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INVERSE METHOD APPLIED TO ADAPTIVE CORE SIMULATION

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ABSTRACT

This paper represents a continuation of an earlier work conducted to gain insight into the applicability of inverse methods to developing adaptive core simulation capabilities. Adaptive simulation utilizes past and current reactor measurements of reactor observables (e.g. core reactivity and incore instrumentation readings) to adapt the simulation in a meaningful way to reduce the calculations' uncertainties. That will increase core design freedom by relaxing design margins and technical specifications. Core simulators introduce errors in the predicted core attributes due to errors in the input data, mathematical models and the numerical solution techniques of the core simulator or the pre-processing codes. In the proposed work, the simulation is adapted by adjusting only the input data to the core simulator producing a new input data set that is statistically consistent with our current input data set, implying that the input data adjustments account for both input data and modeling errors. The validity of this assumption is investigated by checking fidelity and robustness of the adaption. We utilized a virtual core approach, where core observables' data are generated using two versions of a core simulator, one generating actual plant data and the other representing an existing core simulator with known errors introduced into its input data and models. Hundreds of core simulator input data are shown to be successfully simultaneously adapted.

Key Words: Adaptive simulation, inverse theory, regularization, parameter estimation

1. INTRODUCTION

Core simulators are essential components for the successful design and operation of any nuclear reactor system. They can be utilized in an on-line mode, to determine the optimum trajectory in moving the reactor from a current state to a final desired state with all operational and safety limits satisfied, or in an off-line mode, to determine the optimum operating core conditions. The quality of core simulators' predictions will impact the reactor economy through the introduction of design margins on the core design to ensure an operation in which safety and operational limits are satisfied. Large uncertainties result in diminishing core design freedom and hence adversely impact economics. Any reduction in these uncertainties will beneficially impact different aspects

*. The work in this paper is based in part on a Master's thesis by the first author.

of reactor economy, such as reducing fuel and/or operating cost or even initial capital investment for a new plant. To achieve this, in the near term, will require high fidelity and robust adaptive simulation techniques. Fidelity denotes the ability of an adapted simulator to accurately predict the measured observables. Robustness denotes the ability of the adapted simulator to accurately predict core attributes not directly observable and accurately predict observables for core conditions that differ from those at which adaption was completed. Adaptive core simulation is a mathematical algorithm which deals with given a mathematical core model and the associated input data and measured core observables, how can one adjust only the input data and not the core model (in our implementation), and produce a new input data set that is statistically consistent with our current input data set, such as to reduce the disagreement between the measured and predicted observables.

There are different issues and concerns that need to be analyzed when adjusting input data [1]-[2]. How well will the adjusted data perform at different core conditions and how consistent are they with the unadjusted data? Would one obtain the same results utilizing two different data sets and the same measured observables to adapt the core simulator? This type of analysis can also help answer some other questions such as: 1) How well do the existing input data predict the core attributes of interest with the current modeling capabilities? 2) How sensitive are certain core attributes to changes in the input data? 3) How does one identify the sources/causes of discrepancies between the core simulator's predicted and measured core attributes? Answering question (3) will provide direction to those areas of uncertainty where more detailed experimental programs are required to decrease core attributes' uncertainties. The need for adaptive simulation is viewed to be greater for Generation IV versus Generation II and III reactors, since limited applicable experience may be available for Generation IV reactors.

2. CURRENT PRACTICE VERSUS PROPOSED DEVELOPMENT

The art of data adjustments had been extensively utilized during the 1970s for fast reactors' experimentation. Uncertainty in the basic cross-section data affects the uncertainty of different core attributes. Integral experiments, which are critical assemblies operating at zero power and mimic the reactor core behavior as much as possible in regards to composition and geometry, were used in conjunction with differential experiments to adjust cross-sections in order to reduce the uncertainty of the important core attributes (i.e. breeding ratio [3] or k_{eff} eigenvalue). The idea for the proposed work of adaptive core simulation stems from these past experiences [4]-[9] but with two major differences: The first is that instead of using small scale experiments to simulate the operation of the real reactor, one can use the full-scale experiment (e.g. real core) as one's integral experiment [10]. This is advantageous when considering the huge amount of core follow data from our operating experience with current power plants. However, the interpretation of these data is much more complex than the "clean" integral experiments, since the quality of the data is uneven and non-linear feedback effects are present. For integral experiments, the quality of the data is considered to be the same, since there are no depletion effects and no thermal-hydraulic feed-back effects. In addition, for power plants transient phenomena might be occurring at anytime yet a steady state core model is employed, and instrumentation may not be properly calibrated or failed. The second difference is in the mathematical formulation of the problem. The problem at hand can be treated as an "inverse problem". Inverse theory has been extensively developed over the last four decades since the early papers by Tikhonov in 1963 [11]-[12], and

has been applied successfully to many engineering areas such as image enhancement [13] and geophysical applications [14].

Current on-line core simulators employ very limited or no adaption, in a way which is conceptually different from what we are proposing. Sometimes the differences between measurements and predictions are usually fitted using surface response methodologies and are used to predict the discrepancy between future predictions and measurements. Similar versions of this approach were utilized during the course of fast reactors experiments, and are referred to as "bias factor methods" [15]. Other approaches match the calculated and measured responses by adjusting the node-wise k_{eff}, leakage terms or the boundary conditions in the core simulator's diffusion solver [16]-[20]. The "bias operator method" is another elegant mathematical approach that has also been proposed in the past as a method for adaption [21]-[24]. None of these methods will be utilized in our work, since we believe they do not have a physical justification. In the proposed work, the adjustments are done to the input data in a mathematically consistent way in which the physics of the problem are satisfied. That is achieved by first limiting the input data adjustments by the uncertainty information, sometimes propagated through the pre-processors' codes to the core simulator to preserve the proper pre-processor codes' core physics behavior. In doing so, the proposed adjustments will be consistent with their known uncertainties. Second, the input data adjustments will be propagated through the core model satisfying the physics described by the core simulator. Physical justification is required when adapting input data, since if the physics of the core are not satisfied, one would not be able to predict core behavior at different operating conditions than those adapted to, defeating the whole purpose of the adaption. Third, the uncertainties in the core observables will be accounted for during the adaption, assuring that the input data adjustments are consistent with the quality of the experimental core observables being adjusted to.

3. CORE SIMULATORS' MODELS AND ASSOCIATED DATA

The core simulator model consists of models for both reactor physics and thermal-hydraulics in order to account for the non-linear feedback mechanisms through cross-sections. Typically, reactor physics behavior is modeled employing few-group neutron diffusion theory, hydraulics behavior is modeled employing some 1-D approximate form of the Navier-Stokes equations (e.g. drift flux model), and the thermal behavior is modeled using some approximate thermal model (e.g. functionalizing fuel temperature as a function of linear power density).

Input data to the core simulator are enormous and determined by the needs of the models employed within. We include in input data, the coefficients that appear in various correlations, including the thermal-hydraulic correlations since core thermal-hydraulic simulations require user input of core geometry and many empirical parameters, such as local form loss coefficients and heat transfer coefficients. In many cases, input data to the core simulator are determined using codes that model aspects of the core physics in more detail than the core simulator models do, such as with the determination of few-group homogenized cross-sections using lattice physics codes. So, in general, the input data include any data directly passed to the core simulator or indirectly through any pre-processor code.

Core observables include the readings of incore detectors (i.e. LPRMs and TIPs), which are usually positioned throughout the core taking advantage of radial symmetry. Since core simulators are usually based on a steady state model and measured observables are normally taken at steady state conditions, core reactivity could also serve as a basis for adaption.

The sources for the disagreement between the measured and predicted core attributes are due to instrument errors and noise in the measured output, or errors introduced by core simulators in the predicted core attributes. Those latter errors are due to input data and modeling errors of the core simulator or the pre-processing codes. In our implementation, however, only the input data will be adapted to account for those different sources of errors, implying that the main sources of disagreement are assumed to originate due to errors in the input data to the pre-processor codes and pre-processor codes' independent input data to the core simulator. This assumption is likely valid and necessary since the combined adaptation problem of core simulator's input data and models is beyond current and foreseeable capabilities. This assumption is also physically valid since the adaption will be completed such that the a posteriori input data are statistically consistent with their a priori information. However, to justify this assumption, a more generalized approach has to be investigated in this work by checking the fidelity and robustness of the adaption. Note that the proposed adaptive techniques will adjust only the input data to the core simulator and not the input data to any pre-processor code, (i.e. only adjust the few-group homogenized cross-sections to the core simulator and not the multi-group or ENDF point-wise presentation of cross-sections).

4. INVERSE PROBLEM

A forward problem* is one in which, one seeks information about core observables from the knowledge of core input data utilizing a core simulator model. Inverse problems deals with the reverse process of estimating the input data that give rise to the measured core attributes. The number of independent pieces of information about input data which can be inferred from the core observables cannot exceed the number of input data. If less, however, the solution to the inverse problem might not be a single answer, so one of the aspects of the inverse analysis is the ability to determine which answer is reasonable, valid, and acceptable. Mathematically, one can represent the state of lack of information by an ill-posed problem. A problem is said to be ill-posed if it is not well-posed. According to Hadamard [25], for an inverse problem to be well-posed, the input data need to be estimated uniquely, moreover, their estimated values should change smoothly with smooth changes in the values of the measured core observables which are used to adapt the core. The latter condition, if not satisfied, is troublesome since it implies that the values of estimated input data are highly sensitive to the values of observables which normally contain measurement uncertainties; hence no reliable information about input data can be inferred from core observables. The mathematical distinction between a well- and an ill-posed problem is very clear. However, in some practical cases, that distinction reveals itself to be less apparent and one cannot obtain an accurate, unique solution for a well-posed problem due to the finite precision of the computations and the associated errors in the measured observables, forcing us to treat such problems as ill-posed.

*. Discussion is within the context of our application.

Regularization refers to the mathematical methods utilized to incorporate extra information about input data necessary to recast an ill-posed problem into a well-posed one [26]. In our proposed work, an ill-posed problem will be regularized by first realizing that the inverse problem of interest is under-determined in that there are many more input data items that can be adjusted than there are observables. Parametrization of the system is one way to recast the problem from an under to over-determined one. Parametrization is the process of characterizing core simulator input data in terms of a minimal set of parameters, which we shall refer to as core parameters. Care is required in the selection of those parameters since improper choices made will be reflected in poorer fidelity and/or robustness of the adaptive simulator. The space of the solutions allowed will also be restricted by applying additional constraints to confine the adaption to physical cases only. The search for the adjusted (e.g. a posteriori) core parameters will also be restricted to our best known a priori information about these parameters.

5. VIRTUAL APPROACH

The ultimate target in this work is to contrast the predictions of the adapted core simulator to actual plant data, in an effort to assess and validate the proposed techniques. However, since the project is still under development, a virtual approach has been adopted [27]. In this approach, the actual plant data has been produced by an existing version of a core simulator, referred to as a virtual core (VC). The core selected for analysis is a BWR/3 reload core containing 724 fuel bundles with a cycle burnup of 8579.20 MWD/STU. The plant data consists of the readings of 44 strings of incore detectors, where each string contains four detectors located at different axial positions. The VC has been utilized to simulate those detectors' responses, and in doing so the noise on those responses was also simulated by perturbing their signals from their simulated values by sampling them from a Gaussian distribution of standard deviation of 0.04 of the average relative detectors' response, which is a representative value of the actual instruments' noise level. For this exploratory study, a simple detector model is employed, where the response of each detector is assumed to be proportional to the average value of the thermal fluxes in the four neighbouring assemblies. An altered version of the same core simulator has been used after deliberately introducing two major sources of errors, specifically, in its modeling and in its input data. This version of the core simulator is referred to as the design basis core simulator (DC). The size of the introduced errors in the DC were chosen so as to create discrepancies between the predictions of the VC and DC for core reactivity and LPRM detectors' signals RMS errors of the same magnitude as the actual discrepancies found between actual plant data and existing core simulators' predictions. The core simulator utilized in this exploratory study is FORMOSA-B [29]-[31], which has been extensively benchmarked against plant follow data and commercial core simulators.

5.1. Core Parameterization

Modeling errors have been included in the DC by recognizing that the modeling of voids in a BWR core has a large impact on the prediction of different core attributes such as the power distribution and reactivity. Based on that fact, the void-quality correlation was perturbed in the DC. The void-quality* relationship is given by the form identified by Zuber-Findlay [32]:

*. Refer to Moore's PhD thesis for more details [31], all symbols have their standard interpretation.

$$\alpha = x/(C_0[x + (\rho_g/\rho_l)(1-x)] + \rho_g k_3 \sqrt[4]{(\rho_l - \rho_g)\sigma g/\rho_l^2}/G).$$

The DC utilized the Zuber-Findlay void-quality correlation in which two variables, the concentration parameter (C_0) and terminal velocity parameter (k_3), were assumed spatially independent throughout the core and given by their best known values (e.g. $C_0 = 1.13$ and $k_3 = 1.41$). Both variables were selected as input data to be adapted according to the relations:

$$\tilde{C}_0 = f^{c_0} C_0 \text{ and } \tilde{k}_3 = f^{k_3} k_3,$$

where the factors f^{c_0} and f^{k_3} represent the selected thermal-hydraulics core parameters which are to be determined by the adaptive techniques, $\tilde{C}_0$ and $\tilde{k}_3$ are the adapted void-quality constants utilized in the adapted design basis core simulator (AC). The VC, however, utilized the Lellouche-Zolotar EPRI methodology [33] to determine C_0 and k_3, which can be thought of as using spatially dependent C_0 and k_3. Functionalization of the void-quality correlation in these two different manners will help us investigate how well the adaptive technique will perform when the functionalization of the data is not consistent with reality (i.e. employing observables predicted using the Zuber-Findlay void-quality correlation to attempt to match those predicted using the Lellouche-Zolotar void-quality correlation), and how adaptive techniques can account separately for the combined sources of errors due to inconsistent modeling and input data errors.

The second source of errors included in the virtual approach is due to input data errors. The thermal-hydraulics data in the current study consists of only the C_0 and k_3, coefficients of the Zuber-Findlay void-quality correlation. The reactor physics input data consists of all types of few-group homogenized fast and thermal microscopic cross-sections (i.e. absorption, fission, prompt and energy yields per fission) of all nuclides, included explicitly or implicitly in the microscopic depletion model (i.e. actinides, burnable poison isotopes and background pseudo isotopes) of the utilized core simulator. The cross-section representation utilized by FORMOSA-B [34] requires that a number of 'cases' be generated at the lattice physics level. Base micro and/or macroscopic cross-sections are obtained from a lattice physics unit assembly depletion at the nominal hot full power (HFP) average core conditions at different vapor void fractions, from which branch cases are performed to capture the instantaneous effects of perturbing different core conditions such as fuel temperature, coolant void, and control rod insertion. The cross-section is constructed as the summation of a reference term and a set of correction terms given by:

$$\Sigma = \Sigma^{REF} + \Sigma^{CRH} + \Sigma^{TF} + \Sigma^{FP} + \Sigma^{CRD}$$

where the reference term is the first term on the R.H.S. and is a function of fuel exposure, and instantaneous and history void fractions. The remaining terms are correction terms, accounting for control rod history, fuel temperature Doppler broadening effect, fission products poisoning, and instantaneous control rod insertion effect. The reference and the correction terms are constructed

using piece-wise cubic splines and quadratic fitting polynomials, which for correction term *k* of cross-section type *j* for nuclide *n* in fuel color *c* is given by:

$$\Sigma^{n,j,k,c}(Bu) = \sum_i d_i^{n,j,k,c}(Bu)\, y_i(\bar{x}),$$

where $\bar{x}$ is a state variable describing the dependence of the specific cross-section reference or correction term on different core conditions (i.e. fuel temperature, void fraction, etc.), $\{d_i^{n,j,k,c}\}$ are the polynomial coefficients calculated based on the lattice physics data functionalized in terms of the fuel exposure, Bu, and $\{y_i\}$ are polynomial functions. The base or correction terms are adapted according to the relation:

$$\tilde{\Sigma}^{n,j,k,c}(Bu) = \sum_i f_i^{n,j,k,c}(Bu) d_i^{n,j,k,c}(Bu)\, y_i(\bar{x}),$$

where the $f_i^{n,j,k,c}$ factors are functionalized in terms of fuel exposure in the following way:

$$f_i^{n,j,k,c}(Bu) = f_{i,1}^{n,j,k,c} + (1 - f_{i,2}^{n,j,k,c}) Bu / \tilde{Bu},$$

with the factors $\{f_{i,1}^{n,j,k,c}, f_{i,2}^{n,j,k,c}\}$ representing the reactor physics core parameters which are to be determined by the adaptive techniques, and $\tilde{Bu}$ is a scaling factor with exposure units.

The direct approach to simulate input data errors would be one in which each input data from the set studied (e.g. C_0, k_3 and the lattice physics few-group homogenized cross-sections library) is randomly selected from a normalized Gaussian distribution whose standard deviation corresponds to the relative uncertainty for that specific input data. One can then assume that the perturbed values are our current best knowledge of these data. At this time, the uncertainty information is not directly available though, since to obtain it, one would need to propagate the uncertainty information starting from an energy point-wise or resonance parameters presentation like ENDF/B to a few-group, spatially homogenized presentation provided by lattice physics codes. This is not a trivial task to do and requires substantial calculational efforts. For that reason and for the sake of an exploratory investigation, a simpler approach has been utilized. In this approach, the input data are considered to be fully characterized by the selected core parameters*. To simulate input data errors in this approach, each of the core parameters were randomly selected from a normalized Gaussian distribution whose standard deviation is assumed now to be the same for all core parameters. The perturbed core parameters will be assumed to constitute our best knowledge (the a priori information) about the input data which are now given by:

*. Note that the unperturbed "true" values for the selected core parameters are all equal to 1.0.

$$\hat{C}_0 = \hat{f}^{c_0} C_0,\ \hat{k}_3 = \hat{f}^{k_3} k_3 \text{ , and } \hat{d}_i^{n,j,k,c} = (\hat{f}_{i,1}^{n,j,k,c} + (1 - \hat{f}_{i,2}^{n,j,k,c}) Bu/\tilde{Bu}) d_i^{n,j,k,c},$$

where the hatted *f* factors are the perturbed core parameters. A constant standard deviation (1%) is selected, so as to give rise to discrepancies between the predictions of the VC and DC which are representative of the actual magnitude of such discrepancies found between real plant data and existing core simulators (i.e. LPRM RMS error of 6% and criticality 2σ error of 1000 pcm).

Note that each of the reference and corrections terms are functionalized in terms of the void fraction, through $y_i(\bar{x})$, so when one is adapting the void-quality parameters and the polynomial coefficients as well, one is explicitly correcting separately, to a first-order approximation, for the void-quality modeling error and the input data errors. However, if one is not adapting the void-quality correlation, the adjusted reactor physics core parameters will be accounting both for input data and modeling errors as well, but only for those specific core conditions at which the adaption was completed (e.g. certain core average void fraction). That could lead to a less robust adaption when trying to predict core behavior at different core conditions. This issue was investigated in our exploratory study to show how effectively adaption can be utilized to separately adjust for the different sources of prediction errors and gain insight to situations when adaption is performed on the wrong core parameters.

To simplify the adaption, dependencies of the reactor physics core parameters $\{f_i^{n,j,k,c}\}$ has been restricted to nuclides (*n*) and reactions types (*j*) with dependencies on correction terms (*k*), fuel colors (*c*) and fitting polynomials (*i*) dropped. In reality, core parameters are expected to be dependent upon branch cases and fuel color, since the unit lattice flux energy and spatial shapes are dependent upon these attributes. Hence, the few-group homogenized cross-sections errors are also dependent on different branch cases and fuel colors. As this project moves forward, more sophistication will be introduced. Even with this simplification, a total of 108 core parameters were free to adapt.

5.2. Mathematical Model

Let $\bar{d}^m$ and $\bar{d}^c$ be vectors of dimension m whose components are the measured and predicted (calculated) core observables, respectively. Let $\bar{\lambda}^m$ and $\bar{\lambda}^c$ be two vectors of dimension l whose components are the measured and predicted steady state core reactivity (e.g. k_{eff}) at selected l burnup points (time steps) during cycle life. Let the core design basis simulator model be represented by the two vector nonlinear equations:

$$\bar{d}^c = \vec{\Theta}(\bar{p}) \text{ and } \bar{\lambda}^c = \vec{\Pi}(\bar{p}),$$

where $\bar{p}$ is a vector of dimension n whose components are the selected core parameters. The core parameters and their a priori information are given by:

$$\bar{p} = \left[f^{c_3} f^{k_3} \{f_1^{n,j}, f_2^{n,j}\} \right]^T \text{ and } \bar{p}_\infty = \left[\hat{f}^{c_3} \hat{f}^{k_3} \{\hat{f}_1^{n,j}, \hat{f}_2^{n,j}\} \right]^T,$$

respectively. The adjusted core parameters can be selected such as to minimize the difference between the measured and predicted core observables in a weighted least-squares sense. However, this direct approach results in an ill-conditioned and consequently unreliable least-squares estimate of core parameters. The main sources of ill-conditioning were attributed to two factors: 1) Sensitivity coefficients of the adjusted parameters differed by orders of magnitude, so the information about these parameters with low sensitivities cannot be effectively inferred from the core observables, specially when the noise level becomes large compared to this information. 2) Some of the core parameters have similar sensitivity profiles, reducing the number of independent information about core parameters, and hence the effective rank of the associated least-squares matrix. Under these conditions, the estimated core parameters will be very sensitive to the noise, and hence giving rise to an unreliable adaption. Moreover, for rank deficient problems, the solution of core parameters is known not to be unique. Therefore, the search had to be regularized by incorporating extra information. One can show that the information that is effectively recovered from the observables correspond to directions in the parameters' space that has strong sensitivity profiles and large uncertainty information [28]. This is a very desirable situation since on one hand, the range of adaption of a parameter must be consistent with the uncertainty of that parameter and strong sensitivity implies a smaller change in the parameter value (i.e. more likely to be within its uncertainty range) is required to achieve the desired change in response. On the other hand, if certain parameters are adjusted beyond their uncertainty ranges, that will be an indication that certain parts of the core physics model that are associated with these parameters are potentially flawed. The potential power of regularization lies in these simple intuitive concepts, where now, one can adapt all core parameters in a completely systematic way which is consistent with the uncertainty in the core parameters.

Tikhonov regularization is one of the most famous regularization techniques, where the search is now modified to regions where the solution is not far from a priori values, and the adjustments to core parameters are based on the information recovered from the core observables, and for such information which cannot be recovered from the core observables, the adjustments are restricted to our best known values. Tikhonov regularization minimizes [35]-[36] a quadratic form given by:

$$S(\bar{p}) = \left\| \bar{d}^m - \vec{\Theta}(\bar{p}) \right\|^2_{\bar{\bar{C}}_d} + \alpha^2 \left\| \bar{p} - \bar{p}_\infty \right\|^2_{\bar{\bar{C}}_p} + \beta^2 \left\| \bar{\lambda}^m - \vec{\Pi}(\bar{p}) \right\|^2. \qquad (1)$$

where α and β are called regularization parameters, and $\bar{\bar{C}}_d$ and $\bar{\bar{C}}_p$ are selected weights. The core model is linearized and the search for the adjusted core parameters is done iteratively using a Quasi-Newton type approach. For this exploratory study, the sensitivity coefficients of core observables with respect to core parameters were obtained by numerical differentiation since the computational burden is acceptable at this stage. However, for full implementation of the proposed adaptive techniques, alternative methods for obtaining the sensitivity coefficients will be considered.

Different approaches offer different ways to determine the optimum values for the regularization parameters α and β, based upon different criteria. For the current work, we performed a

parametric study to determine their optimum values experimentally, by "trial and error", based upon the characteristic 'L-curve'. The first term in Eq(1) is referred to as the misfit term, and the remaining terms are referred to as the regularized terms. If the selected regularization parameters approach zero, the problem reduces to a regular least-squares case and the parameters' adjustment are mainly determined by the observables; whereas, if the regularization parameter approaches infinity, the data misfit term is negligible with respect to the regularized terms, and the parameters are mainly determined by the a priori information.

In this work, special weights are given to the observables at different burnup steps to satisfy certain additional criteria, as for example: weights are selected to be proportional to the relative amount of burnup increment for the different burnup steps, which if the plant were at full power, is the relative time the error exists. It is also more important to accurately predict core behavior (i.e. core criticality) at EOC than at BOC. To do that, a simple linear proportionality with cycle burnup was assumed for the weights.

6. CASES STUDIED AND RESULTS

Two different approaches for adapting the core have been studied [27], denoted by AC1 and AC2, where in AC1, thermal-hydraulics and cross-sections input data were both adapted to enhance the agreement between the DC and VC; whereas in AC2, only cross-sections input data were adapted. Adaption was always completed at the rated conditions of power and flow rates.

To differentiate initially (e.g. before adapting the core), in a qualitative sense, between the two simulated types of error sources and their respective magnitudes, a comparison was made of the VC to the DC response when input data errors were and were not present, the latter isolating the effect of modeling error. Also compared are the AC to the VC response with and without LPRM signal noise, with the VC associated data used in the adaption always containing noise. That step was taken to see how well adaptive techniques can be used to filter out instrument noise.

Figure 1 compares the LPRM RMS and core reactivity errors of the DC before and after adaption using the two approaches, AC1 and AC2, for three different cases. For case AC1-F, the recorded detectors' signals and criticality constraints at all burnup steps (e.g. l time steps) are included in the adaption, hence denoted 'F' for full. For case AC1-H only the observables recorded at half of the total number of time steps (e.g. $l/2$), selected at random, are utilized to adapt the core, hence denoted 'H' for half. Approach AC2 has been utilized only for case AC2-F. Each of the figures showing the LPRM RMS errors consists of four sets of graphs as now explained: Graph(a) presents the LPRM RMS error between the VC, which includes the instrument noise, and the DC's predictions before adaption, where the errors are calculated based on the simulation of both modeling (M) and input data (I) errors. Graph(b) calculates LPRM RMS errors in the same fashion as Graph(a), except now only modeling errors (Zuber-Findlay void-quality correlation) were introduced in the DC. Graph(c) presents the LPRM RMS error between the VC and the DC after the adaption has been completed. Graph(d) determines the LPRM RMS errors in the same fashion as Graph(c), except now noise is not incorporated in the VC responses. Qualitatively, Graph(a) and Graph(b) show that the initial individual contributions of each source of error, modeling errors and input data errors, are approximately of the same magnitude. Graph(c) shows that the differences in LPRM signals between the AC and the VC are of the order of the noise

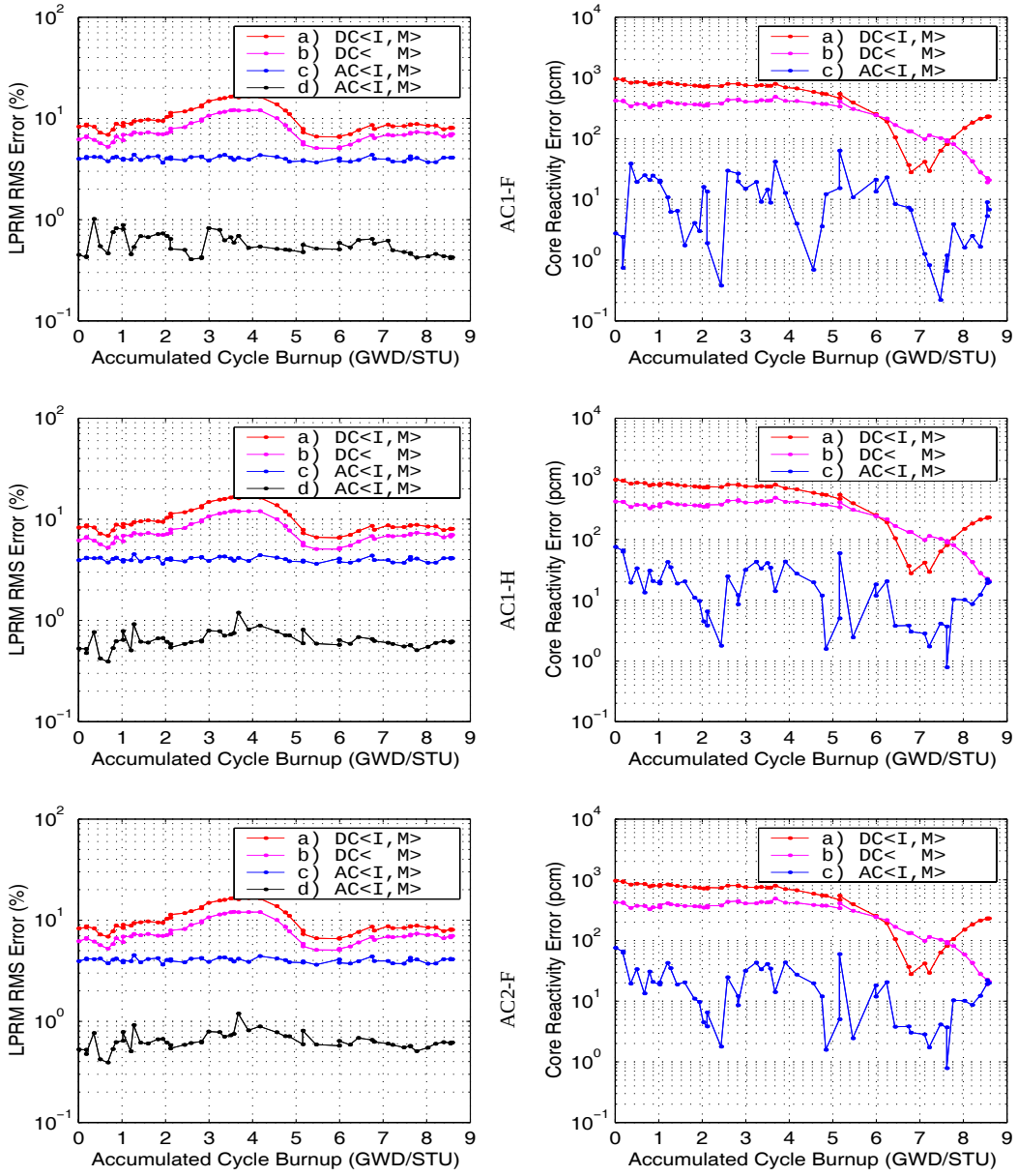

Figure 1. LPRMs' Misfit & Reactivity Error (Rated Conditions).

level. The LPRM RMS errors in Graph(d) are presented without the noise, showing that the AC and the VC LPRM signals agree well, indicating that adaption can be potentially utilized as a powerful noise filter. Core reactivity errors are also presented to show how powerful the adaption is in accurately reducing the prediction uncertainties in the k_{eff} values to about one order of magnitude less than their original values before adaption. By comparing case AC1-F and case AC1-H, one finds that the order of prediction errors for all the detectors' signals and the criticality constraints are the same whether they are or are not included in the adaption, which confirms the physical nature of the adaption. Case AC2-F also shows as good agreement as with either case AC1-F or AC1-H since, as noted before, the adapted core parameters account for both the input

data errors and the modeling errors at which the adaption was completed. However, the agreement between the VC and DC will be degraded when trying to predict core behavior at different flow conditions (i.e. higher or lower void fractions). To attain higher void fractions, denoted by HV, in the core, the flow rates were uniformly reduced throughout the cycle life to 80%, and to attain lower void fractions, denoted by LV, the flow rates and power were uniformly changed to 110% and 90%, respectively, of their rated values, denoted by RV. Figure 2 and Figure 3 compare the LPRM signals' prediction errors for the HV and LV cases, respectively, for two different adaptive approaches: AC1-F and AC2-F. The results show that the LPRM RMS errors obtained using approach AC2-F are at least two times larger than those errors obtained when using approach AC1-F. This confirms that approach AC1-F corrects more explicitly for modeling errors and input data errors than approach AC2-F, giving rise to a more robust adaption.

To further confirm this, we have utilized a more quantitative comparison test in which different criteria are used to calculate the LPRMs RMS and core reactivity errors. This was accomplished by re-calculating the initial and final errors (e.g. before and after adaption) but now utilizing the Lellouche-Zolotar EPRI methodology for the DC. In this manner, the effect of cross-section input data errors could be isolated. Note that when calculating the LPRMs RMS errors, noise was not included in the VC signal. In Figure 4, Graph(a) presents the re-calculated initial errors of the DC, and Graph(b) and Graph(c) present the re-calculated errors of the AC utilizing approaches AC1-F and AC2-F, respectively. Sets of sub-figures present the three previously mentioned cases of core

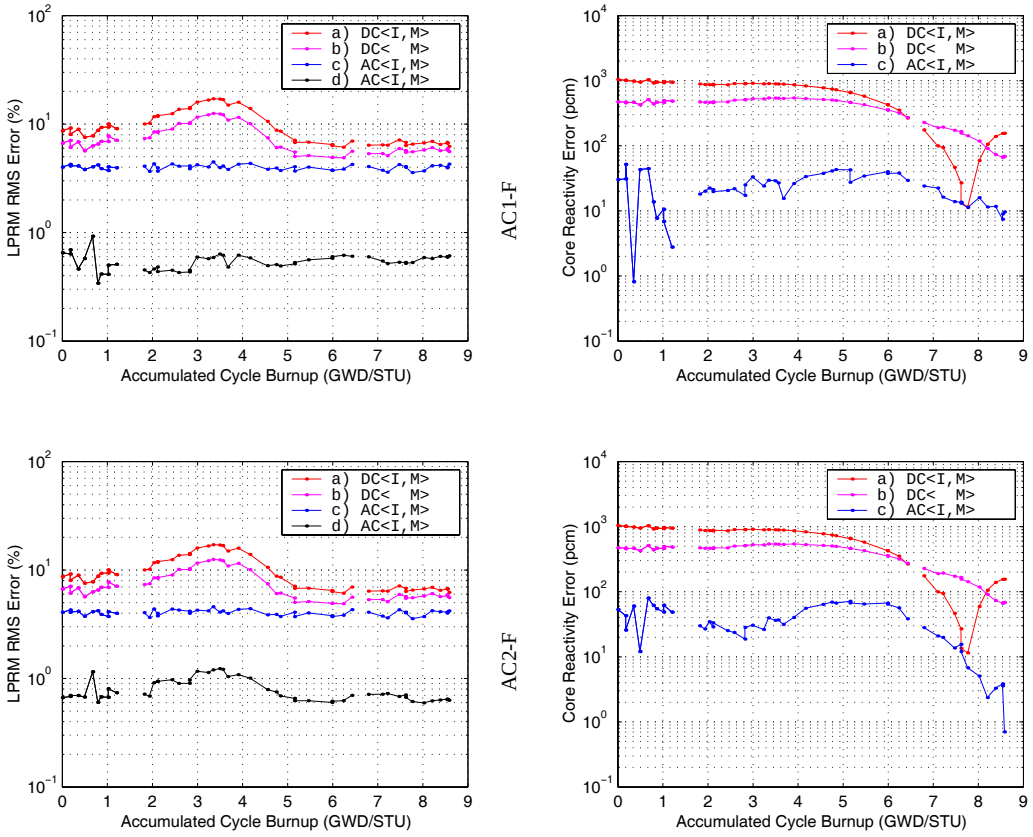

Figure 2. LPRMs' Misfit & Reactivity Error (High Void Fraction).

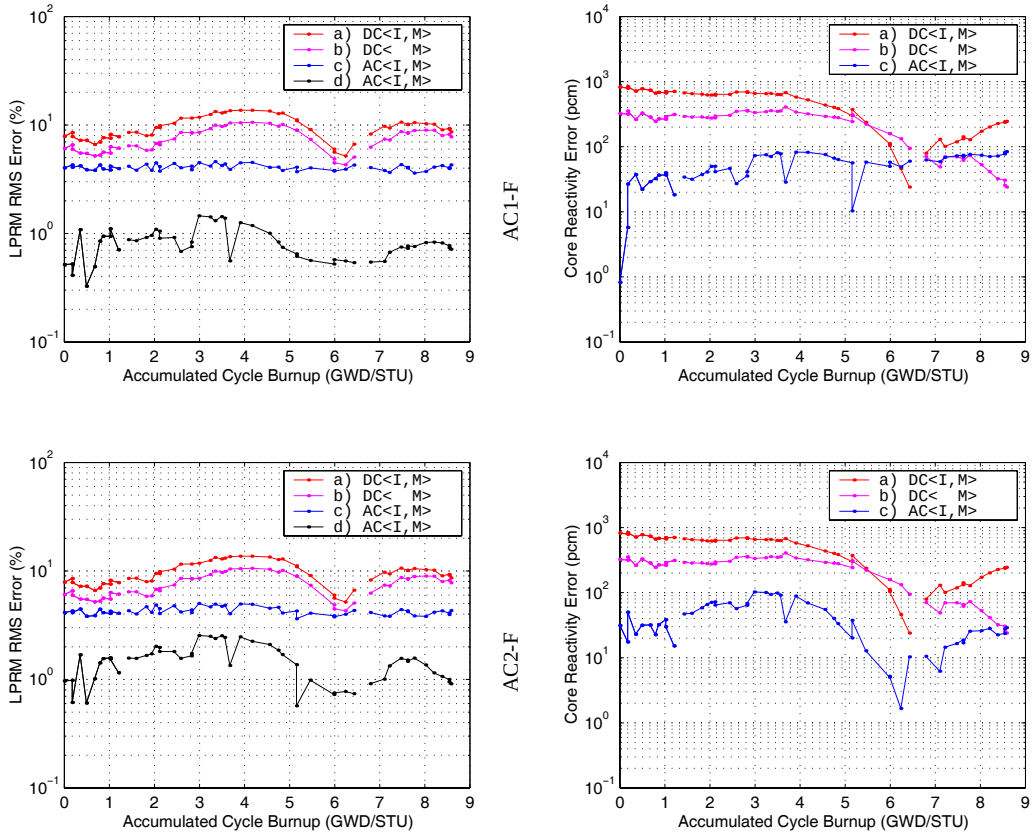

Figure 3. LPRMs' Misfit & Reactivity Error (Low Void Fraction).

conditions (i.e. RV, HV, and LV), respectively. Graph(a) and Graph(c) show that the adaption generally reduces the DC errors due to initial input data errors, even though the adaption is based on the Zuber-Findlay void-quality correlation, and the comparison is based on the Lellouche-Zolotar EPRI methodology. Hence, we can conclude that the adaption is mainly reducing the prediction errors due to input data errors separately from those due to modeling errors. However, when one analyzes Graph(c) where the void-quality parameters are not adapted, one observes that the errors of the AC2 now are even larger than the initial errors, which indicates that the adaption is highly affected by the magnitude and type of the modeling errors at which the adaption was completed. One should also notice that when using DC to predict core behavior at LV, that leads to a reduction in the predictions errors since there is less voiding in the core, and consequently the size of the prediction errors due to modeling errors will be reduced (see Graph(a) and Graph(b), AC2-F in Figure 2 and Figure 3). Comparing the size of the initial prediction errors for the LV case (see Graph(b), LV in Figure 4), it is observed that the prediction errors are of the same magnitude as when using the AC1 to predict core behavior at LV (see Graph(d), AC1-F in Figure 3), with the latter of somewhat smaller size, since adaption reduced input data errors. So, whether one uses the Lellouche-Zolotar EPRI methodology or the Zuber-Findlay void-quality correlation for the AC to predict core conditions at LV, one obtains the same magnitude of LPRMs RMS and

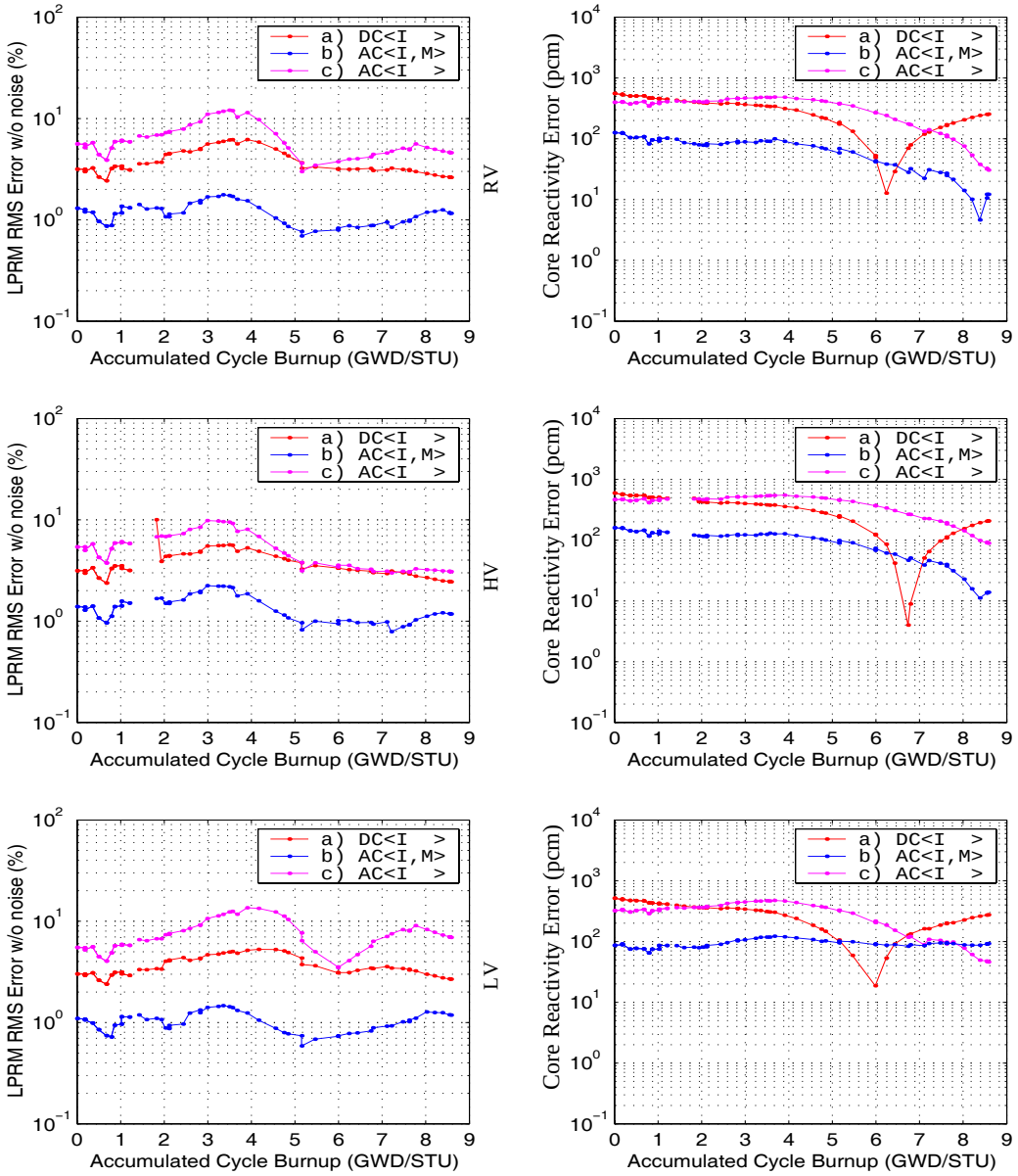

Figure 4. LPRMs' Misfit & Reactivity Error (Different Criteria).

reactivity errors. This confirms also that the adaption only corrects for input data errors, and is not affected by how much voiding is present when the core is adapted at rated conditions.

Figure 5 shows the a priori and a posteriori information about core parameters for three different levels of regularization. The second case corresponds to the highest fidelity and robustness adaption. It is observed that the a posteriori information about core parameters is statistically consistent with the a priori information for case(b) and case(c). For strong regularization, case(c), the a posteriori information approaches the a priori information and effectively no adaption is performed. For weaker regularization, case(a), the a posteriori information, for an ill-conditioned

problem, has larger uncertainty in general, since the adaption is affected by the noise level in the observables' signals. For an intermediate level of regularization, case(b), a compromise is achieved where now, the pieces of information about core parameters which are effectively inferred from the core observables, their uncertainty level is mainly determined by the observables uncertainty; and the remaining pieces of information, their a posteriori uncertainty information is mainly determined by the a priori uncertainty information about core parameters. The thermal-hydraulic parameter (C_0) has an adjusted value that deviates 6% from its nominal Zuber-Findlay value. This is to be expected since the functionalization of the void is not consistent with reality. The thermal-hydraulic parameter (C_0) has a strong sensitivity profile (i.e. its sensitivity profile is large in magnitude and different from other model parameters' sensitivity profiles), and hence its adjustment is mainly determined by the observables for different levels of regularization; however, the second thermal-hydraulic parameter (k_3) has a very weak sensitivity profile, and its adjusted value is mainly determined by the a priori information.

The choice of the regularization parameters affects the trade-off between the regularized terms and the data misfit term of the quadratic function in Eq(1), and consequently how far the a

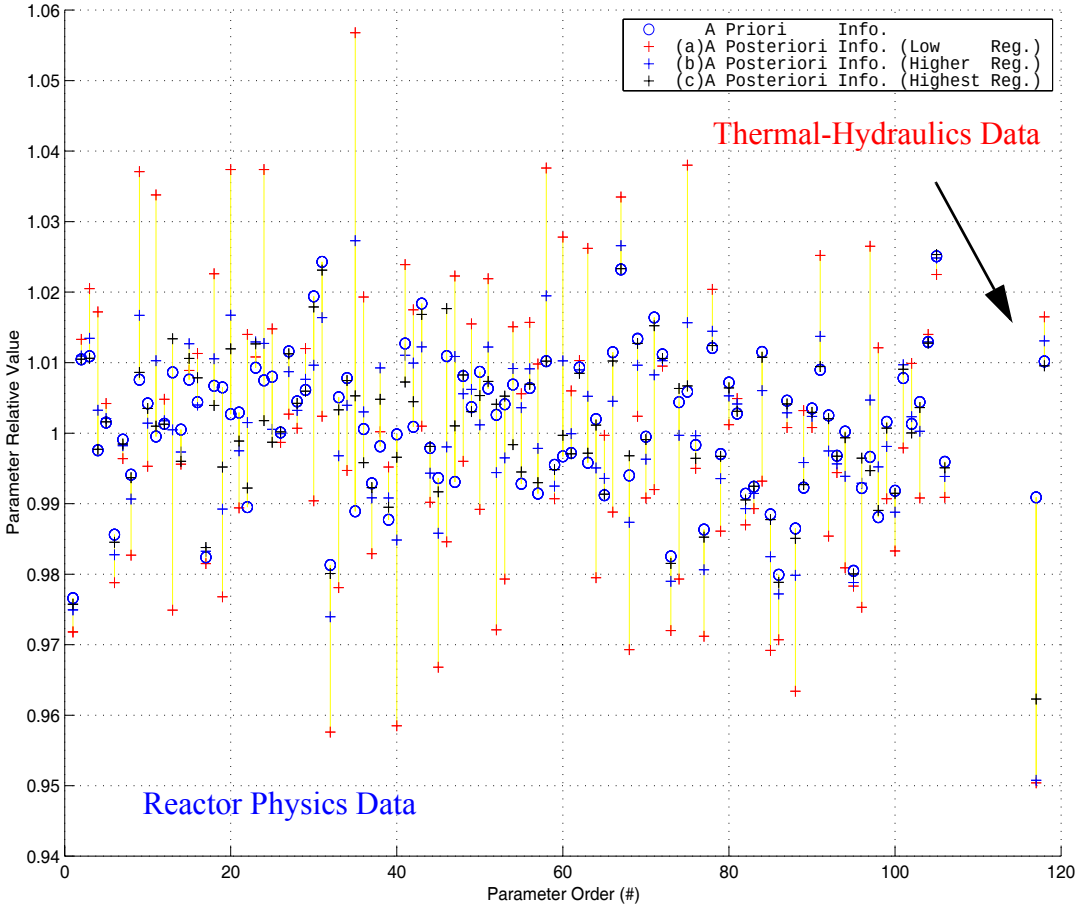

Figure 5. A Priori & A Posteriori Info. about Core Parameters.

posteriori information is from the a priori information about core parameters. Figure 6 plots a representative L-curve for a one-regularization-parameter adaption for one iteration step during a simplified unconstrained search. We utilized the same regularization parameters in our exploratory study to regularize the search for all core parameters. That can be considered as a first logical assumption since the a priori uncertainty information about core parameters is assumed to be of the same magnitude. In general, the trade-off between the data misfit term and the regularized terms should depend on the particular core parameter with respect to strength of its sensitivity profile and the level of uncertainty of its a priori information. Hence parameter-specific regularization parameters are required to reflect the difference between the core parameters and how much information can be recovered from the core observables. This issue is facilitated by the introduction of the full a priori covariance information about core parameters, which will be part of the future development of this project..

7. CONCLUSIONS

This study has served as a vehicle to gain insight into how effectively regularization techniques can be utilized to adapt the DC, giving rise to a meaningful adaption which is reflected in the high fidelity and robustness of the adaptive techniques employed. In these techniques, the different core parameters are adapted according to how much information about them are contained in the observables' signals. If the observables do not contain sufficient information about certain core parameters, their estimates will be confined to our best knowledge of these parameters. It was demonstrated with our choice of core parameters, we are able to separate and identify different

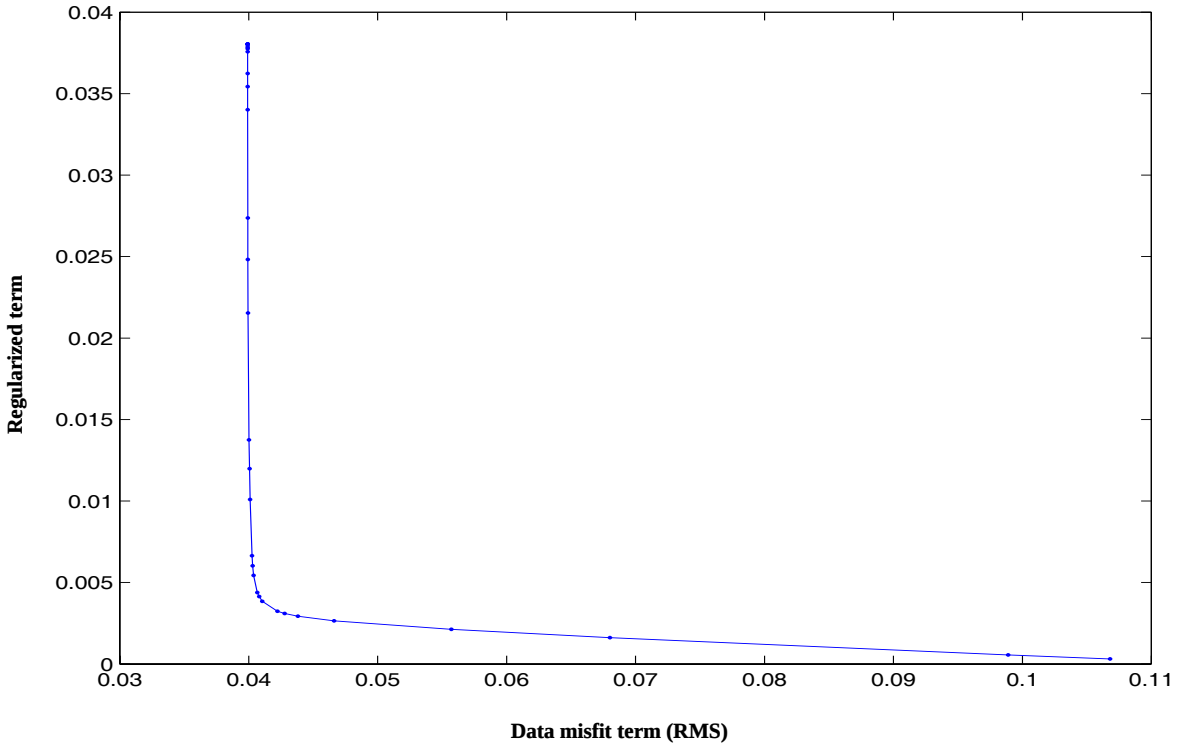

Figure 6. L-Curve

sources of modeling or input data errors and suggest more educated guesses of error functionalization.

The statistical consistence between the a posteriori and a priori information about core parameters has been confirmed for a range of regularization parameters. For this exploratory study, the a priori uncertainty information was simply modeled by diagonal covariance matrices, such that all core observables and core parameters have the same a priori uncertainty, $\sigma = 4\%$ and $\sigma = 1\%$, respectively. In reality, the covariance matrix of the reactor physics input data is not diagonal, and its elements depend on the uncertainty information of the basic cross-section data, (i.e. ENDF/B covariance information). Hence, the uncertainty information of the basic cross-section data has to be propagated starting from a point-wise cross-section or resonance parameters representation as available from ENDF/B and then processed through all the pre-processor codes to the core simulator. These pre-processor codes include multi-group library generation codes and lattice physics codes. This is a nontrivial uncertainty and sensitivity analysis type problem, since the currently available lattice physics codes do not have the capability to calculate the sensitivity coefficients of their output parameters with respect to their input data. This task will have to be carried out once for different lattice, bundle and core designs.

Our future work will mainly focus on utilizing *effective* methods to propagate this uncertainty information, where effective refers to minimizing computational effort without compromising fidelity of the adaption. This effectiveness we believe can be achieved by recognizing that the overall matrix system is expected to be severely ill-conditioned, and that utilization of regularization to address the ill-conditioning will limit the adaption to directions in the parameters' space that have strong sensitivities and high uncertainties. Hence a large part of the sensitivity and uncertainty information propagated through the lattice physics code is not directly required for a high fidelity adaption and hence need not be determined. This implies an approach where random perturbations of multiple parameters can be utilized to extract the sensitivity information for the important parameters. Having that in mind, we believe, the computational burden of sensitivity analysis type calculations can be reduced to be within practical limits.

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