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FORMOSA-P RODDED 3-D/2-D GEOMETRY COLLAPSE METHODOLOGY

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ABSTRACT

A methodology has been developed whereby consistent 3-D/2-D geometry collapse methodology may be applied to models of pressurized water reactors (PWRs) which include insertion of control rods, both for the purpose of rodded depletion models, and for calculations of off-nominal rodded configurations, with negligible loss of fidelity. While the methodology is successful in collapsing macroscopic cross section models for both rodded depletions and off-nominal rodded models, with microscopic cross section models, the effort was only successful for rodded depletion models, with microscopic model-based off-nominal rodded models showing undesirable power distribution errors. The successful rodded collapse methodology typically exhibits RMS pin power collapse errors of approximately $5x10^{-3}$ in normalized pin powers, and mean maximum errors of approximately $1x10^{-2}$ in normalized pin powers.

Keywords: Reactor analysis, incore optimization

1. INTRODUCTION

The incore nuclear fuel management optimization code FORMOSA-P [1] was developed to determine the family of near-optimum loading patterns (LPs) for pressurized water reactors (PWRs). The core simulator originally utilized by FORMOSA-P was a 2-D geometry (radial), nodal expansion method (NEM) core simulator coupled with an assembly power response generalized perturbation theory (APRGPT) LP evaluator used during optimization. Because modern PWR fuel assembly axial heterogeneities have increasingly challenged the fidelity of 2-D core simulators, coupled with the fact that most PWR LP attributes of interest during the optimization, *e.g.,* core life, feed fuel enrichments, peak pin powers, and discharge burnup, are 2-D or lower dimensional based core attributes, a three-dimensional/two-dimensional (3-D/2-D) geometry collapse methodology was developed [2]. This methodology, coupled with APRGPT, provides the capability to accurately duplicate the results of 3-D methods at speed more than 100 times faster than 3-D methods. This 3-D/2-D collapse methodology did not originally include a control rod model, needed to model reactors which are operated with control rods inserted through most of their fuel cycles, as is the case for reactors designed and built by the former Babcock & Wilcox Company. The core models which model these reactors which are depleted with control rods inserted are referred to in this paper as rodded depletion models. The capability to treat control rod insertion is also potentially useful for modeling control rod insertion as an off-nominal condition, which is required if core configurations with control rods inserted must be constrained in some way, most often via a constraint on the rodded F-delta-H power peaking factor. In this context, off-nominal rodded configurations are considered to be any core configuration with control rods at positions differing from the positions at which the core depletion is carried out. Although the off-nominal rodded condition of primary interest in this work is the condition of the lead bank of control rods inserted to the full power insertion limit, this work is also of interest to any reactors operated at considerable power levels with control rods inserted. Of particular applicability to this methodology are the reactors operated by the French national electric utility Electricite de France, some of whose reactors are operated at substantial power levels using complex rodded configurations [3]. In this work, we describe the extension of the 3-D/2-D collapse methodology to include rodded models, both for rodded depletion models, and for off-nominal rodded configurations. The theory of this rodded 3-D/2-D geometry collapse methodology is presented in Section 2, and the results and computational performance are presented in Section 3, followed by conclusions in Section 4.

2. THEORY OF RODDED 3-D/2-D GEOMETRY COLLAPSE

The 3-D/2-D geometry collapse methodology is based upon a reaction rate preserving collapse of the 3-D cross sections, obtained from lattice calculations, to 2-D, and, in the case of the assembly discontinuity factors (ADFs), a surface current preserving collapse. This collapse methodology preserves the problem's eigenvalue as well. In the case of cross section derivatives, both instantaneous derivatives, and cross section history derivatives, the collapse preserves the reaction rate change due to these cross section derivatives, or, in the case of the ADF derivatives, the change in surface current attributable to the ADF derivative. Because the FORMOSA-P three-dimensional reactor simulator includes the capability to model cross sections using either a macroscopic or microscopic depletion model, the collapse methodology includes the capability to collapse from either macroscopic or microscopic-based 3-D cross sections. Because use of the microscopic cross section model during the 2-D collapsed optimization phase remains impractical, when the microscopic model is utilized, the 3-D/2-D collapse methodology collapses from the 3-D microscopic model to a 2-D macroscopic model. This has the effect of "freezing" the history effects modeled by the microscopic model at their reference LP values during the course of the optimization. As will be shown in Section 3, this does result in some minor degradation of the collapse fidelity.

For all rodded models, the instantaneous control rod branch case cross sections $\Delta\Sigma^{i}_{xnk}$, where the subscripts *x, n,* and *k* refer to cross section type *x* for radial node *n* and axial node *k*, and superscript *i* refers to the unrodded or rodded configuration *i*, are collapsed in accordance with:

$$\langle\Delta\Sigma^{i}_{xn}\rangle = \frac{\sum_{k = k^{Bottom}_{Fuel}}^{k^{Top}_{Fuel}} \Delta\Sigma^{i}_{xnk}\phi^{i}_{gnk}\Delta z_k}{\bar{\phi}^{i}_{gn}H}, \tag{1}$$

where ϕ^{i}_{gnk} refers to the flux solution for neutron energy group *g*, Δz_k refers to the node axial height and *H* refers to the active core height. Equation (1) is equally applicable to both the microscopic and macroscopic cross section models, since it is applied to macroscopic cross sections which may be derived either directly from macroscopic lattice cross sections, or to macroscopic cross sections calculated by FORMOSA-P from lattice microscopic cross sections and internally calculated nuclide number densities obtained from the FORMOSA-P microscopic depletion model.

For rodded depletion cases in which a macroscopic control rod history cross section correction are utilized, the control rod history cross section corrections $\Delta\Sigma_{xnk}^{rh}$ are collapsed in accordance with:

$$\langle\Delta\Sigma_{xn}^{rh}\rangle = \frac{\sum_{k = k_{Fuel}^{Bottom}}^{k_{Fuel}^{Top}} \Delta\Sigma_{xnk}^{rh}\phi_{gnk}^{1}\Delta z_k}{\bar{\phi}_{gn}^{1} H} \qquad (2)$$

The superscript 1 on the fluxes indicates this collapse is only performed for the rodded configuration that a rodded depletion is carried out at, using the flux solution that is obtained for that configuration. No control rod history cross section corrections are applied for the off-nominal rodded configurations, since it is assumed that those configurations are not maintained for a period sufficient to have a significant effect on the depletion. The 3-D/2-D collapse methodology also includes a power shaping factor [2], which permits the 2-D collapsed model to account for differences in axial power shapes and preserve the axial average moderator density. For off-nominal rodded configurations, this axial power shaping factor is made configuration dependent, in order to take into account the substantial changes in axial power shape which may result from control rod insertion.

Because the 3-D/2-D collapse methodology is directed towards optimization capability, which requires the possibility of sampling different fuel assemblies in the rodded locations during the course of the optimization sampling process, some means of generating the collapsed rodded cross section data for fuel assemblies not in rodded locations in the reference LP is required. This generation of additional collapsed rodded cross section data is performed by an addition to the 3-D perturbations algorithms described in reference 4.

3. RESULTS OF RODDED 3-D/2-D GEOMETRY COLLAPSE

The resulting collapsed rodded cross sectional data is filtered by the same filtering algorithms used for all other collapsed cross sections [2]. Because this filtering process generates the largest errors associated with the 3-D/2-D collapse process, comparisons of the 3-D and 2-D collapsed power distributions for the reference LP provide the most meaningful presentations of the fidelity of the 3-D/2-D collapse methodology.

3.1 Reference LP Collapse Results

Illustrated in Figure 1 are the fuel management scheme and control rod locations for the lead control bank of Westinghouse Electric-designed 193 assembly core. This reactor is typical of US reactors which are depleted unrodded, but for which the off-nominal condition of the lead control bank at the full power insertion limit is of interest. This off-nominal condition has the lead control withdrawn to a position of 255 cm withdrawn out of the 365 cm active core height. This core was modeled using FORMOSA-P's macroscopic cross section model. Shown in Figure 2 are the differences between the 3-D and 2-D collapsed assemblywise peak pin powers for the conditions of the unrodded configuration and the off-nominal configuration of the lead control bank at the full power insertion limit for this reactor at a cycle average burnup (CAB) of 8000 MWD/MTU. This CAB is approximately the burnup at which the maximum rodded F-delta-H power peaking factor occurs. It may be observed in this figure that the rodded collapse errors are little different from the unrodded collapse errors. The pin power collapse errors shown in Figure 2 are quite comparable to results previously presented in Reference 2, indicating that the fidelity of the rodded 3-D/2-D collapse methodology should be adequate for incore optimization purposes.

The fuel management scheme and control rod locations for a hypothetical rodded model for the second fuel cycle of a Babcock & Wilcox-designed 177 assembly reactor which utilizes rodded depletion are shown in Figure 3. The part-length rods (PLRs) in this core use solid inconel as their active material and have an active length of 180 cm, while the full length lead control bank rods use Ag-In-Cd alloy as their active material. The core has an active fuel height of approximately 360 cm, and the PLRs are maintained at a constant position of 126 cm withdrawn for both the depletion and off-nominal configurations, while the lead control bank rods are positioned at 330 cm withdrawn for the depletion configuration, and 250 cm withdrawn of the single off-nominal configuration considered. This core was again modeled using FORMOSA-P's macroscopic cross section model. The differences between the 3-D and 2-D collapsed assemblywise peak pin powers at a CAB of 100 effective full power days (EFPD) for both configurations for this case are presented in Figure 4. The 100 EFPD CAB was chosen because it is close to the maximum rodded F-delta-H point. It may be observed in Figure 4 that the collapse errors are little different from the previous case, suggesting adequate fidelity for optimization purposes.

While these macroscopic cross section model rodded results appear promising, efforts to extend this rodded collapse methodology to the microscopic cross section model were not as successful. Shown in Figure 5 are the assemblywise peak pin power collapse errors at a CAB of four EFPD for another hypothetical Babcock & Wilcox-designed 177 assembly reactor modeled using

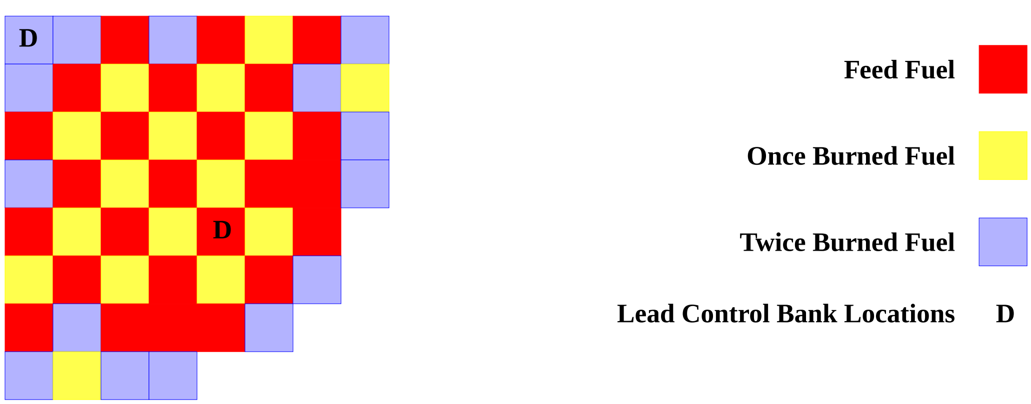

Figure 1. Fuel Management Scheme and Lead Control Bank Locations for Westinghouse Electric-Designed 193 Assembly Core.

	H	G	F	E	D	C	B	A
8-	0.0012	0.0032	-.0000	0.0003	-.0005	-.0010	0.0011	-.0017
7-	0.0032	0.0001	-.0028	0.0012	-.0004	-.0006	-.0010	-.0052
6-	-.0000	-.0026	-.0013	-.0026	-.0005	-.0003	0.0015	-.0020
5-	0.0003	0.0012	-.0025	-.0001	-.0000	0.0007	0.0019	-.0009
4-	-.0005	-.0003	-.0004	-.0000	0.0009	-.0029	0.0019	
3-	-.0010	-.0005	-.0002	0.0008	-.0029	-.0007	-.0032	
2-	0.0011	-.0009	0.0016	0.0019	0.0019	-.0031		
1-	-.0017	-.0051	-.0019	-.0008				

```
RMS Pin Power Difference    :   1.87E-03
Maximum Pin Power Difference:  -5.21E-03   Location:  A  7
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(a.) Unrodded Configuration.

	H	G	F	E	D	C	B	A
8-	0.0045	0.0014	-.0054	-.0030	-.0061	-.0039	-.0023	0.0007
7-	0.0014	-.0047	-.0049	0.0008	-.0020	-.0047	0.0007	-.0000
6-	-.0054	-.0052	-.0048	-.0010	-.0026	-.0010	0.0009	0.0012
5-	-.0030	0.0007	-.0009	-.0031	0.0010	-.0017	0.0015	0.0026
4-	-.0061	-.0019	-.0025	0.0010	0.0011	0.0022	0.0023	
3-	-.0039	-.0046	-.0009	-.0017	0.0022	0.0013	0.0025	
2-	-.0023	0.0007	0.0010	0.0016	0.0023	0.0025		
1-	0.0007	-.0001	0.0012	0.0026				

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RMS Pin Power Difference:       2.90E-03
Maximum Pin Power Difference:  -6.16E-03   Location:  H  4
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(b.) Rodded Configuration.

Figure 2. Comparison of Reference LP 3-D and 2-D Collapsed Unrodded and Rodded Assemblywise Peak Pin Powers for Westinghouse Electric-Designed Reactor at 8000 MWD/MTU, Eq. Xe, Sm.

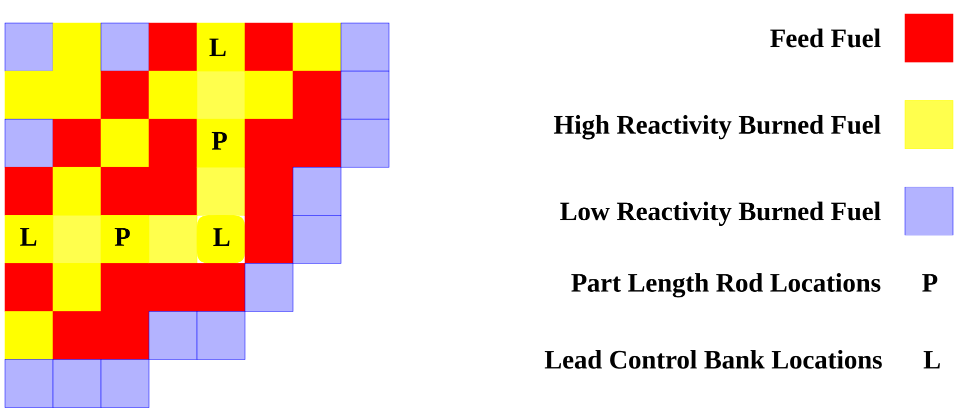

Figure 3. Fuel Management Scheme and Control Rod Locations for Hypothetical Babcock & Wilcox-Designed 177 Assembly Reactor.

	H	G	F	E	D	C	B	A
8-	-.0000	-.0000	0.0008	0.0008	0.0014	0.0006	0.0004	0.0008
7-	-.0000	0.0007	0.0000	-.0003	0.0002	0.0078	0.0001	0.0023
6-	0.0008	-.0000	-.0012	0.0001	-.0040	-.0001	-.0002	0.0028
5-	0.0008	-.0003	0.0000	-.0000	-.0000	-.0007	0.0022	
4-	0.0014	0.0002	-.0040	-.0000	0.0013	-.0000	0.0027	
3-	0.0006	0.0079	-.0001	-.0007	-.0000	0.0030		
2-	0.0004	0.0004	-.0001	0.0022	0.0027			
1-	0.0008	0.0024	0.0029					

RMS Pin Power Difference: 2.14E-03
Maximum Pin Power Difference: 7.97E-03 Location: G 3

(a.) Rodded Depletion Configuration.

	H	G	F	E	D	C	B	A
8-	-.0044	-.0075	-.0014	-.0034	0.0020	-.0014	-.0001	0.0025
7-	-.0075	-.0058	-.0046	-.0040	-.0002	0.0080	-.0034	0.0034
6-	-.0014	-.0048	-.0039	-.0042	-.0022	-.0028	-.0035	0.0035
5-	-.0034	-.0041	-.0045	0.0005	0.0041	-.0003	0.0036	
4-	0.0020	-.0002	-.0022	0.0041	0.0057	0.0036	0.0063	
3-	-.0014	0.0079	-.0028	-.0003	0.0037	0.0077		
2-	-.0001	-.0034	-.0034	0.0036	0.0063			
1-	0.0025	0.0035	0.0035					

RMS Pin Power Difference: 4.08E-03
Maximum Pin Power Difference: 8.01E-03 Location: C 7

(b.) Off-Nominal Configuration.

Figure 4. Comparison of Reference LP 3-D and 2-D Collapsed Assemblywise Peak Pin Powers for Hypothetical Babcock & Wilcox-Designed Reactor at 100 EFPD, Eq. Xe, Sm.

FORMOSA-P's microscopic cross section model. The control rod locations for this case are identical to those used for the case illustrated in Figures 3 and 4. The PLRs utilized in this core, however, use Ag-In-Cd as their active material and have an active length of only 91 cm. They are positioned at 72 cm withdrawn for both the depletion and off-nominal configurations. It may be observed in Figure 5 (a) that the rodded depletion configuration collapse errors are approximately three times higher than either of the two previous cases. This is typical for the 3-D/2-D collapse methodology when the microscopic cross section model is utilized. These larger 3-D/2-D collapse errors which result when the microscopic model is utilized are attributed to the spectral history effect exerted on the microscopic depletion by differing neighboring fuel assemblies. This is an effect which can not be modeled using a macroscopic cross section model, where spectral history effects are only modeled based on state functions such as local fuel or moderator temperature, or soluble boron concentration. Unrodded microscopic model 3-D/2-D collapse results are generally quite similar to those shown in Figure 5 (a).

	H	G	F	E	D	C	B	A
8-	0.0120	0.0111	0.0080	0.0059	-.0025	-.0038	-.0073	-.0044
7-	0.0111	0.0074	0.0110	0.0051	-.0001	-.0040	-.0072	-.0040
6-	0.0080	0.0111	0.0148	0.0084	0.0060	-.0029	-.0058	-.0055
5-	0.0059	0.0049	0.0082	0.0093	-.0014	-.0053	-.0051	
4-	-.0025	0.0000	0.0060	-.0014	-.0058	-.0069	-.0073	
3-	-.0038	-.0037	-.0029	-.0053	-.0069	-.0054		
2-	-.0073	-.0071	-.0057	-.0051	-.0073			
1-	-.0044	-.0042	-.0054					

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RMS Pin Power Error:           6.73E-03
Maximum Pin Power Error:       1.48E-02   Location:  F  6
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(a.) Rodded Depletion Configuration.

	H	G	F	E	D	C	B	A
8-	0.0445	0.0388	0.0372	0.0332	0.0209	0.0022	-.0152	-.0135
7-	0.0388	0.0380	0.0408	0.0278	0.0193	-.0036	-.0164	-.0128
6-	0.0372	0.0408	0.0403	0.0294	0.0198	-.0130	-.0231	-.0184
5-	0.0332	0.0275	0.0290	0.0260	-.0048	-.0242	-.0262	
4-	0.0209	0.0194	0.0199	-.0048	-.0091	-.0215	-.0217	
3-	0.0022	-.0031	-.0130	-.0243	-.0216	-.0174		
2-	-.0152	-.0165	-.0232	-.0263	-.0218			
1-	-.0135	-.0131	-.0185					

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RMS Pin Power Error:           2.46E-02
Maximum Pin Power Error:       4.45E-02   Location:  H  8
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(b.) Off-Nominal Configuration.

Figure 5. Comparison of Reference LP 3-D and 2-D Collapsed Assemblywise Peak Pin Powers for Second Hypothetical Babcock & Wilcox-Designed Reactor at Four EFPD, Eq. Xe, Tr. Sm.

The off-nominal configuration collapse errors shown in Figure 5 (b), nearly three times larger still than those shown for the depletion configuration in Figure 5 (a), are not considered acceptable. Although it is possible that the inferior results of Figure 5 (b) are the result of a coding error, this is considered rather less likely, since a comparison of the assembly averaged k_∞ collapse errors for this case reveals assembly averaged k_∞ errors between the 3-D and 2-D collapsed cases for the off-nominal case which are little different from the two previous cases. The reasons for this apparent failure of the multiple rodded configuration collapse methodology when used with the microscopic cross section model are not known, but it may be related to that fact that the base cross sections and instantaneous cross section derivatives other than the rodded derivatives are collapsed using the depletion configuration, and thus neutron fluxes, whereas the rodded cross section derivatives for the off-nominal configurations are collapsed using the corresponding off-nominal configuration, and its associated neutron fluxes.

3.2 Archived LP Collapse Results

During the course of its stochastic optimization sampling, the FORMOSA-P code saves a number of potentially interesting LPs for presentation to the user as the near-optimal LPs. These LPs are referred to as archived LPs, and the FORMOSA-P code has a provision to forward solve them, in both 2-D and 3-D, after the completion of the optimization sampling process. This provides a means for assessing the fidelity of the 3-D/2-D collapse process, since the 2-D forward solution of the near-optimal LPs utilizes 2-D collapsed cross sections which were generated via collapse of the reference LP, or 3-D perturbations thereof. Table I presents the mean 3-D/2-D collapse errors for the archived LPs from an optimization performed on the Westinghouse-designed 193 assembly reactor presented in Figures 1 and 2. Table I shows the mean collapse errors for the 38 LPs archived during an optimization which maximized the region average discharge burnup of the region of fuel designated for discharge at the end of the fuel cycle. The fuel placement permitted during the optimization sampling process was essentially unrestricted, including the rodded locations, save for the requirement that pseudo-eighth core symmetry be maintained. This tends to maximize any 3-D/2-D collapse errors which might be encountered. Most fuel assembly rotation sampling, which models cross-core shuffles in the modeled quarter core geometry, was deactivated for this optimization, save for the twice burned fuel, whose placement solution can not be decoupled from the rotation solution in this problem. As discussed previously, the off-nominal configuration has the lead control bank at a position of 255 cm withdrawn out of the total active core height of 365 cm. It may be observed in Table I that these mean archived LP collapse errors are little larger than the reference LP collapse errors shown in Figure 2. The single, modest exception is the mean (over the archived LPs) maximum pin power collapse errors for the off-nominal configuration. This slightly increased error is still considered acceptable since closer examination of the larger errors reveals that the larger errors occur in the rodded locations, which, not being limiting as far as power peaking is concerned, are not of concern insofar as the optimization is concerned.

Presented in Table II are the mean 3-D/2-D collapse errors for the first hypothetical Babcock & Wilcox-designed reactor, first presented in Figures 3 and 4 of this paper. Again, the fuel placement permitted during the optimization sampling process was largely unrestricted, with the only exceptions being that pseudo-eighth core symmetry was again required, and shimmed feed fuel could not be placed in rodded locations due to mechanical interference. No rotation sampling was permitted for this problem. It may be observed in Table II that the second rodded configuration collapse errors are only modestly higher than the rodded configuration collapse

Table I: Archived LP collapse errors for optimization of Westinghouse-designed reactor

	Depletion (Unrodded)	Off-Nominal (Rodded)
Cycle RMS RPF Collapse Error	2.1×10^{-3}	3.2×10^{-3}
Cycle Mean Maximum RPF Collapse Error	5.0×10^{-3}	8.9×10^{-3}
Cycle RMS Pin Power Collapse Error	2.7×10^{-3}	3.9×10^{-3}
Cycle Mean Maximum Pin Power Collapse Error	6.5×10^{-3}	9.7×10^{-3}

Table II: Archived LP collapse errors for optimization of Hypothetical Babcock & Wilcox-designed reactor

	Rodded Depletion Configuration	Off-Nominal Configuration
Cycle RMS RPF Collapse Error	1.6×10^{-3}	4.4×10^{-3}
Cycle Mean Maximum RPF Collapse Error	4.1×10^{-3}	1.2×10^{-2}
Cycle RMS Pin Power Collapse Error	2.3×10^{-3}	5.2×10^{-3}
Cycle Mean Maximum Pin Power Collapse Error	6.0×10^{-3}	1.2×10^{-2}

errors for the previous case, indicating that this rodded macroscopic cross section model is capable of providing a useful multiple rodded configuration optimization capability as well.

3.3 Computational Performance

Presented in Table III are the computational times for the optimization performed for the hypothetical Babcock & Wilcox-designed reactor. The CPU times shown are CPU seconds on a 400 MHz Sun workstation, which is approximately one-third faster than an Intel Pentium III processor in the 500-700 MHz range when running this application. The core physics model used in this case used ten time steps to model the 450 EFPD cycle length, and calculations were performed for the single off-nominal rodded configuration at every time step during the optimization. The advantage of APRGPT using 2-D collapsed cross sections is somewhat less than the factor of 200 ratio between the per LP 3-D archive solution time and optimization sampling time shown in Table III. This is due to computational speedup techniques such as limiting pin reconstruction during optimization and simulated annealing early rejection which are applied during the optimization sampling process which could be applied to optimizations using 3-D forward solutions. These techniques would probably reduce 3-D optimization solution times by approximately 1/3, resulting in a net advantage of 2-D APRGPT of somewhat over 100 over 3-D solutions.

4. CONCLUSIONS

A method has been developed for consistent 3-D/2-D geometry collapse of rodded PWR models, for both rodded depletion models and for additional, off-nominal rodded configurations when a macroscopic cross section model is utilized. The rodded depletion model may also be coupled

Table III: FORMOSA-P computational perfomance

3-D Reference LP & 3-D Perturbations Solution (sec)	1970.
Number of LPs Sampled During Optimization	10338
Optimization Sampling Time (sec)	5370
Average Computational Time per LP Sampled During Optimization (sec)	0.52
Average Computational Time per 3-D Archive Solution (sec)	105
Total Computational Time (sec)	12950

with a microscopic cross section model. It has been shown that fidelity adequate for high speed optimization may be obtained from these models. The use of the rodded 3-D/2-D methodology for off-nominal rodded models with the microscopic cross section model was not successful. This may be due to fundamental limitations of the 3-D/2-D geometry collapse methodology.

ACKNOWLEDGEMENT

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