eye to the further development of the soft and hardware for research in the field of nuclear engineering and particle transport theory.

Using the package of programs, it is possible to formulate certain requirements for the initial data, for example in research on transmutation concepts in accelerator-driven systems. But after first version of this package in methodology they has been includes some changes. Most of them are connected with correlation between errors (and sensitivity coefficients) on any nuclides and total capacity of reactor.

Deviation of nuclide concentration is determined by condensed reaction rate and neutron flux. But last dependants from dedicated total power and content of fissile material and rate of reaction on it.

In dependence, which is used in the present consideration, it is supposed, that during all campaign total reactor capacity is kept on some given level. Then any contributions to change of capacity are compensated by changes of a neutron flux. In model accepted for an estimation of an error it is carried out by introduction of the appropriate correction member, which is connected to the contribution of each parameter in energy yield, necessary amendment of flux and influence of change of a flux on concentration of considered nuclide.

Thus, the variation of any nuclide concentration is entered as represents functional derivative received in conditions of a constancy of factors in a matrix of transitions.

$$\frac{\delta N_k}{\delta \varepsilon_m} = \frac{\partial N_k}{\partial \varepsilon_m} - \frac{\partial N_k}{\partial \Phi_0} \cdot \left[\frac{\partial En}{\partial \Phi_0} \right]^{-1} \cdot \frac{\partial En}{\partial \varepsilon_m} \qquad (22)$$

where: $\frac{\delta N_k}{\delta \varepsilon_m}$ - derivation of concentration of k-th nuclide on parameter of m – th one, $\frac{\partial N_k}{\partial \Phi_0}$ derivation of concentration on flux, $\frac{\partial En}{\partial \Phi_0}$ - derivation of total capacity by

variation of parameter of m – th nuclide, $\frac{\partial En}{\partial \varepsilon_m}$ - derivation of total capacity by neutron flux, $\frac{\delta N_k}{\delta \varepsilon_m}$ - complete variation of k-th nuclide concentration.

But in reality measured value will be not rate of reaction but ratio between two reaction rates. Must be noted that for each unit may exist prototype for experimental matching of all reactor parameters. Matching as any other procedures may be carried out also by analysis of reactivity and spectral indexes.

Then it is needed new formulation of resulting error, which is based on sensitivity analysis by spectral indexes.

$$\frac{\delta N_k}{\delta(a_{m,l})} = \frac{\delta N_k}{\delta(\varepsilon_m / \varepsilon_l)} = \frac{\delta N_k}{\delta \varepsilon_m} \cdot \frac{1}{a_{m,l}} - \frac{\delta N_k}{\delta \varepsilon_l} \cdot \frac{1}{\varepsilon_l} \cdot \frac{1}{a_{m,l}} \qquad (23)$$

where: $\frac{\delta N_k}{\delta(a_{m,l})}$ - sensitivity coefficient by spectral index ($a_{m,l} = \varepsilon_m / \varepsilon_l$).

2.1 Modeling of lifetime performance with "reactivity breeding".

Let's consider the reactor with minimal bias of reactivity during all operation period (*E.O. Adamov and other 1999*). This requirement (small stock of reactivity) is one of significant parameter in supplying of inherent safety of reactor.

But in ensuring of such ideology of nuclear engineering it is needed to solve the problem of justification of an opportunity to develop explicit balance of reactivity when burning level in each period between partial reloading is significant.

Even in permitting of absolutely accurate calculation there are source of possible errors connected with uncertainties in sizes and masses of constructive elements of reactor (very small but not zero uncertainties). Then problem of analysis of concept of design is reduced into problem of studying of influence of small perturbation of initial data in slow transients (lifetime performance) modeling.

For estimation of uncertainties of reactivity stock (and therefore of required control bodies efficiency) they are used sensitivity coefficients of nuclides concentrations to deviations of spectral indexes and sensitivity coefficients of multiplication factor to nuclides concentrations.

Influence of error in reaction rate on multiplication factor can be obtained by next formulae:

$$\frac{\delta K_{эф}}{\delta a_{m,l}} = \sum_k \frac{\delta K_{эф}}{\delta N_k} \cdot \frac{\delta N_k}{\delta a_{m,l}} \qquad (24)$$

Table 1 Initial contents of zones of reactor (10^{24} cm^{-3}).

Nuclides	Regions				
	1	2	3	4	5
U^{235}	$2.56\ 10^{-5}$	$2.92\ 10^{-5}$	$3.56\ 10^{-5}$		
U^{238}	0.006355	0.007263	0.008859		
Pu^{239}	0.000672	0.000768	0.000937		
Pu^{240}	0.000294	0.000336	0.00041		
Pu^{241}	$3.28\ 10^{-5}$	$3.75\ 10^{-5}$	$4.57\ 10^{-5}$		
Pu^{242}	$1.72\ 10^{-5}$	$1.97\ 10^{-5}$	$2.4\ 10^{-5}$		
Am^{241}	0.000022	$2.51\ 10^{-5}$	$3.07\ 10^{-5}$		
Am^{242m}	$7.19\ 10^{-6}$	$8.22\ 10^{-6}$	$1\ 10^{-5}$		
Pb	0.02062	0.01949	0.01732	0.02745	0.02745
Fe	0.00659	0.006874	0.007835	0.007987	0.007987
Cr	0.001162	0.001149	0.001315	0.001185	0.001185
Ni	$2.09\ 10^{-5}$	$2.18\ 10^{-5}$	$2.5\ 10^{-5}$	$2.25\ 10^{-5}$	$2.25\ 10^{-5}$
Mo	0.000138	0.000144	0.000164	0.000148	0.000148
N	0.007445	0.008508	0.001038		

Table 2 Placement of zones by radius and height

Height, cm	Mixtures			
310	4	4	4	5
210	1	2	3	5
155	1	2	3	5
100	4	4	4	5
Radius, cm	63.7	95.9	114.8	214.8

Table 3 Related values of nuclides per zones

Nuclides	Regions				
	1	2	3	4	5
U^{235}	0.00912	0.00467	0.00916	9.21 10-9	1.12 10-8
U^{238}	-.0708	-.083	-.0248	0	0
Pu^{239}	0.24	0.205	0.119	1.83 10-8	3.34 10-8
Pu^{240}	0.00327	0.0116	0.000937	0	0
Pu^{241}	0.012	0.0162	0.0163	0	0
Pu^{242}	-.0041	-.0016	0.0014	0	0
Am^{241}	0.00225	-.0042	0.00707	0	0
Am^{242m}	0.00555	-.00327	0.0111	0	0
Pb	-.0353	-.0304	-.00116	0.00541	0.0194
Fe	-.00534	-.0058	0.000828	-.00223	0.00354
Cr	-.00753	-.00336	0.00199	0.0044	-.00072
Ni	0.00269	0.00349	-.00112	0.0106	0.00699
Mo	-.00042	0.0129	-.00304	0.00454	-.00369
N	-.0198	-.0261	-.00207	0	0

Appropriate deviation in this case is product of dividing of beta effective on reactivity of each nuclide:

$$\delta a_{m,l} = \beta_{эф} \Big/ \left(\frac{\delta K_{эф}}{\delta a_{m,l}} \right) \qquad (25)$$

Table 7 Evolution of average content of fuel (10^{24} cm^{-3}).

Nuclide	Time of operation, years				
	0	1	2	3	4
U^{233}	0	$3.01\ 10^{-15}$	$1.4\ 10^{-14}$	$3.38\ 10^{-14}$	$6.28\ 10^{-14}$
U^{234}	0	$1.59\ 10^{-9}$	$9.54\ 10^{-9}$	$2.5\ 10^{-8}$	$4.77\ 10^{-8}$
U^{235}	$3\ 10^{-5}$	$2.78\ 10^{-5}$	$2.57\ 10^{-5}$	$2.37\ 10^{-5}$	$2.2\ 10^{-5}$
U^{236}	0	$5.2\ 10^{-7}$	$9.94\ 10^{-7}$	$1.42\ 10^{-6}$	$1.82\ 10^{-6}$
U^{237}	0	$1.79\ 10^{-10}$	$3.43\ 10^{-10}$	$4.9\ 10^{-10}$	$6.22\ 10^{-10}$
U^{238}	0.007466	0.007394	0.007323	0.007253	0.007184
U^{239}	0	$4.1\ 10^{-9}$	$4.06\ 10^{-9}$	$4.01\ 10^{-9}$	$3.97\ 10^{-9}$
Np^{237}	0	$4.34\ 10^{-8}$	$9.11\ 10^{-8}$	$1.42\ 10^{-7}$	$1.96\ 10^{-7}$
Np^{238}	0	$1.74\ 10^{-11}$	$3.48\ 10^{-11}$	$5.32\ 10^{-11}$	$7.25\ 10^{-11}$
Np^{239}	0	$5.92\ 10^{-7}$	$5.85\ 10^{-7}$	$5.78\ 10^{-7}$	$5.72\ 10^{-7}$
Np^{240}	0	$6.07\ 10^{-12}$	$5.97\ 10^{-12}$	$5.87\ 10^{-12}$	$5.78\ 10^{-12}$
Pu^{238}	0	$5.55\ 10^{-7}$	$1.52\ 10^{-6}$	$2.52\ 10^{-6}$	$3.48\ 10^{-6}$
Pu^{239}	0.000789	0.000794	0.000799	0.000802	0.000805
Pu^{240}	0.000346	0.000347	0.000348	0.000349	0.00035
Pu^{241}	$3.85\ 10^{-5}$	$3.87\ 10^{-5}$	$3.88\ 10^{-5}$	$3.89\ 10^{-5}$	$3.9\ 10^{-5}$
Pu^{242}	$2.02\ 10^{-5}$	$2.05\ 10^{-5}$	$2.08\ 10^{-5}$	$2.11\ 10^{-5}$	$2.13\ 10^{-5}$
Pu^{243}	0	$2.32\ 10^{-10}$	$2.33\ 10^{-10}$	$2.35\ 10^{-10}$	$2.36\ 10^{-10}$
Am^{241}	$2.58\ 10^{-5}$	$2.61\ 10^{-5}$	$2.63\ 10^{-5}$	$2.65\ 10^{-5}$	$2.67\ 10^{-5}$
Am^{242}	0	$3.65\ 10^{-9}$	$3.66\ 10^{-9}$	$3.67\ 10^{-9}$	$3.69\ 10^{-9}$
Am^{242M}	$8.45\ 10^{-6}$	$7.34\ 10^{-6}$	$6.38\ 10^{-6}$	$5.54\ 10^{-6}$	$4.82\ 10^{-6}$
Am^{243}	0	$3.65\ 10^{-7}$	$7.06\ 10^{-7}$	$1.03\ 10^{-6}$	$1.33\ 10^{-6}$
Am^{244}	0	$1.86\ 10^{-11}$	$3.58\ 10^{-11}$	$5.16\ 10^{-11}$	$6.63\ 10^{-11}$
Cm^{242}	0	$5.92\ 10^{-7}$	$7.17\ 10^{-7}$	$7.43\ 10^{-7}$	$7.49\ 10^{-7}$
Cm^{243}	0	$3.22\ 10^{-9}$	$8.89\ 10^{-9}$	$1.49\ 10^{-8}$	$2.08\ 10^{-8}$
Cm^{244}	0	$3.65\ 10^{-11}$	$1.56\ 10^{-10}$	$3.45\ 10^{-10}$	$5.94\ 10^{-10}$
Cm^{245}	0	$3.85\ 10^{-13}$	$3.05\ 10^{-12}$	$9.92\ 10^{-12}$	$2.26\ 10^{-11}$
FP	0	0.000119	0.00024	0.000359	0.000478

Approximately nuclear reactor can be described by multi-zones cylinder. Table 1 describes contents of zones and table 2 presents their placement in model. Table 3 contains distribution of reactivity weights of nuclides.

Estimated value of effective part of decay neutrons few greater then one per third part of percent β=0.00352. Data given by this normalizing multiplier are shown in tables 5 and 6. The values of errors can be used as requirement for accuracy of new nuclear data.

Table 5 Influence on resulting error and allowable uncertainties.

Nuclide 1	Nuclide 2	Sensitivity			Uncertainty		
		σ_c/σ_c	σ_f/σ_c	σ_f/σ_f	σ_c/σ_c	σ_f/σ_c	σ_f/σ_f
Am^{242M}	U^{238}	5.7%	12.0%	105.0%	6.2%	2.9%	0.3%
Pu^{241}	U^{238}	6.6%	7.4%	64.9%	5.3%	4.7%	0.5%
U^{235}	U^{238}	8.7%	5.9%	51.5%	4.1%	6.0%	0.7%
B^{10}	U^{238}	41.2%	0.0%	0.0%	0.9%	0.0%	0.0%
Pu^{238}	U^{238}	11.3%	3.7%	32.2%	3.1%	9.6%	1.1%
Np^{239}	U^{238}	29.4%	1.5%	13.4%	1.2%	22.9%	2.6%
Am^{241}	U^{238}	27.1%	0.9%	7.4%	1.3%	40.9%	4.7%
Np^{237}	U^{238}	22.7%	1.1%	9.3%	1.5%	32.9%	3.8%
Pu^{239}	U^{238}	7.7%	3.1%	16.5%	4.5%	11.3%	2.1%
Am^{243}	U^{238}	15.5%	0.7%	5.8%	2.3%	52.8%	6.1%
Cm^{244}	U^{238}	13.1%	1.4%	11.8%	2.7%	26.1%	3.0%
Pu^{239}	U^{236}	0.4%	1.8%	12.2%	83.0%	19.4%	2.9%
Pu^{240}	U^{238}	7.8%	1.5%	9.5%	4.5%	23.1%	3.7%
U^{234}	U^{238}	9.2%	1.1%	9.4%	3.8%	32.8%	3.8%
Pu^{239}	Cm^{242}	0.6%	2.8%	7.7%	54.4%	12.7%	4.6%
FP35	U^{238}	7.7%	0.0%	0.0%	4.6%	0.0%	0.0%
FP39	U^{238}	7.6%	0.0%	0.0%	4.6%	0.0%	0.0%
Pu^{242}	U^{238}	6.9%	0.8%	6.7%	5.1%	45.5%	5.2%
U^{236}	U^{238}	6.6%	0.3%	2.4%	5.3%	127.2%	14.6%
Pu^{239}	Am^{243}	0.2%	0.8%	5.1%	193.9%	45.2%	6.9%
Am^{242M}	Pu^{239}	0.3%	5.1%	1.2%	114.4%	6.9%	29.8%
U^{238}	Cm^{242}	4.6%	0.5%	0.2%	7.7%	66.4%	186.0%
Cm^{242}	U^{238}	4.4%	0.4%	3.8%	8.1%	80.0%	9.2%

Additionally it can be considered variant with analysis of influence of uncertainties of fuel rods manufacturing. All of deviations are compensated by position of control bodies. It is obtained that when deviation of fuel density on 1% compensated by control system the total deviation in reactivity bias will be about 1.980%, that more than in two times greater than beta effective. Thus it means that deviations in geometric and density parameters must be less than 0.5%.

Table 6 Influence of errors and uncertainties on structure materials

Nuclide 1	Nuclide 2	Sensitivity		Uncertainty	
		σ_c/σ_c	σ_f/σ_c	σ_c/σ_c	σ_f/σ_c
U^{238}	Pb	26.8%	0.6%	1.3%	60.1%
Pu^{239}	Pb	3.8%	23.8%	9.3%	1.5%
U^{238}	Fe	12.6%	0.3%	2.8%	127.5%
Pu^{239}	Fe	1.8%	11.2%	19.8%	3.1%
U^{238}	Cr	8.8%	0.2%	4.0%	182.7%
Pu^{239}	Cr	1.2%	7.8%	28.4%	4.5%
U^{238}	N	6.1%	0.1%	5.8%	265.7%
Pu^{239}	N	0.9%	5.4%	40.8%	6.5%

These numerical results (presented in table 7) show that when we develop new nuclear system with prediction of reactivity bias less then safety level (of β_{ef}) we have to know nuclear data with uncertainties less then ones transpired in table 7. By another words if in the future we want to achieve complete using of fission energy of natural uranium with low level of nuclear threats it is needed to significantly increase requirements to accuracy of nuclear data.

2.2 Modeling of lifetime performance autonomic space nuclear system.

Deep space penetration in the future obviously will require very long-lived power generating systems without any human management. Here it is considered nuclear reactor on fast neutrons with ultra deep burning of fuel. In table 8 they are shown evolution of nuclides concentrations.

Table 8 Changes of nuclides concentrations during lifetime, 10^{24} cm^{-3}.

Nuclide	Time of operation, years				
	0	1	4	7	10
U^{234}	0	$6.16\ 10^{-7}$	$1.72\ 10^{-6}$	$2.76\ 10^{-6}$	$3.79\ 10^{-6}$
U^{235}	0.030314	0.029795	0.028825	0.027854	0.026884
U^{236}	0	$8.4\ 10^{-5}$	0.00024	0.000396	0.000552
U^{238}	0.001595	0.00159	0.001582	0.001573	0.001563
Np^{237}	0	$1.33\ 10^{-7}$	$9.55\ 10^{-7}$	$2.54\ 10^{-6}$	$4.95\ 10^{-6}$
Pu^{239}	0	$2.67\ 10^{-6}$	$7.83\ 10^{-6}$	$1.32\ 10^{-5}$	$1.87\ 10^{-5}$

As in previous case we can construct complete solution (of task of estimation of an error) by using of values of each of nuclides in dependence of multiplication factor and sensitivity coefficients of influence of deviations of initial data on nuclide's concentrations during lifetime performance. It was considered lifetime performance of reactor and changes of its parameters. Calculated values are shown in tables 9-10.

Table 9 Related values of nuclides

Nuclides	Region				
	1	2	3	6	8
U^{235}	0.645				
U^{238}	0.117				
B^{10}		0.141			0.17
Fe		0.313			0.886
Cr		0.196			-.94
Ni		0.15			-.795
W		-.113			-.352
C		-.866			0.73
Be				0.468	
B^{11}		-.245			-.473
Na			0.157		0.746
N	0.488				

In this example there is problem of influence of initial accuracy of reactor elements manufacturing on predicted value of loss of reactivity during lifetime. But there is next source of uncertainties such as stochastic behavior of coolant density and changes of cladding's thickness by corrosion.

Table 10 Influence of errors on fuel materials

Nuclide 1	Nuclide 2	Sensitivity		
		σ_c/σ_c	σ_f/σ_c	σ_f/σ_f
U^{235}	U^{238}	-.1109	-.5521	1.961
U^{235}	U^{235}	0	-1.078	0
U^{235}	U^{236}	-.07958	-.4133	0.876
U^{235}	FP	-.09663	-.4886	0
U^{234}	U^{235}	0.08113	-.3343	-.06612
U^{235}	U^{234}	-.05292	-.2676	0.3295
B^{10}	U^{235}	0.3182	0	0
U^{236}	U^{235}	0.05434	-.1239	-.0242
U^{235}	B^{10}	-.01349	-.0682	0
U^{238}	U^{235}	0.03884	-.05648	-.01125
FP	U^{235}	0.04442	0	0
U^{236}	U^{238}	0.000181	0.001623	0.006429
U^{234}	U^{236}	-.00036	-.00556	-.00544
U^{234}	U^{238}	-.00024	0.003446	0.005388
U^{236}	U^{236}	0	-.001817	0
B^{10}	U^{236}	-.00146	0	0
U^{238}	U^{236}	0	-.000888	-.00133
B^{10}	U^{238}	-.00097	0	0
U^{236}	U^{234}	0.000162	0.0001574	0.000749

Both of them are presented in table 11 as $\Delta K/\Delta J$ and $\Delta K/\Delta I$ and characterized property of this reactor which can be named as sensitivity with manufacturing parameters.

Table 11 Parameters of reactor during lifetime performance

Parameter	Operating time, years				
	0	1	4	7	10
FP concentration, 10^{24} cm^{-3}	0	0.000828	0.002379	0.003931	0.005483
ΔK, %	0.00	-.93	-2.91	-4.30	-6.55
Fima, %	0.00	2.60	7.46	12.32	17.18
$\Delta K/\Delta G$		0.36	0.39	0.35	0.38
$\Delta K/\Delta I$[1]		0.526	0.861	1.18	1.19
Error in K_{ef}, %	2.09	2.06	1.92	1.80	1.70
Correction, %		-.02	-.04	-.06	-.07
$\Delta K/\Delta J$[2]		0.0541	0.0841	0.113	0.108

[1] ΔI – magnitude of unsteady process

[2] ΔJ – deviation in fuel density

CONCLUSIONS

1. It is presented method of analysis of uncertainties of slow processes in reactor modeling based on linear response approximation.

2. A method and algorithm are developed for computing the sensitivity coefficients and calculating the effect of small perturbations in the context of analytical solution of nuclide kinetics equations.

3. An algorithm and methodology are proposed for extending the method of analytical solution of nuclide kinetics equations (in the form of Bateman equations) to problems with weak non-linearity and the presence of neutron and photon radiation.

4. It was shown that in analysis of nuclear systems with increased value of safety and very long lifetime performance it can be useful to estimate so-called sensitivity of reactor parameters to constructive parameters uncertainties.

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