Nuclear Mathematical and Computational Sciences: A Century in Review, A Century Anew
Gatlinburg, Tennessee, April 6-11, 2003, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2003)

STOCHASTIC NEUTRONICS WITH PANDA DETERMINISTIC CODE

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ABSTRACT

The master equation for the neutron probability distribution induced by one initial neutron in a multiplicative medium is a Kolmogorov Backward equation. From this equation, the generating function methodology is used to derive the first two moments and survival probability equations. They are time dependent adjoint transport equations. The first moment or mean number equation is the classic linear adjoint transport equation. The second moment and survival probability equations present an additional term. Nevertheless, they can be solved with standard numerical methods used in deterministic transport theory. A neutron source can be taken into account using the same methodology. The generating function method is applied to the master equation derived for the source induced neutron population probability distribution. By this way, the source induced mean number, variance and initiation probability expressions are derived. For nuclear safety applications, the deterministic 2D S_N code PANDA has been adapted to solve these linear and nonlinear time dependent adjoint multigroup transport equations and to take into account a neutron source. A numerical application is presented. This space and energy dependent numerical test will be used in the validation process for intercode comparison.

Key Words: Stochastic Neutronics, Neutron Transport Code

1. INTRODUCTION

Stochastic neutronics theory is used for applications like fast burst reactor initiation[1], criticality accidents analysis[2] or neutron noise experiment interpretation[3]. Classical deterministic neutronics is concerned with the mean neutron density which is solution of a linear transport equation. Stochastic neutronics is based on neutron number probability distribution which is solution of a master equation, the Kolmogorov Backward equation[4].

Regarding the solution of this equation, analytical and numerical solutions for transition probabilities are derived within the stochastic neutronics point model approximation[5]. For more realistic problems, the Monte Carlo analog methods is used in neutron noise analysis applications[6] and estimation of non extinction probabilities[2]. Considering deterministic methods, moments of the transition probability distribution[10] and survival probability[11] equations can be solved using standard S_N methods[8].

This paper is based on the general theory of stochastic neutron transport theory developed by Bell[4]. To be self consistent we will recall the derivation of the equations of interest but the real novelty of the paper is the numerical solution of stochastic neutronics equations with deterministic methods using the PANDA code. This new code originates from the ORPHEE code [9]. The study is restricted to prompt neutron population problems. We do not take into account the detection process and delayed neutrons emission whose theory has been developed by Muñoz-Cobo et al[12, 13].

In a first part we consider the neutron distribution induced by a single initial neutron. Starting from the master equation we give the equations of the first two moments and survival probabilities using the generating function methodology[12]. In a second part we consider the same quantities for a neutron distribution induced by a random source. The same methodology is applied to the master equation with source. In the last part we consider the numerical solution of the multigroup equations with the PANDA code and we present a numerical application.

2. STOCHASTIC NEUTRONICS WITHOUT NEUTRON SOURCE

2.1. Definitions

- To describe the neutron-matter interaction we define the following macroscopic cross-sections :
  - $\sigma_i(\vec{r}, v)$ is the macroscopic cross section associated with total reaction of multiplicity i.

$$\sigma_i(\vec{r}, v) = \int d^3v_1 \ldots \int d^3v_i \; \sigma_i(\vec{r}, \vec{v}, \{\vec{v}_1, \ldots, \vec{v}_i\}) \tag{1}$$

  - $\sigma_T(\vec{r}, v)$ is the total cross section ($\sigma_T = \sum_{i=0}^{M} \sigma_i$).
  - $\sigma_C(\vec{r}, v)$ is the capture cross section.
  - $\sigma_S(\vec{r}, \vec{v}, \vec{v}')$ is the differential scattering cross section.
  - $\sigma_i^F(\vec{r}, v)$ is the fission macroscopic cross section ($\sigma_F = \sum_{i=0}^{M} \sigma_i^F$).
  - Considering fission reactions, the neutron velocities for incident and produced neutrons are uncorrelated and the fission neutrons are emitted isotropically, we have :

$$\sigma_i^F(\vec{r}, \vec{v}, \{\vec{v}_1, \ldots, \vec{v}_i\}) = \sigma_i^F(\vec{r}, v) \left[\frac{\chi(v_1)}{4\pi} \ldots \frac{\chi(v_i)}{4\pi} \right] \tag{2}$$

  - $\overline{\nu}$ is the mean neutron number produced by one fission ($\overline{\nu}\sigma_F = \sum_{i=1}^{M} i\sigma_i^F$)
  - The fission parameters c_q are defined by :

$$c_q(\vec{r}, v) = \sum_{i=q}^{M} \frac{i!}{q!(i-q)!} \sigma_i^F(\vec{r}, v) \tag{3}$$

- We define the following conditional probabilities for a single initial neutron:
  - $K(n, T|1, \vec{r}, \vec{v}, t)$ is the conditional probability that one neutron injected at initial time t with characteristics $(\vec{r}, \vec{v})$ will produce n neutrons at final time T in the system without neutron source. We will consider the first two statistical moments of this neutron probability distribution.

$$\overline{n}(T|1, \vec{r}, \vec{v}, t) = \sum_{n=0}^{\infty} nK(n, T|1, \vec{r}, \vec{v}, t) \tag{4}$$

$$\overline{n^2}(T|1, \vec{r}, \vec{v}, t) = \sum_{n=0}^{\infty} n^2 K(n, T|1, \vec{r}, \vec{v}, t) \tag{5}$$

– $K(n, T|i, \vec{r}, \{\vec{v}_1, ...\vec{v}_i\}, t)$ is the conditional probability that i neutrons injected at initial time t and point $\vec{r}$ with velocities $\{\vec{v}_1, .., \vec{v}_i\}$ will produce n neutrons at final time T in the system without neutron source.

– Regarding these conditional probabilities we have the following relation :

$$K(n, T|i, \vec{r}, \{\vec{v}_1, ..., \vec{v}_i\}, t) = \underbrace{\sum_{n_1} ... \sum_{n_i}}_{n=n_1+...+n_i} \prod_{j=1}^{i} K(n_j, T|1, \vec{r}, \vec{v}_j, t) \tag{6}$$

– The extinction probability $e(T|1, \vec{r}, \vec{v}, t)$ is the probability that one neutron injected at initial time t with characteristics $(\vec{r}, \vec{v})$ will produce 0 neutrons at final time T in the system without neutron source. The survival probability $s(T|1, \vec{r}, \vec{v}, t)$ is the complementary event. The initiation $a(\vec{r}, \vec{v}, t)$ probability is the asymptotic value of the survival probability when $T \to \infty$.

$$e(T|1, \vec{r}, \vec{v}, t) = K(0, T|1, \vec{r}, \vec{v}, t) \tag{7}$$

$$s(T|1, \vec{r}, \vec{v}, t) = 1 - K(0, T|1, \vec{r}, \vec{v}, t) \tag{8}$$

$$a(\vec{r}, \vec{v}, t) = 1 - K(0, \infty|1, \vec{r}, \vec{v}, t) \tag{9}$$

- Extinction probabilities and statistical moments can be computed using the generating function methodology.

  – The generating functions associated with the previous conditional probabilities are defined by :

$$G(x, T, |1, \vec{r}, \vec{v}, t) = \sum_{n=0}^{\infty} x^n K(n, T|1, \vec{r}, \vec{v}, t) \tag{10}$$

$$G(x, T, |i, \vec{r}, \{\vec{v}_1, ..., \vec{v}_i\}, t) = \sum_{n=0}^{\infty} x^n K(n, T|i, \vec{r}, \{\vec{v}_1, ...\vec{v}_i\}, t)$$

  – Regarding these generating functions we have the following relation :

$$G(x, T|i, \vec{r}, \{\vec{v}_1, ..., \vec{v}_i\}, t) = \prod_{j=1}^{i} G(x, T|1, \vec{r}, \vec{v}_j, t) \tag{11}$$

  – The relation between generating function and extinction probability is :

$$e(T|1, \vec{r}, \vec{v}, t) = [G(x, T|\vec{r}, \vec{v}, t)]_{x=0}$$

  – The generating function G associated with the probability distribution K is related to the first two statistical moments of K by :

$$\overline{n}(T|1, \vec{r}, \vec{v}, t) = \left[\frac{\partial G}{\partial x}(x, T|1, \vec{r}, \vec{v}, t)\right]_{x=1} \tag{12}$$

$$\overline{n(n-1)}(T|1, \vec{r}, \vec{v}, t) = \left[\frac{\partial^2 G}{\partial x^2}(x, T|1, \vec{r}, \vec{v}, t)\right]_{x=1} \tag{13}$$

2.2. Master Equation

The conditional probability K is solution of a balance or master equation. We consider the evolution of K with the initial time t for a fixed final time T so that the master equation is a backward Kolmogorov equation. To derive this equation we consider the probability balance within an infinitesimal time interval $[t, t + \Delta t]$. The probability is then expressed as a sum of elementary mutually exclusive probability events. Considering the events : no interaction, capture and reactions of multiplicity up to M we derive the following balance equation :

$$\begin{aligned} K(n, T|1, \vec{r}, \vec{v}, t) &= [1 - \sigma_T(\vec{r}, \vec{v}) v \Delta t]\, K(n, T|1, \vec{r} + \Delta\vec{r}, \vec{v}, t + \Delta t) + \sigma_0(\vec{r}, v)\delta_{n,0} + \\ &+ \sum_{i=1}^{M} \int d^3v_1 \ldots \int d^3v_i\ \sigma_i(\vec{r}, \vec{v}, \{\vec{v}_1, \ldots, \vec{v}_i\}) K(n, T|i, \vec{r} + \Delta\vec{r}, \{\vec{v}_1, \ldots, \vec{v}_i\}, t + \Delta t) \end{aligned} \tag{14}$$

After some terms rearrangement and using the limit process $(\Delta t \to 0)$ we get the backward Kolmogorov equation.

$$\begin{aligned} \left(-\frac{1}{v}\frac{\partial}{\partial t} - \vec{\Omega}.\vec{\nabla} + \sigma_T\right) K(n, T|1, \vec{r}, \vec{v}, t) &= \int \sigma_S(\vec{r}, \vec{v}, \vec{v}') K(n, T|1, \vec{r}, \vec{v'}, t)\ d^3v' + \\ &+ \sigma_0(\vec{r}, v)\delta_{n,0} + \sum_{i=1}^{M} \sigma_i^F(\vec{r}, v) \underbrace{\sum_{n_1} \ldots \sum_{n_i}}_{n = n_1 + \ldots + n_i} \prod_{j=1}^{i} \int \frac{\chi(v_j)}{4\pi} K(n_j, T|1, \vec{r}, \vec{v}_j, t)\ d^3v_j \end{aligned} \tag{15}$$

This equation is subject to terminal and outward boundary conditions.

$$\begin{aligned} K(n, T|1, \vec{r}, \vec{v}, T) &= \delta_{n,1} \\ K(n, T|1, \vec{r}_B, \vec{v}, t) &= \delta_{n,0}\ for\ \vec{n}_B.\vec{v} \geq 0 \end{aligned} \tag{16}$$

2.3. Generating Function Equation

Using the generating function definition (10) in the master equation (15) and using property (11) we derive the generating function equation :

$$\begin{aligned} \left(-\frac{1}{v}\frac{\partial}{\partial t} - \vec{\Omega}.\vec{\nabla} + \sigma_T\right) G(x, T|1, \vec{r}, \vec{v}, t) &= \int \sigma_S(\vec{r}, \vec{v}, \vec{v}') G(x, T|1, \vec{r}, \vec{v'}, t)\ d^3v' + \\ &+ \sum_{i=1}^{M} \sigma_i^F(\vec{r}, v) \left(\int \frac{\chi(v)}{4\pi} G(x, T|1, \vec{r}, \vec{v}, t)\ d^3v\right)^i \end{aligned} \tag{17}$$

This equation is subject to terminal and outward boundary conditions.

$$\begin{aligned} G(x, T|1, \vec{r}, \vec{v}, T) &= x \\ G(n, T|1, \vec{r}_B, \vec{v}, t) &= 1\ for\ \vec{n}_B.\vec{v} \geq 0 \end{aligned} \tag{18}$$

2.4. Extinction, Survival and Initiation Probabilities

Using the generating function equation (17), fission parameters definition (3) and survival definition (8) we derive the survival probability equation :

$$\left(-\frac{1}{v}\frac{\partial}{\partial t} - \vec{\Omega}.\vec{\nabla} + \sigma_T\right) s(T|1,\vec{r},\vec{v},t) = \int \sigma_S(\vec{r},\vec{v},\vec{v'}) s(T|1,\vec{r},\vec{v'},t)\ d^3v' + \\ + \int \overline{\nu}\sigma_F(\vec{r},v,v') s(T|1,\vec{r},\vec{v'},t) d^3v' - \\ - \sum_{q=2}^{M} c_q(\vec{r},v)\left(-\int \frac{\chi(v)}{4\pi} s(T|1,\vec{r},\vec{v},t) d^3v\right)^q \tag{19}$$

This equation is subject to terminal and outward boundary conditions.

$$s(T|1,\vec{r},\vec{v},T) = 1 \\ s(T|1,\vec{r}_B,\vec{v},t) = 0\ for\ \vec{n}_B.\vec{v} \geq 0 \tag{20}$$

The survival probability equation is an adjoint time dependent transport equation with a right hand side specific nonlinear third term. The initiation probability equation is the stationary survival equation :

$$\left(-\vec{\Omega}.\vec{\nabla} + \sigma_T\right) a(\vec{r},\vec{v},t) = \int \sigma_S(\vec{r},\vec{v},\vec{v'}) a(\vec{r},\vec{v'},t)\ d^3v' + \int \overline{\nu}\sigma_F(\vec{r},v,v') a(\vec{r},\vec{v'},t) d^3v' - \\ - \sum_{q=2}^{M} c_q(\vec{r},v)\left(-\int \frac{\chi(v)}{4\pi} a(\vec{r},\vec{v},t) d^3v\right)^q \tag{21}$$

This equation is subject to outward boundary conditions.

$$a(\vec{r}_B,\vec{v},t) = 0\ for\ \vec{n}_B.\vec{v} \geq 0 \tag{22}$$

2.5. Moments of the Neutron Number Probability Distribution

These moments are calculated using the generating function methodology. Applying the first partial derivative operator to both sides of the generating function equation (17) and setting $x = 1$ we get the first moment or mean neutron number equation.

$$\left(-\frac{1}{v}\frac{\partial}{\partial t} - \vec{\Omega}.\vec{\nabla} + \sigma_T\right) \overline{n}(T|1,\vec{r},\vec{v},t) = \int \sigma_S(\vec{r},\vec{v},\vec{v'}) \overline{n}(T|1,\vec{r},\vec{v},t)\ d^3v' + \\ + \int \overline{\nu}\sigma_F(\vec{r},v,v') \overline{n}(T|1,\vec{r},\vec{v},t) d^3v' \tag{23}$$

This equation is subject to terminal and outward boundary conditions.

$$\overline{n}(T|1,\vec{r},\vec{v},T) = 1 \\ \overline{n}(T|1,\vec{r},\vec{v},t) = 0\ for\ \vec{n}_B.\vec{v} \geq 0 \tag{24}$$

One can verify that the mean neutron number is solution of the classic linear adjoint Boltzmann neutron transport equation.

Applying the second partial derivative operator to both sides of the generating function equation (17) and setting $x = 1$ we get the second moment equation.

$$\left(-\frac{1}{v}\frac{\partial}{\partial t} - \vec{\Omega}.\vec{\nabla} + \sigma_T\right)\overline{n^2}(T|1,\vec{r},\vec{v},t) = \int \sigma_S(\vec{r},\vec{v},\vec{v'})\overline{n^2}(T|1,\vec{r},\vec{v},t)\ d^3v' + \tag{25}$$

$$+\int \overline{\nu}\sigma_F(\vec{r},v,v')\overline{n^2}(T|1,\vec{r},\vec{v},t)d^3v' +$$

$$+\overline{\nu(\nu-1)}\sigma_F(\vec{r},v)\left(\int \frac{\chi(v')}{4\pi}\overline{n}(T|1,\vec{r},\vec{v'},t)d^3v'\right)^2$$

This equation is subject to terminal and outward boundary conditions.

$$\overline{n^2}(T|1,\vec{r},\vec{v},T) = 1 \tag{26}$$

$$\overline{n^2}(T|1,\vec{r}_B,\vec{v},t) = 0\ for\ \vec{n}_B.\vec{v} \geq 0$$

The second moment equation is also an adjoint transport equation coupled to the first moment equation by an additional source term.

3. Stochastic Neutronics With a Neutron Source

3.1. Definitions

- Regarding the neutron source we have the following definitions :
  - $S_i(\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t)$ is the probability per unit time that i neutrons are produced at point $\vec{r}$ and time t with velocities $\{\vec{v}_1,...,\vec{v}_i\}$ by the neutron source.
  - p_i is the probability of emission of i neutrons per source events.

$$S_i(\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t) = p_i\chi_i(\vec{v}_1,...,\vec{v}_i)S(\vec{r},t) \tag{27}$$

  - For a fission source :

$$\chi_i(\vec{v}_1,...,\vec{v}_i) = \prod_{j=1}^{i}\frac{\chi_S(v_j)}{4\pi} \tag{28}$$

  $\overline{\nu}_S$ is the average neutron source multiplicity ($\overline{\nu}_S = \sum_{i=1}^{M} ip_i$). $\chi_S(v)$ is the fission spectrum of the source.
  - For an uncorrelated source : $\overline{\nu}_S = 1\ p_i = \delta_{i,1}$

$$S_i(\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t) = \delta_{i,1}S(\vec{r},\vec{v},t) \tag{29}$$

- We define the following conditional probabilities for a neutron source :
  - $P(n,T|0,t)$ is the probability to have n neutrons at final time T produced by a neutron source introduced at initial time t when there are no neutron in the system. We will consider the first two statistical moments of this neutron probability distribution :

$$\overline{N}(T|0,t) = \sum_{n=0}^{\infty} nP(n,T|0,t) \tag{30}$$

$$\overline{N^2}(T|0,t) = \sum_{n=0}^{\infty} n^2P(n,T|0,t) \tag{31}$$

The variance is defined by :

$$V(T|0,t) = (\overline{N^2} - \overline{N}^2)(T|0,t) \tag{32}$$

- $P(n,T|i,\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t)$ is the probability to have n neutrons at final time T generated by i neutrons produced at point $\vec{r}$ with velocities $\{\vec{v}_1,...,\vec{v}_i\}$ at initial time t with a neutron source.

- Initiation probabilities and statistical moments can be computed using the generating function methodology.
  - The generating function associated with the previous conditional probabilities are defined by :

$$G_S(x,T|0,t) = \sum_{n=0}^{\infty} x^n P(n,T|0,t) \tag{33}$$

$$G_S(x,T|i,\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t) = \sum_{n=0}^{\infty} x^n P(n,T|i,\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t) \tag{34}$$

  - Regarding the generating function we have the following relation :

$$G_S(x,T|i,\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t) = \prod_{j=1}^{i} G(x,T|1,\vec{r},\vec{v}_j,t) G_S(x,T|0,t) \tag{35}$$

  - The relation between the initiation probability with source $A(t)$ and the generating function G_S is :

$$A(t) = 1 - \lim_{T\to\infty} [G_S(x,T|0,t)]_{x=0} \tag{36}$$

  - The partial derivatives of the generating function G_S associated with the probability distribution P is related to the first two statistical moments by :

$$\overline{N}(T|0,t) = \left[\frac{\partial G_S}{\partial x}(x,T|0,t)\right]_{x=1} \tag{37}$$

$$\overline{N(N-1)}(T|0,t) = \left[\frac{\partial^2 G_S}{\partial x^2}(x,T|0,t)\right]_{x=1} \tag{38}$$

3.2. Master Equation

To derive the master equation for P we consider the probability balance within an infinitesimal time interval $[t, t+\Delta t]$. The probability is then expressed as a sum of elementary exclusive probability events. Considering no source decay in the time interval, source disintegration without neutron emission and source event with production of i neutrons we derive the following balance equation :

$$P(n,T|0,t) = \left(1 - \Delta t \int S(\vec{r},t)\, d^3r\right) P(n,T|0,t+\Delta t) + \Delta t p_0 P(n,T|0,t) \int d^3r\; S(\vec{r},t) + \Delta t \sum_{i=1}^{M} \int d^3r \int d^3v_1 ... \int d^3v_i\; S_i(\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t) P(n,T|i,\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t+\Delta t) \tag{39}$$

After rearrangement of terms and using the limit process $(\Delta t \to 0)$ we get :

$$-\frac{dP}{dt}(n,T|0,t) = (p_0-1)P(n,T|0,t)\int S(\vec{r},t)d^3r + \tag{40}$$

$$+\sum_{i=1}^{M} p_i \int d^3r S(\vec{r},t)\int d^3v_1 ... \int d^3v_i \; \chi_i(\vec{v}_1,...,\vec{v}_i)P(n,T|i,\vec{r},\{\vec{v}_1,...,\vec{v}_i\},t)$$

$$P(n,T|0,T) = \delta_{n,0}$$

3.3. Generating Function Equation

The generating function equation is deduced from the master equation (40) using the generating function definition (33) and property (35).

$$-\frac{dG_S}{dt}(x,T|0,t) = G_S(x,T|0,t)\int S(\vec{r},t)\left[\sum_{i=0}^{M} p_i\left(\int \frac{\chi_S(v)}{4\pi}\; G(x,T|1,\vec{r},\vec{v},t)\; d^3v\right)^i - 1\right]\; d^3r \tag{41}$$

$$G_S(x,T|0,T) = 1$$

The solution of this generating function equation is :

$$G_S(x,T|0,t) = \exp\left\{\int_t^T d\tau \int d^3r\; S(\vec{r},\tau)\left[\sum_{i=0}^{M} p_i\left(\int \frac{\chi_S(v)}{4\pi}\; G(x,T|1,\vec{r},\vec{v},t)\; d^3v\right)^i - 1\right]\right\} \tag{42}$$

3.4. Initiation Probability

The initiation probability is derived from the generating function expression (42) using relation (36). In case of a fission source we have :

$$A(t) = 1 - \exp\left\{-\int_t^\infty d\tau \int d^3r\; S(\vec{r},\tau)\left[\sum_{i=0}^{M} p_i\left(\int \frac{\chi_S(v)}{4\pi}\; (1-a(\vec{r},\vec{v}))\; d^3v\right)^i - 1\right]\right\} \tag{43}$$

The initiation probability with a neutron source is :

$$A(t) = 1 - \exp\left\{-\int_t^\infty d\tau \int\int S(\vec{r},\vec{v},\tau)a(\vec{r},\vec{v})\; d^3r\; d^3v\right\} \tag{44}$$

The initiation probability is expressed in terms of single initial neutron initiation probabilities.

3.5. Moments of the Neutron Number Probability Distribution

The first two moments are calculated using the generating function methodology. Applying the first derivative operator to the generating function expression (42) and setting $x = 1$ we get the first moment or mean neutron number expression. In case of a fission source we have :

$$\overline{N}(T|0,t) = \overline{\nu}_S \int_t^T d\tau \int d^3r\; S(\vec{r},\tau)\int d^3v\; \frac{\chi_S(v)}{4\pi}\; \overline{n}(T|1,\vec{r},\vec{v},\tau) \tag{45}$$

For an uncorrelated source the mean neutron number is :

$$\overline{N}(T|0,t) = \int_t^T d\tau \int\int S(\vec{r},\vec{v},\tau)\ \overline{n}(T|1,\vec{r},\vec{v},\tau)\ d^3r\ d^3v \tag{46}$$

Applying the second partial derivative operator to both sides of the generating function expression (42) and setting $x = 1$ and using the mean neutron number expression we get the variance expression. In case of a fission source we have :

$$\begin{aligned} V(T|0,t) = \overline{\nu}_S \int_t^T d\tau \int d^3r\ S(\vec{r},\tau) \int d^3v\ \frac{\chi_S(v)}{4\pi}\overline{n^2}(T|1,\vec{r},\vec{v},\tau) + \\ +\overline{\nu_S(\nu_S-1)} \int_t^T d\tau \int d^3r\ S(\vec{r},\tau) \left(\int d^3v\ \frac{\chi_S(v)}{4\pi}\overline{n}(T|1,\vec{r},\vec{v},\tau)\right)^2 \end{aligned} \tag{47}$$

For an uncorrelated source the neutron number variance is :

$$V(T|0,t) = \int_t^T d\tau \int\int S(\vec{r},\vec{v},\tau)\overline{n^2}(T|1,\vec{r},\vec{v},\tau)\ d^3r\ d^3v \tag{48}$$

The moments of the probability distribution with source are expressed in terms of single initial neutron probability distribution moments.

4. SOLUTION OF MULTIGROUP STOCHASTIC EQUATIONS WITH PANDA

4.1. Numerical Solving Methodology

PANDA is a general purpose transport code with stochastic neutronics options. The transport equation is solved using standard discrete ordinates and multigroup methods. One and two dimensional geometries with cartesian and curvilinear coordinate systems are treated using orthogonal structured spatial grids and angular S_N discretization of the unit sphere. The time independent transport equation is used for fixed source and eigenvalue problems (α and k). In addition to classic criticality applications PANDA is used for time dependent survival probabilities, mean and variance of the neutron number and asymptotic extinction probability calculations.

The single initial neutron mean number equation (23) is the time dependent homogeneous adjoint transport linear equation. This equation is routinely solved by standard transport numerical methods like the classic deterministic S_N method. The second moment equation (25) and the survival probabilities equation (19) satisfy transport equations with the same kernel used for the adjoint linear transport problem. Therefore, these equations should be solved by the S_N method devised for numerical solution of the direct and adjoint linear transport equations.

The additional term of the second moment equation depends on the first moment, these equations are coupled. The backward solution, starts form the terminal time. On each time step the mean number equation is solved first, then the second moment equation is solved using the first moment result.

The additional terms for the survival probability and second moment equation use the fission parameters c_q. These parameters are computed using definition (3) and Terrell fission multiplicity distribution[14].

4.2. Multigroup Equations

The mean number, second moment and survival equations for the single initial neutron are solved using the multigroup method for the energy variable and Legendre expansion of the scattering kernel in two dimensions. Using the following differential operator definition :

$$L_g^{\dagger} = \left(-\frac{1}{v_g}\frac{\partial}{\partial t} - \vec{\Omega}.\vec{\nabla} + \sigma_g^T(\vec{r}) \right) \tag{49}$$

the generic multigroup adjoint transport equation with terminal and boundary conditions can be written as :

$$\begin{aligned} L_g^{\dagger}\varphi_g^{\dagger}(\vec{r},\vec{\Omega},t) = \sum_{g'=g}^{G}\sum_{l=0}^{L}\sum_{n=0}^{l}(2l+1)R_{l,n}(\vec{\Omega})\sigma_{g,g'}^{(l)}(\vec{r})\varphi_{g'}^{\dagger(l)}(\vec{r},t) + \\ + \sum_{g'=0}^{G}\sigma_{g,g'}^{F}(\vec{r})\varphi_{g'}^{\dagger(0)}(\vec{r},t) + Q_g^{\dagger}(\vec{r},\vec{\Omega},t) \end{aligned} \tag{50}$$

$$\begin{aligned} \varphi_g^{\dagger}(\vec{r}_B,\vec{\Omega},t) = 0 \;\; for \;\; \vec{n}_B.\vec{v} \geq 0 \\ \varphi_g^{\dagger}(\vec{r},\vec{\Omega},T) = 1 \end{aligned}$$

The specific values of the generic equation source term for first moment, second moment and survival equation are given in table (I). The initiation probability is solution of the stationary survival probability

Table I. Generic adjoint transport equation additional source definitions.

$\varphi_g^{\dagger}(\vec{r},\vec{\Omega},t)$	$Q_g^{\dagger}(\vec{r},\vec{\Omega},t)$
$\overline{n}_g(T\|1,\vec{r},\vec{\Omega},t)$	0
$\overline{n^2}_g(T\|1,\vec{r},\vec{\Omega},t)$	$2c_g^{(2)}(\vec{r})\left[\sum_{g'=1}^{G}\chi_{g'}\overline{n}_{g'}^{(0)}(T\|1,\vec{r},t)\right]^2$
$s_g(T\|1,\vec{r},\vec{\Omega},t)$	$-\sum_{q=1}^{M}c_g^{(q)}(\vec{r})\left[-\sum_{g'=1}^{G}\chi_{g'}s_{g'}^{(0)}(T\|1,\vec{r},t)\right]^q$

equation. The mean and variance neutron number and the initiation probability of a neutron source induced population are computed using the single initial neutron first, second moment and initiation probability results. For an uncorrelated isotropic multigroup source $S_g(\vec{r},t)$ the resulting mean, variance and initiation probabilities are :

$$\overline{N}(T|0,t) = \int_t^T d\tau \int_V d^3r \left(\sum_{g=1}^{G} S_g(\vec{r},\tau) \overline{n}_g^{(0)}(\vec{r},\tau) \right) \tag{51}$$

$$V(T|0,t) = \int_t^T d\tau \int_V d^3r \left(\sum_{g=1}^{G} S_g(\vec{r},\tau) \overline{n^2}_g^{(0)}(\vec{r},\tau) \right) \tag{52}$$

$$A(t) = 1 - exp\left\{ \int_t^\infty d\tau \int_V d^3r \left(\sum_{g=1}^{G} S_g(\vec{r},\tau) a_g^{(0)}(\vec{r},\tau) \right) \right\} \tag{53}$$

For validation of the method a number of test problems were run using PANDA and compared against stochastic neutronics point model results. A good agreement was found.

4.3. Numerical Application

An application to the stochastic behavior of a super-critical HEU cylinder is considered.

- The cylinder characteristics are :
  - height H=30cm, radius R=7.5cm,
  - atomic composition 95% ^{235}U and 5% ^{238}U,
  - density ρ=19 g/cm^3.
- The computed quantities are :
  - the k_{eff} and α eigenvalues,
  - the survival (s) and initiation probability (a) for a single initial fission neutron,
  - the mean $(\bar{n})$ and standard deviation (σ) of the neutron distribution induced by a neutron source of one fission neutron per nanosecond.
- The PANDA calculation conditions are :
  - Reflective boundary conditions at Z=0,
  - 1568 cells, S_{16}, P_4,
  - 16 groups ENDF/B6-VI nuclear data.

The numerical results are summarized in table II. The initiation probability map in figure 1 gives for each cell the initiation probability for one fission neutron emitted in this cell. In figure 2 we have plotted the time dependent single initial neutron survival probability which converges to the stationary initiation probability. We also verify in figure 3 that the logarithmic time derivatives of mean neutron number and standard deviation converge to the alpha eigenvalue. Such numerical tests in finite dimensions will be used in the validation process for intercode comparison.

Table II. Criticality and stochastic neutronics numerical results for the test problem.

k_{eff} (total)	k_{eff} (prompt)	α	a	$\bar{n}(T = 100ns)$	$\sigma(T = 100ns)$
1.09777	1.09142	$15.196\ 10^6\ s^{-1}$	0.0582	212.3	115.7

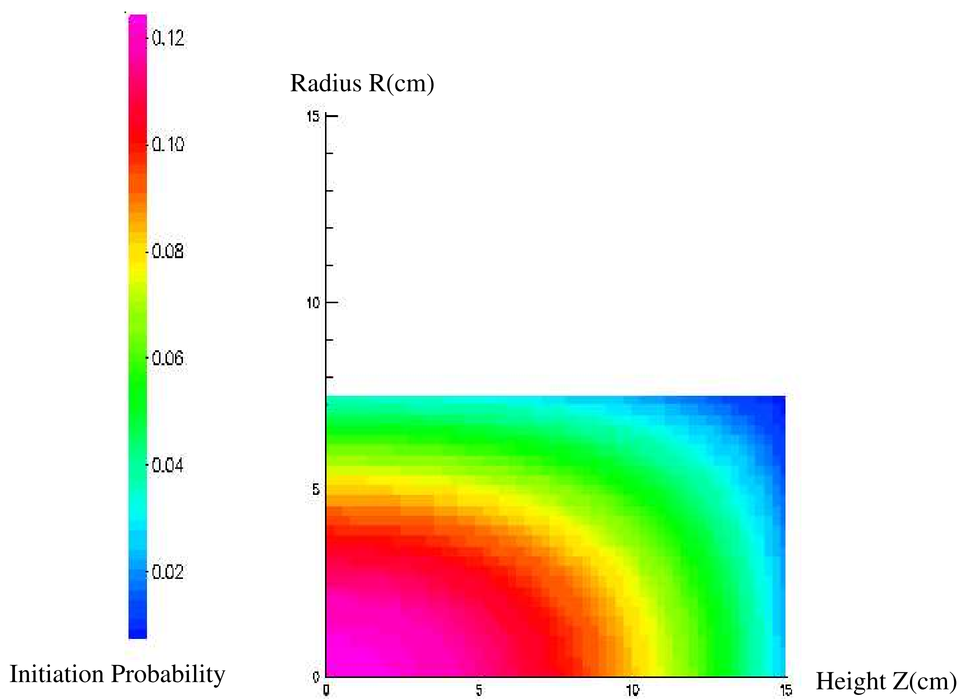

Figure 1. Initiation probability map for a single initial fission neutron.

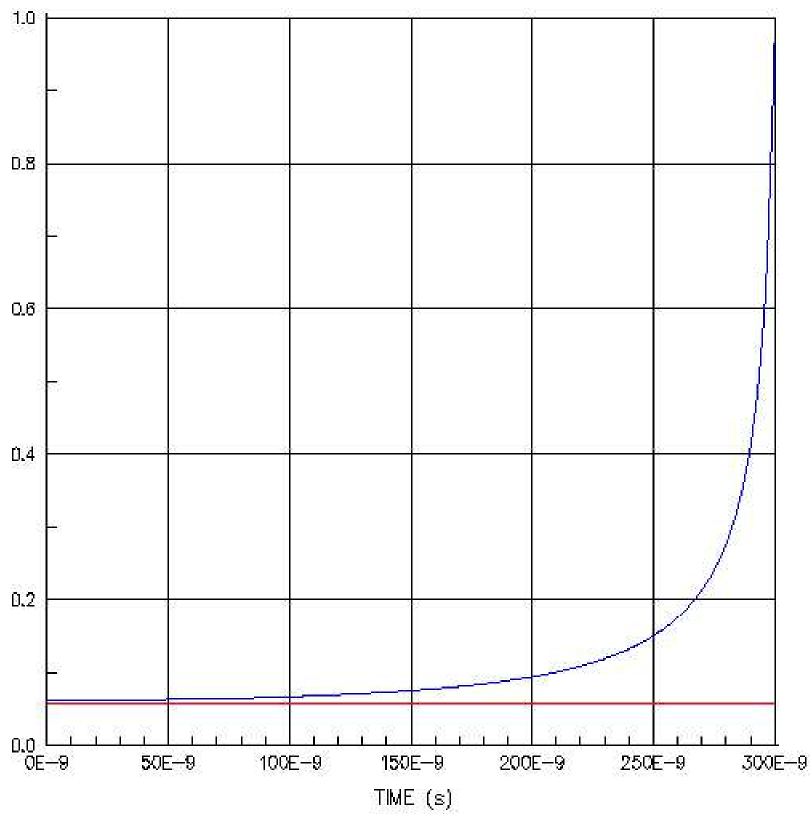

Figure 2. Time dependent survival probability (blue) and initiation probability (red).

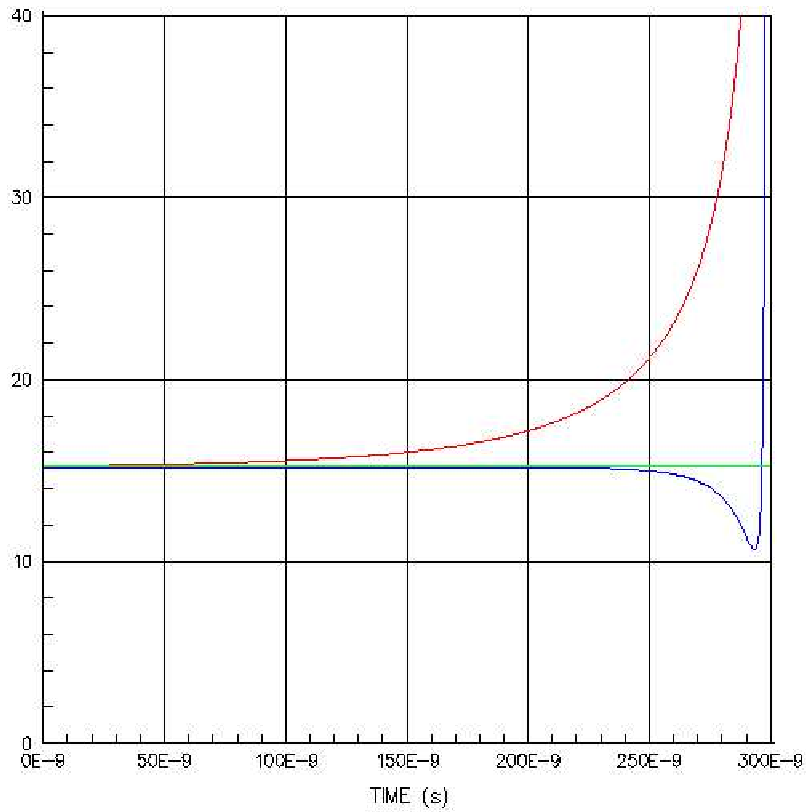

Figure 3. Stationary α eigenvalue (green) and Logarithmic time derivative of mean neutron number (blue) and standard deviation (red) (α unit is $10^6\ s^{-1}$)

5. CONCLUSIONS

The first and second moment of the neutron number probability distribution and the survival probability for a single initial neutron were derived from the Kolmogorov Backward equation. The corresponding equations behave as time dependent adjoint transport equations. The second moment and survival probability equations involve an additional source term. Nevertheless these equations can be solved using standard S_N methods. Moreover these single initial neutron quantities can be used to compute the neutron source induced distribution's mean, variance and initiation probability.

The deterministic 2D S_N code PANDA has been adapted to solve these adjoint time dependent linear and nonlinear multigroup transport equations. These stochastic developments of this code will be used for nuclear safety applications. In the future, neutron noise measurements applications will also be considered.

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