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FIELD RECONSTRUCTION FROM MEASURED VALUES USING SYMMETRIES

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ABSTRACT

A frequently encountered problem in engineering, for example, is that of determining the distribution of a physical quantity in a large, symmetric volume from a finite number of measured values. In this context, we address two model problems: Under what conditions can we reconstruct a field from a few known values? What can we say about the field from an estimate of random values at fixed positions? Our investigation is based on invariance properties of the system equation from which the measured physical quantities are derived. No specific form of the system equation is used. We show that there are cases when the missing values can be inferred without information loss. Criteria are given for the properties of the measured values to avoid information loss.

1. INTRODUCTION

Let us consider the following two problems. *Problem 1.* Given a symmetric volume V, and a smooth function $F(x)$, $x \hat{I} V$, we know only the values of F at certain positions $F(x_i)$, $i=1, ...,n$. Under what conditions is it possible to reconstruct $F(x)$ in volume V? *Problem 2.* As in Problem 1, but we do not know $F(x_i)$ but a function $F(x_i,x_i)$, where x_i is a random variable. Knowing the distribution of $F(x_i, x_i)$, what can we say about $F(x)$?

The above problems emerge in particular in connection with the operation of nuclear power plants. For the safe and economic operation of nuclear power plants, early detection of any anomaly and careful analysis of all the measurements reporting the actual core state are necessary. Processing of in-core measurements has a paramount role, because these measurements report phenomena occurring in the core, where the energy is being released.

Every power plant has algorithms for processing in-core measurements. Most of these algorithms include a calculation model, which supplements the information missing

from the measurements. Our goal is to provide estimates for values at non-measured assemblies without relying on additional information. Why is this approach necessary? We believe that a calculational reactor physical model can be an error source because:

- The calculational model is a compromise between relevant and irrelevant things. For example, a part of the input to the model is the actual core state (including fuel burnup, fuel and moderator temperatures) that should be averaged over a certain time interval. The averaging interval depends on the phenomenon to be investigated: burnup, readings of thermocouples, certain neutron detectors require different averaging periods. The noise from an industrial ambiance may result in an input that depends on the averaging period.
- The approximations in the calculational model inevitably introduce errors. These errors are then established in a validation and verification process. Since this process needs measured values, situations arise where the model error is hardly known, e.g. when a new fuel type is introduced, or when a new load pattern is investigated.

In view of the above, it would be beneficial to determine the actual core status based on the measured values alone. Our aim is to exploit all the available information on the measured values. We assume that the measurements take place in a symmetric volume V, the measured quantities are linear functionals of the neutron flux, which satisfies an equation. This equation is invariant under the symmetries of volume V. This is true for the multigroup diffusion equation as well as for the Boltzmann transport equation (Henry, 1975). At the same time the actual form of the equation (synthetic coefficients, averaged values, etc.) is not utilized. In the following, we deal with nuclear power reactors, but our considerations apply to a number of other industrial devices as well.

We assume that the measured fields are governed by an equation; or, the measured values are functionals of a function governed by an equation. We call this equation the system equation. We assume the system equation to have a unique solution, hence, the measured values are unique. This is the case with neutron diffusion (Habetler and Martino, 1961) and transport (Shihov, 1976). In those cases the measured values stem from the neutron flux (power, energy release). We assume furthermore that symmetries of V leave the system equation invariant. These assumptions hold for a number of physical problems. We also assume that early anomalies can be considered as perturbations.

Let us subdivide volume V into congruent nodes, and when we speak of a function, distribution, etc. it suffices to specify one value per node. This is a generally accepted practice even if the nodes are too large to be described by a single average value.

2. PROPOSED APPROACH

Our analysis is based on the general features of the system equation. Under the conditions stipulated in Section 4, the solution is decomposed into orthogonal components called irreducible components (Makai, 1996), (Makai and Orechwa, 1999). We assume the system equation to be an eigenvalue problem:

$$\mathbf{A}_0\Phi_0(x) = l_0\Phi_0(x). \qquad (1)$$

Here $\mathbf{A}_0$ is a linear operator, $\Phi_0(x)$ is called the system function, from which the measured physical quantity *F(x)* can be determined. We assume $\Phi_0(x)$ and *F(x)* to have the same symmetry properties. This assumption usually holds because the transformation is often a multiplication by a constant, which is a characteristic of the measuring device.

The assumption of a symmetric volume implies volume *V* has transformations mapping *V* into itself; these transformations are called the symmetry group of *V*. We assume operator **A** commutes with the symmetries. The existence of transformations commuting with the system equation is exploited in the same way as the existence of a commuting Hermitian operator (Falicov, 1996). Commuting operators have a common eigenvalue set, hence we can expand the solution of the system equation in terms of the eigenvectors of the given Hermitian operator. Details are given in section Terminology and Notation; we use the terminology introduced there.

Let *G* stand for the symmetry group of *V*, $|G|$ is the number of symmetries in group *G*, and the orthogonal bases spanning subspaces invariant under group *G* are called irreducible representations or irreps. The ground is a part of volume *V*. When elements of *G* are applied to it, the transforms cover *V*. In order to introduce the technique, we investigate the most concise storage of a distribution in a symmetric volume *V* when *V* is composed of congruent nodes and the distribution is characterized by one value per node.

Contraction Theorem Let function *F(x)* be given by one value per node in *V*. If *F(x)* has less than $|G|$ irreducible components, the most concise storage of *F* is when one value is stored per node in the ground for each irreducible component and the index of the irreducible components are also stored.

The Contraction Theorem (Makai and Orechwa, 2000) will be exploited in the subsequent sections in such a way that if the measured field is not contractible, the missing measurements are filled up on account of increasing error. The error enlarges because we do not know which irrep should be taken. If the measured field is contractible, then the missing values are supplemented with no error increase. The original intention behind the limited number of measurements in a symmetric device is the assumption that the measured field belongs to the unit representation, i.e. the same shape can be seen in each sector. If this assumption does not hold, then either we see noise of small amplitude or something has happened and the symmetry is no longer true.

The second, well-known technique we use is linear perturbation theory. At an early departure from the nominal state, the disturbance is small enough to be considered as a perturbation. Using the linear perturbation theory, we get simple equations where the disturbance occurs as a source. Without knowing the exact form of the system equation, we are able to deduce properties of the perturbed solution from the structure of the perturbation equations.

Perturbation Theorem Let us consider the perturbation caused by disturbance $\mathbf{A}_1$, in the equation

$$(\mathbf{A}_0 + e\mathbf{A}_1)(\Phi_0(x) + e\Phi_1(x)) = (l_0 + e\,l_1)(\Phi_0(x) + e\Phi_1(x)) - \qquad (2)$$

Then, the following statements hold:

1. $\mathbf{A}_1$ contributes to the change of the eigenvalue in the first order perturbation theory if and only if $\mathbf{A}_1$ has a component transforming as the unit representation.

2. If disturbance $\mathbf{A}_1$ can be considered as a perturbation and the second and higher order terms may be neglected, the solution of the perturbed problem transforms, as does $\mathbf{A}_1$.

3. If $\mathbf{A}_1$ is known except from a multiplicative constant, and the perturbed solution transforms as does the perturbed solution, then the perturbation is weak enough to neglect the second and higher order terms in the solution.

4. The second order term of the solution always contains a non-singular, symmetric component, irrespective of the symmetry properties of $\mathbf{A}_1$. There is always a second order term in the eigenvalue whatever the symmetry of $\mathbf{A}_1$.

The perturbation theorem (Makai and Orechwa, 2000) has non-negligible consequences. In agreement with Item 1, if the first order perturbation formalism indicates a change in the eigenvalue (in nuclear reactors this is a reactivity change) then the disturbance has to have a symmetric component. Otherwise, the disturbance has to have only asymmetric components. According to Item 2, the disturbance and the perturbation have the same symmetry properties. This is key to excluding possible disturbances based on the measured values alone.

In order to determine $F(x)$, we measure it at certain locations x_k, $k=1, ...,m$. The measured value, however, has also a random component. In order to account for that, the measured value at x_k is written as $F(x_k,x_k)$, where x_k is a random variable. The expectation value of $F(x_k, x_k)$ is written as $\mathsf{E}(F(x_k, x_k))$ the variance of the measured value is s_k. The solution to Eq. (1) may contain parameters. The exact solution may depend on parameters in operator $\mathbf{A}_0$ such as a synthetic cross-section, or an effective parameter. The approximate solution depends on coefficients of the trial functions or free parameters in the trial functions. We use the notation $F_{0p}(x)$ or $F_p(x)$ to refer to the parameters explicitly.

We assume the measured values to have a Gaussian distribution, independent at each measured location. We assume the maximum likelihood estimation

$$Q_{\min} = \min_p \sum_{k=1}^{m} \frac{1}{\mathbf{s}_k^2} \left(F(x_k, x_k) - F_p(x_k)\right)^2 \tag{3}$$

to give a non-biased estimate for parameter p.

The following theorem summarizes the statistical aspects (Seber and Wild, 1989) of linear regression utilized throughout our analysis. We put forth some notation. We form a matrix from the elements of irreducible vectors e_{ik} as $\boldsymbol{X}_{i,j}=e_{ij}$, $i=1, ...,m$; $j=1, ...,n$. We assume $\boldsymbol{X}$ is of full rank n. The measured values are also contracted into a vector:

$$\underline{F} = \left(F(x_1),...,F(x_m)\right). \tag{4}$$

Least Square Fit Theorem. Let us consider the least squares problem

$$Q = \min_{\underline{\Phi}} (\underline{F} - \mathbf{X}\underline{\Phi})^2 \tag{5}$$

in connection with which the following statements hold:

1. Let $\underline{F}= \mathbf{X}\underline{\Phi}+\underline{\varepsilon}$, where $\underline{\varepsilon}\approx N_m(\mathbf{0},\sigma^2 \mathbf{I}_m)$ and $\underline{\Phi}$ plays the role of the parameter to be fitted. The least square estimate $\hat{\underline{\Phi}}$, which minimizes $Q_\Phi=|\underline{F}- \mathbf{X}\underline{\Phi}|^2$, is the solution of the normal equations $\mathbf{X}'\mathbf{X}\hat{\underline{\Phi}} = \mathbf{X}'\underline{F}$, namely $\hat{\underline{\Phi}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\underline{F}$. Then,
$$\hat{\underline{\Phi}} \approx N_m\left(\underline{\Phi}, s^2(\mathbf{X}'\mathbf{X})^{-1}\right) \tag{6}$$
Here $N_m(\underline{a}, \boldsymbol{A})$ is an m-dimensional normal distribution with mean $\underline{a}$ and dispersion matrix $\boldsymbol{A}$.
2. An unbiased estimate of σ^2 is $s^2=Q(\hat{\underline{\Phi}})/(m-n)$. Since $\mathbf{X}\hat{\underline{\Phi}} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\underline{F}= \mathbf{P}_X\underline{F}$ where $\mathbf{P}_\mathbf{X}$ is a projection matrix, we have
$$Q(\hat{\underline{\Phi}}) = \left\|\underline{F} - \mathbf{X}\hat{\underline{\Phi}}\right\| = \underline{F}'(\mathbf{I}_m - \mathbf{P}_X)\underline{F} = \underline{F}'\mathbf{R}_X\underline{F}. \tag{7}$$
Also $\hat{\underline{\Phi}}$ is statistically independent of $Q(\hat{\underline{\Phi}})$ and $Q(\hat{\underline{\Phi}})/\sigma^2 \approx \chi_{m-n}^2$.
3. The residual vector for the linear model is $\hat{\underline{e}} = \underline{F} - \mathbf{X}\underline{\Phi} = (\mathbf{I}_m - \mathbf{P}_X)\underline{e}$. as $\mathbf{P}_\mathrm{X}$ X=X. From the normal equations we have $\mathbf{X}'\hat{\underline{e}} =\mathbf{0}$. Also, $\mathsf{E}(\hat{\underline{e}})=0$ and $\mathsf{D}(\hat{\underline{e}}) =\sigma^2(\mathbf{I}_m - \mathbf{P}_\mathrm{X})$.
4. From the normal equations it can be shown that
$$Q(\underline{\Phi}) = \left\|\underline{F} - \mathbf{X}\hat{\underline{\Phi}} + \mathbf{X}\left(\hat{\underline{\Phi}} - \underline{\Phi}\right)\right\|^2 = Q(\hat{\underline{\Phi}}) + \left(\hat{\underline{\Phi}} - \underline{\Phi}\right)\mathbf{X}'\mathbf{X}\left(\hat{\underline{\Phi}} - \underline{\Phi}\right).$$
5. When $\boldsymbol{X}$ does not have full rank n, the normal equations do not have a unique solution. However, the solution with minimum norm, i.e. that which minimizes $\|\underline{\Phi}\|$, is $\hat{\underline{\Phi}} = \mathbf{X}^+\underline{F}$, where $\boldsymbol{X}^+$ is the Moore-Penrose generalized inverse of $\boldsymbol{X}$.

By means of the above observations (Seber and Wild, 1989), we derive the framework of an in-core signal processing method based on first principles. Our method relies on no additional assumption. For an illustration, we provide a case study. The processing of assembly outlet temperature measurements is discussed in a hexagonal VVER-440 core, and we present the basic techniques at work.

3. RECONSTRUCTION

When field $F(x)$ is known only in a fraction of the nodes and we try to find out the missing values, we are actually creating information. The source of the created information is usually in a calculation model. In order to base the processing of measured data on measured values only, we have to avoid the use of a calculational model. First we ask whether the information carried by the finite number of values in the metered positions is sufficient to determine the missing values. For this, we rely on the Compression Theorem, which gives the conditions for the existence of fields $F(x)$ such that it is possible to supplement the missing values without error increase. We need to determine if the measured field is compressible. Hence, we need to determine the number of irreducible components present in the measured field. This is achieved below.

Points in V are arranged into equivalence classes. Let $x_1, x_2, ...,x_n \in V$, they are considered as identical if there is a symmetry $\mathbf{O}_i$ for every x_i so that $x_i=\mathbf{O}_i x_1$ with x_1 lying on the ground. These points are said to form an orbit $O \in V$.

First we deal with *Problem 1*. In agreement with Eq. (28), the measured field $F(x)$ transforms according to the k^{th} irrep of group G if

$$F(x) = j_k(x_0)e_k(\arg(x)) \quad (8)$$

where $arg(x) \in 2p$, x_0 is a point in the ground of the orbit passing through point x. Since the function $e_k(J)$ takes only integer values, see Terminology and Notation, the above expression gives $F(x)$ as a product of two terms. The first term $j_k(x_0)$ varies in the ground with varying x_0, the second term (e_k) varies with the sector to which x belongs. So, if $F(x)$ is expressed as

$$F(x) = \sum_{k=1}^{n} j_k(x_0)e_k(J), \quad (9)$$

then, $F(x)$ has n irreps. Here x determines both x_0 and J, the former being the position in V_0 corresponding to x, the latter being the phase of x. Since $e_k(J)$ takes the same value in a sector, we can check if (9) is satisfied by fitting a sum of ground function $j_k(x_0)$ and sector amplitude $e_k(J)$ to the measured field $F(x)$. Since $e_k(J)$ is the same inside a sector, we replace it by e_{ki}. So, in principle, if the minimum of

$$Q = \min_{j_k} \sum_i \left[F(x_i) - \sum_{k=1}^{n} j_k(x_{0i})e_{ki} \right]^2 \quad (10)$$

is zero, where subscript i runs over the measured positions, then the field $F(x)$ is compressible provided that $n<½G½$. On the other hand, whenever $Q>0$, the field $F(x)$ is only approximated by (9). In a given orbit O, j_k is a constant and differentiating (10) with respect to j_k, we get

$$\sum_{i \in O} \left(F(x_i) - \sum_{k=1}^{n} j_k e_{ki} \right) e_{k'i} = 0 \quad (11)$$

The summation runs over those positions of the orbit where there are measurements. Since vector $\underline{e}_k$ is orthogonal to vector $\underline{e}_{k'}$, $k' \neq k$ when each position of the orbit is measured, j_k is only a transformation of the measured values:

$$j_k = \frac{\sum_{i=1}^{n_s} F(x_i)e_{ki}}{\sum_{i=1}^{n_s} e_{ki}e_{ki}}. \quad (12)$$

If not all the positions are measured in orbit O, we introduce

$$q_{k'} = \sum_{i \in O} F(x_i)e_{k'i} \quad (13)$$

compress the js and the qs into $\underline{j}=(j_1, ..., j_n)$ and $\underline{q}=(q_1, ..., q_n)$, respectively. Matrix $\boldsymbol{X}$ is given by

$$\mathbf{X}_{kk'} = \sum_{i=1}^{n} e_{ki} e_{k'i} \ , \tag{14}$$

where the summation runs over the measured positions of orbit O. Matrix $\boldsymbol{X}$ is invertible when the "$\underline{e}$" vectors are linearly independent in the measured positions. We exclude those combinations which would lead to a singular $\boldsymbol{X}$ matrix. Because we have no indication which selection of the "$\underline{e}$" vectors is preferable, we carry out every possible fitting, and determine the maximum, minimum and average values of the fitted parameters.

The values of the field $F(x)$ are often positive and almost of the same magnitude. This is the case in the core of a nuclear reactor, for the power released in the assemblies, or, the average coolant temperature in an asembly. Let $\underline{f}=(f_1, ..., f_{ns})$ stand for the known values along an orbit O. If the $\underline{e}_i$ vectors are orthonormal, we have

$$\underline{f} = \sum_i \left(\underline{e}_i^{+} \underline{f}\right) \underline{e}_i = \sum_i c_i \underline{e}_i \ . \tag{15}$$

Let us order $\underline{e}_i$ in decreasing order: $c_i > c_j$ if $i<j$. To account for possible non-metered positions in orbit O, we write $\underline{f}^*=\boldsymbol{P}\underline{f}$ for the known values along O. Vector $\underline{e}_i$ projected to the metered positions is written as $\underline{e}_i^*=\boldsymbol{P}\underline{e}_i$. We have to remember, however, that $(\underline{e}_i^*, i=1, ..., n_s)$ may be linearly dependent and the coefficients c_i^* in the expansion

$$f^* = \sum_i c_i^* \underline{e}_i^* \tag{16}$$

can not be determined by multiplying both sides with $\underline{e}_j^*$. If $F(x)$ is reducible, then $c_i^*=0$ when $i>n$. In this case we are able to reconstruct field $F(x)$ exactly provided $\boldsymbol{P}$ projects to a $k \cdot n$ dimensional subspace. We may assume the first n irrep to be used in the reconstruction. If $F(x)$ is not reducible, we may assume $c_i^*=e_i \, c_1^*$, for $i>1$ where $e_i<1$.

The selected approximation satisfies (17), i.e. the known values in orbit O are preserved. Therefore, $\mathbf{P}\left(\underline{F} - \sum_i c_i^{appr} e_i\right) = 0$. From this, we get

$$0 = \sum_{i=1}^{n} \left(c_i - c_i^{appr}\right) \mathbf{P}\underline{e}_i + \sum_{i>n} \left(c_i - c_i^{appr}\right) \mathbf{P}\underline{e}_i \ , \tag{17}$$

but in the second sum $c_i=0$ under the stipulated conditions, thus, we have

$$\sum_{i=1}^{n} \left(c_i - c_i^{appr}\right) \mathbf{P}\underline{e}_i \le + \sum_{i>n} c_i^{appr} \mathbf{P}\underline{e}_i = \sum_{i>n} e_i \mathbf{P}\underline{e}_i \ , \tag{18}$$

that we formulate as follows. Let $\underline{F}=\{F(x): x \in O\}$ the field along orbit O, its decomposition into irreps is

$$\underline{F} = \sum_{i=1}^{n} c_i \underline{e}_i \ , \tag{19}$$

and an approximation obtained for $F(x)$ is $F^{appr} = \sum_i c_i^{appr} \underline{e}_i$, where c_i^{appr} is a solution to Eq. (16). We also assume $c_i^{appr} = e_i c_1^{appr}$. Then $\underline{F} - \underline{F}^{appr} \le (e_i \mathbf{P} \underline{e}_i) c_1^{appr}$.

The theorem assures the reconstructed field (19) to be close to the exact field. The theorem sets an upper limit, but the elements of $\mathbf{P}\underline{e}_i$ are positive as well as negative values, this implies that the exact field is between the range spanned by the available estimates.

If $F(x)$ is a measured field, we encounter *Problem 2*, and then Eq. (10) never holds. In this case we resort to Assumption 1. Now, instead of Eq. (10), we have to find the following minimum

$$Q(x_1, \cdots, x_m) = \min_{j_k} \sum_{i=1}^{n} \left[F(x_i, x_i) - \sum_{k=1}^{n} j_k (x_{0i}) e_{ki} \right]^2 . \tag{20}$$

According to item 2 in Least Square Fit Theorem, Q_{min} is random, its distribution is chi-square with $(m-n)$ degree of freedom provided Assumption 1 holds. Its expectation value is (m-n). In order to see if Assumption 1 holds we perform a hypothesis test. From Eq. (5) and item 3 of Least Square Fit Theorem, we can estimate the error of the fitted parameter (Szatmáry, 1977):

$$j_k = \frac{Q_{\min}}{n-m} \mathbf{M}^{-1}_{kk'}, \ \mathbf{M}_{kk'} = \sum_i e_{ki} e_{k'i} \ . \tag{21}$$

Here the summation runs over all measured positions. When the number of fitted parameters (m) is close to the number of measured points n, the error increases and when $n=m$ we have zero in the nominator (then $Q=0$) and also in the denominator (then $n=m$) of Eq. (20). The estimation for the field $F(x)$ is

$$F(x) = \sum_{k=1}^{n} j_k (x_0) e_k (\arg(x)) . \tag{22}$$

The variance of this expression is estimated by the general error propagation formula. For arbitrary $f(x,y)$ we get:

$$df = \partial_x f(x,y) dx + \partial_y f(x,y) dy , \tag{23}$$

where df denotes the variance of , and f depends on variables x and y with variances dx and dy, respectively. If $F(x)$ is not compressible, a number of fits can be carried out. The different fitted values give rise to the error of $F(x)$. This term is missing for compressible fields. We can do more than supplementing the values for the non-metered positions. Whenever we detect the symmetry of the field $F(x)$, we know that the disturbance $\mathbf{A}_1$ causing the departure from the nominal state possesses the same symmetries. This observation may give us a chance to identify the possible disturbance if we have a small

number of choices. In practice, we do have a small number of disturbances in the technology, and if we have no matching cause we may suspect either something new (e.g. appearance of crud) or something is wrong with the electronics of the measurement. As to the statistics of j_k, let us consider an orbit O, comprising points $(x_1, ..., x_n)$.We write the measured values at x_i as $F(x_i)=y_i+h_i(x_i)$, where random variable ξ accounts for the randomness of the measured value. We assume, furthermore, the measurement to be unbiased: $E(h_i(x_i)) = 0$. Here $E(\xi)$ forms the average of ξ. According to Eq. (12), we have

$$j_k = \sum_i e_{ik} F(x_i) = \sum_i e_{ik} \left(y_i + h_i(x_i) \right)$$

and

$$E\left(j_k\right) = \sum_i e_{ik} y_i \tag{24}$$

$$E\left(\left(j_k - E\left(j_k\right)\right)^2\right) = \sum_{i,j} e_{ik} e_{jk} E\left(h_i(x) h_j(x)\right). \tag{25}$$

Thus, the expectation value of j_k is a linear expression of the expectation values of the measurements on orbit O, whereas variance of j_k depends on the correlation matrix of the measured values on orbit O.

4. CASE STUDY

Let us apply the above considerations to a VVER-440 in-core measurement. The radial structure of the VVER-440 core (it is our volume *V* in the sequel) is shown in Fig. 1.

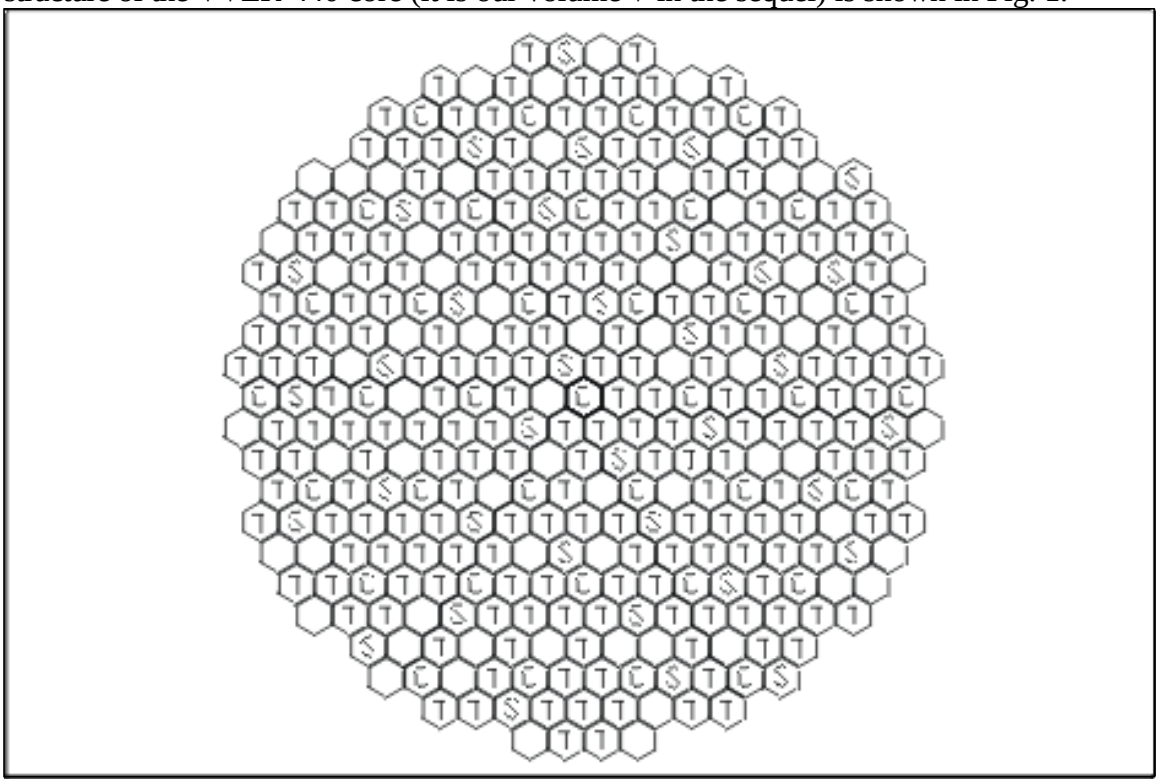

Fig.1. VVER-440 core and instrumentation
C-control assembly, T-thermocouple, S-SPND detectors

The core is composed of 349 hexagonal fuel assemblies. Two kinds of in-core measurements are implemented, there are 210 outlet temperature measurements and 36 fuel assemblies are equipped with self powered neutron detector sets, each set containing 7 detectors at 7 axial elevations. The core height is 250 cm. The outlet temperature is assumed to be the average coolant temperature at the exit from the assembly. The core pattern, which depends on the material properties of the assemblies loaded into symmetrical positions, is assumed to have 60 deg symmetry. The assemblies are consecutively numbered from 1 to 349. The central assembly has number 175. Assembly No. 1 is the uppermost left assembly, the numbering goes from left to right in a row. The ground now is a 60 deg sector. For simplicity's sake the central assembly belongs to sector No. 1 only. There are 59 positions in the ground. Although the core might have 30 deg symmetry, the load usually has only 60 deg symmetry. For this reason, we speak of 6 sectors and the "$\underline{e}$" vectors have 6 components.

First, we have to explore the symmetries of V. There are 6 sectors of 60 deg, which are transformed into each other by k*60 deg rotations, k=1,...,6. There are 6 symmetry planes in the core crossing the corners and mid-side points of the central assembly. We see altogether 12 symmetries, they form a group called C_{6v}, its character table is given in Table 1.

From (Atkins, 1970), we know that group C_{6v} has 6 classes: E, C_2, C_3, C_6, {d_1, d_2, d_3}, {v_1, v_2, v_3}. In the column labels of Table 1, the number of symmetries in the given class is also indicated. Here C_n denotes a rotation by n times $\pi/3$, d is a reflection through a plane crossing face centers, v represents a reflection through a plane crossing vertices. The symmetric, one-dimensional representations are labeled A, the asymmetric ones B, the two dimensional representations E. There are 4 one-dimensional, and two, two-dimensional irreducible subspaces, each having an equivalent subspace. Hence, there are 12 independent bases in them. Thus, we have exhausted the symmetry properties of the core.

Irrep	E	C_2	C_3	C_6	d_1,d_2,d_3	v_1,v_2,v_3
A_1	1	1	1	1	1	1
A_2	1	1	1	1	-1	-1
B_1	1	-1	1	-1	1	-1
B_2	1	-1	1	-1	-1	1
E_1	2	-2	-1	1	0	0
E_2	2	2	-1	-1	0	0

Table 1. Character Table for a Regular Hexagon (Group C_{6v})

The 12 element vectors can be reduced to 6 element vectors allowing for even and odd pairs in the 6 positions of the vector. We will use 6 element vectors. Accordingly, we subdivide the core into 6 sectors, each having 59 positions. The next step is to derive the $\underline{e}_i$ vectors characterizing the irreps. This is done by means of the character table and Eq. (26). The obtained vectors are given in Table 2.

We point out that there is an alternative basis vector set which is obtained by the principal component method (Makai and Temesvári, 1992). In this method, a 6 by 6 matrix is derived from the 210 measured values and the eigenvectors of that matrix are used as basis vectors in Eq. (27).

Irrep	Subspace	Vector elements
$\underline{e}_1$	A_1	(1,1,1,1,1,1)
$\underline{e}_2$	A_2	(1,-1,1,-1,1,-1)
$\underline{e}_3$	E_1	(2,-1,-1,2,-1,-1)
$\underline{e}_4$	E_1	(0,1,-1,0,1,-1)
$\underline{e}_5$	E_2	(2,1,-1,-2,-1,1)
$\underline{e}_6$	E_2	(0,1,1,0,-1,-1)

Table 2. Basis Vectors for Group C_{6v}

We show the estimated values for non-metered assemblies in Table 3 and 4. There are 59 orbits, 7 of which have no measurement. These orbits include only control assemblies, which have no meter. Other orbits have at least one measurement. An example is the orbit (155, 156, 174, 176, 194, 195) with measured values in assemblies Nos. 156, 176, 194 and 195. The measured field is not compressible, hence, we have to try every reasonable fit involving 4 vectors. This means 24 possible fits. The expectation value, and the maxima and minima along with the measured values are given in Table 3. Similarly for the orbit with 5 measured positions is shown in Table 4.

Orbit	Exp.	Max.	Min.	Measured
155	297.7	298.3	297.3	--
156	297.0	297.7	296.8	296.8
174	297.7	298.5	297.1	--
176	298.0	298.2	297.6	298.1
194	297.9	298.4	297.3	297.9
195	297.9	298.3	297.4	298.1

Table 3. Features of orbit (155, 156, 174, 176, 194, 195)

Orbit	Exp.	Max.	Min.	Measured
65	295.4	295.7	295.3	295.3
77	296.1	296.3	296.0	296.1
161	296.0	296.3	295.6	--
189	296.3	296.5	296.0	296.4
273	296.1	296.2	296.1	296.2
285	295.9	296.0	295.8	295.9

Table 4. Features of orbit (65, 77, 161, 189, 273, 285)

Comparing Table 3 and Table 4, we see large differences in the intervals of the non-measured positions. In Table t4, the intervals are (0.4, 0.3, 0.3, 0.5, 0.1, 0.2) whereas in Table 3 (1.0, 0.9, 1.4, 0.6, 1.1, 0.9). This clearly shows the missing measurements to have differing impact on the accuracy of the field in different positions.

We do not present qualitative investigations concerning perturbations. Since the most easily triggered perturbations are the first order perturbations, which display dipole type behavior, they are easy to detect. In a nuclear reactor, the two most frequent causes of such kind of perturbations are a single control rod motion, or disturbance in coolant flow (Siltanen and Anttila, 1984). The associated technique is simple and known: there is a selected trial function set, each one describing a given disturbance. The evaluation process checks if the share of any of the trial functions exceeds the noise level. If so, we are able to identify the underlying cause: a departure from normal core state (Makai and Szatmáry, 1986), (Adorján, 1992).

5. CONCLUDING REMARKS

Most technological devices have a symmetric geometrical form V. Nuclear and chemical reactors, and containers should have as few measurements as possible. In most cases, the physical equation describing the process in the device has a unique, symmetric solution and is invariant with respect to the transformations mapping V into itself. The method presented in this work is based on the following assumptions:

1. There is a system equation from which the system function is determined. In nuclear reactors, the system function is the neutron flux.
2. The system equation (1) is invariant under the symmetries of V.
3. The transformation between the system function and the measured field does not alter the symmetry properties. In nuclear reactors this transformation is between the neutron flux and power release.
4. In the analysis the explicit form of the system equation is never used.

Assumptions 1-4 are generally valid, hence, no additional information is needed in the evaluation process. The measured field determined by Eq. (2), carries only that information which is present in the measured values. Hence, there is no need to repeat the validation and verification process when a modification (new core load or fuel type) is introduced. The approach to measurement processing exploiting the symmetry properties of V has a number of advantages. We have shown that:

1. There exist fields where missing measurements do not increase the variance of the measured distribution. Such a field must satisfy Eq. (3). If the criterion fixed by Eq. (3) is not met, we have given a formula, Eq. (9), enabling one to calculate the increase of the variance because of the missing measured values.
2. We propose that one estimates the missing measured values by fitting Eq. (5). If $Q_{min}=0$, the non-measured values can be supplemented without error increase, but when $Q_{min}>0$ the missing measurement leads to an increase of the error. Besides Q_{min}, we must also pay attention to the variance of the fitted values. When the number of fitted parameters is close to the measured values, the increase may be considerable.
3. If we have a limited set of possible disturbances, we can select from them the actual one by the observation that the symmetry of the perturbation is the same as the symmetry of the disturbance.

4. The error of the reconstructed field is described by an interval which is obtained by performing a fit via Eq. (5) with the available choices of the indices of ($\underline{e}_k$, $k=1,...,n$) vectors in Eq. (3).

We have only touched on the problem of using the Perturbation Theorem, but it obviously leads to the identification of possible disturbances from a limited choice, and exclusion of other possible disturbances. Furthermore, to early detection of flow pattern anomalies. Our method is applicable in a number of devices as well as in a number of geometries precisely because the system equation remained unspecified.

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TERMINOLOGY AND NOTATION

Below we give the terminology and notation applied in the technique of utilizing symmetries of a finite volume *V*, in which a function *f(x)* is given.

Basic information on symmetry groups can be found in character tables available in a number of monographs (Atkins, 1970), (Schönert, 1995). The character table is a square table, its rows show how elements of an irreducible subspace transform under the elements of the group. The number of independent irreducible subspaces equals the number of conjugate classes or classes denoted by n_c. The table can also be used to project out bases of the irreducible subspace from an arbitrary function *f(x)*:

$$f_{ik}^{a} = \frac{h_a}{|G|}\sum_{O\in G}\Gamma_{ik}^{a}(O)\mathbf{O}f(x)\,, \quad \text{i=1,...,h}_\alpha\text{; k=1,...,h}_\alpha. \tag{26}$$

Here $G^{a}_{ik}(O)$ is the regular representation of symmetry O, h_α is the dimension of subspace α. If the basis vectors are numbered consecutively in the subspaces $\alpha=1,...n_c$, we get altogether $|G|$ basis vectors. These are called irreps. More precisely, a basis in an irreducible subspace is called an irrep. If $f(x)$ is defined for $x \hat{I} V$, and G is the symmetry group of V, then irreps of $f(x)$ are given by

$$f_i(x) = w_i(x_0) * e_i(\arg(x)) \tag{27}$$

where $\underline{e_i}$ is an irreducible vector of the regular representation of G. Since the $\underline{e_i}$ vectors are normalized arbitrarily and are the eigenvectors of matrices with integer coefficients, the associated eigenvalue is also an integer, its values can be selected as integers. Orthogonal $\underline{e_i}$ vectors are derived with the help of low order terms of Fourier series. If V is a regular body with n_F faces, the following vectors are regular representations of the symmetry group of V:

$$e_{1k} = \cos(0 * p / n_F * k)\,, \text{ k=1,...,n}_\text{F}. \tag{28}$$

$$e_{2i,k} = \cos(i * p / n_F * k)\,,\ \text{i=1, ..., } n_F/2;\ \text{k=1, ..., } n_F. \tag{29}$$

$$e_{2i+1,k} = \sin(i * p / n_F * k)\,,\ \text{i=1, ..., } n_F/2;\ \text{k=1, ..., } n_F. \tag{30}$$

The set $V_0=V/G$ is called ground. Its transforms under group G cover V. Let $x_0 \hat{I} V_0$. The set $\Omega=\{x: x=O_i x_0, \boldsymbol{O}_i \hat{I} G\}$ is called an orbit. An orbit has at most $|G|$ distinct points. The symmetry of volume V is an invertible automorphism of V, i.e. an invertible mapping of V into itself. Symmetry of an equation is a transformation leaving the equation invariant. In other words, if **AO**=**OA** then **O** is a symmetry of the equation

$$\mathbf{A}\Phi(x) = l\Phi(x) \tag{31}$$

Then, $\boldsymbol{OF}(x)$ is also a solution to (31). It is not necessary (Kawohl, 1998) that the invariance of the equation should entail the invariance of the solution unless we have additional conditions, too. Such an additional condition is, among others, the uniqueness of the solution, which is true for the fundamental solution of the linear Boltzmann equation (Shihov, 1976) and also for the multigroup diffusion equation (Habetler and Martino, 1961).

In Section 3, we introduced random variables to account for experimental error. Variables x and h (with or without subscript) are random variables, E and D are expectation value and variance, respectively.