

ROD DROP EXPERIMENT ANALYSIS EQUATIONS

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ABSTRACT

In this paper are summarized methodological works on rod drop experiment analysis provided in SKODA JS.

It is given theoretical analysis and applications of reactor point kinetics to the analysis of rod drop experiments. There are derived basic point kinetics equations and reviewed different definitions of reactivity based on Henry's approach. Formulation of local kinetics equation is given and correct ion chamber algorithm is defined.

1. INTRODUCTION

In VVER's reactors rod drop measurements are typically performed for all rods scram and for scram with a stuck control rod. Former investigations about this experiments based on comparison of reactimeter (reactivity meter) readings and static core calculations failed namely in the case of stucked control rod. During the last three years ŠKODA presented the broadest analysis in which was stated that control rods introduce into core strong distortions of neutron flux distribution and it has worth to compare only Henry's (Henry, 1958) time dependent reactivity calculated by dynamical macrocodes provided with influence functions for ion chamber signal simulation. Nevertheless there exists questions about the algorithm of reactimeter and about used kinetics data.

During the rod drop experiments the natural reactivities are related to real flux distributions in the subcritical reactor and to the importance function (adjoint flux) in the reference critical reactor. In 1958, Henry (Henry, 1958) defined so - called time - dependent reactivity which is based on presumption that core properties changes continually. Henry derivations were based on partition the transport solution into shape function and time dependent function. For diffusion approximation, which is used in this paper, the Henry's partition of group g fluxes φ_g means

$$\varphi_g(\vec{r}, t) = \psi_g(\vec{r}, t) \cdot n(t) \quad (1.1)$$

where $\psi_g(\vec{r}, t)$ is time dependent shape function of group g and $n(t)$ is pure time dependence.

In this paper is assumed that this shape function is in some time interval time independent and on this presumption are formulated point kinetics equations and inverse point kinetics algorithm. Finally is theoretically provided application of time dependent kinetics parameters on the time dependent reactivity calculation algorithm and its extension to the cores with external source. A new formulation of point kinetics equation for part of core is

provided. It is shown that signals of ion chambers can be used for direct calculation of core time dependent reactivity with the core kinetics parameters calculated during run of the core fluxes.

2. THE KINETICS EQUATIONS

Perturbed equations for the reactor transient problem may be written in the standard n_g multigroup diffusion form:

$$\begin{aligned} \frac{1}{v_g} \frac{\partial \varphi_g}{\partial t} - \nabla D_g \nabla \varphi_g + \Sigma_{v_g} \varphi_g &= \sum_{\substack{g^{\circledast}=1 \\ g^{\circledast} \neq g}}^{n_g} \Sigma_S^{g^{\circledast} \rightarrow g} \varphi_{g^{\circledast}} + \\ \sum_q \chi_g^q \sum_{g'=1}^{n_g} \left(1 - \sum_{j=1}^6 \beta_{j,g'}^q \right) v \sum_{f_{g'}}^q \varphi_{g'} + \\ \sum_{j=1}^6 \sum_{g'=1}^{n_g} \sum_q \lambda_{j,g'}^q C_{j,g'}^q \chi_{j,g}^{Dq} + S_g \end{aligned} \quad g = 1, \dots, n_g \quad (1.2)$$

and

$$\begin{aligned} \frac{\partial C_{j,g'}^q}{\partial t} &= \beta_{j,g'}^q v \sum_{f_{g'}}^q \varphi_{g'} - \lambda_{j,g'}^q C_{j,g'}^q, \\ j &= 1, \dots, 6, \quad g' = 1, \dots, n_g \quad q = \text{isotope} \end{aligned} \quad (2.2)$$

where

- n_g = number of neutron groups
- \sum_q = sum over all fissionable isotopes
- $\varphi_g(\bar{r}, t)$ = real neutron flux of group g in point \bar{r} at time t
- Σ_{v_g} = removal macroscopic cross section from group g
- D_g = diffusion coefficient of group g
- $\Sigma_S^{g \rightarrow g'}$ = scattering macroscopic cross section from group g' to group g
- $v \sum_{f_{g'}}^q$ = fission isotope q macroscopic fission cross section of group g' multiplied by the number of fission neutrons
- v_g = mean neutron velocity of neutrons of group g
- χ_g^q = prompt fission neutron spectrum of group g produced by fissioning of isotope q

$\chi_{j,g}^{Dq}$	=	delayed fission neutron spectra of group j for a fission of an isotope q with an incident neutron group g
S_g	=	external source density
$\lambda_{j,g}^q$	=	decay constant for precursors of neutron group j for a fission of isotope q with an incident neutron of group g'
$\beta_{j,g}^q$	=	fractions of delayed neutrons of group j for a fission of isotope q with an incident neutron of group g'
$C_{j,g}^q$	=	delayed neutron j group precursor (isotope q) density for fission of isotope q by neutrons of group g'

3. STATIC REACTIVITY AND ITS CALCULATION

Static reactivity of common (critical or noncritical systems) is defined as

$$\rho_\lambda = 1 - \lambda \quad (1.3)$$

where λ , the reciprocal of the effective multiplication factor, is eigenvalue of the static (and without source) equation to the equation (1.1), which can be expressed in real or adjoint formalism. For example in adjoint formalism we have static equation

$$0 = \nabla D_g \nabla \phi_g^* - \Sigma_{v_g} \phi_g^* + \sum_{g'=1}^{n_g} \Sigma_S^{g' \rightarrow g} \phi_{g'}^* + \frac{1}{k_{eff}(t)} \sum_q \nu \sum_{f_g} \sum_{g'=1}^{n_g} \left(\chi_{g'}^q \left(1 - \sum_{j=1}^6 \beta_{j,g}^q \right) + \sum_{j=1}^6 \beta_{j,g}^q \chi_{j,g'}^{Dq} \right) \phi_{g'}^* \quad g = 1, \dots, n_g \quad (2.3)$$

where $\phi_g^*(\bar{r}, t)$ = adjoint flux of group g , in point \bar{r} and at time t

$k_{eff}(t)$ = perturbed multiplicative factor at time t

which can be written in operators' notation

$$(L^* + \lambda Q^*) \phi_\lambda^* = 0 \quad (3.3)$$

and static eigenvalue λ can be expressed as a variational functional

$$\lambda[\phi_\lambda^*, \phi_\lambda] = \frac{\langle \phi_\lambda^*, L \phi_\lambda \rangle}{\langle \phi_\lambda^*, Q \phi_\lambda \rangle} \quad (4.3)$$

where ϕ_λ is solution of static real equation

$$(L + \lambda Q)\phi_\lambda = 0 \quad (5.3)$$

and L, L^*, Q and Q^* are linear operators satisfying

$$\begin{aligned} \langle \phi_\lambda^*, L\phi_\lambda \rangle &= \langle \phi_\lambda, L^*\phi_\lambda^* \rangle \\ \langle \phi_\lambda^*, Q\phi_\lambda \rangle &= \langle \phi_\lambda, Q^*\phi_\lambda^* \rangle \end{aligned} \quad (6.3)$$

and symbol \langle, \rangle represents integration over all space (volume and energy (groups)).

From the practical considerations, solving equation (5.3) for the reference critical configuration (with subscript zero) seldom produces exactly $\lambda_0 = 1$ even though the reactor is known to be critical (due to the different calculations models, etc.). For this reason, the static reactivity is often defined as $\rho_\lambda = \lambda_0 - \lambda$ with corresponding perturbation expression

$$\rho_\lambda = \frac{\langle \phi_{\lambda_0}^*, (\delta L + \lambda_0 \delta Q)\phi_\lambda \rangle}{\langle \phi_{\lambda_0}^*, \lambda_0 Q\phi_\lambda \rangle} \quad (7.3)$$

where $\phi_{\lambda_0}^*$ is adjoint solution of static critical nonperturbed equation

$$(L_0^* + \lambda_0 Q_0^*)\phi_{\lambda_0}^* = 0 \quad (8.3)$$

and ϕ_λ is static solution of perturbed equation (5.3) and it is valid

$$\delta L = L - L_0 \quad \text{and} \quad \delta Q = Q - Q_0. \quad (9.3)$$

The perturbation expression (7.3) is exact perturbation formula.

4. GENERAL FORMULATION OF TIME DEPENDENT REACTIVITY (Henry's type) AND POINT KINETIC EQUATIONS

In adopting Henry's methodology for diffusion approximation in simplest way we suppose full partition of time and space dependence of flux ϕ_g solution of equation (1.1) such that

$$\phi(\bar{r}, t) = \phi(\bar{r}, t_n) \cdot^n n(t) = \phi(\bar{r}) \cdot^n n(t) \quad (1.4)$$

is valid in time interval $\langle t_n, t_{n-1} \rangle$ with constant macroscopic cross sections in this interval.

The operations with equations (1.1), (2.2) and (8.3)

$$\langle \phi_{\lambda_0}^*, (1.2) \rangle - {}^n n(t) \langle \phi, (8.3) \rangle \quad (2.4)$$

$$\langle \phi_{\lambda_0}^*, \chi^D (2.2) \rangle \quad (3.4)$$

(where χ^D is operator representing delayed neutron fission spectra ($\chi_{j,g}^{Dq}$)) yield the known point kinetics equations

$$\frac{\partial {}^n n(t)}{\partial t} - {}^n \Lambda = {}^n n(t) \left[\rho_d - {}^n \beta_{eff} \right] + {}^n \Lambda \sum_{j=1}^6 {}^n \lambda_{eff j} {}^n C_{eff j} + \frac{\langle {}^0 \phi^*, [S] \rangle}{{}^n DVN} \quad (4.4)$$

$$\frac{\partial {}^n C_{eff j}}{\partial t} = {}^n n(t) \frac{{}^n \beta_{eff j}}{{}^n \Lambda} - {}^n \lambda_{eff j} {}^n C_{eff j}, \quad (5.4)$$

where for effective delayed neutron precursors

$${}^n C_{eff j} = \frac{\left\langle \sum_q \left(\sum_{g=1}^{n_g} {}^0 \phi_g^* \chi_{j,g}^{Dq} \right) \left(\sum_{g'=1}^{n_g} C_{j,g'}^q \right) \right\rangle}{\left\langle \sum_{g=1}^{n_g} \phi_g^* \frac{1}{v_g} {}^n \phi_g \right\rangle} \quad (6.4)$$

and with fundamental definitions as follows:

$$\beta_{eff} = \sum_{j=1}^6 {}^n \beta_{eff j}$$

${}^n \beta_{eff j}$ = effective fractions of delayed neutron group j for time interval $\langle t_n, t_{n+1} \rangle$

${}^n \Lambda$ = effective inverse velocity for time interval $\langle t_n, t_{n+1} \rangle$

${}^n \lambda_{eff j}$ = effective decay constants for time interval $\langle t_n, t_{n+1} \rangle$

$\frac{\langle {}^0 \phi^*, [S] \rangle}{{}^n DVN}$ = effective outer source for time interval $\langle t_n, t_{n+1} \rangle$

Calculation of kinetics parameters (for time interval $\langle t_n, t_{n+1} \rangle$) is provided by formulas

$${}^n \beta_{eff j} = \frac{\left\langle \sum_{g=1}^{n_g} {}^n \phi_g \sum_q v^n \Sigma_{f_g}^q \sum_{g'=1}^{n_g} \beta_{j,g}^q \chi_{j,g'}^* {}^0 \phi_{g'}^* \right\rangle}{{}^n DVN} \quad (7.4)$$

$${}^n\Lambda = \frac{\left\langle \sum_{g=1}^{n_g} \varphi_g^* \frac{1}{v_g} {}^n\varphi_g \right\rangle}{{}^nD VN} \quad (8.4)$$

$${}^n\lambda_{eff j} = \frac{\left\langle \sum_q \left(\sum_{g=1}^{n_g} \varphi_g^* \chi_{j,g}^{Dq} \right) \left(\sum_{g'=1}^{n_g} \lambda_{j,g'}^q C_{j,g'}^q \right) \right\rangle}{\left\langle \sum_q \left(\sum_{g=1}^{n_g} \varphi_g^* \chi_{j,g}^{Dq} \right) \left(\sum_{g'=1}^{n_g} C_{j,g'}^q \right) \right\rangle} \quad (9.4)$$

where

$${}^nD VN = \left\langle \sum_{g=1}^{n_g} {}^n\varphi_g^0 \sum_q v^n \Sigma_{f_g}^q \sum_{g'=1}^{n_g} \left(\chi_{g'}^q \left(1 - \sum_{j=1}^6 \beta_{j,g'}^q \right) + \sum_{j=1}^6 \beta_{j,g}^q \chi_{j,g'}^{Dq} \right) {}^0\varphi_{g'}^* \right\rangle. \quad (10.4)$$

Finally we get analytical expression of so - called time dependent (dynamical) reactivity ρ_d in the form

$$\begin{aligned} & - \left\langle \sum_{g=1}^{n_g} \nabla^0 \varphi_g^* (\delta^n D) \nabla^n \varphi_g \right\rangle - \left\langle \sum_{g=1}^{n_g} {}^0\varphi_g^* (\delta^n \Sigma_{v_g})^n \varphi_g \right\rangle + \left\langle \sum_{g=1}^{n_g} {}^0\varphi_g^* \sum_{g'=1}^{n_g} (\delta^n \Sigma_S^{g' \rightarrow g})^n \varphi_{g'} \right\rangle \\ & + \left\langle \sum_{g=1}^{n_g} {}^0\varphi_g^* \sum_q \chi_g^q \sum_{g'=1}^{n_g} \left(1 - \sum_{j=1}^6 \beta_{j,g'}^q \right) (\delta v^n \Sigma_{f_{g'}}^q)^n \varphi_{g'} \right\rangle + \left\langle \sum_{g=1}^{n_g} {}^0\varphi_g^* \sum_q \chi_g^{Dq} \sum_{g'=1}^{n_g} (\delta v^n \Sigma_{f_{g'}}^q)^n \varphi_{g'} \right\rangle \end{aligned}$$

$$\rho_d(t_n) = \frac{\text{---}}{{}^nD VN} \quad (11.4)$$

which can be represented in matrix notation

$$\rho_d = \frac{\langle \phi_{\lambda_0}^*, (\delta L + \lambda_0 \delta Q) \phi \rangle}{\langle \phi_{\lambda_0}^*, \lambda_0 Q \phi \rangle}, \quad (12.4)$$

where ρ_d is reactivity at time t_n , ϕ is real flux solution of time dependent equation (1.2) for time t_n and δ are perturbations:

$$\delta L = {}^nL - L_0 \quad \text{and} \quad \delta Q = {}^nQ - Q_0,$$

where nL and nQ are operators representing the perturbed core at time t_n .

Important part of presented algorithm are initial conditions. at the beginning of each interval $\langle t_n, t_{n+1} \rangle$. I_n system with external source the solution ${}^n\varphi$ can not be arbitrary normalized and due to this should be

$${}^n n(t_n) = 1. \quad (13.4)$$

Also effective concentrations of precursors must satisfy the initial condition

$${}^n C_{eff\ j}(t_n) = {}^{n-1} C_{eff\ j}(t_n^-) / {}^{n-1} n(t_n^-) \quad (14.4)$$

But in some cases partition (1.4) and presumption of constant cross sections are not valid and then exists some discrepancy between C_{eff} calculated by point kinetics $({}^n \tilde{C}_{eff\ j})$ and correct 3D calculation $({}^n \tilde{C}_{eff\ j}^C)$ which can be expressed for each time t_n by the factor SEP_n

$$SEP_n = \frac{{}^n \tilde{C}_{eff\ j}^P(t_n)}{{}^n \tilde{C}_{eff\ j}^C(t_n)}. \quad (15.4)$$

It should be noted that expression (12.4) is formally equal to perturbation formula (7.3) but with real fluxes from dynamical equations (1.2) and (2.2) and adjoint solution $\phi_{\lambda_0}^0$ of static nonperturbed equation (8.3).

Presumption of partition (1.4) within time interval $\langle t_n, t_{n+1} \rangle$ induces problem of discontinuity $\partial n(t) / \partial t$ at time points t_n . Fortunately during the rod drop experiment analysis the large negative reactivity insertion causes that the first member in equation (1.2) has negligible impact on the reactivity calculation.

Derivation of point kinetics equations provided by Keepin (Keepin, 1965) is based on presumption of small perturbations in power distribution during dynamical process and then for this case we usually have values of static reactivity and time dependent reactivity very close. This model however is not suitable for large perturbations like rod drop.

Calculations of many NPP Dukovany (VVER type reactor) core loadings has shown that dynamical reactivity ρ_d for full control rod drop is in average by 15% higher then static reactivity ρ_λ and in the case of stucked one control rod by 45% higher.

5. CALCULATION OF TIME DEPENDENT KINETICS PARAMETERS AND POINT KINETICS EQUATIONS

Change of real flux distribution during dynamical process causes according formulas (7.4), (8.4) and (9.4) changes in kinetics parameters $\beta_{eff\ j}, \Lambda$ and $\lambda_{eff\ j}$. The calculation procedure of time dependent kinetics parameters (based on macrocode MOBY-DICK (Svarny, 1998) is as follows:

A. Real flux distribution (${}^n\varphi$) at time point t_n is calculated by dynamical macrocode (e.g. MOBY-DICK-SK (Krysl, 1999)).

B. In time interval $t_n \leq t < t_{n+1}$ we suppose that space and time partition is valid

$$\varphi(\bar{r}, t) = \varphi(\bar{r}, t_n) \cdot {}^n n(t) = {}^n \varphi \cdot {}^n n(t) \quad (1.5)$$

is valid and in each of this interval are cross sections constant in time.

C. Adjoint flux weighting function (${}^0 \varphi^*$) is calculated by static adjoint equation (8.3) at the beginning (nonperturbed) state.

D. Substitution ${}^n \varphi$ and ${}^0 \varphi^*$ into equations (7.4), (8.4) and (9.4) get effective point parameters ${}^n \beta_{eff j}$, ${}^n \Lambda$ and ${}^n \lambda_{eff j}$ which are valid in the time interval (t_n, t_{n+1}) .

In practical calculations of rod drop processes in VVER reactors was found that time dependences of kinetics parameters was negligible.

6. OTHER DEFINITIONS OF REACTIVITY

The Henry's definition of reactivity (11.4) or (12.4) is natural reactivity which is related to real (flux distributions in subcritical reactor and to importance function in the critical reactor. The two other definitions of reactivity (Greespan, 1976) which are in focus of our interest are asymptotic-period reactivity and source multiplication reactivity.

a) Asymptotic - period reactivity

In this case a source - free subcritical reactor runs on a asymptotic period, i.e. for flux and precursors in equation (1.2) and (2.2) we have in operator's form (see Chapter 3):

$$\varphi = \phi_\omega \cdot e^{-\omega t} \quad (1.6)$$

$$C_j = C_{\omega j} \cdot e^{-\omega t} \quad (2.6)$$

For this after insertion into (1.2) and (2.2) we have in operator form asymptotic equation

$$\left[-\frac{\omega}{\nu} + L + Q(1 - \beta) + \sum_{j=1}^6 \frac{\lambda_j}{(\lambda_j + \omega)} \beta_j Q_j \right] \phi_\omega = 0 \quad (3.6)$$

where Q_p = fission operator for prompt neutrons with only prompt fission neutron spectrum χ

Q_j = fission operator with delayed neutron group j spectrum χ_j^D

β_j = fraction of delayed neutrons of group j and $\beta = \sum_{j=1}^6 \beta_j$

Solution ϕ_ω of equation. (3.6) is called shape function, which with importance function $\phi_{\lambda_0}^*$ (of static critical unperturbed equation given by equation (8.3)) according general Henry's definition (12.4) can form asymptotic-period reactivity:

$$\rho_\omega = \frac{\langle \phi_{\lambda_0}^*, (\delta L + \lambda_0 \delta Q) \phi_\omega \rangle}{\langle \phi_{\lambda_0}^*, \lambda_0 Q \phi_\omega \rangle}. \quad (4.6)$$

During rod drop we get large negative reactivity change which causes large decreasing of flux and dominancy of precursors density. This effects causes that importance of the first member in equation (3.6) decreases and ϕ_ω approaches to static solution ϕ_λ of eq. (5.3).

The methodology of point kinetic equations and kinetics parameters derivation presented in Chapter 4 is valid also for reactor running on a asymptotic period and ${}^n\beta_{eff j}$, ${}^n\Lambda$ and ${}^n\lambda_j$ are calculated by formulae (7.4), (8.4) and (9.4) with replacing real flux ${}^n\phi_g$ by $\phi_{g,\omega}$ (group values of ϕ_ω). Using this kinetics parameters into inverse kinetics algorithm (see Chapter .9) will give asymptotic - period reactivity ρ_ω .

b) Source-multiplication reactivity

For steady state source S is neutron flux distribution calculated by solution of the inhomogenous equation (in operators notation of Chapter 3).

$$(L + Q)\phi_s = S \quad (5.6)$$

Like in the case of asymptotic-period we can define source-multiplication reactivity:

$$\rho_s = \frac{\langle \phi_{\lambda_0}^*, (\delta L + \lambda_0 \delta Q) \phi_s \rangle}{\langle \phi_{\lambda_0}^*, \lambda_0 Q \phi_s \rangle} \quad (6.6)$$

This formula (6.6) can be obtained by means of the unperturbed equation (8.3) and perturbed equation (5.6) and standard process like (2.4).

Also for source - multiplication reactivity can be calculated kinetics parameters ${}^n\beta_{eff j}$, ${}^n\Lambda$ and ${}^n\lambda_j$ by formulae (7.4), (8.4) and (9.4) with replacing real flux ${}^n\phi_g$ by $\phi_{g,s}$ (group values of ϕ_s from (5.6)).

After rod drop with external source S the precursor concentrations decreases so much that flux will be stabilized on the level ϕ_s and in point kinetics equations $\partial n / \partial t \doteq 0$, ${}^n C_{eff j} \doteq 0$ and

$$\rho_s = \frac{\langle \phi_{\lambda_0}^*, [S] \rangle}{{}^n DVN.^\infty n(t_\infty)}, \quad (6.7)$$

where $\infty n(t_\infty)$ is saturated decreasing of fluxes (power) during the rod drop process which according (13.4) will have value $\infty n(t_\infty) = 1$.

7. POINT KINETICS EQUATIONS FOR PART OF CORE

We shall limit ourselves for simplicity to the core equations without external source. Extension to the external source presence will be straightforward.

In common time dependent behavior of neutron flux in part volume of core (e.g. nod) V_n can also be described by the point kinetics equations. Derivation of the equations we will do in closer analogy to the previous derivation described in Chapter 4, but integration $\langle \rangle_n$ will be provided only over part volume V_n of core. Finally we can get analogous equations to (4.4) and (5.4):

$$\Lambda_n \frac{\partial n(t)}{\partial t} = n(t) [\rho_n - \beta_{eff\ n}] + \Lambda_n \sum_{j=1}^6 \lambda_{eff\ j,n} C_{eff\ j,n} \quad (1.7)$$

$$\Lambda_n \frac{\partial C_{eff\ j,n}(t)}{\partial t} = n(t) \beta_{eff\ j,n} - \Lambda_n \lambda_{eff\ j,n} \cdot C_{eff\ j,n} \quad (2.7)$$

In equation (1.7) ρ_n represents „modified“ reactivity defined by

$$\rho_n = \frac{k_{eff}(t) - 1}{k_{eff}(t)} - L_n(t) \quad (3.7)$$

where $k_{eff}(t)$ is core multiplicative factor at time t

$L_n(t)$ = represents leakage of neutrons from volume V_n

$$L_n(t) = \frac{\int_{S_n} (\varphi(t) D \nabla \varphi^*(t) - \varphi^*(t) D \nabla \varphi(t)) dS}{D V N_n(t)} \quad (4.7)$$

Equation (4.7) was derived standardly after application of Gauss's theorem and

$$D V N_n = \langle \varphi, [\chi(1 - \beta) v \Sigma_f] \varphi^* \rangle_n + \langle \varphi, [v \Sigma_f \beta \chi^D] \varphi^* \rangle_n \quad (5.7)$$

where S_n = is surface of volume V_n .

The expressions (7.4), (8.4) and (9.4) for kinetics parameters introduced in Chapter 4 for whole core can be easily used to calculation of local kinetics parameters $\Lambda_n, \beta_{eff\ n}, \beta_{eff\ j,n}$

and $\lambda_{eff\ j,n}$ by replacing whole core volume integration with local volume V_n (i.e. $\langle \cdot, \cdot \rangle_n$) and using DVN_n (see (5.7)) as a normalization.

It is clear that processing of time dependent local (averaged over V_n) fluxes /or powers by standard inverse kinetics algorithm in general will give reactivity volume V_n dependent due to the leakage effect L_n impact. This theoretical result was practically approved by analysis of simulation of local measurements (theoretical placement of ion chamber into different local positions of core) of rod drop processes. From this follows that correct measurement of dynamical reactivity can be reached only by insertion of ion chambers into all fuel assemblies.

8. CORRECT PRESENTATION OF ION CHAMBER READING

Ion chamber is sensitive neutron detector placed out of core and based on reaction rates of given type. Point kinetics equations derived in Chapter 4 describe the core behavior in average and strictly physically has not sense to talk about reactivity and delayed neutrons in connection with ion chamber readings. We can only disclose mathematical trick which will explain what ion chamber reading represents.

As was mentioned above the ion chamber current I is given by reaction rates of given type which can be expressed by influence w for this type reaction rates:

$$I = \left\langle \left(\Sigma_f \varphi \right), w \right\rangle + \langle S, w \rangle \quad (1.8)$$

where

$\Sigma_f(\bar{r}, t)\varphi(\bar{r}, t)$ = local power q in point \bar{r} and at time t

$w(\bar{r}, t)$ = influence function in point \bar{r} and at time t

$\langle \cdot, \cdot \rangle$ = integration over whole core volume

Lets for the simplicity rewrite point kinetics equations into the base form with time independent kinetics parameters:

$$\frac{\partial n(t)}{\partial t} \Lambda = n(t) [\rho - \beta_{eff}] + \Lambda \sum_{j=1}^6 \lambda_{eff\ j} C_{eff\ j} + \tilde{S} \quad (2.8)$$

$$\frac{\partial C_{eff\ j}}{\partial t} = n(t) \frac{\beta_{eff\ j}}{\Lambda} - \lambda_{eff\ j} C_{eff\ j} \quad (3.8)$$

where $\rho = \text{reactivity} = \frac{k_{eff} - 1}{k_{eff}}$

$$\tilde{S} = \frac{\langle \varphi^*, [S] \rangle}{DVN}$$

As was shown in Chapter 4, this point kinetics equations were derived on presumption of separability (1.4) and constant cross sections in given time interval. Let these presumptions

are valid in time interval $\langle t_0, t_1 \rangle$. Suppose that this time and space separability is valid also for power distribution :

$$q(\bar{r}, t) = q^0(\bar{r}, t_0).n(t) , \quad (4.8)$$

with normalization

$$\langle q^0 \rangle = 1. \quad (5.8)$$

Multiply eq. (2.8) and (3.8) by expression $\langle q^0 w \rangle$ and after substitution

$$n\langle q^0 w \rangle = \langle nq^0 w \rangle = \langle qw \rangle = I(t) - \langle S, w \rangle = \tilde{I}(t)$$

$$C_{eff}\langle q^0 w \rangle = \tilde{C}_{eff}(t)$$

we can rewrite core point kinetics equations as follows

$$\Lambda \cdot \frac{\partial I}{\partial t} = \tilde{I}[\rho - \beta_{eff}] + \Lambda \sum_{j=1}^6 \lambda_{eff j} \tilde{C}_{eff j} + \tilde{S} \quad (6.8)$$

$$\frac{\partial \tilde{C}_{eff j}}{\partial t} = \tilde{I} \frac{\beta_{eff}}{\Lambda} - \lambda_{eff j} \tilde{C}_{eff j}. \quad (7.8)$$

These equations (6.8) and (7.8) are point kinetic equations of ion chamber (for time interval $\langle t_0, t_1 \rangle$). The key is new definition of effective precursor concentration and effective source:

$$\tilde{C}_{eff j} = C_{eff j} \cdot \theta_0, \quad (8.8)$$

$$\tilde{S} = \tilde{S} \cdot \theta_0 \quad (9.8)$$

on conditions

$$\theta_0 = \frac{\langle q^0 w \rangle}{\langle q^0 \rangle}. \quad (10.8)$$

Very important is that in ion chamber point kinetics equations (6.8) and (7.8) are used core kinetics parameters and core reactivity (time dependent/or dynamical reactivity).

In the case of unit influence function $w \equiv 1$, $\tilde{C}_{eff j} = C_{eff j}$, $\tilde{S} = \tilde{S}$ and $I = n$ and ion chamber equations (6.8) and (7.8) changes into core point kinetics equations.

9. DIFFERENCE APPROXIMATION OF INVERSE KINETICS AND ITS ANALYSIS

According Chapter 4 and 8 the difference approximation of the new ion chamber point kinetics equations (6.8) and (7.8) in time interval $\langle t_n, t_{n+1} \rangle$ can be written as follows

$$\frac{\rho(t_\ell)}{{}^n\beta_{eff}} = 1 + \frac{1}{\tilde{I}(t_\ell)} \left[\tau \frac{\partial \tilde{I}(t_\ell)}{\partial t} - \sum_{j=1}^6 {}^n\lambda_j a_j X_j(t_\ell) - \frac{{}^n\tilde{S}\theta_n}{{}^n\beta_{eff}} \right] \quad (1.9)$$

$$X_j(t_k) = X_j(t_{k-1}) \cdot \theta_n + \left[\tilde{I}(t_\ell) - {}^n\lambda_j X_j(t_{k-1}) \theta_n \right] \Delta t \quad (2.9)$$

where

t_k is refinement of coarse time step t_ℓ ($t_k \in (t_\ell, t_{\ell+1})$) and

$$t_n \leq t_\ell \leq t_k < t_\ell < t_{n+1}$$

$$\tilde{I}(t_\ell) = (I(t_\ell) - \langle s, w \rangle) / \tilde{I}(t_n)$$

$$X_j(t_0) = \tilde{I}(t_0)^0 \lambda_j$$

$$X_j(t_n) = \frac{X_j(t_n^-)}{{}^{n-1}n(t_n^-)} \cdot \theta_n \cdot SEP_n$$

$$\tau = {}^n\Lambda / {}^n\beta_{eff}$$

$$a_j = {}^n\beta_{eff j} / {}^n\beta_{eff}$$

$$\theta_n = \frac{\langle q^n \cdot w \rangle}{\langle q^n \rangle} \quad (\text{see (10.8)})$$

$$SEP_n \quad (\text{see (15.4)})$$

and it is seen that

$$X_j(t_k) = \frac{{}^n\Lambda}{{}^n\beta_{eff j}} C_{eff j}(t_k) \cdot {}^n\theta_n$$

Standard algorithm of reactimeter can be derived from (1.9) and (2.9) by substitution $SEP_n = 1$, $\theta_n = 1$, and $\tilde{S} = 0$:

$$\frac{\rho^{IK}(t_\ell)}{\beta_{eff}} = 1 + \frac{1}{I(t_\ell)} \left[\tau \frac{\partial I(t_\ell)}{\partial t} - \sum_{j=1}^6 \lambda_j a_j X_j^{IK}(t_\ell) \right] \quad (3.9)$$

$$X_j^{IK}(t_k) = X_j^{IK}(t_{k-1}) + [I(t_n) - \lambda_j X_j^{IK}(t_{k-1})] \Delta t \quad (4.9)$$

Slow decay of precursors allows to derive simple relation between time dependent reactivity from ion chamber reading to reactivity from core power development (for approximation (3.9) and (4.9))

$$\rho^{IK}(t_E) / \beta_{eff} \doteq 1 - \frac{I(t_0)}{I(t_E)} \sum_{j=1}^6 (1 - a_j) \quad (5.9)$$

or

$$\rho^{IK}(t_E) / \beta_{eff} \doteq 1 - \frac{I(t_0)}{I(t_E)} \alpha, \quad \alpha \doteq 1 \quad (6.9)$$

when t_0 is beginning of rod drop process and t_E is end of rod dropping. In the same manner we can write for core power development

$$\rho(t_E) / \beta_{eff} \doteq 1 - \frac{q(t_0)}{q(t_E)} \alpha \quad (7.9)$$

and from eq. (6.9) and (7.9) we have

$$\rho^{IK}(t_E) / \beta_{eff} \doteq 1 - \frac{\gamma^{IK}}{\gamma} \left[1 - \rho(t_E) / \beta_{eff} \right], \quad (8.9)$$

where γ represents rate of decreasing of core power (q) or ion chamber readings (I^{IK})

$$\gamma^{IK} = \frac{I(t_0)}{I(t_E)} \quad \gamma = \frac{q(t_0)}{q(t_E)}$$

Analysis of equation (8.9) was provided on the basis of MOBY-DICK-SK calculations [5] of experiments realized on NPP Dukovany and is presented in Table 1.

Tab.1 Analysis of eq. (8.9)

ρ - dynamical reactivity from core power, ρ^{IK} - dynamical reactivity from in chamber reading

Unit 1, Cycle 15	$\rho = -0.0969$		$\beta_{eff} = 0.006231$	
	IK7	IK6	IK4	IK3
γ^{IK} / γ	1.0727	0.8347	0.5860	0.7886
a) ρ^{IK} acc. M-D [5]	-0.1039	-0.0809	-0.0568	-0.0764
b) ρ^{IK} acc. (8.9)	-0.1039	-0.0808	-0.0566	-0.0763
$\left(\frac{ b }{ a } - 1 \right) * 100$ [%]	-0.0 %	-0.1 %	-0.2 %	-0.1 %
Unit 2, Cycle 14	$\rho = -0.0987$		$\beta_{eff} = 0.006304$	
	IK7	IK6	IK4	IK3
γ^{IK} / γ	1.044	0.836	0.598	0.777
a) ρ^{IK} acc. M-D [5]	-0.1033	-0.0819	-0.0570	-0.0758
b) ρ^{IK} acc. (8.9)	-0.1033	-0.0815	-0.0565	-0.0753

$\left(\frac{ b }{ a } - 1\right) * 100$ [%]	-0.0 %	-0.5 %	-0.9 %	-0.6 %
Unit 3, Cycle 13	$\rho = -0.1165$		$\beta_{eff} = 0.006243$	
	IK7	IK6	IK4	IK3
γ^{IK} / γ	1.0256	0.8038	0.5184	0.6973
a) ρ^{IK} acc. M-D [5]	-0.1195	-0.0922	-0.0585	-0.08
b) ρ^{IK} acc. (8.9)	-0.1196	-0.0924	-0.0574	-0.0793
$\left(\frac{ b }{ a } - 1\right) * 100$ [%]	+0.1 %	+0.2 %	-1.9 %	-0.1 %

10. CONCLUSIONS

In the paper was shown that presumption of time independence of shape function in small time intervals will lead to point kinetics equation with time dependent kinetic parameters.

Correct rod drop experiment analysis with large power distribution analysis needs dynamic macrocode system provided by so called influence functions (functions representing impact of the core on reactimeter readings).

In general power development of the part of core during the time can be also represented by point kinetics equations, but with modified time dependent reactivity values.

Ion chamber reading represent only mathematical transformation of the core behavior and correct rod drop experiment analysis needs correct core dynamic parameters and correct definition of core impact on ion chamber reading (influence functions).

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