MULTIPOINT METHODS IN NUCLEAR REACTOR KINETICS

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ABSTRACT

The paper presents a derivation of a multipoint model for neutron kinetic problems, which can be useful for the study of new concepts of multiplying systems, such as source-driven subcritical reactors and highly decoupled configurations. The multipoint equations are obtained by extending the classic separation-projection procedure employed to derive point kinetics. The procedure is general and it uses a subdivision of the phase space of the problem in a given number of zones, applying a separation hypothesis on each zone and projecting on a weight function which is chosen as solution of a proper adjoint problem. The numerical results presented highlight many physical situations for which the use of the multipoint model can be advantageous with respect to point kinetics, giving accurate results as compared to reference full calculations.

1 INTRODUCTION

The proposal of advanced reactor concepts is putting new challenges to reactor physics, for the assessment and development of adequate models and computational methods for the description of their physical behavior. A great deal of research work is being carried out in the field of subcritical accelerator driven systems (ADS) for energy production, long-lived radioactive product transmutation and other interesting technological applications. The physics of these systems is dominated by the presence of the external source injecting neutrons at energies higher than the typical values for fission neutrons and it requires new approaches, especially for the study of time-dependent phenomena. Moreover, some proposed designs foresee space-energy decoupled configurations, posing new difficulties to the calculations of spatial-spectral transients. These decoupled systems are constituted by two distinct regions characterized by very different neutron spectra (Kukharchuk, 2000). The central region of the system where the high-energy neutron source is located presents a fast spectrum, while the external region presents a thermal spectrum. The two regions are neutronically separated for low neutron energies by a shell of material highly absorbing for thermal neutrons, which prevents them to flow back into the central region.

The present work is devoted to some theoretical aspects and numerical studies and applications of the so-called multipoint method for reactor neutronics. The physics of a multiplying system is modelled by the neutron Boltzmann equation in its most general integro-differential form (Bell, 1970). Any discretization algorithm over the phase (geometrical space and velocity) space generates a multipoint problem, i.e., a system of time first-order equations coupling space and velocity point values of the neutron flux. However, the multipoint approach refers to a model which involves only a reasonably small number of points. Therefore, on one side one looks at the full discretized Boltzmann (or one of its lower level approximations) equation and on the other at the simplest of all (one-) point model.

The first multipoint model was developed by Avery (Avery, 1958). In its original work he presented both static and dynamic applications giving interpretations to the equations developed on physical grounds and he proposed a technique to evaluate coupling coefficients involving a modal representation of the neutron flux and a projection process on an adjoint function.

More recently, intense and significant contributions to the field have been given by Kobayashi. He has been developing the original idea by Avery and showing its application merits. In a series of papers (Kobayashi, 1990; Kobayashi, 1991; Kobayashi, 1992; Nagaya 1995; Kobayashi, 1996) he derived the multipoint model using a projection on the Green-function for the fission emission function of the problem and performed several applications in the time-dependent field, also proposing possible extensions to the quasi-static framework, and giving physical interpretations to the kinetic parameters.

The work here presented is based on Kobayashi's philosophy. Both Avery's and Kobayashi's multipoint models are to be regarded as multi-space point approaches. For some applications to accelerator-driven systems, however, it can be important also to keep more than one point in energy, to account for important spectral effects, which in some recent applications have been considered by a multigroup model in the point formulation (Rosselet, 2000). Therefore, we generalize to the phase space the multipoint concept, using proper importance functions to be determined as solutions of adjoint problems. The approach is quite general and it is particularly suitable for use in conjunction with studies concerning ADS.

Some results are presented for the validation of the procedure, in comparison with full numerical or analytical calculations on one side and with point-modelizations on the other. The model is particularly suitable for decoupled fast-thermal systems and it looks worthwhile for introduction into a generalized quasi-static scheme.

2 MULTIPOINT KINETICS

The deduction of the classic point-kinetic model (Henry, 1958) is based on the factorization of the neutron density in the product of a time-constant shape function for the whole reactor and of a time-dependent amplitude function. The full neutron balance equations are then projected on a weight function in order to eliminate the dependence on the phase-space point and to construct a system of equations in the time domain only, describing the evolu-

tion of the bulk of the neutron population, which is adequate in transients characterized by only small distortions of the neutron distribution. The natural extension of the method is the quasi-static scheme (Ott, 1985), in which the shape function for the neutron density is updated along the evolution of the transient to account for the changes which have occurred in the meantime. The normalization condition for the shape function is such as to let the amplitude function account for the fastest phenomena, so that only a few shape recalculation are needed. Two time scales are thus identified. The suitable weighting function is the adjoint function for the reference (initial) system, either critical or source-injected as the case may be.

Many situations cannot be adequately described by a point model, or, on the other hand, quasi-statics may require frequent shape recalculations. This is the case of decoupled systems as the ones previously described, which present attracting features for many applications. For these configurations it may be highly advantageous to use a technique which is still very simple and reduces to a calculation in the time domain only, while allowing for some space distortions in the phase space of the problem. The idea underlying the procedure herewith proposed to generalize Avery's procedure further exploited by Kobayashi relies on the subdivision of the domain of the phase space (in general, space and velocity) in a suitable number of subdomains, applying then a shape-amplitude separation in each of these subdomains. Henceforth, this multiple factorization is introduced into the balance equation and a projection operation is performed, in order to obtain a system of coupled differential equations in the time variable for the amplitude functions. Of course coupling is coming forward owing to the physical fact that neutrons born in one of the subdomains can generate fission neutrons in other subdomains, due to streaming in space and collision with energy and direction redistribution in velocity. Alternative formulations of the model can be attained using different weighting importance functions. In the following some examples are given using different approaches for the subdivision of the phase space to highlight specific aspects and using different adjoint problems to derive the weighting function.

2.1 Energy multipoint model

A useful introduction of the philosophy of the multipoint method can be given considering the energy problem. For instance a multi-group model is considered and to better describe spectral transient effects, different amplitudes are allowed for each group. Models of this type were recently developed and applied (Rosselet, 2000). To present the derivation of the model, a problem in diffusion theory is considered for a source-driven system:

$$\begin{cases} \dfrac{1}{v_g}\dfrac{\partial}{\partial t}\phi_g = \mathcal{L}_g\phi_g + \sum\limits_{g'}^{G}\Sigma_{gg'}\phi_{g'} + (1-\beta)\chi_{p,g}\sum\limits_{g'}^{G}\nu\Sigma_{fg'}\phi_{g'} + \sum\limits_{\alpha}^{R}\chi_{\alpha,g}\lambda_\alpha C_\alpha + S_g, \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad g = 1,...,G \\ \dfrac{\partial C_\alpha}{\partial t} = -\lambda_\alpha C_\alpha + \beta_\alpha\sum\limits_{g'}^{G}\nu\Sigma_{g'}\phi_{g'}, \quad \alpha = 1,...,R \text{ and } g = 1,...,G \end{cases} , \tag{1}$$

where, as usual: $\phi_g = \phi_g(\mathbf{r},t)$, $S_g = S_g(\mathbf{r},t)$ and $C_\alpha = C_\alpha(\mathbf{r},t)$; the space diffusion operator appearing above is defined according to: $\mathcal{L}_g = \boldsymbol{\nabla}\cdot D_g\boldsymbol{\nabla} - \Sigma_{Rg}$, and the fission spectrum

is $\chi_g = (1-\beta)\chi_{p,g} + \sum_{\alpha}^{R}\beta_\alpha\chi_{\alpha,g}$. The phase space of the problem is constituted by space and the energy groups. Initial conditions are determined as the distributions corresponding to a steady-state system characterized by properties which are denoted in the following by subscript 0.

To obtain the multipoint equations on the energy axis, it is assumed:

$$\phi_g(\mathbf{r},t) = A_g(t)\varphi_g(\mathbf{r}), \tag{2}$$

where the spatial shapes φ_g are assumed to coincide with the neutron fluxes of the steady source-driven initial configuration. A scalar product between functions f and g of direct and adjoint spaces, respectively, is defined as integration on the space variable over the physical system:

$$\langle f|g\rangle = \int_V d\mathbf{r} f(\mathbf{r})g(\mathbf{r}). \tag{3}$$

The weighting functions for the projection process are determined solving the system of equations:

$$\mathcal{L}_{g,0}\varphi_g^+ + \sum_{g'}^{G}\Sigma_{g'g,0}\varphi_{g'}^+ + \nu\Sigma_{g,0}\sum_{g'}^{G}\chi_{g'}\varphi_{g'}^+ + \nu\Sigma_{g,0} = 0,\ g = 1,...,G. \tag{4}$$

The solution for each group has the physical meaning of *importance* of a neutron injected in a specific energy group at a point in space, i.e., the total number of fission neutrons that are to be generated by it through the multiplication process. Each of the Eqs. (1) are now scalar-multiplied by φ_g^+ . Letting the following definitions:

$$I_f = \sum_{g}^{G}\sum_{g'}^{G}\left\langle\varphi_g^+|\chi_g\nu\Sigma_{g'}\varphi_{g'}\right\rangle, \tag{5}$$

$$\Lambda_g = \frac{1}{I_f}\frac{1}{v_g}\left\langle\varphi_g^+|\varphi_g\right\rangle, \tag{6}$$

$$\overline{\beta}_{\alpha gg'} = \frac{1}{I_f}\left\langle\varphi_g^+|\beta_\alpha\chi_g\varphi_{g'}\right\rangle, \quad \overline{\beta}_{gg'} = \sum_{\alpha}^{R}\overline{\beta}_{\alpha gg'}, \tag{7}$$

$$\rho_{gg',0} = \frac{1}{I_f}\left\langle\varphi_g^+|\left[\mathcal{L}_{g',0}\delta_{gg'} + \Sigma_{g'g,0} + \chi_g(\nu\Sigma_{fg',0})\right]\varphi_{g'}\right\rangle, \tag{8}$$

$$\rho_{gg',\delta} = \frac{1}{I_f} \left\langle \varphi_g^+ \right| \left[\delta \mathcal{L}_{g'} \delta_{gg'} + \delta \Sigma_{g'g} + \chi_g \delta(\nu \Sigma_{fg'}) \right] \varphi_{g'} \rangle \,, \tag{9}$$

$$\rho_{gg'} = \rho_{gg',0} + \rho_{gg',\delta}, \tag{10}$$

$$c_{\alpha g} = \frac{1}{I_f} \left\langle \varphi_g^+ | \chi_{\alpha g} C_\alpha \right\rangle , \tag{11}$$

$$s_g = \frac{1}{I_f} \left\langle \varphi_g^+ | S_g \right\rangle , \tag{12}$$

the system of equations for the G group-amplitude functions A_g is obtained:

$$\begin{cases} \Lambda_g \dfrac{dA_g}{dt} = \displaystyle\sum_{g'}^{G} (\rho_{gg'} - \overline{\beta}_{gg'}) A_{g'} + \sum_{\alpha}^{R} \lambda_\alpha c_{\alpha g} + s_g, \quad g = 1, ..., G \\ \dfrac{dc_{\alpha g}}{dt} = -\lambda_\alpha c_{\alpha g} + \displaystyle\sum_{g'}^{G} \overline{\beta}_{\alpha g g'} A_{g'}, \quad \alpha = 1, ..., R \text{ and } g = 1, ..., G \end{cases} , \tag{13}$$

involving also the $(G \times R)$ new unknowns $c_{\alpha g}$ connected to the delayed emission process. The kinetic parameters listed above can be easily interpreted physically. With the previous assumption for the shape function, the initial conditions for the above system are:

$$\begin{cases} A_g = 1, \quad g = 1, ..., G \\ c_{\alpha g} = \frac{1}{\lambda_\alpha} \displaystyle\sum_{g'}^{G} \overline{\beta}_{\alpha g g'}, \quad \alpha = 1, ..., R \text{ and } g = 1, ..., G. \end{cases} \tag{14}$$

2.2 Space multipoint model

When treating the space variable by the multipoint approach, different formulations are obtained using different weighting functions in the projection operation. In principle, the proper adjoint problem could be used, defining the importance as the total number of fission neutrons to be produced by a single neutron injected at a point in space with a given energy and, in transport theory, also direction. The projection is then performed by multiplying the balance equations by the importance function and integrating successively over each spatial region in which the system is subdivided. The equations for the amplitudes of each region are coupled through the appearance of boundary terms introduced by the streaming operator. Alternatively, several adjoint problems can be introduced for each region, by defining the corresponding importance as the number of fission neutrons that are produced in the chosen region by the injection of a single neutron at any point in phase space. This procedure recalls Kobayashi's proposal and would require a projection operation with integration over the whole system volume after successive multiplication of the balance equations by each importance function. However, this will prevent one from choosing a subdivision

of space which includes a non-fissile region, since the corresponding importance function would thus be identically zero.

In the following a space multipoint model for a monokinetic problem is proposed, using the second option for the importance function as illustrated above. Therefore, after subdivision of the space domain in any number I of volumes V_i, a set of adjoint problems is introduced for the reference system, as:

$$\mathcal{L}_0\varphi_i^+ + \nu\Sigma_{f,0}\varphi_i^+ + S_i^+ = 0, \tag{15}$$

with the adjoint source defined by:

$$S_i^+ = \begin{cases} \nu\Sigma_{f,0} \text{ for } \mathbf{r} \in V_i \\ 0 \text{ for } \mathbf{r} \notin V_i \end{cases} . \tag{16}$$

The projection process requires a definition of the following region-wise scalar product:

$$\langle f|g\rangle_i = \int_{V_i} d\mathbf{r} f(\mathbf{r})g(\mathbf{r}). \tag{17}$$

The separation formula to be used in this case is:

$$\phi(\mathbf{r},t) = A_i(t)\varphi(\mathbf{r}) \quad \text{for } \mathbf{r} \in V_i, \tag{18}$$

which is introduced into the balance equations that are then projected over each importance function φ_i^+. Using the definitions:

$$I_f = \sum_{i,j} \left\langle \varphi_i^+ | \nu\Sigma_{f,0}\varphi \right\rangle_j , \tag{19}$$

$$\Lambda_{ij} = \frac{1}{I_f}\frac{1}{v} \left\langle \varphi_i^+ | \varphi \right\rangle_j , \tag{20}$$

$$\overline{\beta}_{\alpha ij} = \frac{1}{I_f} \left\langle \varphi_i^+ | \beta_\alpha \nu\Sigma_f\varphi \right\rangle_j , \quad \overline{\beta}_{ij} = \sum_\alpha^R \overline{\beta}_{\alpha ij}, \tag{21}$$

the balance equations for the neutrons and the precursors are now projected on each adjoint function, and the separation formula (18) is used. The following definitions need to be introduced:

$$\sum_j \left\langle \varphi_i^+ | \mathcal{L}_0\phi + \nu\Sigma_{f,0}\phi \right\rangle_j = \sum_j \left\langle S_i^+ | \varphi \right\rangle_j A_j + \{\text{space boundary terms}\} = \sum_j \rho_{ij,0} A_j, \tag{22}$$

$$\sum_j \left\langle \varphi_i^+ | \delta\mathcal{L}\phi + \delta\mathcal{F}\phi \right\rangle_j = \sum_j \rho_{ij,\delta} A_j, \tag{23}$$

$$\rho_{ij} = \rho_{ij,0} + \rho_{ij,\delta}, \tag{24}$$

$$c_{\alpha i} = \frac{1}{I_f} \sum_j \left\langle \varphi_i^+ | C_\alpha \right\rangle_j , \tag{25}$$

$$s_i = \frac{1}{I_f} \sum_j \left\langle \varphi_i^+ | S \right\rangle_j . \tag{26}$$

Finally, the multipoint system of equations is obtained:

$$\left\{ \begin{array}{l} \sum_j \Lambda_{ij} \frac{d}{dt} A_j = \sum_j (\rho_{ij} - \overline{\beta}_{ij}) A_j + \sum_\alpha^R \lambda_\alpha c_{\alpha i} + s_i, \quad i = 1, ..., I \\ \frac{d}{dt} c_{\alpha i} = -\lambda_\alpha c_{\alpha i} + \sum_j \overline{\beta}_{\alpha ij} A_j, \quad \alpha = 1, ..., R \text{ and } i = 1, ..., I \end{array} \right. , \tag{27}$$

or, in vector form:

$$\left\{ \begin{array}{l} \widehat{\Lambda} \frac{d}{dt} \mathbf{A} = (\widehat{\rho} - \widehat{\overline{\beta}}) \mathbf{A} + \sum_\alpha^R \lambda_\alpha \mathbf{c}_\alpha + \mathbf{s} \\ \frac{d}{dt} \mathbf{c}_\alpha = -\lambda_\alpha \mathbf{c}_\alpha + \widehat{\overline{\beta}}_\alpha \mathbf{A}, \quad \alpha = 1, ..., R \end{array} \right. . \tag{28}$$

Of course, both separations in space and energy can be combined in the same procedure. In the following, the procedure is synthesized in a unitary structure.

2.3 General formulation

The multipoint-approach can be given a general formulation. The balance equations for neutrons and precursors are as usual considered after discretization in the phase space. Hence they can be cast into the following form:

$$\left\{ \begin{array}{l} \frac{1}{v_m} \frac{d}{dt} \phi_{nm} = \sum_{n'} \sum_{m'} k_{nm,n'm'} \phi_{n'm'} + \sum_\alpha \lambda_\alpha \chi_{\alpha,m} C_{\alpha,n} + S_{nm} \\ \frac{d}{dt} C_{\alpha,n} = -\lambda_\alpha C_{\alpha,n} + \beta_\alpha \sum_{m'} f_{nm'} \phi_{nm'}, \quad \alpha = 1, ..., R \end{array} \right. , \tag{29}$$

where index n and m denote a point in phase space, and

$$\phi_{nm}(t) = \phi(\mathbf{r}_n, \mathbf{v}_m, t), \quad C_{\alpha,n}(t) = C_\alpha(\mathbf{r}_n, t). \tag{30}$$

The phase space is now subdivided into a number of subdomains Γ_{NM} and a separation formula is adopted, as:

$$\phi_{nm} = A_{NM}\varphi_{nm} \quad \forall(\mathbf{r}_n, \mathbf{v}_m) \in \Gamma_{NM}. \tag{31}$$

A weighting function w_{nm} is now introduced, and it can be chosen as solution of a suitable adjoint problem. The scalar product is defined through the sum:

$$\sum_n \sum_m w_{nm} g_{nm} \quad \forall(\mathbf{r}_n, \mathbf{v}_m) \in \Gamma_{NM}. \tag{32}$$

By the projection procedure, from Eqs. (29) it is straightforward to obtain the following multipoint system of coupled equation in the time domain:

$$\begin{cases} \dfrac{d}{dt}A_{N'M} = \displaystyle\sum_{N'}\sum_{M'} K_{NM,N'M'}A_{N'M'} + \sum_{\alpha}\lambda_\alpha C_{\alpha,NM} + S_{NM} \\ \dfrac{d}{dt}C_{\alpha,NM} = -\lambda_\alpha C_{\alpha,NM} + \beta_\alpha \displaystyle\sum_{M'} F_{NM'}A_{NM'}, \quad \alpha = 1, ..., R \end{cases}. \tag{33}$$

3 RESULTS

Some results are now presented in order to test the performance of multipoint kinetics in comparisons with one-point model and with rigorous analytical and numerical results. Simple configurations are considered and applications to fast-thermal systems will be presented in a following paper.A first set of results concerns transients in a symmetrically placed localized source-driven system constituted by a homogeneous slab described by two-group diffusion, where a step perturbation of the absorption properties causes a transient with adjustment of the power at a different level. The material data are reported in Table 1. The initial subcriticality corresponds to $k_0 = 0.98$ and the perturbations with associated inserted reactivities are: a) $\delta\Sigma_{R1}/\Sigma_{R1} = \delta\Sigma_{R2}/\Sigma_{R2} = 0.01$, -1500 pcm; b) $\delta\Sigma_{R1}/\Sigma_{R1} = 0.01$,$-840$ pcm; c) $\delta\Sigma_{R2}/\Sigma_{R2} = 0.01$,$-660$ pcm; d) $\delta\Sigma_{R1}/\Sigma_{R1} = \delta\Sigma_{R2}/\Sigma_{R2} = -0.01$, 1500 pcm. Delayed neutron data are characteristic of a Plutonium fueled system.

Table 1. Material parameters for a two-groups slab system.

g	1	2
D_g	1.5	0.5
Σ_g	0.026	0.18
$\nu\Sigma_{fg}$	$9.9731{\cdot}10^{-3}$	$1.9946{\cdot}10^{-1}$
χ_g	0.9	0.1
v_g	$1{\cdot}10^{7}$	$3{\cdot}10^{5}$

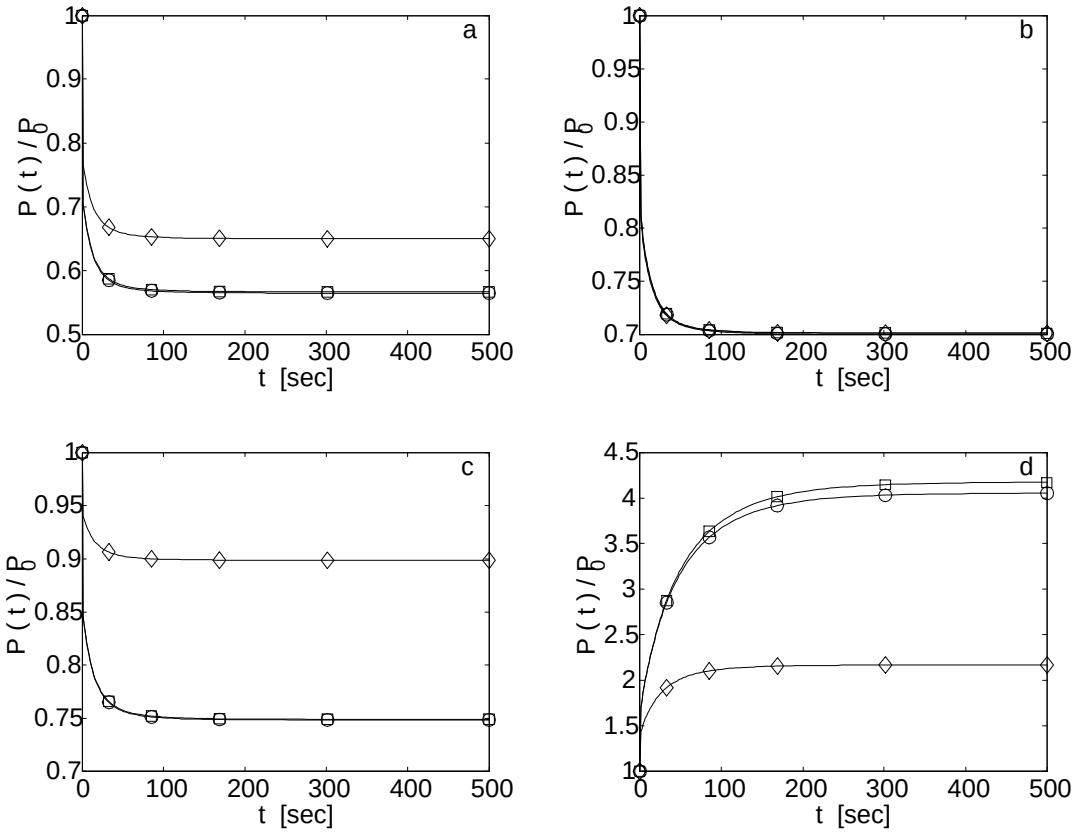

Fig.1 Transients caused by different variations of the absorption cross section. Diamonds (♢) indicate the classical point kinetics, squares (□) indicate the two energy- point kinetics and circles (◯) indicate the exact analytic solution.

Table 2. Material parameters for a two-region slab system.

	zone 1	zone 2
thickness	$75\ cm$	$25\ cm$
D	1.5	0.5
Σ	0.026	0.18
$\nu\Sigma_f$	$2.7311{\cdot}10^{-2}$	$1.0504{\cdot}10^{-2}$

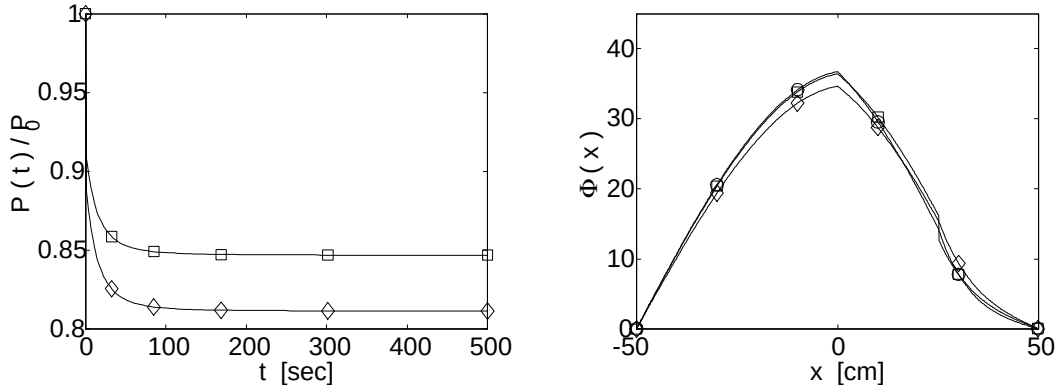

Fig.2 Power transient for variation of the absorption cross section (left). Fluxes at the end of the transient (right). Diamonds (♢) indicate the result of the classical point kinetics, squares (□) indicate the result of the two-space point kinetics and circles (◯) indicate the full numerical solution.

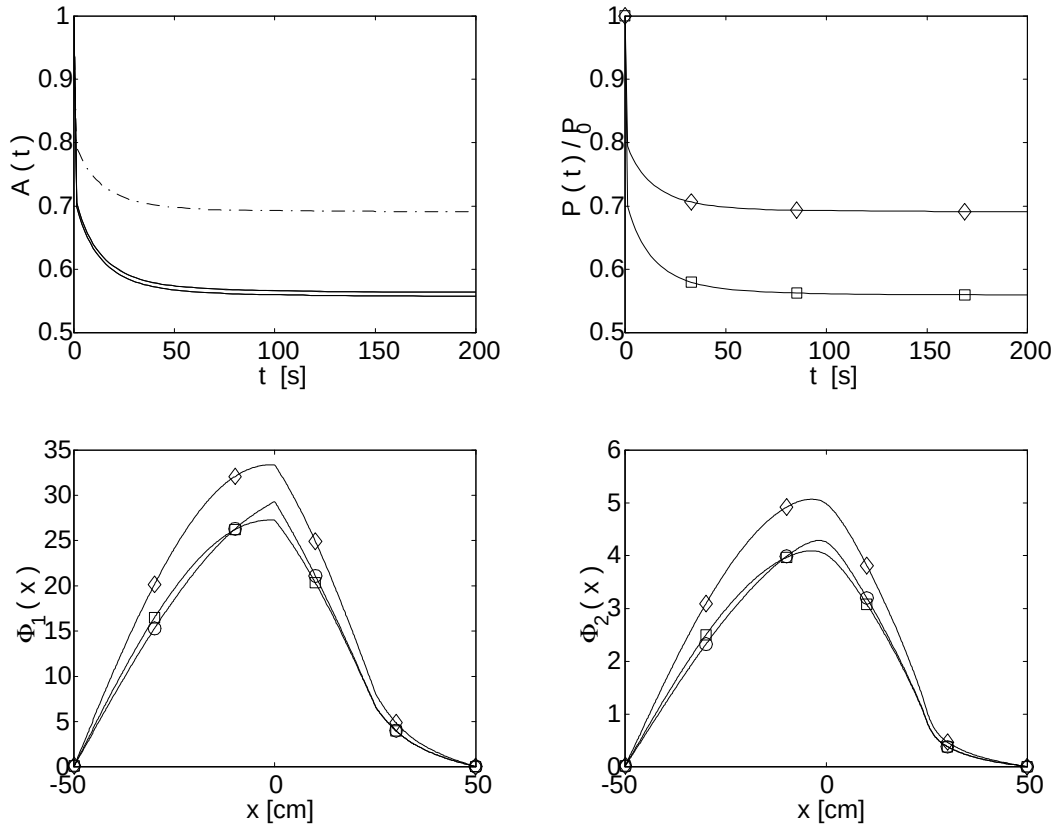

Fig.3 Amplitudes (dash-dotted line: point kinetics; upper solid line: amplitudes A_{11} and A_{12}, and lower solid line: amplitudes A_{21} and A_{22}) and powers (♢, one-point kinetics; □, four-point kinetics) for the first transient in a two-zone two-group system. The fast and thermal fluxes are also shown at the end of transient (◯, full numerical solution).

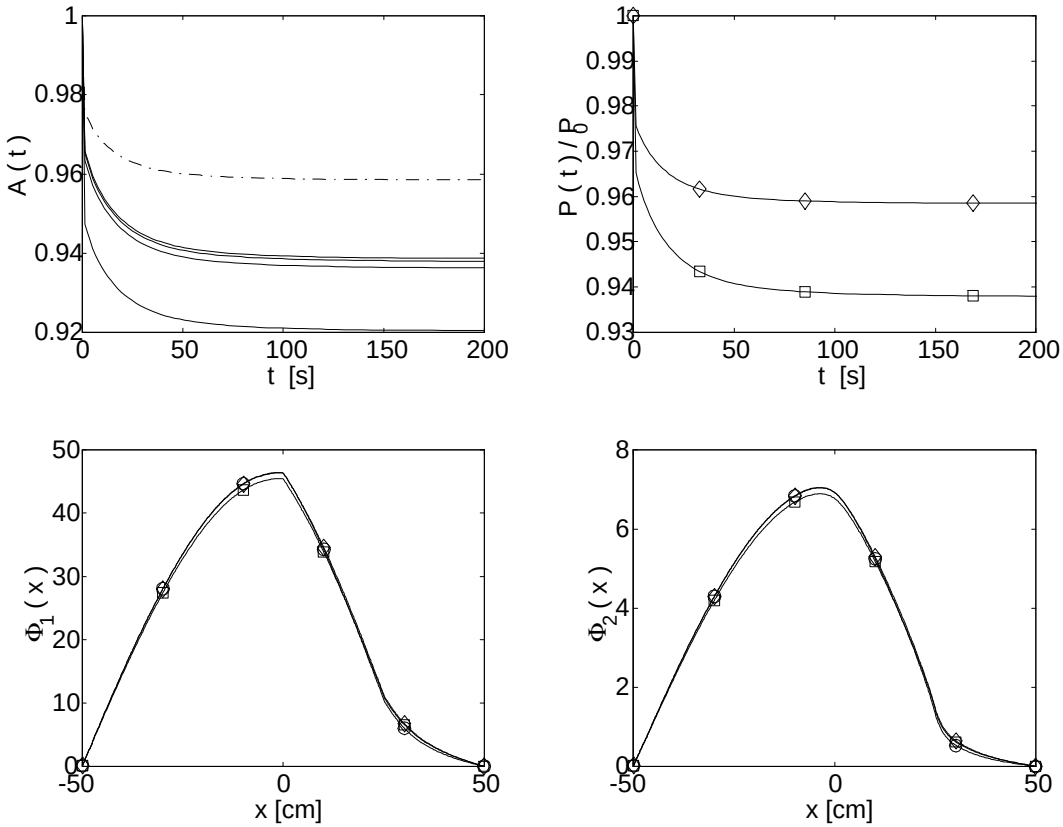

Fig.4 Amplitudes (dash-dotted line: point kinetics; solid line: amplitudes A_{11}, A_{12}, A_{21} and A_{22} from the top) for the second transient in a two-zone two-group system. The fast and thermal fluxes are also shown at the end of transient (◯, full numerical solution).

Table 3. Material parameters for a two-group, two-region slab system.

	zone 1		zone 2	
thickness	$75\ cm$		$25\ cm$	
g	1	2	1	2
D_g	1.5	0.5	1.0	0.5
Σ_g	0.026	0.02	0.02	0.08
$\Sigma_{g\to g'}$	0.015	0.0	0.01	0.0
$\nu\Sigma_{fg}$	$5.7970{\cdot}10^{-3}$	$1.1478{\cdot}10^{-1}$	$1.1594{\cdot}10^{-2}$	$1.7391{\cdot}10^{-2}$
χ_g	0.9	0.1	0.9	0.1
v_g	$1{\cdot}10^{7}$	$3{\cdot}10^{5}$	$1{\cdot}10^{7}$	$3{\cdot}10^{5}$

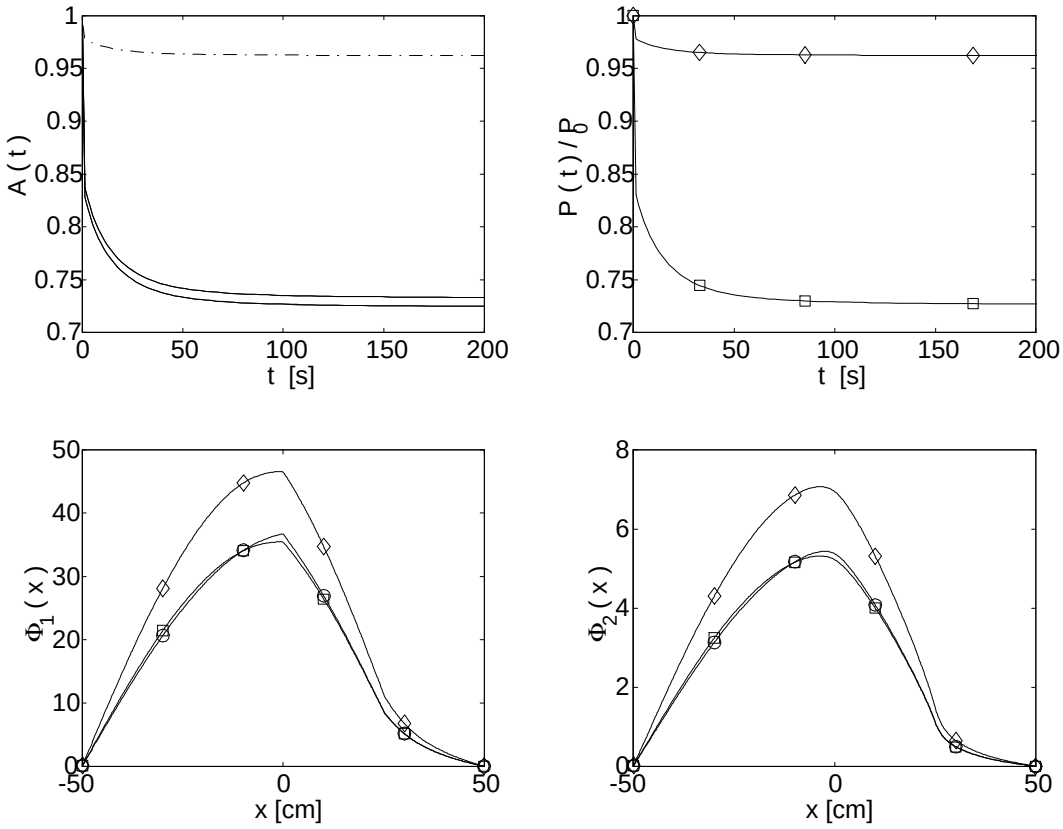

Fig.5 Amplitudes (dash-dotted line: point kinetics; upper solid line: amplitudes A_{11} and A_{12}, and lower solid line: amplitudes A_{21} and A_{22}) and powers ($\diamondsuit$, one-point kinetics; $\square$, four-point kinetics) for the third transient in a two-zone two-group system. The fast and thermal fluxes are also shown at the end of transient ($\bigcirc$, full numerical solution).

The problem allows a fully analytic solution. The multipoint model is derived using a single adjoint function. The graphs in Fig. 1 report comparisons of the power evolutions computed using different models. The multipoint model is using two energy points. As can be seen, for some transients classic point kinetics is inadequate and non-conservative, while it is rather efficient for the case involving a perturbation only in the fast group. For all cases considered, the performance of two-energy-point kinetics looks satisfactory.A spatial problem is now considered. A one-diffusion-group problem is analysed for a two-region slab, characterized by data given in Table 2 and injected by a symmetrically localized source. The initial effective multiplication constant is 0.98 and the transient is started by changing the removal cross section in zone 2 $\delta\Sigma_a/\Sigma_a = 0.25$ (-363 pcm). Comparison of point-kinetic and two-space-point (one per each zone, using two adjoint functions) kinetics is illustrated in Fig. 2. The figure reports also the space distortions for the neutron flux at the end of the transient. Clearly, the two-point model shows a discontinuity at the border between the two zones, but it is approaching the full solution very well. A two-group, two-zone application is finally considered, for the system having the characteristics reported in Table 3. The initial system presents $k_0 = 0.98$ and the first transient is started by a modification of 1% of the removal cross sections in all groups and zones, implying a

negative reactivity of 1510 pcm. Multipoint is using one point per group and one point per zone (four-point kinetics, using one adjoint function). The following Figs. 4 and 5 report other transients for the same system. In Fig. 4 the second transient follows a 10% modification of the removal cross sections in zone 2, implying only a 52 pcm reactivity. In that case the transient is not particularly severe and point kinetics performs satisfactorily. In Fig. 5 an increase of 1% of the removal cross sections for the second group is introduced into both zones. Although spatial distortions are not particularly evident, the distortion of the neutron spectrum causes large discrepancies in the transient foreseen by point kinetics, while multipoint accounts rather well for the spectral transient.

4 CONCLUSIONS

The paper discusses a general approach to derive a multipoint model suitable to treat the neutron kinetics of strongly decoupled systems, such as fast-thermal source-driven reactors today proposed for energy and technological applications. Test results highlight the limits of applicability of standard point kinetics, while multipoint performs adequately for a large class of subcritical transients, involving both spatial and spectral deformation of the flux shape. A final application of the method is carried out for a decoupled system.

Many improvements to widely used techniques employed in reactor kinetics can be attained using the multipoint model. In particular, a quasi-static scheme using multipoint equations can be implemented in a straightforward manner, thus highly enhancing the efficiency of the scheme.

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