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HEXAGONAL CMFD FORMULATION EMPLOYING TRIANGLE-BASED POLYNOMIAL EXPANSION NODAL KERNEL

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ABSTRACT

A triangle-based polynomial expansion nodal (TPEN) method is developed such that it can be readily implemented into a hexagonal coarse mesh finite difference (CMFD) formulation. This method combines the higher order polynomial expansion nodal (HOPEN) method and the nodal expansion method (NEM) by decoupling the three-dimensional neutron diffusion equation into a radial and an axial one. For the implementation into the hexagon-based CMFD formulation, the TPEN method is derived such that it solves one hexagonal node problem consisting of six triangular nodes. The coupled solution for the six triangles is obtained concurrently by algebraic direct elimination. The inefficient point flux computation scheme of the original HOPEN method is improved by using a 13 term polynomial expansion for the intranodal flux within a hexagon. The computational speed of the TPEN based CMFD is about 5 times faster than the original HOPEN accelerated with coarse mesh rebalancing. The accuracy of this method is very good in that the errors in the eigenvalue and power distribution are within 15 pcm and 1%, respectively, for various eigenvalue benchmark problems, and in that the transient core power behaviors for two VVER rod ejection benchmark problems also agree well with the references.

1. INTRODUCTION

The coarse mesh finite difference (CMFD) formulation is widely used as an efficient implementation of the advanced nodal method (Chao et al., 2000; Joo et al., 1998). In this formulation, corrective nodal coupling coefficients are iteratively determined at each node interface by higher order nodal calculations for local one- or

two-node problems. In the work here, a CMFD formulation is derived for hexagonal-z geometry by taking the hex-octahedron as the base node and by employing a polynomial expansion method for local higher order nodal calculations.

The higher order polynomial expansion nodal (HOPEN) method (Cho and Kim, 1998) is an advanced nodal method to solve neutron diffusion problems in hexagonal geometry. It employs a polynomial expansion consisting of 15 terms in a three-dimensional *prism* node. These expansion coefficients are uniquely determined by the nodal coupling relations including first-order weighted residual nodal balance conditions. Compared to other hexagonal nodal methods, HOPEN has the advantages of an easy mesh refining feature and multigroup extension. However, the use of three-dimensional prism nodes, each of which contains one node average flux, three directional moments (x, y and z), five surface average fluxes and six corner point fluxes as the unknowns, results in complicated nodal coupling relations, especially in corner point balance equation, which reduces the computational efficiency. This complexity motivates an alternative polynomial expansion formulation employing two transverse-integrated diffusion equations, one for the radial direction and the other for the axial direction. The new formulation involves a two-dimensional polynomial expansion within a *triangle* and a one-dimensional polynomial expansion in the axial direction.

In the hexagonal CMFD formulation, a hexagon is chosen as the base node radially to make the radial mesh sufficiently coarse. Thus the smallest unit of the local problem is a hexagon. Since a hexagon contains six triangles, six polynomial expansions are required to describe the intranodal flux distribution within a hexagon in case that triangle-based polynomial expansion is employed. Thus a coupled solution of the six polynomials satisfying the boundary conditions specified at the hexagon boundaries is required, and it can be derived by a direct elimination process. In the next section, a hexagonal one-node problem is formulated and the derivation of the coupled solution is presented. Although the problem to be solved is a hexagonal problem, the solution method is named triangle-based polynomial expansion nodal (TPEN) method because the basic entity for which the solution is described is a triangle.

The CMFD formulation requires alternate calculations of the global and local problems. The global problem should provide the boundary conditions to the local problems that will determine the nodal coupling relations to be used in the global problem. For a convergent iteration logic, the boundary conditions to the local one-node problems should be adequately chosen and properly specified to the local problem. The methods for determining the local node boundary conditions such as the transverse leakage distribution and the corner point fluxes are presented in the later part of Section 2. In Section 3, the nonlinear iteration scheme to control the global and local calculations are presented for the eigenvalue and transient problems. The performance of this hexagonal CMFD formulation with the TPEN kernel is examined for several eigenvalue benchmark and transient problems. The solution accuracy and computational speed are compared with other formulations in Section 4.

2. TRIANGLE-BASED POLYNOMIAL EXPANSION NODAL METHOD

Within a computational domain represented by a node number *n,* the axially integrated multigroup neutron diffusion is given by the following equation:

$$-D_g^n\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi_g^n(x,y)+\Sigma_{rg}^n\phi_g^n(x,y)$$

$$=\frac{\chi_g}{k_{eff}}\sum_{g'}\nu\Sigma_{fg'}^n\phi_{g'}^n(x,y)+\sum_{g'}\Sigma_{sgg'}^n\phi_{g'}^n(x,y)+S_{gE}^n(x,y), \quad (1)$$

where

$$S_{gE}^n(x,y)=S_{g,TFS}^n(x,y)-L_{gz}^n(x,y)\ ,$$

$$L_{gz}^n(x,y)=\frac{1}{h_z^n}\left(j_{gz}^{n,T}(x,y)-j_{gz}^{n,B}(x,y)\right)\ .$$

Here, $L_{gz}^n(x,y)$is the axial leakage distribution and $S_{g,TFS}^n(x,y)$ is the independent source distribution that appears in the formulation of the transient fixed source problem to solve the time-dependent problem. The leakage and transient fixed sources are combined together to form the effective source. The functional form of the effective source is assumed to be known when solving the radial problem. Determining the functional form of the effective source is considered in the section 2.2. For the axial direction, the radially integrated equation, which is similar to Eq. (1) other than it has only one independent variable *z*, can be setup. For the solution of the axial one-dimensional transverse-integrated equation, the NEM (Finneman, 1977) will be used so that the axial flux distribution is described by a fourth order polynomial. In the next section, the solution of the radial problem only is presented.

2.1 Polynomial Expansion Nodal Method on Hexagonal Geometry

In the two-dimensional HOPEN method, the intranodal flux distribution within a triangular domain is approximated by a two-dimensional third order polynomial:

$$\phi_g^m(x,y)=c_{g0}^m+a_{gx}^m x+a_{gy}^m y+b_{gx}^m x^2+b_{gu}^m u^2+b_{gp}^m p^2+c_{gx}^m x^3+c_{gu}^m u^3+c_{gp}^m p^3, \quad (2)$$

where

$$u = -\frac{1}{2}x - \frac{\sqrt{3}}{2}y \quad \text{and} \quad p = -\frac{1}{2}x + \frac{\sqrt{3}}{2}y.$$

The nine coefficients above can be expressed in terms of nodal unknowns defined for each triangle, namely, node average flux, x and y moments, three surface average fluxes, and three corner point fluxes. In order to determine the nodal unknowns uniquely, nine constraints are required and they are nodal neutron balance, *x*-, *y*-weighted residual equations and current continuity, corner point balance equations. This solution approach is the same in the TPEN method except that the node average value and the first moments of the effective source must be provided for each triangular node. The node average source and source moments are defined as:

$$\bar{S}_{gE}^{n} = \frac{1}{A^n}\int_{A^n} S_{gE}^{\mathrm{n}}(x,y)\, dA \quad , \tag{3}$$

$$\tilde{S}_{gEx}^{n} = \frac{1}{A^n}\frac{2\sqrt{3}}{3h}\int_{A^n} x\, S_{gE}^{\mathrm{n}}(x,y)\, dA \quad , \text{ and} \tag{4-a}$$

$$\tilde{S}_{gEy}^{n} = \frac{1}{A^n}\frac{2}{h}\int_{A^n} \mathrm{y}\, S_{gE}^{\mathrm{n}}(x,y)\, dA. \tag{4-b}$$

The node average, *x*- and *y*-moment of the effective source can be easily calculated once the radial distribution of the source is known.

In the TPEN, the boundary conditions for the local problem are given for a hexagon. A response matrix that relates the outgoing partial currents to the incoming partial currents is thus derived for a hexagon. Fig. 1 shows a hexagonal node consisting of 6 triangular nodes. For the hexagonal node, 6 incoming partial currents and 6 corner point fluxes need to be specified as the boundary conditions for a unique solution represented by Eq. (2) for each triangular node. The incoming partial current is determined from the net current and the surface flux by the following equation that states the P1 approximation:

$$j_{in} = \frac{1}{4}\phi_s + \frac{1}{2}j_{net}. \tag{5}$$

Note that the net current and surface flux are determined at each interface using the flux distribution obtained from the global CMFD calculation. The determination of the other boundary condition, namely, corner point fluxes, is discussed in Section 2.3.

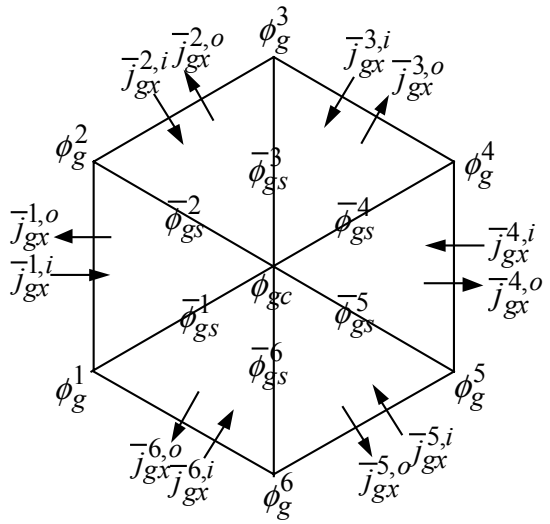


Fig. 1 Boundary Conditions and Nodal Unknowns in a Hexagonal Node for TPEN

In the TPEN point of view, there are 31 unknowns defined within the hexagon per group: 6 triangular node average fluxes, 6 *x*-moments, 6 *y*-moments, 6 inner surface fluxes, 6 outgoing partial currents and 1 center point flux. To determine these unknowns, 6 nodal balance equation, 6 x- and 6 y-weighted residual equations for each triangular node, 6 net current continuity conditions at inner surfaces, 6 net current condition at outer surfaces and one corner point leakage balance equation at the center point are used. The resulting linear system to determine these unknowns can be expressed as:

$$\begin{bmatrix} \mathbf{A_1} & \mathbf{0} & \mathbf{0} & \mathbf{A_2} & \mathbf{A_3} & \mathbf{A_4} \\ \mathbf{0} & \mathbf{X_1} & \mathbf{0} & \mathbf{X_2} & \mathbf{X_3} & \mathbf{X_4} \\ \mathbf{0} & \mathbf{0} & \mathbf{Y_1} & \mathbf{Y_2} & \mathbf{0} & \mathbf{0} \\ \mathbf{C_1} & \mathbf{C_2} & \mathbf{0} & \mathbf{C_3} & \mathbf{0} & \mathbf{C_4} \\ \mathbf{S_1} & \mathbf{S_2} & \mathbf{S_3} & \mathbf{0} & \mathbf{S_4} & \mathbf{S_5} \\ \mathbf{0} & \mathbf{P_1} & \mathbf{0} & \mathbf{0} & \mathbf{P_2} & \mathbf{P_3} \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{\varphi}} \\ \tilde{\boldsymbol{\varphi}}_\mathbf{x} \\ \tilde{\boldsymbol{\varphi}}_\mathbf{y} \\ \bar{\mathbf{j}}_\mathbf{o} \\ \bar{\boldsymbol{\varphi}}_\mathbf{s} \\ \boldsymbol{\varphi}_\mathbf{p} \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{S}}_\mathbf{a} \\ \tilde{\mathbf{S}}_\mathbf{x} \\ \tilde{\mathbf{S}}_\mathbf{y} \\ \bar{\mathbf{S}}_\mathbf{j} \\ \bar{\mathbf{S}}_\mathbf{s} \\ \mathbf{S}_\mathbf{p} \end{bmatrix}, \tag{6}$$

where $\bar{\boldsymbol{\varphi}}$, $\tilde{\boldsymbol{\varphi}}_\mathbf{x}$, $\tilde{\boldsymbol{\varphi}}_\mathbf{y}$, $\bar{\boldsymbol{\varphi}}_\mathbf{s}$, $\bar{\mathbf{j}}_\mathbf{o}$ are the solution vectors of triangle average fluxes, x- and y-moments, inner surface fluxes, outgoing currents with a dimension of 6xG (G: the number of energy group), respectively and $\boldsymbol{\varphi}_\mathbf{p}$ is a center point flux with a dimension of G. The specific feature of the above linear system is that $\mathbf{A_1}$, $\mathbf{X_1}$ and $\mathbf{Y_1}$ are energy-block diagonal matrices and $\mathbf{C_3}$ is also a diagonal matrix. Therefore the first step to reduce the above linear system is the elimination of triangle average fluxes and moments, and then the elimination of outgoing partial current using the inverse of diagonal matrices. The reduced linear system contains only inner surface fluxes and center point flux as the unknowns, which can be expressed as:

$$\begin{bmatrix} \mathbf{S}_4' & \mathbf{S}_5' \\ \mathbf{P}_2' & \mathbf{P}_3' \end{bmatrix} \begin{bmatrix} \bar{\boldsymbol{\varphi}}_s \\ \boldsymbol{\varphi}_p \end{bmatrix} = \begin{bmatrix} \bar{\mathbf{S}}_s' \\ \mathbf{S}_p' \end{bmatrix}. \tag{7}$$

Here $\mathbf{S}_4'$ is the transformed matrix of $\mathbf{S}_4$ maintaining the block structure. The following shows the elements of the two matrices:

$$\mathbf{S}_4 : \begin{bmatrix} \begin{vmatrix} s_1 & 0 \\ 0 & s_2 \end{vmatrix} & \begin{vmatrix} s_3 & 0 \\ 0 & s_4 \end{vmatrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \begin{vmatrix} s_3 & 0 \\ 0 & s_4 \end{vmatrix} \\ \begin{vmatrix} s_3 & 0 \\ 0 & s_4 \end{vmatrix} & \begin{vmatrix} s_1 & 0 \\ 0 & s_2 \end{vmatrix} & \begin{vmatrix} s_3 & 0 \\ 0 & s_4 \end{vmatrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \begin{vmatrix} s_3 & 0 \\ 0 & s_4 \end{vmatrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \begin{vmatrix} s_3 & 0 \\ 0 & s_4 \end{vmatrix} & \begin{vmatrix} s_1 & 0 \\ 0 & s_2 \end{vmatrix} \end{bmatrix}$$

$$\rightarrow \mathbf{S}_4' : \begin{bmatrix} \begin{vmatrix} s_1' & s_5 \\ s_6 & s_2' \end{vmatrix} & \begin{vmatrix} s_3' & s_7 \\ s_8 & s_4' \end{vmatrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \begin{vmatrix} s_3' & s_7 \\ s_8 & s_4' \end{vmatrix} \\ \begin{vmatrix} s_3' & s_7 \\ s_8 & s_4' \end{vmatrix} & \begin{vmatrix} s_1' & s_5 \\ s_6 & s_2' \end{vmatrix} & \begin{vmatrix} s_3' & s_7 \\ s_8 & s_4' \end{vmatrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \begin{vmatrix} s_3' & s_7 \\ s_8 & s_4' \end{vmatrix} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \begin{vmatrix} s_3' & s_7 \\ s_8 & s_4' \end{vmatrix} & \begin{vmatrix} s_1' & s_5 \\ s_6 & s_2' \end{vmatrix} \end{bmatrix}.$$

Note that each row of blocks in the above matrices has only two off diagonal blocks meaning that a inner surface flux is related with only two neighboring inner surface fluxes. The solution of Eq. (7) can be easily obtained because the inverse matrix of $\mathbf{S}_4'$ can be expressed algebraically in a simple form. Once the solution of Eq. (7) is obtained, the other unknowns defined in Eq. (6) can be determined in terms of the inner-surface fluxes and point flux. Note that the TPEN solution also updates the hexagon average flux, which is defined as the average of the 6 triangle average fluxes.

2.2 Source Shape Approximation in Hexagonal Node

In order to obtain the source moments, the distribution of the effective source within the triangle should be known. Here, the source distribution is approximated by a two-dimensional second-order polynomial. As shown in Fig. 2, the hexagon of interest is surrounded by 6 hexagons. The radial dependence of the effective source within the

central hexagon can be approximated employing a polynomial consisting of 7 independent terms as:

$$S_{gz}(x,y) = d_{g0} + d_{g1}x + d_{g2}y + d_{g3}x^2 + d_{g4}u^2 + d_{g5}p^2 + d_{g6}xup \quad (8)$$

The 7 coefficients of Eq. (8) is determined by conserving 7 node average effective sources.

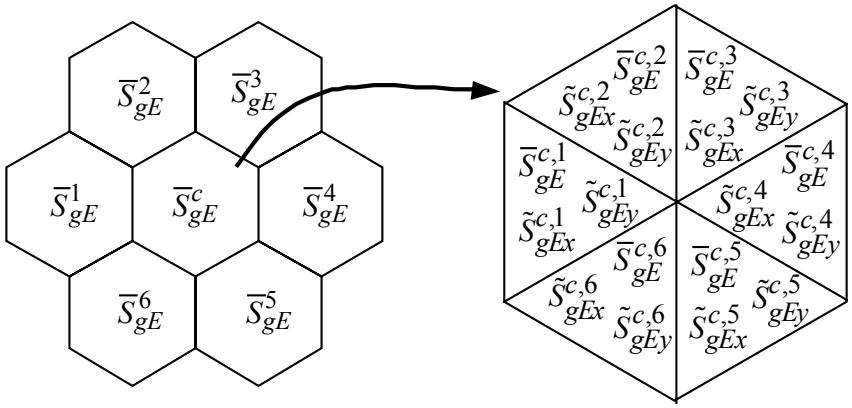


Fig. 2 Effective Sources and Moments for the Central Hexagon

As explained in Section 2.1, the TPEN method requires three source moments at each triangular node of the hexagonal node. By inserting Eq. (8) to (3) and (4), the node average source and the source moments of each triangular node can be expressed by 7 hexagon average source values. For example, the node average source and the source moments for the 1st triangle of center hexagon of Fig. 2 are:

$$\bar{S}^{c,1}_{gE} = \bar{S}^c_{gE} + \frac{1}{540}\left(83\ \bar{S}^1_{gE} + 17\ \left(\bar{S}^2_{gE} + \bar{S}^6_{gE}\right) - 37\ \left(\bar{S}^3_{gE} + \bar{S}^5_{gE}\right) - 43\ \bar{S}^4_{gE}\right), \quad (9\text{-a})$$

$$\tilde{S}^{c,1}_{gEx} = \frac{1}{3240}\left(-60\ \bar{S}^c_{gE} + 59\ \bar{S}^1_{gE} + 14\ \left(\bar{S}^2_{gE} + \bar{S}^6_{gE}\right) - 10\ \left(\bar{S}^3_{gE} + \bar{S}^5_{gE}\right) - 7\ \bar{S}^4_{gE}\right), \quad (9\text{-b})$$

$$\tilde{S}^{c,1}_{gEy} = -\frac{1}{40}\left(\bar{S}^2_{gE} - \bar{S}^6_{gE}\right) - \frac{1}{360}\ \left(\bar{S}^3_{gE} - \bar{S}^5_{gE}\right). \quad (9\text{-c})$$

2.3 Corner Point Flux Calculation

In order to determine the corner point fluxes directly following the original HOPEN method employing corner point balances, it is required to update the flux moments and inner surface fluxes. Since it is not straightforward to update the moments and inner surface fluxes based on the node average flux information available from the CMFD solution, an alternative corner flux determination method (Cho, 1998) is employed. This alternative method approximates the hexagonal flux distribution with a 13-term polynomial and the coefficients are expressed in terms of 6 surface and 6 corner point fluxes as well as the hexagon average flux. The surface fluxes and the hexagon average are available from the CMFD calculation and the unknown corner point fluxes are determined by imposing the corner point balance (CPB) conditions at each point which yields a coupled linear system on corner point fluxes. Under this scheme, the leakage balance equation consistent with the HOPEN expansion is used only for the determination of the center point flux of the hexagon.

2.4 Axial NEM Calculation

The axial intranodal flux distribution is determined from the conventional NEM method with a quadratic transverse leakage approximation. The radial leakage of each hexagonal node is determined directly from the CMFD results or from the preceding radial TPEN solution. The NEM calculation is performed for one node for which the incoming currents are specified at both top and bottom boundaries. The one-node NEM solution determines the outgoing partial currents as well as the node average flux. However, only the outgoing axial currents are taken from the radial TPEN calculation discarding the NEM average flux.

3. NON-LINEAR ITERATION LOGIC

The TPEN method described in Section 2 is used as a nodal kernel in the CMFD framework to update the corrective coefficient for the interface current. In this section, the nonlinear iteration logic involving alternate solutions of the local and global problems is presented for the solution of eigenvalue and transient problems.

3.1 Eigenvalue Calculation

Once the TPEN solution is obtained, the outgoing partial current as well as the surface fluxes can be determined. The net current can be obtained as the difference of the two partial currents (positive and negative directions) at each surface. Since the interface current in CMFD method is defined in terms of right and left node average fluxes as:

$$J = -\tilde{D}(\bar{\phi}_R - \bar{\phi}_L) - \hat{D}(\bar{\phi}_R - \bar{\phi}_L), \tag{10}$$

the corrective coefficient at the interface should then be obtained using the following equation as:

$$\hat{D} = -\frac{J + \tilde{D}(\bar{\phi}_R - \bar{\phi}_L)}{\bar{\phi}_R + \bar{\phi}_L}. \tag{11}$$

Similarly, by defining the interface flux in CMFD method is defined as:

$$\phi_s = \alpha\bar{\phi}_R + (1-\alpha)\bar{\phi}_L + \hat{\alpha}(\bar{\phi}_R + \bar{\phi}_L), \tag{12}$$

the surface flux correction factor α can be calculated for each interface by the following equation:

$$\hat{\alpha} = \frac{\phi_s - \alpha\bar{\phi}_R - (1-\alpha)\bar{\phi}_L}{\bar{\phi}_R + \bar{\phi}_L}, \tag{13}$$

CMFD calculation is performed with the corrective coefficient given by Eq. (11) to determine a new hexagon average flux distribution. Then, the updated hexagon average fluxes from CMFD calculation are used to update interface current and fluxes for the next set of one-node nodal problems.

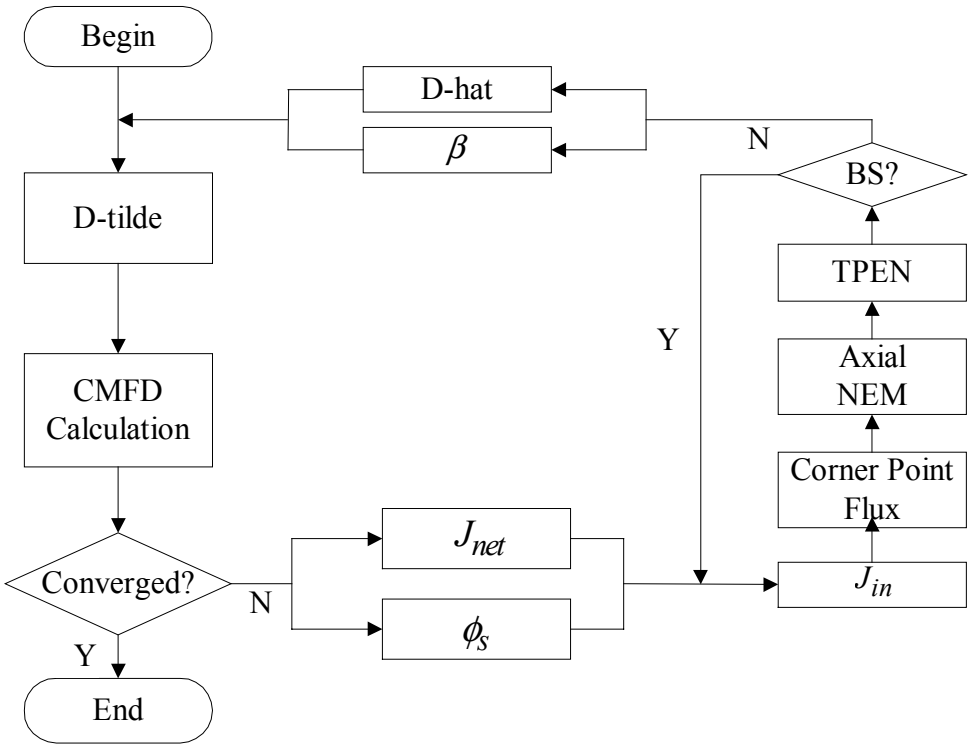
Fig. 3 shows a brief view of the nonlinear iteration logic for the eigenvalue problem. The first step for the TPEN calculation is to determine the incoming partial currents from the interface currents and fluxes, and the next step is then to determine the hexagonal corner point fluxes. The hexagonal corner point fluxes are obtained by solving the CPB equations in the Gauss-Seidel manner. Then the axial NEM calculation is performed to determine axial currents. The new axial currents are used to update hexagon average axial leakage sources for the subsequent radial TPEN solver. The radial TPEN calculation updates the radial currents and also the node-average fluxes. The axial and radial one-node calculations are performed twice by sweeping over all the hex-octahedron nodes two-times, i.e. forward and backward sweeps. This dual sweep scheme is to achieve a stable convergence behavior. After the dual sweeps are completed, the net currents are updated and also the D-hat's accordingly. This completes the TPEN updates and the next CMFD calculation is to be continued. This iterative scheme between the nodal and CMFD calculation is repeated until obtaining a converged nodal solution.

3.2 Transient Calculation

Temporal discretization of the time-dependent neutron balance equation yields a transient fixed source problem and the resulting CMFD linear system can be represented in the operator form as:

$$\left(M + \frac{1}{v\Delta t} I - \left(1 - \beta + w\right) F \right) \phi_i = \frac{1}{v\Delta t} \phi_{i-1} + S_d^i , \tag{14}$$

where β, S_d^i, and w represent the fraction, source, and current contribution fraction of delayed neutrons at time step i.



BS: Backward Sweep

Fig. 3 Logical Logic Control for TPEN Calculation within CMFD Framework

During the transient calculation, TPEN kernel can in principle solve the continuous form of Eq. (14). In such case, however, an accurate representation of the spatial dependence of the fixed source term is required. Since it is nontrivial to obtain such functional dependence, an alternate form of Eq. (14) is solved in the TPEN kernel, that has the non steady-state terms on the right-hand side (RHS) (Engrand, 1998). Bringing such terms from the left-hand side (LHS) makes the magnitude of the source term smaller so that the error in spatial dependence of the source term has less impact on the solution. This scheme also helps achieve a constant solution for a null transient because the LHS remains the same as the steady-state calculation while the RHS becomes nil. More specifically, the effective source to be used in the TPEN kernel appears as:

$$S_{eff} = S_d^i + \frac{1}{v\Delta t}\left(\phi_{i-1} - \phi_i\right) - \left(1 - \beta_p\right) F \phi_i . \tag{15}$$

Since the effective source term includes the current time step flux, Eq. (15) has to be iteratively updated. Once the node-average effective source terms are obtained, they are added to the transverse leakage term and thus the same spatial representation as the transverse leakage is used for the effective source.

The transient calculation flow is somewhat similar to the steady-state one in that the CMFD and nodal updates are alternatively performed and the same TPEN kernel is used. The major difference lies in that the transient calculation solves a fixed source problem and thus there is no eigenvalue update part and also the fixed source must be considered in the TPEN solution. Another difference is that the corrective coefficient is updated only when there are substantial changes in total cross sections. This means that if the total cross section change is small, the CMFD alone using the corrective coefficient determined at the old time step determines the nodal solution.

4. NUMERICAL COMPUTATION AND RESULTS

The TPEN based CMFD formulation has been implemented into the MASTER code (Cho, et al., 1999) and the solution performance was examined for several eigenvalue benchmark problems (Chao, 1995) and two transient problems, i.e., a realistic VVER1000 rod ejection benchmark (Podlazov, et al., 1997; Podlazov, et al., 1998) and VVER440 rod ejection benchmark problem. The computational speed was also compared with other hexagonal solution methods.

4.1 Eigenvalue Calculation Results

Fig. 4 summarizes the eigenvalue and maximum assembly power errors for the eigenvalue problems. As shown in the table, the maximum eigenvalue error is 15 pcm and the maximum power error is less than 1%, which confirms that the solution accuracy of the TPEN solver is excellent. Table 1 compares the computing times of methods for a large hexagonal full core consisting of 421 hexagonal nodes and 12 planes (total 5052 nodes). The TPEN CMFD computing time is only 4.5 seconds on a 733 Mhz Pentium-III PC. This indicates that the computing speed of TPEN CMFD is also very fast compared with the AFEN/NEM hybrid method (Cho, et al., 1998) and HOPEN method in Coarse Mesh Rebalancing (CMR) framework and also compared with TPEN with CMR

Table 1 Computing Time for 3-Dimensional Eigenvalue Problems
(733 MHz Pentium-III PC, seconds)

Problems	TPEN/CMFD	TPEN/CMR	AFEN/CMR	HOPEN/CMR
VVER-1000	1.7	4.3	4.1	7.5
VVER-440	4.5	13.9	13.8	26.1

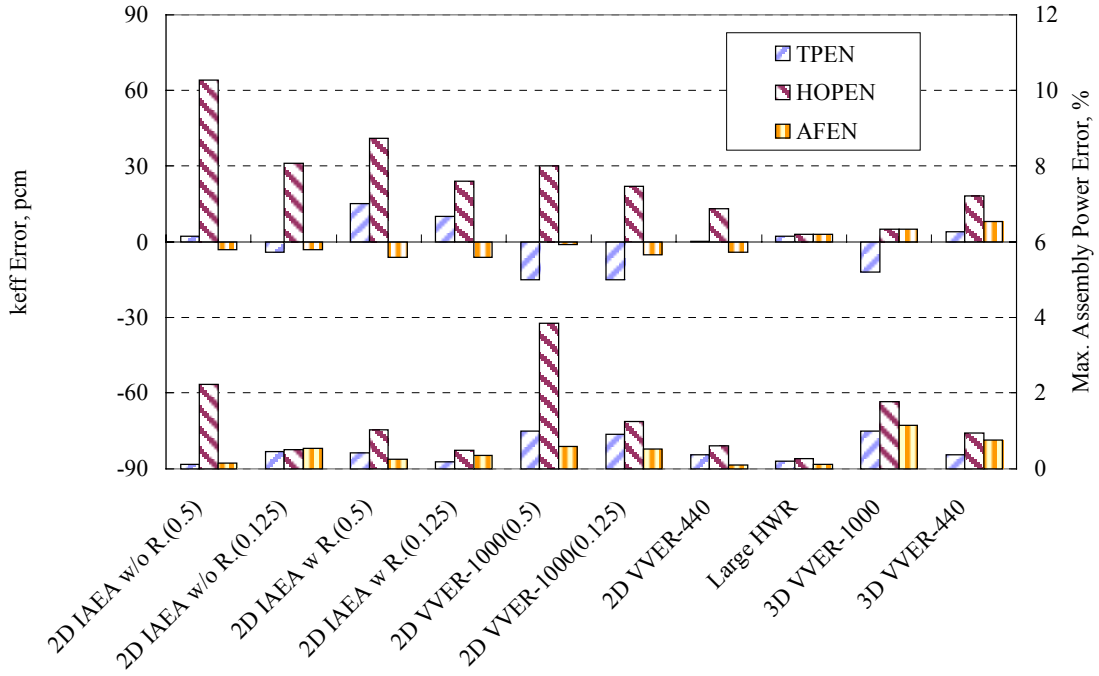


Fig. 4 TPEN Solution Accuracy for Hexagonal Eigenvalue Benchmark Problems

4.2 VVER1000 Rod Ejection Benchmark Result

This benchmark solves a 3-rod ejection problem for a hexagonal core consisting of 163 fuel assemblies. The initial power of the core is 10% of nominal rated power and the transient core power reaches about 90% after the rod ejection. The cross sections, kinetics parameters, boundary conditions, T/H inlet conditions and fuel property data are all given and thus the problem is self-sustaining.

The steady-state critical boron concentration search at the initial 10 % power yields 1374 ppm. This value is a little bit off from 1405 ppm given in the report (Podlazov, et al., 1997). However, about 30 ppm is not considered significant considering that there are many modeling discrepancies between the two codes including T/H modeling as well as neutronics.

The transient core power behaviors obtained for the rod ejection are given in Fig. 5 and compared with the results of SAS-DIF3DK and TENAR (Podlazov, et al., 1998). Two results from TPEN transient calculations were obtained, one using 2 msec time step size and the other using 10 msec as the primary time step size for the rapid power change region. Fig. 5 shows that the solution accuracy of the two cases from TPEN calculation are essentially the same. The TPEN transient core power behaviors around the peak power range are very similar with TENAR but about 10 % higher than SAS-DIF3DK. The core powers of TPEN at the end of core transient are more similar with SAS-DIF3DK than TENAR. The computing time data for each MASTER solution module is

also given in Table 2 for the 10 msec time step case. The total computation time is about 50 seconds for the 5 sec transient and the spread of the computing time of the three major solution modules is quite reasonable.

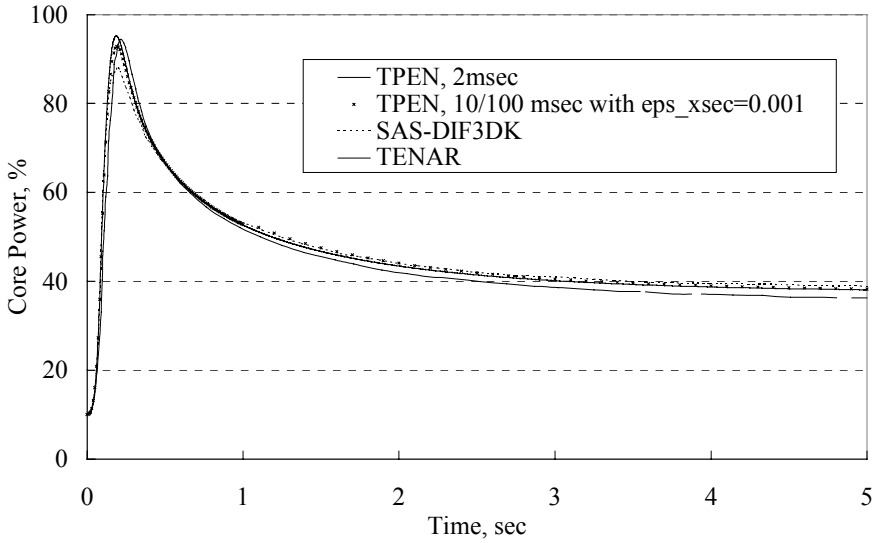


Fig. 5 Transient Core Powers for the VVER1000 Rod Ejection Benchmark

Table 2 Computing Time for Transient Problems(733 MHz Pentium-III PC, sec)

Breakup	CMFD	Nodal	T/H	Miscellaneous	Total
VVER-1000	20.8	10.2	12.1	6.3	49.4
VVER-440	67.1	39.2	78.4	8.9	193.6

4.3 VVER440 Rod Ejection Benchmark Result

This benchmark problem is available at the following web site:

http://www.kfki.hu/~aekihp/AER_home/bench_book

This three-dimensional dynamic benchmark(Test ID: AER-DYN-003) in hexagonal core geometry concerns a control rod ejection accident in a VVER-440 core. It includes the modeling of thermal hydraulics in the core and of the resulting reactivity feedback effects. Initially the core is at the hot zero power state(10^{-6} of full power) of end-of-cycle and the transient occurs by ejecting one control rod in 0.16 sec. There is no reactor scram and after a power excursion the reactor stabilizes at a power level determined by the feedback effects. This benchmark is a further development of the benchmark AER-DYN-001 and –002.

The transient core power behaviors obtained for the rod ejection are given in Figure 6. One curve in the plot was obtained using 5 millisec time step size and another curve using 10 msec for the rapid power change region for more practical case. The solution accuracy of the two cases are essentially the same and very similar with DYN3D result which is available in the above web site. The computing time data for each MASTER solution module is given Table 2 for the 10 msec case. The total computation time is about 194 seconds and the spread of the computing time of the three major solution modules is quite reasonable.

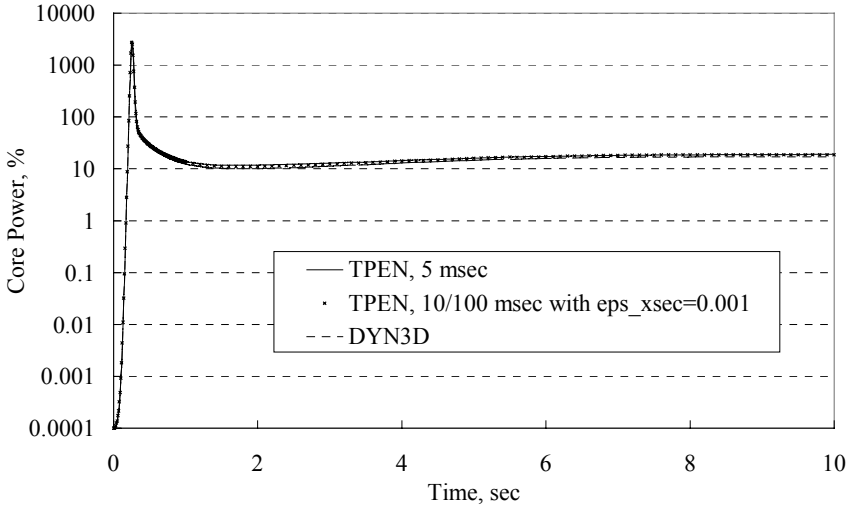


Fig. 6 Transient Core Powers for the VVER440 Rod Ejection Benchmark

5. CONCLUSIONS

A hexagonal CMFD formulation was established employing the triangle-based polynomial expansion nodal (TPEN) method that combines the higher order polynomial expansion nodal (HOPEN) method and the nodal expansion method (NEM) by decoupling the three-dimensional neutron diffusion equation into a radial and an axial one. The TPEN method solves one hexagonal node problem consisting of 6 triangular nodes concurrently by algebraic direct elimination. The corner point calculation scheme was simplified by using a hexagon based intranodal expansion that requires information only from the CMFD results. The axial leakage and transient fixed source are combined and approximated by a 2-dimensional polynomial whose coefficients are determined by 7 hexagon average values. The examination results obtained for several eigenvalue benchmark problems and two transient problems demonstrated that the computational speed of this method is measured to be about 5 times faster than the original HOPEN method with CMR acceleration. This method is also very accurate showing less than 15 pcm of eigenvalue errors and less than 1% maximum assembly power errors for the all eigenvalue benchmark problems. The transient core power behaviors of the rod ejection benchmark problems of TPEN method are also reasonable comparing with references.

Thus it can be concluded that the CMFD formulation with the TPEN kernel is an efficient and accurate hexagonal calculation scheme.

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