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INVERSE PROBLEM FOR THE ELECTRON DEPTH-DOSE CURVE

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ABSTRACT

We discuss accuracy of the electron spectra reconstructed from measured central axis depth-dose distributions produced by medical electron beams. This reconstruction technique is an inverse radiation transport problem which is poorly conditioned, in the sense it may become unstable due to small perturbations in the input data. Predicting the very sharp peak in the electron spectra provides an additional challenge for the reconstruction technique because of the general problem numerical techniques have modeling sharp (delta-like) functions. To improve efficiency and robustness of the reconstruction technique, we developed an algorithm based on a separation of the electron spectrum into singular and regular components. The new robust reconstruction algorithm allowed us to estimate the influence of the initial angular spread of incident electrons on the reconstructed electron spectra. We show that this influence is significant and must be taken into account. Effectiveness of the reconstruction techniques is demonstrated by numerical results.

Key Words: Electron, transport equation, inverse problem, depth-dose curve

1. INTRODUCTION

High-energy electron beams (6-20 MeV) are used for treating superficial tumors (less that 5 cm deep) such as skin and lip cancers, head and neck cancers and breast cancer. Currently, the Hogstrom pencil beam algorithm that was proposed in 1981 is the only commercial tool available for electron beam treatment planning [1]. Being fairly accurate in homogeneous tissue, this algorithm can produce up to 40% errors in the presence of tissue inhomogeneities such as bone, lung or air cavities.

In the near future, more advanced Monte Carlo treatment planning systems may appear in clinics which will simulate the 3D electron transport more accurately using CT-defined patient geometry [2]. Commissioning of Hogstrom's pencil beam algorithm requires only measurements of depth dose distributions in water and some beam profiles while the 3D Monte Carlo simulation additionally requires an explicit form of the energy spectra and angular distribution of electrons from medical accelerators incident upon the patient's skin. Magnetic spectrometry [3]

or Monte Carlo treatment head simulation [2] to obtain the electron energy-angular distributions are not commonly implemented into routine clinical praxis because of the complexity of measurement and the knowledge base required for completing the Monte Carlo simulations. We discuss an alternative approach based on inverse reconstruction of electron spectra from measured depth dose distributions (Fig.1). This approach does not require any supplementary equipment and can be easily implemented in a busy clinic [4].

The identification of the energy spectrum from a radiation source with a limited number of external detectors is akin to inverse radiation transport problems. Inverse problems are poorly conditioned, in the sense they may become unstable to small changes in the input data. Moreover, computational finite arithmetic may impose additional practical limitations to the calculation algorithm. Conventional unfolding techniques usually cannot reproduce the FWHM (full-width-at-half-maximum) of the spectral peak and they exhibit large fluctuations throughout the low energy region of the spectrum [4]. Nevertheless, it was shown that these poor reconstruction of the electron spectra is able to reproduce the depth dose curve in water with good accuracy. What is not clear is whether these poor reconstruction results will be accurate enough to allow 3D distributions in heterogeneous phantoms to be accurately predicted. Therefore, we feel that more efficient and robust solution algorithms are desired [5,6].

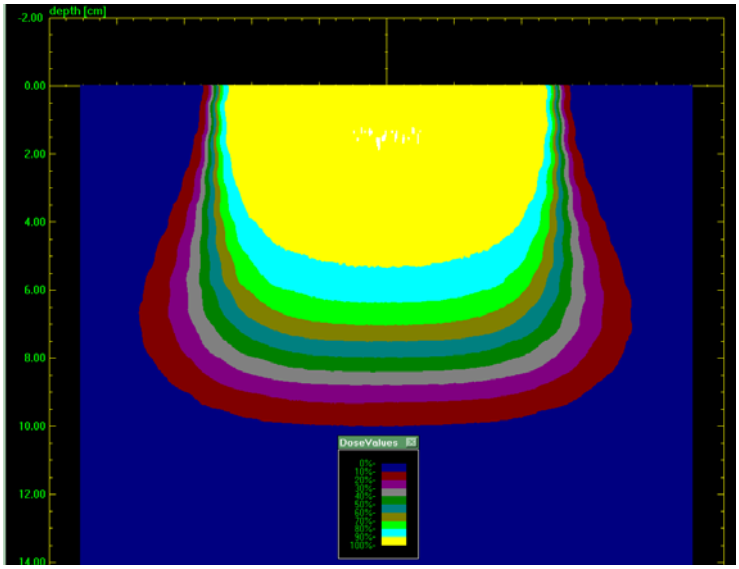

Figure 1. Measured dose distribution in water from a 18 MeV clinical electron beam

2. RECONSTRUCTION TECHNIQUE

Reconstruction of electron spectra is based on numerical solutions to the equation

$$D(z) = \sum_{n=1}^{N} w_n d(z, E_n) \ , \qquad (1)$$

where w_n , $n = 1,...,N$ are the spectral weights, $d(z, E_n)$, $n = 1,...,N$ are the monoenergetic depth-dose distributions called response functions and $D(z)$ is the measured central axis depth-dose distribution. Equation (1) is a discretized form of the integral Fredholm equation of the first kind.

If the response functions are known, the spectral weights may be found using one of the methods of statistical or deterministic optimization with different modifications to improve the quality of numerical solutions. The accuracy of reconstructed spectra depends on (1) the optimization method and (2) the accuracy of response functions.

The response functions $d(z, E_n)$ are usually simulated in RZ geometry for a point source or in parallel broad beam geometry with application of an inverse square factor (Fig. 2). The deterministic discrete ordinates method [7] in parallel plane geometry was used to generate input monoenergetic depth dose kernels over the energy interval 0.5 MeV to 25 MeV in energy steps of 125 keV (Fig. 3). The accuracy of the calculated monoenergetic depth dose curves was 0.01%, which corresponds to the convergence criterion of the discrete ordinates method.

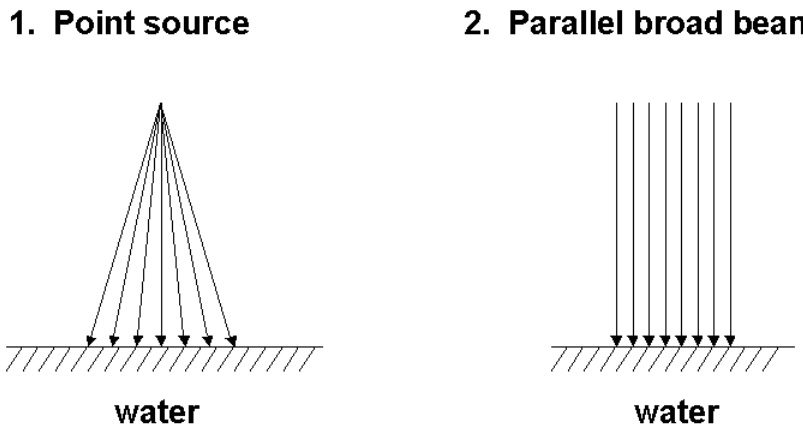

Figure 2. Models used for calculation of monoenergetic response functions

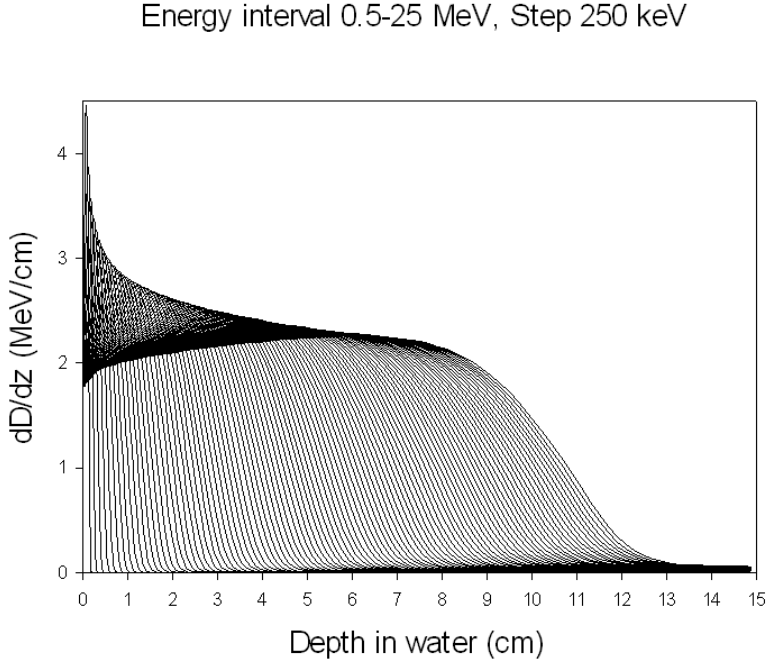

Figure. 3. Monoenergetic response functions

2.1. Variational method with regularization

We developed an unfolding technique where the integral Fredholm equation of the first kind is solved by a variational method combined with different regularization techniques to correct for the ill-conditioned property. The objective function is formulated in terms of constrained least squares and the minimum is sought using a version of the quasi-Newton method. A spatial Tichonov's regularization function is included into objective function to achieve stability of the reconstructed spectrum [8]. The final equation for the objective function is given by

$$\Theta_m[f(E)] = \int_0^{z_{\max}} \left(\overline{D}(z) - D(z)\right)^2 dz \; + \alpha \Omega_m[f(E)] \;, \tag{2}$$

where $\overline{D}(z)$ is the measured depth dose distribution, $\Omega_m[f(E)]$ is the regularization function and α is the regularization parameter. The non-negative source $f(E)$ that will minimize the objective function $\Theta_m[f(E)]$ will give a solution to the inverse problem of the depth-dose curve. We write this problem as

$$\min \Theta_m[f(E)], \quad \text{subject to} \quad f(E) \geq 0 \;. \tag{3}$$

The regularization function $\Omega_m[f(E)]$ in Eq. (2) is given by

$$\Omega_m[f(E)] = \int_0^{E_{\max}} dE \sum_{k=0}^{m} p_k(E) \left(\frac{d^k f(E)}{dE^k} \right)^2 \;, \tag{4}$$

where $p_k(E)$ is a positive continuous weighting function which is usually $p_k(E) \equiv 1$. It can be shown that the variational problem (1)-(4) for any $\alpha > 0$ is equivalent to the integral Fredholm equation of the *second kind* that is well-conditioned.

The minimum of the objective function given by Eq. (2) is found using the L-BFGS-B code which has been developed at the Optimization Technology Center, a joint venture of Argonne National Laboratory and Northwestern University. The code is based on the L-BFGS-B algorithm which is a limited-memory version of the quasi-Newton method with simple constraints on the variables [9]. The method is very convenient for practical use because it requires only an objective function and its derivatives as input to perform the minimization search. Examples of the iterative numerical reconstruction of electron spectra are shown in Figs. 4 and 5.

2.2. Singular component decompostion

The singular sharp peak present in the electron spectra from medical linear accelerators is another major problem for the reconstruction technique. This singular component has steep gradients that are difficult to reproduce numerically. Also this peak contains most of the electrons and so forces the regular component close to the numerical "noise" of the reconstruction technique. These problems motivated us to use a hybrid technique that reconstructs the low-energy part of the spectrum numerically but models the peak of the spectrum using an analytical approximation. We consider the electron energy spectrum as a sum of regular and singular components

$$f(E) = f^{\text{sing}}(E) + f^{reg}(E) \,. \tag{5}$$

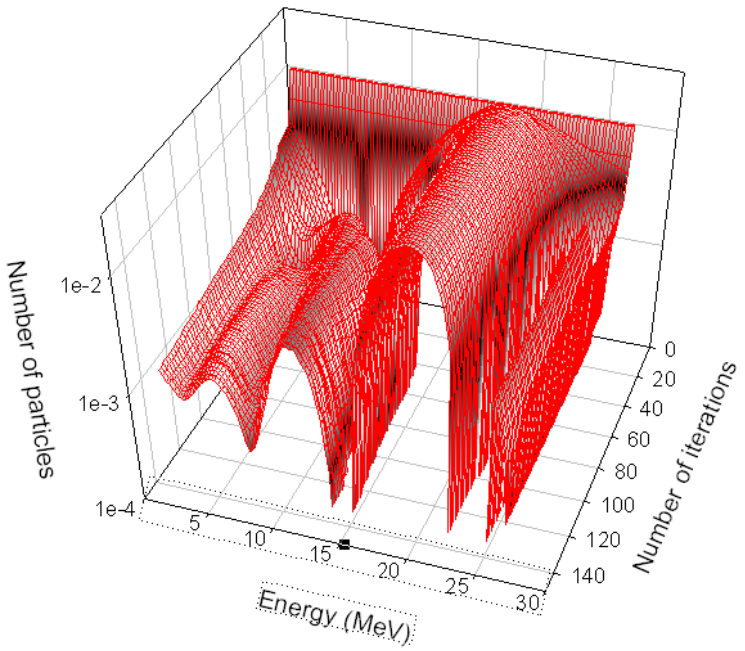

Figure 4. Iterative numerical reconstruction without regularization

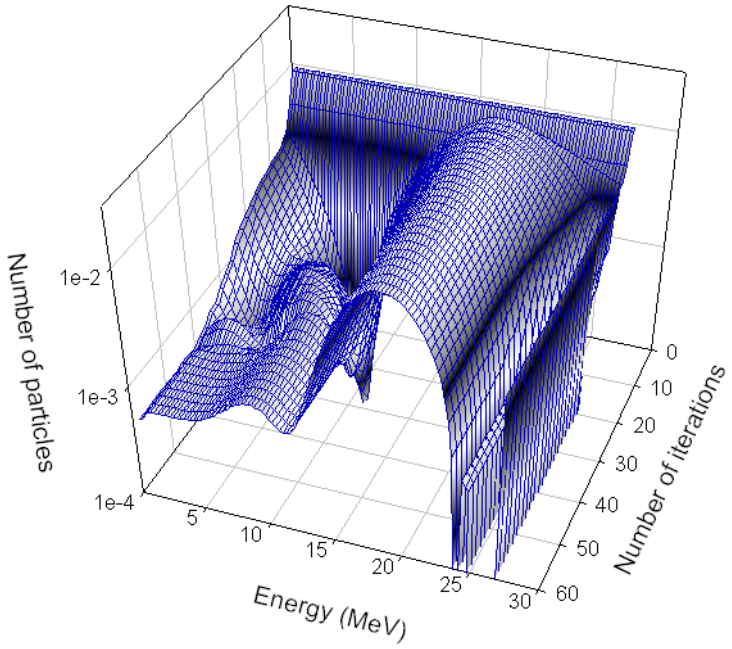

Figure 5. Iterative numerical reconstruction with regularization

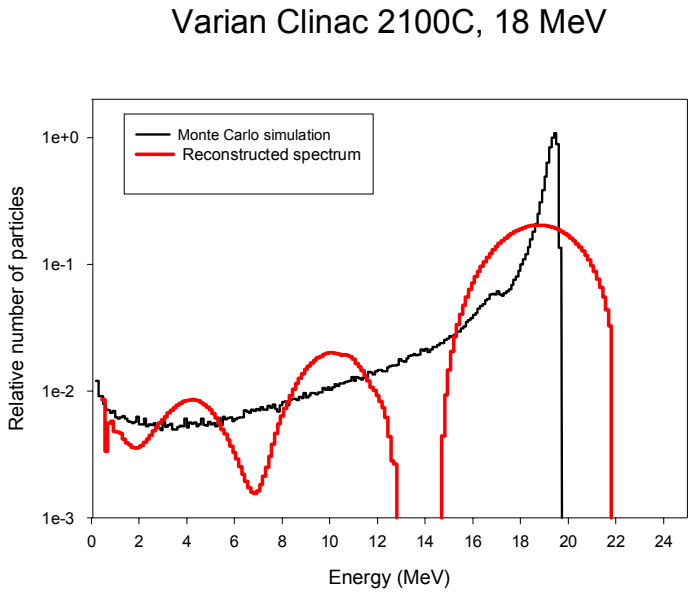

Figure 6. Comparison of reconstruscted and Monte Carlo calculated incident spectra. Reconstruction technique does not include regularization and singular component decomposition techiques.

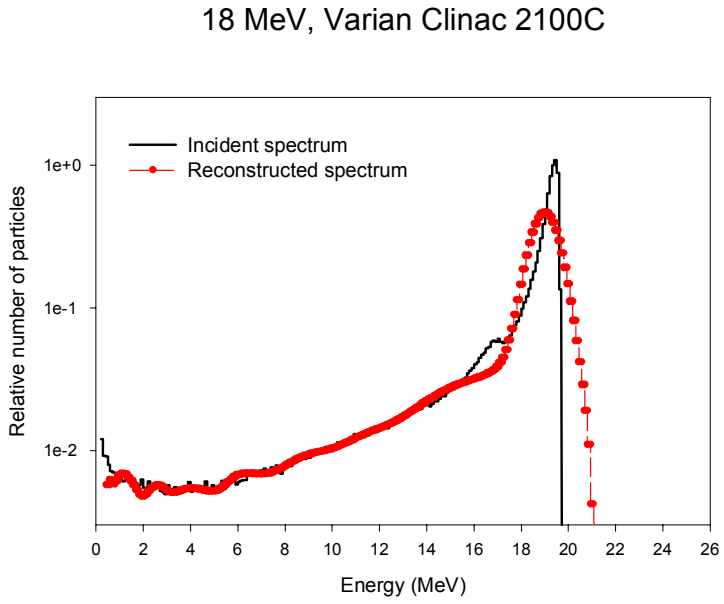

Figure 7. Comparison of reconstructed and Monte Carlo calculated incident spectra. Reconstruction technique includes both regularization and singular component decomposition technigues.

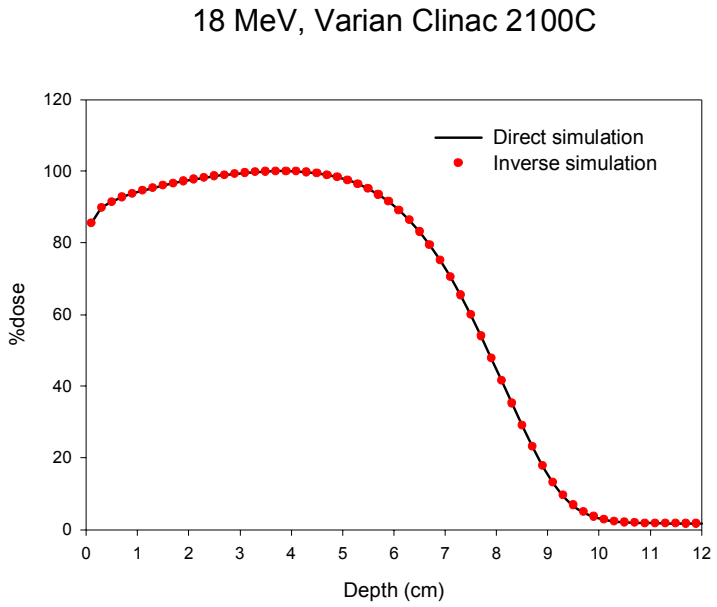

Figure 8. Comparison of the depth-dose distributions obtained with the energy spectra presented in Fig. 7.

In this article, we approximate the singular component of the spectrum by a narrow weighted Gaussian function

$$f^{\text{sing}}(E) = w\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(E-E_0)^2}{2\sigma^2}\right), \qquad (6)$$

where E_0 and σ are the mean energy and variance and w is a weighting factor. If the spectrum is normalized to one particle the weight w provides the fractional number of particles in the singular component and the weight (1-w) provides the fractional number of particles in the regular component. The parameters E_0, σ and w in Eq. (6) are found from the condition that the singular depth dose component $D^{\text{sing}}(z)$ approximates the fall-off and toe regions of the total depth dose $D(z)$. The computational algorithm for solving this problem is based on the minimization of a least squared objective function without the regularization term. The minimization problem is solved for only three variables E_0, σ and w using the same L-BFGS-B optimization routine which is used for spectral weights. The gradients of the objective function are obtained by analytical differentiation of the objective function on the parameter (E_0, σ or w).

The regular dose component $D^{\text{reg}}(z)$ is found as a difference of the total depth dose $D(z)$ and the singular depth dose component $D^{\text{sing}}(z)$. Then, applying the numerical reconstruction technique to the regular depth dose component $D^{\text{reg}}(z)$ we find the regular component of the spectra $f^{\text{reg}}(E)$.

The unfolding technique was applied to "benchmark" depth dose distribution data simulated for different medical linear accelerators using the Monte Carlo BEAM code [10]. The results of unfolding were compared to the known Monte Carlo energy spectra that produced the original depth dose curves. Figures 6 and 7 show computational results for the 18 MeV beam of the Varian Clinac 2100C accelerator. These figures demonstrate that the accuracy in the low energy part of electron spectrum can be improved using regularization.

Accuracy of the singular component is worse primarily due to theoretical limits of reconstruction. This, however, does not affect the accuracy of the depth-dose distributions calculated with the reconstructed spectra. These depth-dose distributions have relative accuracy less than 1%. In Figure 8, we present the depth dose distributions calculated with the Monte Carlo and reconstructed energy spectra. The depth-dose distributions were calculated using the CEPXS/ONEDANT coupled electron-photon discrete ordinates package [7].

3. ANGULAR CORRECTION IN THE RECONSTRUCTION TECHNIQUE

Accuracy in the low energy part of the electron spectra reconstructed from the central axis depth dose distributions is dependent upon the angular distribution of electrons incident at the surface. Reconstruction procedures used to date have not incorporated angular data explicitly. This leads to the electron angular distribution being interpreted by reconstruction algorithms as the presence of an increased number of low energy electrons in the spectrum. Thus the low energy part of the spectrum is overestimated. We present a method to correct for this deficiency.

The response functions $d(z,E_n)$ are usually simulated in RZ geometry for a point source or in parallel broad beam geometry with application of an inverse square factor. These models correspond to *normal* incidence of electrons at the central beam axis. This is an acceptable approximation for direct electrons with small angular spread. However, low-energy electrons from the treatment head and collimation system may have large angles of incidence that affect the depth dose distribution significantly at shallow depths since more energy is deposited close to the surface. Reconstruction methods used to date have ignored the incident angular distribution of electrons. This leads to the angular distribution masquerading as an increased low-energy component to the reconstructed energy spectrum. The use of such a reconstructed spectrum as input to the phase space of a Monte Carlo calculation may lead to a reasonable prediction of depth-dose distribution. However, it may not accurately predict the 3D dose distribution or the dose perturbations caused by shallow-depth heterogeneities since a knowledge of both incident energy and incident angular distribution are required.

We use response functions $d(z,E_n)$ corrected for the angular distribution of incident electrons. For an azimuthally symmetric model of the angular distribution at the beam axis, the corrected response functions may be calculated as

$$d(z,E_n)=\int_{-1}^{1}\chi_n(\mu)\Delta(z,\mu,E_n)d\mu \ , \qquad (7)$$

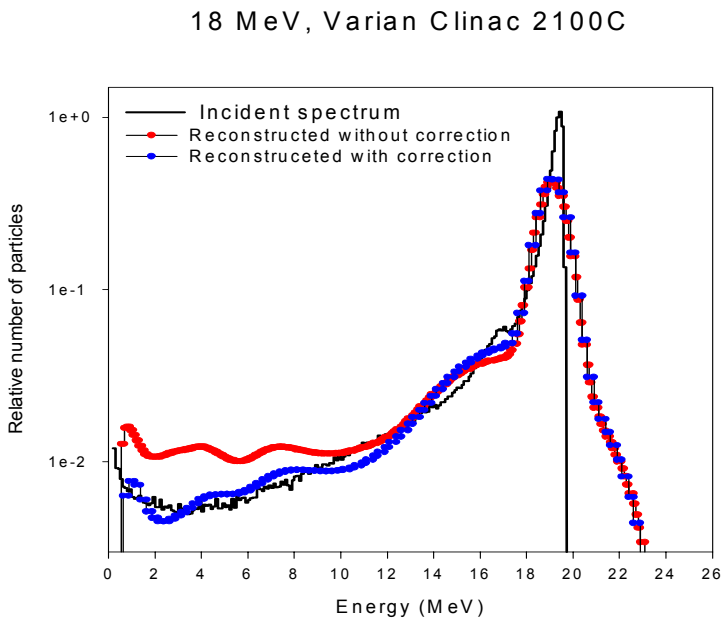

Figure 9. Comparison of energy spectra reconstructed with and without angular correction

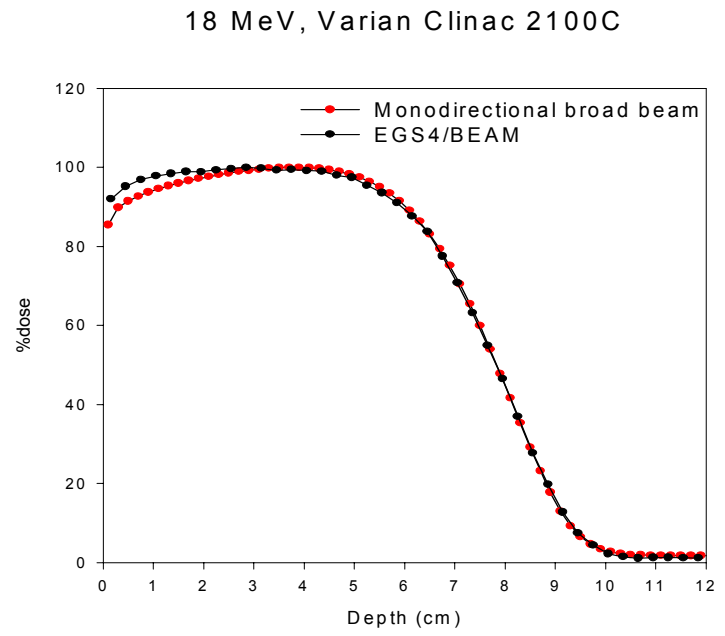

Figure 10. Influence of initial anular spread on the depth-dose distributions

where $\mu = \cos\theta$, θ is the angle between the normal to the surface and the direction of the incident electon. Variable $\chi_n(\mu)$ is the angular distribution of the incident electons. The energy-angular response functions $\Delta(z,\mu,E_n)$ represent the depth-dose distributions from a monoenergetic elementary source $q(\mu,E)$ with an energy $E = E_0$ and an angle of incidence $\mu = \mu'$

$$q(\mu,E) = \delta(E - E_n)\delta(\mu - \mu') \ . \tag{8}$$

Expressing equation (7) in discrete form we obtain

$$d(z,E_n) = \sum_{m=1}^{M} w_m \chi_n(\mu_m)\Delta(z,\mu_m,E_n) \ . \tag{9}$$

The functions $\Delta(z,\mu_m,E_n)$ were calculated in the parallel plane geometry and corrected for the divergence of the electron beam. We used 107 discrete energies between 0.5 and 27 MeV and within each discrete energy 16 angular intervals between 0 and 90° were chosen so the total number of depth-dose distributions to approximate the function $\Delta(z,\mu,E)$ was 1712 (i.e. 107x16). These energy-angular response functions were folded with the incident angular distribution $\chi_n(\mu)$ that, at this stage, was estimated from Monte Carlo simulations [10].

An example of the reconstructed spectrum with corrected response functions is presented in Fig. 9 for the 18 MeV beam of a Varian Clinac 2100C accelerator. Energy spectrum reconstructed without angular correction is also shown in Fig. 9. The influence of angular spread on the depth-dose curve can be understood by comparing a complete Monte Carlo simulation that includes angular distribution of incident electrons to the depth-dose distribution for a monodirectional broad beam (Fig. 10).

3. CONCLUSIONS

We have developed an algorithm for reconstructing the electron spectra of medical accelerators from measured central axis depth-dose distributions. Significant improvement of the robustness and accuracy of reconstructed spectra was achieved using a separation of the singular and regular components of the electron spectrum and the angular weighting of the monoenergetic response functions. The algorithm is based on an effective variational method with regularization technique and a Gaussian function as an approximation for the singular peak of the electron spectrum. The regular component of the spectrum reconstructed is in close agreement with direct Monte Carlo simulations. Accuracy of the singular component is worse primarily due to the theoretical limit of reconstruction. This, however, does not affect the accuracy of the predicted depth-dose distributions. Reconstructed electron spectra do not have the non-physical oscillations typical of other reconstruction techniques and they reproduce the input depth-dose distributions with relative accuracy less than 1%. In order to be applied for commissioning of a Monte Carlo treatment planning system, the reconstruction algorithm presented in this article must be supplemented by a method for determining the angular distribution of incident electrons.

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