HIGHER HARMONICS INFLUENCE FOR THE NOISE CALCULATION OF THE ACCELERATOR DRIVEN SYSTEM

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ABSTRACT

The design of a subcriticality monitoring system is mandated for the development of an accelerator driven system. Design of subcriticality monitors are typically based upon the use of reactivity measurement methods that provide a measure of the prompt neutron decay constant of the system. The purpose of this paper is to investigate the space dependence of neutron noise in an accelerator driven system and its affect on the prompt neutron decay constant. The prompt neutron decay constant can be obtained from the measurement of the cross spectrum between the source and a detector that measures the fluctuations in the neutron population. The source-detector cross spectrum from Monte Carlo calculations is shown to be dependent on the detector location because of the influence of higher modes of the neutron flux. One group diffusion theory is used to substantiate these results and to determine the α eigenvalues and eigenfunctions of the fundamental and higher modes for a simplified model of the accelerator driven system.

INTRODUCTION

Accelerator driven systems (ADS) are an attractive alternative to the production of power because these systems are inherently safer than conventional nuclear power plants and will produce minor actinides than conventional nuclear power plants (Lung, 1997). Development of the ADS will require methods to monitor the subcriticality of these systems to assure safe operating conditions. The development of such systems can be based on the stochastic nature of the fluctuations of neutrons produced in the ADS.

The observation of the fluctuations of the detector counts due to the stochastic nature of neutron transport in the ADS can be used to measure the subcritical reactivity. The analyses of the dynamics of such systems have been reported and various problems have been studied (Behringer and Wylder,1998; Pazsit and Yamane,1998). These analyses investigated the use of the increased fluctuations in the neutron multiplicity from a spallation source as a means to quantify the reactivity of the ADS using the Feynmann variance-to-mean method. The approach taken in this paper is to use the cross spectrum between the spallation source and a detector located in the system to determine the prompt neutron decay constant. The source-detector cross spectrum can be fitted to determine the prompt neutron decay constant which is directly related to the subcritical reactivity.

Monte Carlo simulations of the ADS have been performed to compute the source-detector cross spectrum for a particular design of an ADS. The results of the simulations indicated that the source-detector cross spectrum was dependent on the location of the detector in the ADS. The strong dependence of the cross spectrum between the proton beam and a detector, denoted as the cross power spectral density (CPSD), with the detector location will be discussed. The CPSD was shown to be dependent on the detector location from the detailed Monte Carlo simulations. The purpose of this paper is to study the spatial dynamic behaviour of neutrons in an ADS similar to investigations that have already been performed (Takashi et al,.1998 and Pazsit and Arzhanov,1999). An analytical model was developed for a simplified model of the ADS to quantify the behaviour of the CPSD. The system spatial dependence will be defined through the neutron flux expansion in a -modes.

This paper provides a brief description of the Monte Carlo codes developed to simulate the ADS followed by a brief discussion defining the source-detector CPSD. The results of the Monte Carlo and analytical calculations are presented next. Finally, some concluding remarks concerning this work are presented.

LAHET/MCNP-DSP

The code used in the modelling of the system was the coupling of the two Monte Carlo codes, LAHET developed at LANL, Prael (1997) and MCNP-DSP developed by Valentine (1995) at ORNL. The coupled code is described in the article by Valentine et al (2000). The LAHET (PHT or MRGNTP) file is read by MCNP-DSP as the source for the subsequent calculation. MCNP-DSP treats each batch as a pulse from the proton injector. The number of source particles per pulse (per batch) is specified in the LAHET calculation. Likewise, the initial time distribution of the proton pulse is specified in LAHET. The user must supply the frequency of the proton injector. The simulation assumes that the injector behaves as a pulsed source with a fixed period between the pulses. The time of the pulse depends on the number of proton pulses that occur in each data block. The data block is the time over which the detector responses are accumulated and is equal to the number of detector time bins divided by the sampling rate as specified in the MCNP-DSP extra input file. If there is only one proton pulse per block the starting time of the injector signal is sampled uniformly in the data block. This simulates the use of an internal clock of a processor to start the data accumulation. If more than one proton pulse occurs per block, then the starting time (t_s) of the pulse is sampled uniformly between 0 and T_{pp} where T_{pp} is the period between pulses and is equal to the inverse of the proton pulse frequency. The time of the subsequent pulses is determined by t_s+i*T_{pp} where i is the ith pulse after the first. An option is included in MCNP-DSP to allow the proton injector to serve as a trigger resulting in only one pulse per block occurring at time zero.

THE CROSS POWER SPECTRAL DENSITY

The behaviour of neutrons and gamma rays in a nuclear reactor or configuration of fissile material can be represented as a stochastic process. A stochastic transport theory for neutrons was formulated by Muñoz-Cobo et al (2001) to develop a measurement technique for subcriticality monitoring of an accelerator system assuming that the dynamics of the system would be measured using fission detectors. The parameter to be measured in that methodology was the cross correlation function between a source detector and a system neutron detector. In this paper, the space dependence of the neutron flux produced for the contribution of the higher spatial harmonics and how this dependence contributes to the source-detector CPSD, Φ_{12}, is investigated. Because of the spatial effects, the reactivity estimates depend on the detector location.

In the Muñoz-Cobo methodology, the approximation of the fundamental mode for the source-detector CPSD is given by

$$\left(\Phi_{12}\right)_o = S_o g_{n/p} \sum_n \exp(iw(tf_1 - nT))h^{D1}(tf_1 - nT)\ \frac{(h_n \sum_D ,\Phi_0)}{(iw-\alpha_0)(\frac{1}{v}\Phi_o^+,\Phi_o)}\overline{\Phi}_{0,sp}^+ . \qquad (1)$$

In deriving this expression the source has been considered to be a periodic source with S_o protons per pulse occurring every T seconds. The weighting factor W_n^{D1} depends on the pulse number and the detector D_1 impulse response function and is given by

$$W_n^{D1} = \int dw_1 \exp(iw_1(tf_1 - nT))h^{D1}(w_1) = h^{D1}(tf_1 - nT) . \qquad (2)$$

Note that if $nT > tf_1$ then by the causality principle h^{D1} becomes zero. The space, direction, and velocity weighted m^{th} adjoint eigenfunction is given by

$$\overline{\Phi}_{m,sp}^+ = \int d^3r r_s(\vec{r}) \int dv \int d\Omega f_{sp}(v,\hat{\Omega})\Phi_m^+(\vec{r},v,\hat{\Omega}) . \qquad (3)$$

$g_{n/p}$ is the average number of spallation neutrons produced per proton collision and is given by $g_{n/p} = \sum_{j=0}^{Isp} je_j^{sp}$. $h_p(tf_1 - t)$ is the response function of detector D_1 where tf_1 is the final counting time of detector D_1 and $h_p(w)$ is its Fourier transform.

The reactivity, r, of the system can be obtained from the neutron decay constant using the relationship $r = \frac{a_0}{\Lambda}$ where a_0 is the fundamental mode prompt neutron decay constant and L is the prompt neutron generation time. Using a Bode diagram the fundamental mode decay constant is obtained from the phase of the CPSD from the frequency at $-\pi/4$. The neutron multiplication constant is directly related to the reactivity.

LAHET/MCNP-DSP SIMULATIONS

To study the spatial dependence, LAHET/MCNP-DSP calculations of the CPSD, Φ_{12}, were performed using different neutron detector locations in a model of the ADS. The design parameters used in this paper are from the LAESA prototype of the Energy Amplifier proposal. The driving neutron source is spallation neutrons produced by an intense cyclotron proton beam (about 11mA and 380MeV). The enrichment of ^{233}U is approximately 11.5 wt%. The moderator, spallation target, and reflector are comprised of a Pb-Bi compound that is 44.5 wt% Pb and 55.5wt% Bi. The neutron lifetime of this design is approximately $1\,ms$, Rugama et al (2000). A sketch showing the hexagonal fuel configuration for the energy amplifier is provided in Fig. 1. The numbers in the hexagonal fuel positions correspond to the location of the detectors for the Monte Carlo simulations. In these calculations, a fuel assembly was treated as the fission detector to provide sufficient counting statistics.

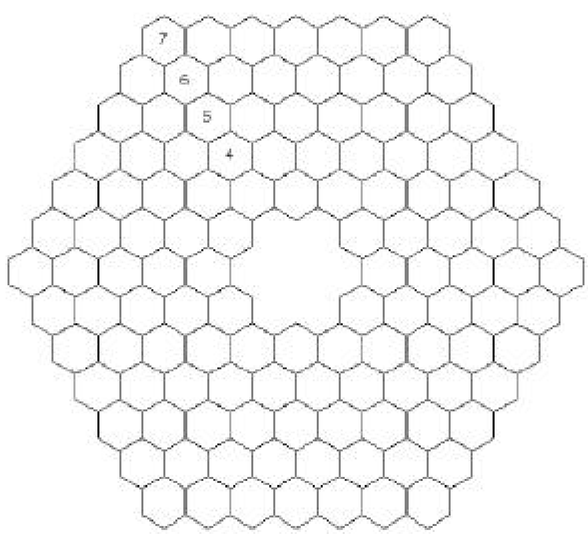

Figure 1. Hexagonal distribution of the fuel assemblies in the ADS.

The amplitude and phase of the CPSD, Φ_{12}, obtained from the calculations is represented in Figs. 2 and 3. The four positions correspond to the four fuel cells that are labelled in Fig. 1. The signal amplitude is a function of the neutron population and increases when the detector position is close to the spallation target. In Fig. 3, a displacement of the $a_0 = 2p \times f_b$, where the break-frequency f_b is calculated from the phase diagram at -π/4, to higher frequencies is observed when the detector location is closer to the target.

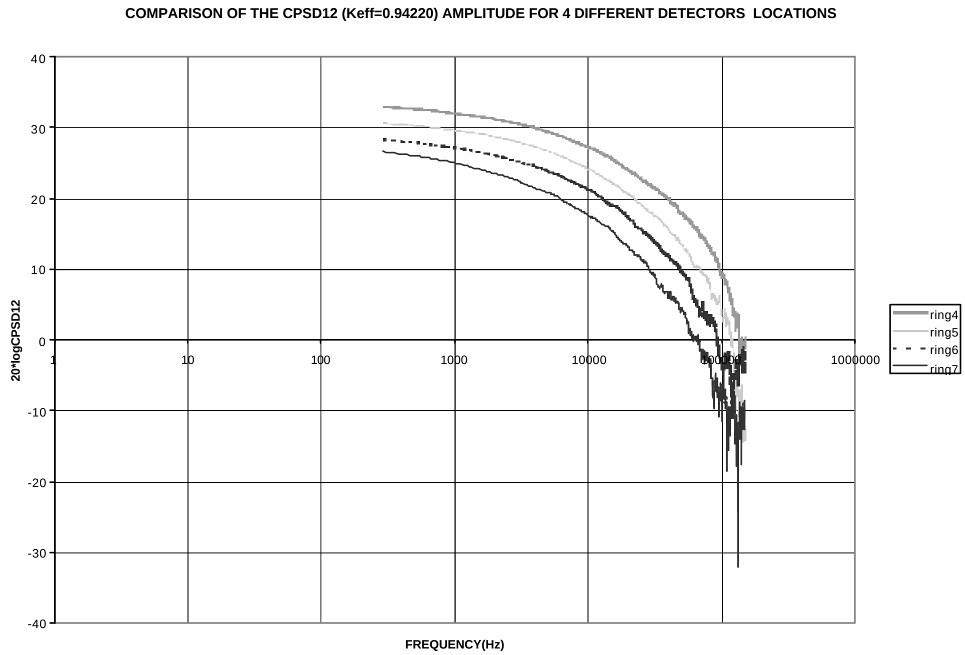

Figure 2. Amplitude of the source-detector CPSD.

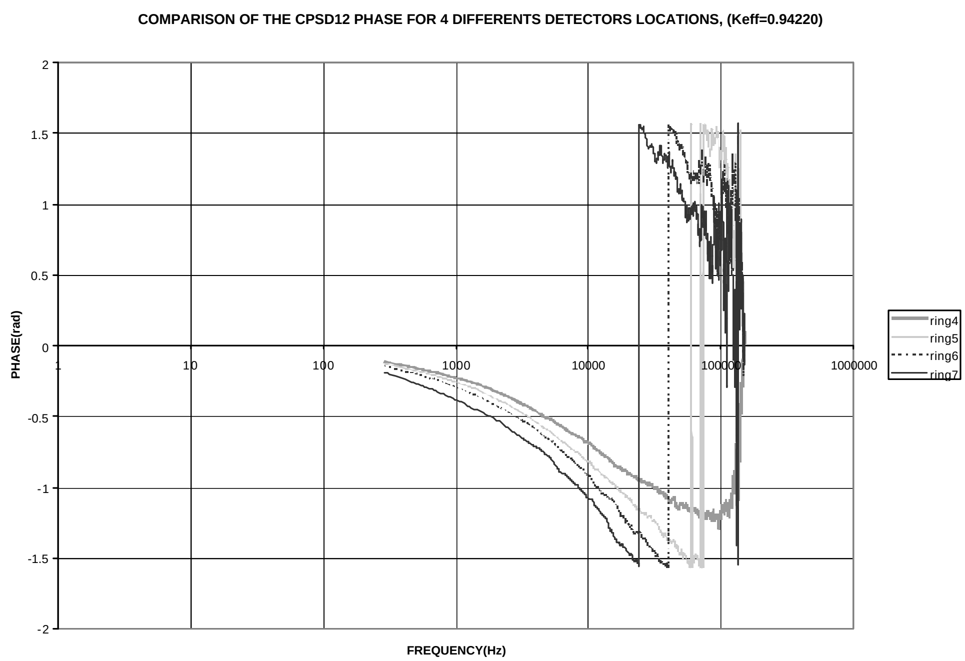

Figure 3. Phase of the source-detector CPSD.

ANALYTICAL SOLUTION SPACIAL-TIME FLUX DEPENDENCE

To explain the break-frequency displacement in Fig. 3, the contribution of the higher modes on the neutron flux, $\Psi(\vec{r},\hat{\Omega},E,t)$, must be examined. To study the space-dependent reactor dynamics, the neutron flux is expanded in the normal modes of the systems (Bell & Gladstone, 1970). The solutions are expanded as a series of eigenfunctions of a homogeneous time dependent problem. The neutron flux is defined as

$$\Psi(\vec{r},\hat{\Omega},E,t)=\sum_i T_i(t)\times\Phi_i(\vec{r},\hat{\Omega},E)\,. \tag{4}$$

The functions $\Phi_i(\vec{r},\hat{\Omega},E)$, are the eigenfunctions of the time-independent problems corresponding to the period eigenvalues $\{a_i\}$ that come from the time-independent approximations of the flux shape factor.

The $\{\Phi_i\}$ eigenfunctions of the higher harmonics were determined using one-group diffusion theory with fixed boundaries with an approximated geometrical description of the subcritical system. In diffusion theory the relationship between some of the eigenvalues $(a$ or $l)$ is clear. This produces an analytical description of the space-dependence of the neutron flux. The problems deriving from the approximations used to analytically describe the system are quite important in that they do not permit a good estimate of the prompt neutron decay constant to be obtained from one-group diffusion theory. On the other hand, it allows for an analytical description of the neutron flux with enough accuracy to explain the form of the space dependence. Slowing down effects, which require energy dependent theory, are neglected. Because the goal of this paper is to explain the detector location dependence, the reactor physics model used to obtain the analytical neutron flux is kept rather simple. Contributions of the delayed neutrons are neglected. The delayed neutrons only contribute to the source-detector cross spectrum at low frequencies that are well below the frequency for prompt neutrons. The general description will be fully three-dimensional although in the application of the simplified model we will focus only the radial dependence where the reactor is described as cylindrical.

For expediency, the heterogeneous system used in the LAHET/MCNP-DSP calculation was simplified to get an analytical description of the space and time dependence of the neutron flux. The geometrical description was reduced to three concentric cylinders representing the three regions of the ADS (target, fissile region, reflector) with an equivalent radius and homogeneous material instead of the heterogeneous description used in the LAHET/MCNP-DSP calculations. In Fig. 4, a representation of a three-region reactor core is shown, and it will explain approximately some of the special spatial dependencies. For instance, three homogeneous cylindrical regions describe the reactor core, and one-group diffusion theory is used in order to obtain the eigenvalues of the system.

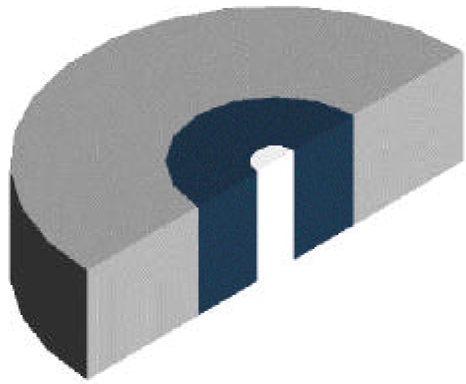

Figure 4. Sketch of simplified ADS.

The fissile zone region 2 was homogenized in the CASIX sequence of the SCALE (Landers et al, 1993) code using a simple pitch description. The system is described by the materials parameter (D, Σ_a, $n\Sigma_f$) of each zone. These material parameters were obtained from the results of the XSDRNPM (Greene, 2000) module of the SCALE code. The homogenized, one-group cross section data were obtained from this module from 44-group ENDFB-V cross-section libraries. XSDRN solves the transport equation with the discrete ordinates approximation in one dimension.

The one-group diffusion equation in cylindrical coordinates is expressed as:

$$D^k\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}\right)\Phi(r,z)+\left[n\Sigma_f{}^k-\Sigma_a{}^k-\frac{a_n}{v}\right]\Phi(r,z)=0, \tag{5}$$

with k=1,2 or 3. The one-group parameters D^k, $n\Sigma_f{}^k$, and $\Sigma_a{}^k$ are all uniform in the z direction for each region k, because the reactor is uniform in the z direction for all r. Applying the boundary conditions $\Phi(r,z)=0$ for $\begin{cases} z=0 \\ z=L \\ r=R_{ext} \end{cases}$, the general solution of the differential Eq. 5 is written as

$$\Phi(r,z)=\left[C_1^k J_0\left(P_n{}^k r\right)+C_2^k J_0\left(P_n{}^k r\right)\right]\times\sin\left(\frac{p}{L}z\right), \tag{6}$$

with k=1,2 or 3. The parameter $P_n{}^k$ is defined as

$$\left(P_n^k\right)^2=\frac{n\Sigma_f{}^k-\Sigma_a{}^k-D^k B_1^2-v^{-1}a_n}{D^k} \quad . \tag{7}$$

In this case, $B_1^2 = \left(\frac{p}{L}\right)^2$ is called the geometrical buckling and is defined by the boundary condition of the z function that $\sin\left(\frac{p}{L}z\right)$ must vanish outside of the cylinder (z=0 and z=L). The condition to have another non-trivial solution for the coefficients C_1^k and C_2^k is that the determinant of these coefficients vanishes. The mathematical solution of this determinant gives us the a_n eigenvalues.

A calculation of the analytical radial function was performed to compare the higher modes influence on the theoretical CPSD given by the Eq. 6 with Figs. 2 and 3 from the LAHET/MCNP-DSP calculation. The reactor description data used in this calculation is as close as possible to the heterogeneous one. The simplified model was a cylinder of height L=165 cm and had ^{233}U enrichment about 11.5 wt%. The first a-eigenvalue of this system is calculated as the least negative solution of the determinant using the boundary conditions applied to the radial function of Eq. 6. The eigenfunctions of this first eigenvalue are written for each region as:

1. Region 1 (Spallation target)

$$R_n(r) = C_1^1 I_0(P_0^1 r) \tag{8}$$

2. Region 2 (Fuel zone)

$$R_n(r) = C_1^2 J_0(P_0^2 r) + C_2^2 Y_0(P_0^2 r) \tag{9}$$

3. Region 3 (Reflector)

$$R_n(r) = C^3\left[I_0(P_0^3 r)K_0(P_0^3 R_{ext}) - K_0(P_0^3 r)I_0(P_0^3 R_{ext})\right] \tag{10}$$

J_n and Y_n are Bessel functions and I_n and K_n are modified Bessel functions.

The value of a_0 obtained from the cross section and diffusion constants, with the conditions of the simplified model, is $a_0 = -10276.5 rad$. The graphical description of the fundamental mode eigenfunction described in Eqs. 8, 9, and 10 is shown in Fig. 5.

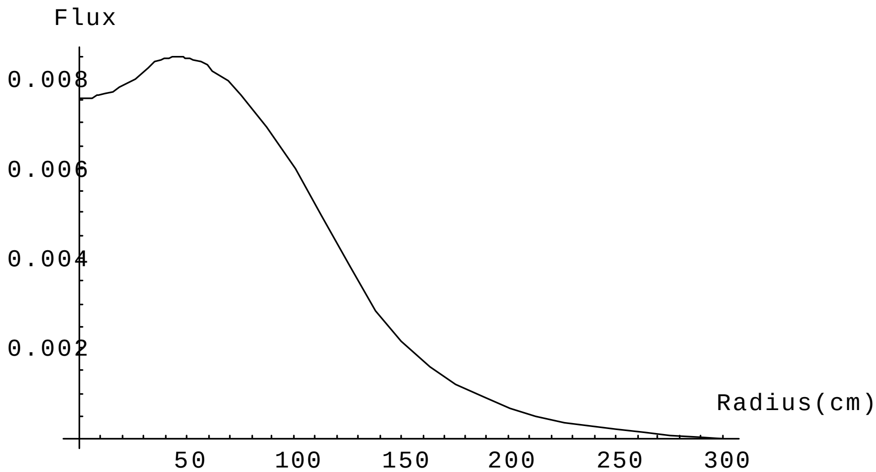

Figure 5. Eigenfunction of the fundamental mode.

The neutron flux distribution does not peak in the central region due to the scattering produced on the Pb-Bi and the fission on the fissile zone. The absorption in the Pb-Bi composite is relatively small, and the neutron population decays slowly in this region.

The first harmonic is the least negative solution of the determinant received from the boundary and continuity conditions of the radial function described as:

4. Region 1 (Spallation target)

$$R_n(r) = C_1^1 J_0(P_1^1 r) \tag{11}$$

5. Region 2 (Fuel zone)

$$R_n(r) = C_1^2 J_0(P_1^2 r) + C_2^2 Y_0(P_1^2 r) \tag{12}$$

6. Region 3 (Reflector)

$$R_n(r) = C^3 \left[J_0(P_1^3 r) Y_0(P_1^3 R_{ext}) - Y_0(P_1^3 r) J_0(P_1^3 R_{ext}) \right] \tag{13}$$

The a_1 eigenvalue takes the value –56122.5 rad and the graphical representation is given on Fig. 6.

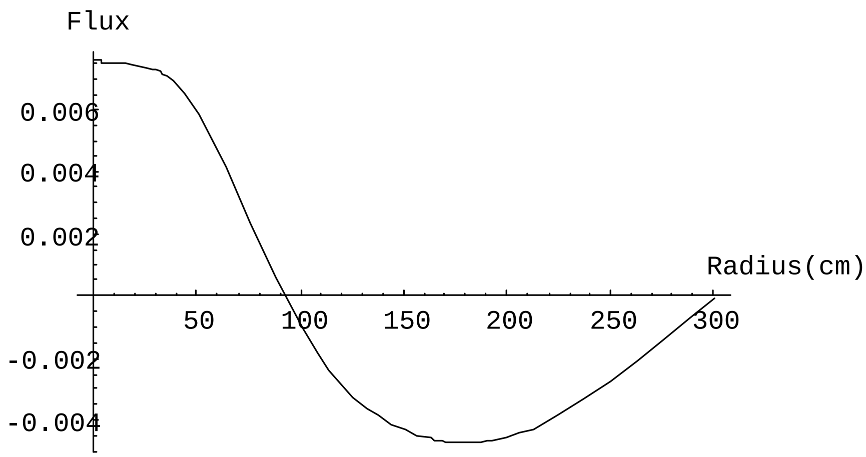

Figure 6. First harmonic eigenfunction

From the eigenfunctions of a_0 and a_1, considering $\Sigma_D(\vec{r}) = \Sigma_D d(\vec{r} - \vec{r}_1)$ in Eq. 6, the space time dependencies of the detector position is proportional to the following shape function:

$$\begin{cases} \dfrac{\Phi_0(\vec{r}_1)}{\left(\Phi^+{}_0(\vec{r}_1)\dfrac{1}{v}, \Phi_0(\vec{r}_1)\right)} \overline{\Phi}_{0,sp}{}^{+} \exp(a_0 t) \\ \dfrac{\Phi_1(\vec{r}_1)}{\left(\Phi^+{}_1(\vec{r}_1)\dfrac{1}{v}, \Phi_1(\vec{r}_1)\right)} \overline{\Phi}_{1,sp}{}^{+} \exp(a_1 t) \end{cases} \quad (14)$$

The Φ_{12} with the contribution of the fundamental mode and first harmonic can be described using Eq. 14 with the cross power spectral density described by Eq. 1.

In the next two figures, we represent the detector location influence by the comparison of the Φ_{12} calculated at different distances (r_i) from the reactor centre. In Fig. 8 the same displacement of the break frequency is observed as shown in Fig. 3. The higher-mode influence is observed in the Φ_{12} phase comparisons shown in Figs. 8 and 3, from the analytical description of the flux and from the LAHET/MCNP-DSP calculation. Observe that when the detector is located close to the reflector region the first harmonic appears and changes phase by p radians.

The importance of the higher modes contribution decreases as k_{eff} increases. For systems that are close to critical, the modal influence will be neglected. To probe this hypothesis, the Φ_{12} for a reactor height $L = 200cm$ was calculated. The k_{eff} value increases as the reactor height increases. With this reactor height, the influence of the higher harmonics is very small and the detector location effect vanishes.

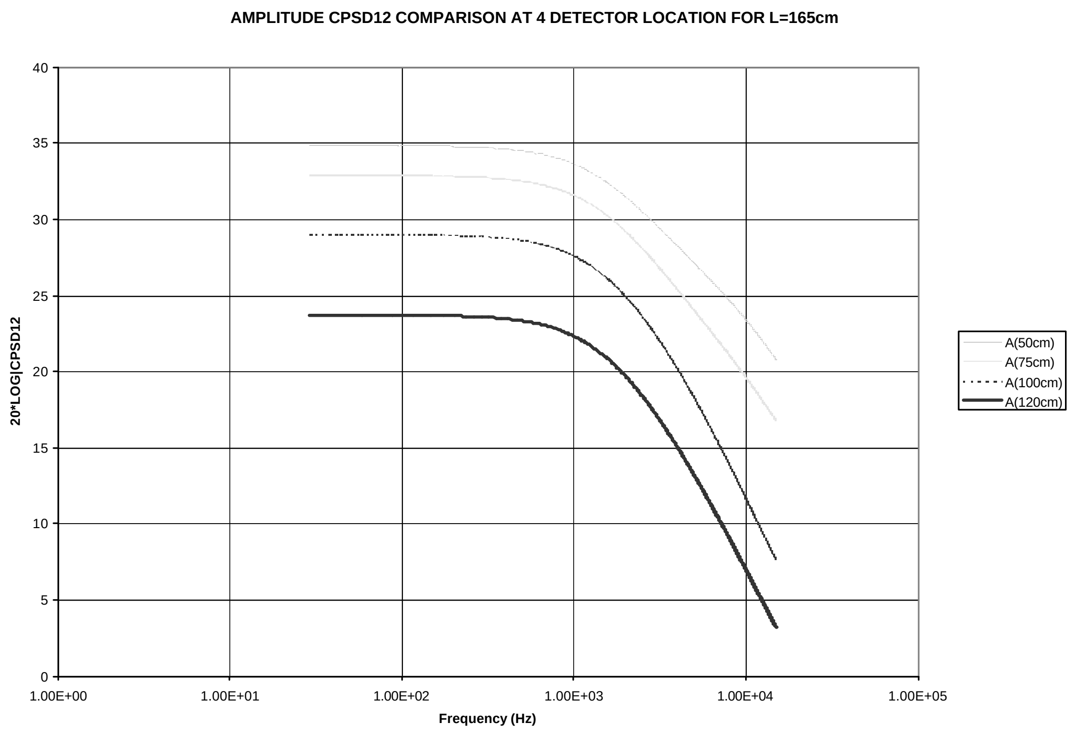

Figure 7. $|CPSD_{12}|$ comparison at 4 detector locations.

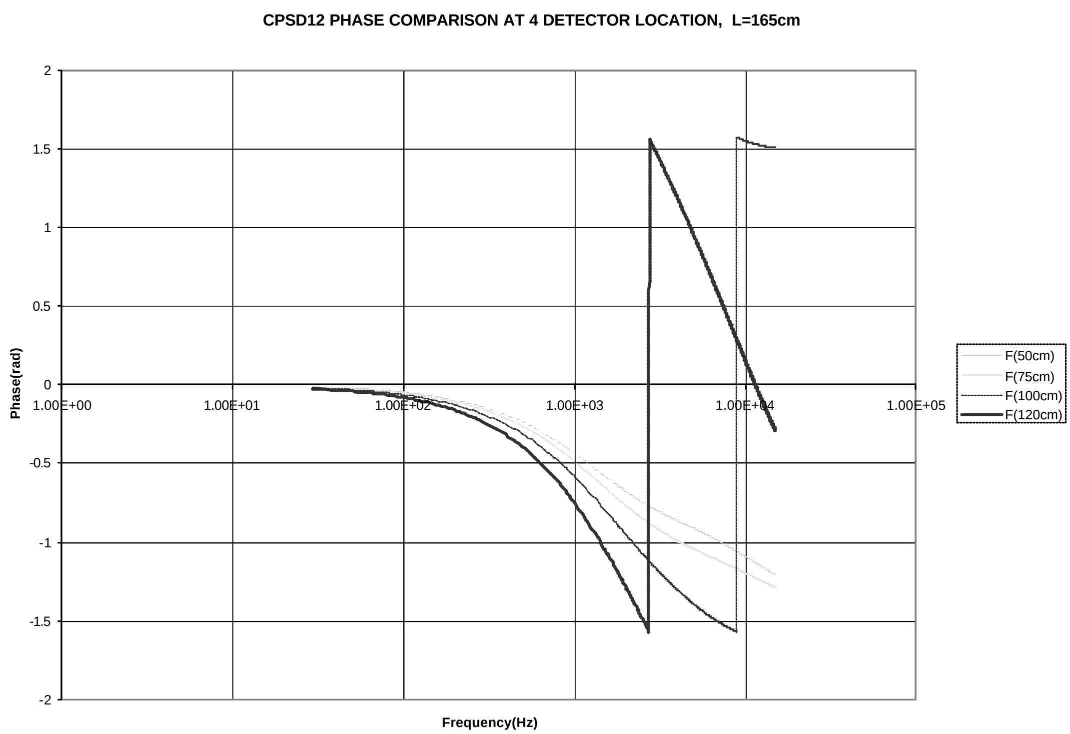

Figure 8. CPSD12 theoretical phase comparison at 4 detector locations.

The second and third harmonic has been considered to study the convergence of the solution. In Figs 9, 10 a comparison of the $CPSD_{12}$ phase using first, second and third harmonics are studied for the detector location at 75cm and 120cm from the centre. The best convergence is observed at 75 cm detector location, where the solution using the fundamental and first and second mode will give as an acceptable solution. The contribution of the higher modes increases with the distance.

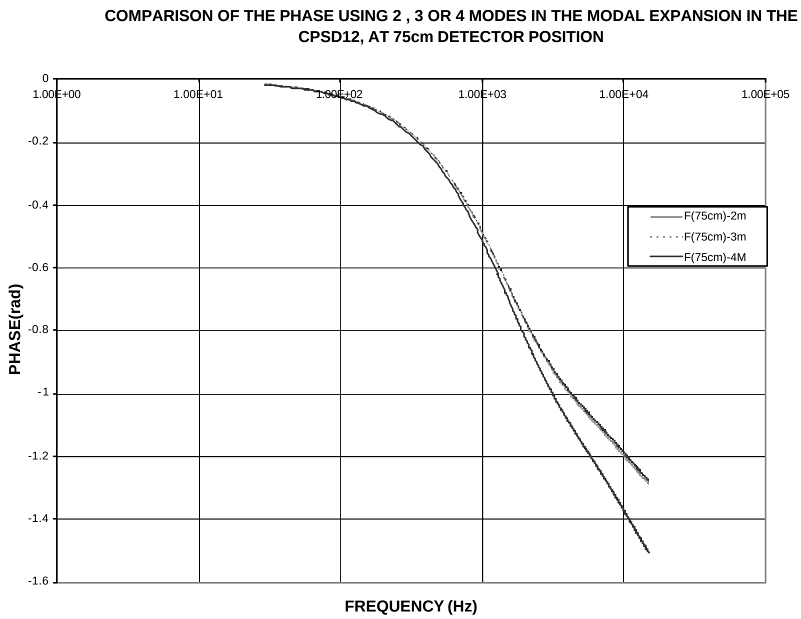

Figure 9 $CPSD_{12}$ phase comparison at 75cm

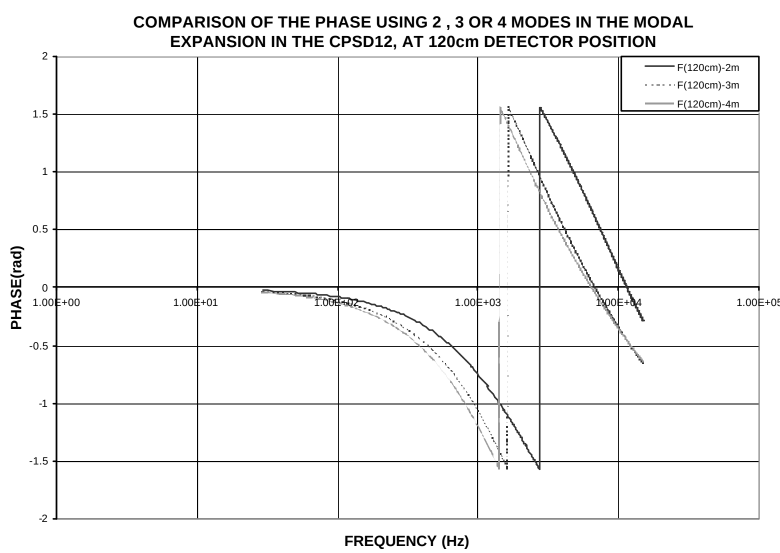

Figure 10 $CPSD_{12}$ phase comparison at 120cm

CONCLUSIONS

This paper has shown that the prompt neutron decay constant obtained from the cross spectrum between the proton beam and a neutron detector contained in an accelerator driven system is dependent on the detector location. This was evident from the coupled LAHET/MCNP-DSP simulations and was also shown using a simplified one-group, two-dimensional model of the ADS. Both the Monte Carlo calculations and the analytical solution demonstrated that the higher modes of the neutron flux contribute to the detector response. This contribution is most significant for lower k_{eff} values and is almost negligible for a fuel configuration near critical. The results of this work demonstrates that care must be taken when interpreting any subcritical measurement that rely upon the prompt neutron decay constant to determine the reactivity of an ADS. The contribution of the higher moments of the neutron flux must be considered and the impact of these higher moments must be investigated.

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