Nuclear Mathematical and Computational Sciences: A Century in Review, A Century Anew
Gatlinburg, Tennessee, April 6-11, 2003, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2003)

MIXED DUAL FINITE ELEMENT METHOD FOR THE SOLUTION OF THE 3D KINETIC SIMPLIFIED PN TRANSPORT EQUATIONS

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ABSTRACT

The accurate knowledge of the time-dependent spatial flux distribution in nuclear reactor is required for nuclear safety and design. The flux distribution on the core reactor is given by the time-dependent neutron transport equation, but usually, for reason of computing time, the time-dependent neutron diffusion equation is used to study the transient behavior of the flux. The mixed dual finite element method has been applied successfully to the treatment of 3D diffusion and simplified transport equations ([3], [5] and [9]) for Cartesian geometries. A specific solver, named Minos, based on this method, has been implemented in the Cronos code developed at the French Atomic Energy Commission (CEA). Until now, for kinetics calculations, Minos solves only the 3D diffusion equations. The previous method is very efficient but in order to take into account the transport effects, we have now extended the kinetic solver to the simplified PN equations with two kinds of time approximation. For these two cases, the time discretization consists in a finite element representation of order 1. We obtain a standard mono-step difference q -scheme with an exact resolution of the precursors equation assuming a linear variation of the cross sections over the time interval like for the kinetic diffusion solver. Numerical results presented show that we can effectively see a transport effect when there is a quick variation of the core configuration such as a rod ejection.

Key Words: 3D-kinetic, simplified PN transport, mixed finite elements

1 INTRODUCTION

The Cronos code [6] is the computer tool devoted to core computation in the SAPHYR system of CEA. Within Cronos, the Minos solver is used to carry out assembly-homogenized fast core calculations for Cartesian geometries. The first version of the MINOS solver [7] was limited to the treatment of the diffusion approximation in 3D Cartesian geometry for eigenvalue calculations. In the particular case of rectangular geometries, the Raviart-Thomas-Nedelec finite element family ([10] and [11]) produces nice structured sparse linear systems. A block Gauss-Seidel method on the three components of the current vector was implemented. This method can be viewed as an alternating direction method and is rapidly converging.
In more recent developments in the context of eigenvalue calculations, the solver was extended to solve simplified PN equations [8] and a kinetic diffusion solver Minos was implemented in the Cronos code. This kinetic diffusion solver was then applied in the improved quasi-static IQS context [2]. The speed of calculation, which is the main advantage of the Minos method, added to the possibility to take large time steps for shape flux calculation with the IQS method, allowed

us to reduce considerably the computing time. A recent development of the Minos solver has been to extend the 3D-kinetic solver to the solution of the simplified PN transport equations in order to study if, as for eigenvalue calculations, this transport approximation can improve results for multigroup transient problems. The work completed for this development is described in this paper. The employed method is similar to that used in the context of the diffusion approximation. Two kinds of time approximations are possible. The first one is to extend the Minos kinetics diffusion solver supposing that the odd components of the flux are constant within each time step. This assumption is consistent with the standard kinetics diffusion approximation and can be straightforwardly implemented. The second approximation consist of taking into account the time derivative of the odd components of the flux to fully incorporate transport effects during the transient. This approximation has required a considerable amount of implementation work so, at this moment, numerical results are only presented on a simplified test configuration.

2 THE SIMPLIFIED PN TRANSPORT EQUATIONS

The neutron dynamics of a nuclear system (in absence of any external source) can be modeled by the multigroup time-dependent Boltzmann transport equation (1) associated with the time-dependent precursor equations (2):

For each energy group $g = 1,...G$ and $\vec{r} \in \Lambda \subset \mathbf{R}^3$

$$\left(\frac{1}{\mathrm{V}^{\mathrm{g}}}\frac{\partial}{\partial \mathrm{t}} + \vec{\nabla}.\vec{\Omega} + \Sigma_t^g(\vec{r},t)\right)\Phi^g(\vec{r},\vec{\Omega},t) = \sum_{g'=1}^{G}\int_{4p}\Sigma_s^{g'\to g}(\vec{r},\vec{\Omega}'.\vec{\Omega},t)\Phi^{g'}(\vec{r},\vec{\Omega}',t)\mathrm{d}\vec{\Omega}' + \frac{1}{4p}S^g(\vec{r},t) \tag{1}$$

where

$$S^g(\vec{r},t) = c_p^g\sum_{g'=1}^{G}(1-b^{g'})n\Sigma_f^{g'}(\vec{r},t)\int_{4p}\Phi^{g'}(\vec{r},\vec{\Omega}',t)d\vec{\Omega}' + \sum_{l=1}^{L}l_l C_l(\vec{r},t)c_l^g(\vec{r})$$

And for each precursor group $l = 1,...L$

$$\frac{\partial}{\partial \mathrm{t}}C_l(\vec{r},t) = -l_l C_l(\vec{r},t) + \sum_{g'=1}^{G}b_l^{g'}n(\vec{r})\Sigma_f^{g'}(\vec{r},t)\int_{4p}\Phi^{g'}(\vec{r},\vec{\Omega}',t)\mathrm{d}\vec{\Omega}' \tag{2}$$

V^g is the neutron speed of energy group g

Φ^{g} is the angular flux of energy group g

Σ_t^g is the macroscopic total cross section of energy group g

$\Sigma_s^{g'\to g}$ is the macroscopic scattering cross section from energy group g' to g

Σ_f^g is the macroscopic fission cross section of energy group g

n is the total number of neutrons emitted per fission

c_p^g is the energy spectrum of prompt neutrons of energy group g

c_l^g is the energy spectrum of precursor group l and energy group g

b_l^g is the delayed neutron fraction in energy group g of precursor group l

b^g is the total delayed neutron fraction of energy group g ($b^g = \sum_{l=1}^{L} b_l^g$)

l_l is the decay constant for delayed neutron precursor group l

C_l is the precursor concentration for delayed neutron precursor group l

$\vec{r}$ is the position

$\vec{\Omega}$ is the direction represented by the cosinus m and the azimuthal angle f (Figure 1)

t is the time

In order to simplify the description of this problem, we associate a zero even flux condition on energy group g defined by:

$$\Phi^g\left(\vec{r},\vec{\Omega},t\right)+\Phi^g\left(\vec{r},-\vec{\Omega},t\right)=0 \qquad on\ \partial\Lambda \tag{3}$$

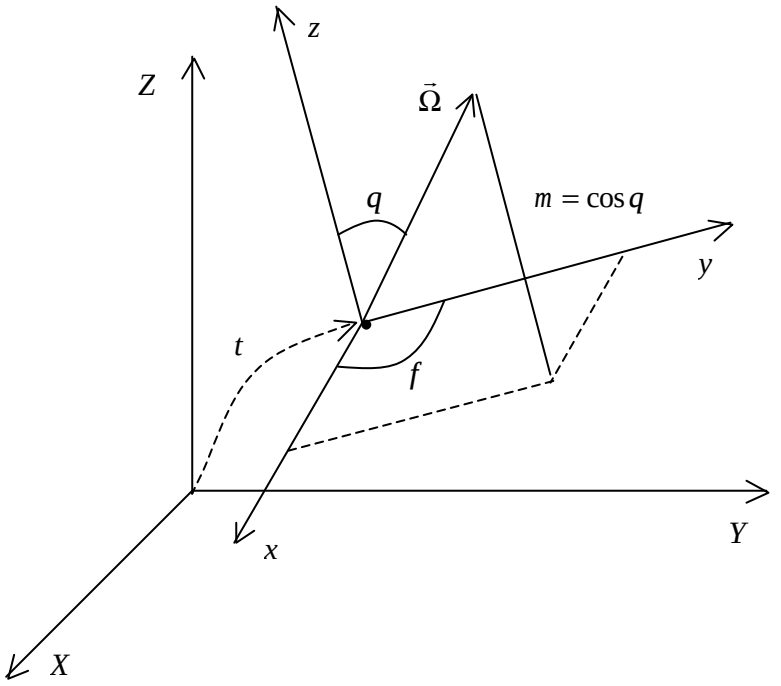


Figure 1. Local and global coordinate systems

The simplified PN transport theory [9] is based on two fundamental hypotheses. The first consist to say that in all point of the core reactor, it exists a coordinate system such that the angular flux is a slowly varying function of x, y and of the azimuth angle f. This hypothesis means that locally, the angular flux exhibits a planar symmetry. Thus, in each point $\vec{r}$, we can choose an axe's system such as the direction z is perpendicular at the plane of symmetry. We can write:

$$\Phi(x,y,z,m,f)=\tilde{\Phi}(z,m)+e\Phi_e(x,y,z,m,f) \tag{4}$$

where e is a little parameter.

Second hypothesis consists to say that between two points $\vec{r}$ and $\vec{r}' = \vec{r} + d\vec{r}$, this coordinate system varies slowly in space.
Substituting the expression (4) of the flux in the equation (1) and by expanding the scattering section on the Legendre polynomials according to:

$$\Sigma_s^{g'\to g}\left(\vec{\Omega}'.\vec{\Omega}\right) = \sum_{k=0}^{\infty}\left(\frac{2k+1}{4p}\right)\Sigma_{sk}^{g'\to g} P_k\left(\vec{\Omega}'.\vec{\Omega}\right) \tag{5}$$

we obtain the new equation:

$$\left(\frac{1}{\mathrm{V}^{\mathrm{g}}}\frac{\partial}{\partial t} + \mu\frac{\partial}{\partial z} + \Sigma_t^g\right)\tilde{\Phi}^g(z,m,t) = \sum_{g'=1}^{G}\int_{-1}^{+1} d\mu' \sum_{k=0}^{\infty}\left(\frac{2k+1}{2}\right)\Sigma_{sk}^{g'\to g} P_k(\mu)P_k(\mu')\tilde{\Phi}^g(z,\mu',t) + \frac{S^g(\vec{r},t)}{4p} + O(e) \tag{6}$$

where $O(e)$ is the measure of the error [9]. When we limit the development at $k = n$, we speak about a Pn approximation of the scattering cross section.
By neglecting the $O(e)$ terms, equation (6) reduces to a 1D transport equation in the local coordinate system. The transverse variables x, y and f in this equation are considered like parameters. A classical method to solve these equations is to use the PN approximation where the flux Φ^g is locally developed on the Legendre polynomial (N is generally chosen $\geq n$):

$$\Phi^g(\vec{r},m,t) = \frac{1}{4p}\sum_{m=0}^{N} a_m \Phi_m^g(\vec{r},t)P_m(m) \tag{7}$$

with $\Phi_m^g(\vec{r},t) = 2p\,\frac{2m+1}{a_m}\int_{-1}^{1}\Phi^g(\vec{r},m,t)P_m(m)dm$ and a_m normalization coefficients which can be chosen later in order to simplify the equations ($a_0 = 1$).
Thus, we can multiply the equation (6) by each Legendre polynomial P_m and integrate on the angular space. By using the recurrence formula for Legendre polynomials, we obtain for each energy group g and each order $0 \leq m \leq N$:

$$\frac{r_m}{\mathrm{V}^{\mathrm{g}}}\frac{\partial}{\partial \mathrm{t}}\Phi_m^g(\vec{r},t) + h_{m+1}\frac{\partial}{\partial \mathrm{z}}\Phi_{m+1}^g(\vec{r},t) + h_m\frac{\partial}{\partial \mathrm{z}}\Phi_{m-1}^g(\vec{r},t) + r_m\Sigma_t^g\Phi_m^g(\vec{r},t) = \sum_{\mathrm{g}'=1}^{\mathrm{G}} r_m\Sigma_{sm}^{g'\to g}\Phi_m^{g'}(\vec{r},t) + d_{m0}S^g(\vec{r},t) \tag{8}$$

where $r_m = \frac{a_m^2}{2m+1}$ and $h_m = \frac{m a_{m-1}a_m}{(2m-1)(2m+1)}$ (9)

In order to rewrite the previous equation in the global coordinate system, we multiply each odd equation by the vector $\vec{k}$ (the unit vector along Z direction). Thus, we get a coupled system where the harmonics of the flux are either scalar for the even index e or vectors for the odd index o. When the expansion of the scattering cross section is limited to order n, we speak about a time

dependent SPN-Pn approximation (the truncation parameter N is chosen odd in order to get an even number of equations).
The mixed dual formulation is obtained by projecting the two family equations on two different spaces, integrating over the domain Λ. After that, we use the Green's formula on the odd equations for each energy group g with the boundary condition (3):

$$\int_\Lambda \vec{\nabla}\Phi_e^g .\vec{\Phi}_o^g \, d\vec{r} = -\int_\Lambda \Phi_e^g \, \vec{\nabla}.\vec{\Phi}_o^g \, d\vec{r}$$

We adopt the notation $[.]$ for the operators after the spatial and energy approximations. B_d ($d = x, y \text{ or } z$) will be the spatial operator of the partial derivative in the direction d, Φ_e and $\Phi_{d,o}$ the multigroup vectors of even fluxes e and of the component in the direction d of the odd fluxes o. So, we obtain the time dependent SPN multigroup system:

$$r_e[\mathrm{V}]^{-1}\frac{\partial \Phi_e}{\partial t} + \sum_o h_{eo}\left(\sum_d B_d^T \Phi_{d,o}\right) + r_e[\Sigma_t]\Phi_e = S_e \qquad \textit{for e even } (e < N) \tag{10}$$

$$r_o[\mathrm{V}]^{-1}\frac{\partial \Phi_{d,o}}{\partial t} - \sum_e h_{oe}(B_d \Phi_e) + r_o[\Sigma_t]\Phi_{d,o} = S_{d,o} \qquad \textit{for o odd } (o \le N) \tag{11}$$

$$\frac{\partial C_l}{\partial t} = -l_l C_l + \sum_{g'=1}^{G} b_l^{g'}[n\Sigma_f^{g'}]\Phi_0^{g'} \qquad \textit{for l precursor} \tag{12}$$

where S_e and $S_{d,o}$ are the vectors defined by:

$$\begin{cases} S_e = r_e[\Sigma_{se}]\Phi_e + d_{e0}\left(\left[c_p(1-b)n\Sigma_f\right]\Phi_0 + \sum_{l=1}^{L} l_l[c_l]C_l\right) \\ S_{d,o} = r_o[\Sigma_{so}]\Phi_{d,o} \end{cases} \tag{13}$$

h_{oe} (respectively h_{eo}) correspond to the terms of a matrix H (respectively H^T) coupling the odd and even components of the flux [8]. These terms are defined by:

$$h_{oe} = \begin{cases} h_o & \textit{if} \quad e = o-1 \\ h_e & \textit{if} \quad e = o+1 \\ 0 & \textit{otherwise} \end{cases}$$

3 TIME APPROXIMATION

The time approximation used in the Cronos Code for the solution of the kinetic diffusion equation with the classical method or the improved quasi-static method [2] is based on an integral q-scheme or a difference q-scheme. In this work, we solve the simplified P_N transport kinetics equations with a time discretization using a difference q-scheme on the angular flux equations and an integral q-scheme on the precursors equations (exact integration of the

precursor equations with a linear expansion of the cross sections and polynomial representation of the flux).
The monostep scheme on the flux harmonics can be written like:

$$\begin{cases} \Phi_e(t) = \Phi_{e,p} w_p(t) + \Phi_{e,p+1} w_{p+1}(t) \\ \Phi_{d,o}(t) = \Phi_{d,o,p} w_p(t) + \Phi_{d,o,p+1} w_{p+1}(t) \end{cases} \quad \text{for } t \in [t_p, t_{p+1}] \tag{14}$$

where $w_p(t)$ and $w_{p+1}(t)$ are second order polynomial such as :

$$w_p(t_p) = 1,\ w_p(t_{p+1}) = 0 \text{ and } \int_{t_p}^{t_{p+1}} w_p(t)dt = (1-q)\Delta t$$

$$w_{p+1}(t_p) = 0,\ w_{p+1}(t_{p+1}) = 1 \text{ and } \int_{t_p}^{t_{p+1}} w_{p+1}(t)dt = q\,\Delta t$$

The order of convergence of the explicit scheme ($q = 0$) and the implicit scheme ($q = 1$) is 1. The choice $q = 1/2$ (Crank-Nicholson scheme) leads to a scheme of order 2.

3.1 Time approximation of the precursors equations

The precursor equations (12) are solved with the integral q-scheme numerical method [4]. The first step of this method consists to an exactly integration of the equation:

$$C_l(t) = C_l(0)e^{-l_l t} + \int_0^t e^{-l_l(t-s)} \sum_{g'=1}^{G} b_l^{g'} \left[\nu\Sigma_f^{g'}(s)\right]\Phi_0^{g'}(s)ds \tag{15}$$

In the Cronos Code, on each time interval $[t_p, t_{p+1}]$, we suppose that the scalar flux vary like the expression (14) and that the fission cross section has a linear variation:

$$\left[\nu\Sigma_f^{g'}(t)\right] = F_p^{g'} l_p(t) + F_{p+1}^{g'} l_{p+1}(t) \tag{16}$$

where $l_p(t) = \dfrac{t_{p+1} - t}{\Delta t}$, $l_{p+1}(t) = \dfrac{t - t_p}{\Delta t}$ and $F_q^{g'} = \left[\nu\Sigma_f^{g'}(t_q)\right]$ *for* $q = p, p+1$
So, for the precursor equations, we have the time recurrence relation:

$$\begin{aligned} C_l(t_{p+1}) &= e^{-l_l \Delta t} C_l(t_p) \\ &+ \sum_{g'=1}^{G} b_l^{g'} \left\{ b_{pp}^l F_p^{g'} \Phi_{0,p}^{g'} + b_{pp+1}^l F_p^{g'} \Phi_{0,p+1}^{g'} + b_{p+1p}^l F_{p+1}^{g'} \Phi_{0,p}^{g'} + b_{p+1p+1}^l F_{p+1}^{g'} \Phi_{0,p+1}^{g'} \right\} \end{aligned} \tag{17}$$

where

$$b_{mq}^l = e^{-l_l \Delta t} \int_{t_p}^{t_{p+1}} e^{-l_l (t_p - s)} l_m(s) w_q(s) ds \tag{18}$$

3.2 Time approximation of the simplified PN transport equation

3.2.1 Approximation of the even equations

We applied a classical difference q-scheme with the parameter q on each even equation *e*. So, the time approximation of the equation (10) leads us to the following system:

$$T_{e,p+1}\Phi_{e,p+1} + \sum_o h_{eo}\left(\sum_d B_d^T \Phi_{d,o,p+1}\right) = T_{e,p}\Phi_{e,p} - \frac{1-q}{q}\sum_o h_{eo}\left(\sum_d B_d^T \Phi_{d,o,p}\right) + d_{e0}S_{C_p} \tag{19}$$

where the matrices $T_{e,p+1}$ and $T_{e,p}$ are the full multigroup matrices defined by:

$$T_{e,p+1} = \frac{r_e}{q\,\Delta t}\left\{ [V]^{-1} + [\Sigma_t(t_{p+1})] - [\Sigma_{se}(t_{p+1})] - d_{e0}[c n\Sigma_f(t_{p+1})] + d_{e0}\sum_{l=1}^{L}\left(\sum_{i=p}^{p+1}\left(d_{ip+1} - l_l b_{ip+1}^l\right)[c_l b_l n\Sigma_f(t_i)]\right)\right\} \tag{20}$$

$$T_{e,p} = \frac{r_e}{q\,\Delta t}\left\{[V]^{-1} - \frac{1-q}{q}\left([\Sigma_t(t_p)] - [\Sigma_{se}(t_p)] - d_{e0}[c n\Sigma_f(t_p)]\right) - d_{e0}\sum_{l=1}^{L}\left(\sum_{i=p}^{p+1}\left(d_{ip}\frac{1-q}{q} - l_l b_{ip}^l\right)[c_l b_l n\Sigma_f(t_i)]\right)\right\} \tag{21}$$

and the source vector S_{C_p} is:

$$S_{C_p} = \sum_{l=1}^{L} l_l c_l \left(\frac{1-q}{q} + e^{-l_l \Delta t}\right) C_l(t_p) \tag{22}$$

In the previous expressions, c corresponds to the vector of components c^g, each component corresponding to the total production cross section of energy group g defined by:

$$c^g n\Sigma_f^{g'}(\vec{r},t) = c_p^g\left(1 - b^{g'}\right) n\Sigma_f^{g'}(\vec{r},t) + \sum_{l=1}^{L} c_l^g b_l^{g'} n\Sigma_f^{g'}(\vec{r},t) \tag{23}$$

3.2.2 Time approximation of the odd equations

For the time discretization of the odd equations, we also apply a difference q-scheme with the same parameter q used for the even equations. We obtain the following system on each direction *d* and on each time step:

$$R_{d,o,p+1}\Phi_{d,o,p+1} - \sum_e h_{oe}\left(B_d\Phi_{e,p+1}\right) = R_{d,o,p}\Phi_{d,o,p} + \frac{1-q}{q}\sum_e h_{oe}\left(B_d\Phi_{e,p}\right) \tag{24}$$

where

$$R_{d,o,p+1} = \frac{r_o}{q\,\Delta t}[V]^{-1} + r_o\left(\left[\Sigma_t\left(t_{p+1}\right)\right] - \left[\Sigma_{so}\left(t_{p+1}\right)\right]\right) \tag{25}$$

$$R_{d,o,p} = \frac{r_o}{q\,\Delta t}[V]^{-1} - r_o\frac{1-q}{q}\left(\left[\Sigma_t\left(t_p\right)\right] - \left[\Sigma_{so}\left(t_p\right)\right]\right) \tag{26}$$

A first development was to neglect the time derivative for the odd components. This approximation is an extension of the hypothesis done for the kinetic diffusion equations. In this case, the matrices in (25) and (26) do not contain the velocity matrix and the odd system (24) is time independent and reads:

$$-\sum_e h_{oe}\left(B_d\Phi_e\right) + r_o\left(\left[\Sigma_t\right] - \left[\Sigma_{so}\right]\right)\Phi_{d,o} = 0 \qquad \textit{for o odd } (o \le N) \tag{27}$$

Thereafter, when the time variation of the odd components are not taken into account, we speak about an even SPN transport solution. In the other case, we speak about a complete SPN transport solution.

3.2.3 Resolution of the kinetic system equations

In the two cases, on each time step, we get the following linear system:

$$\begin{pmatrix} -R_{o,p+1} & B\otimes H \\ \left(B\otimes H\right)^T & T_{e,p+1} \end{pmatrix}\begin{pmatrix} \vec{\Phi}_{o,p+1} \\ \Phi_{e,p+1} \end{pmatrix} = \begin{pmatrix} -R_{o,p} & \frac{q-1}{q}B\otimes H \\ \frac{q-1}{q}\left(B\otimes H\right)^T & T_{e,p} \end{pmatrix}\begin{pmatrix} \vec{\Phi}_{o,p} \\ \Phi_{e,p} \end{pmatrix} + \begin{pmatrix} \vec{0} \\ d_{e0}S_{C_p} \end{pmatrix} \tag{28}$$

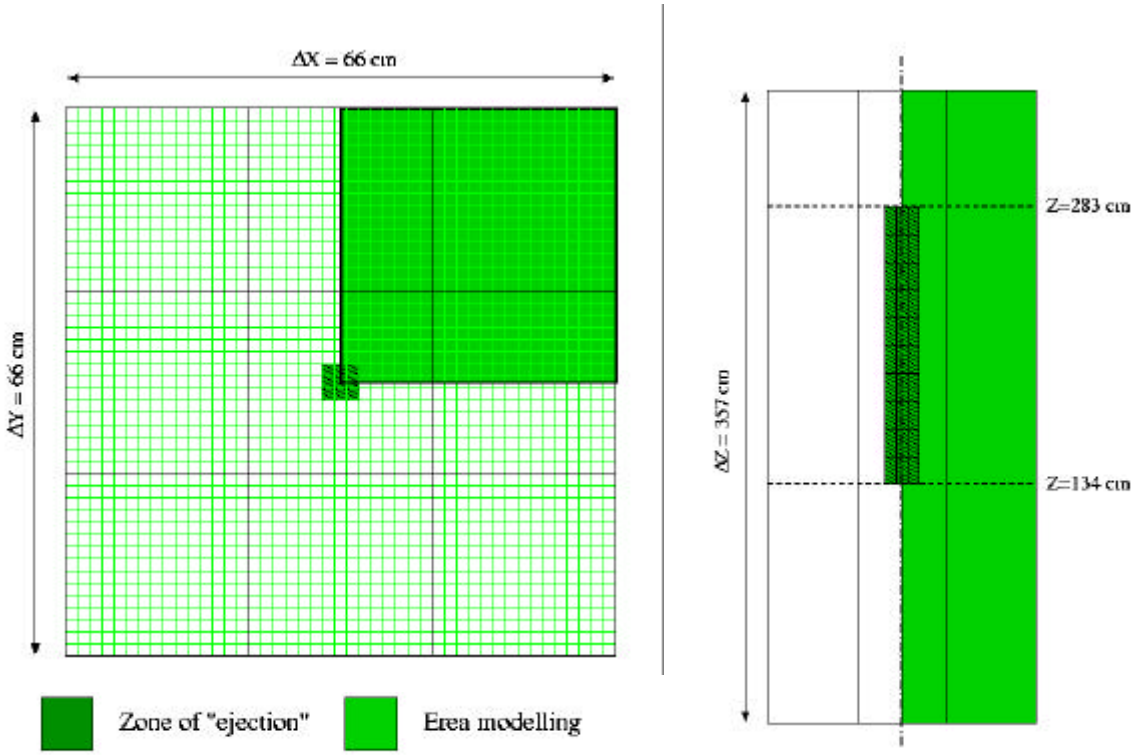
To solve the general system (28), we perform a Gauss-Seidel algorithm on the energy groups. However the matrix for each energy group g is not a positive definite matrix. So, we transform the system (28) by substituting the even flux into the odd equations (this step is very easy because each of matrix $T^{gg}_{e,p+1}$ is diagonal in space). The definite positive system which has been obtained on the odd components can be solved by a Gauss-Seidel algorithm on the direction d like in the case of classical source problem. [8].

4 APPLICATION, NUMERICAL RESULTS

Numerical tests were performed to prove consistency between diffusion and simplified PN transport approximations, and to measure the real impact of the SPN solver. For that purpose, a RIA (Reactivity Initiated Accident) configuration was chosen and performed by using the SAPHYR code system developed by the CEA/DEN [1].
RIA consists to a rapid insertion of reactivity due to a control rod ejection. This event induces a power increase limited first of all by Doppler feedback. The energy deposited in the core is not uniform. We assist to a deformation of the power distribution during the transient. So, because of time and space dependency, a RIA is particularly interesting for testing neutronic solver.

4.1 RIA configuration

The figure 2 shows the three dimensional cell-by-cell design used to test solvers. It covers a 3×3 UOX fuel assemblies geometry. Reflexion condition is imposed on radial and axial faces. Isotopic composition is the same in all cells and corresponds to a fissile/moderator medium characterized by scattering sections developed to order 5.
Sections depend on fuel temperature and moderator density that allows simulating feedbacks. Another parameter permits to control addition of AIC absorbent in the medium. AIC(*) composition is commonly used to adjust reactivity in a core during irradiation cycles. This absorbent is placed in control rods and introduced in the core. A control rod ejection is at the origin of a RIA.



Radial section **Axial section**

Figure 2. design used to analyze solver effects in a dynamic configuration (RIA)

(*) Silver (80.6%) Inconel (14.5%) Cadmium (4.9%)

In the present case, the accident is initiated by a local (several mesh in the center of the design) and quick (during 0.1s) modification of AIC absorption that is equivalent to a rod ejection. At the beginning of the transient, the core is in a hot zero power configuration.

Furthermore, thermal description is heterogeneous that guaranty local Doppler feedbacks. Analysis is carried out from integral and local quantities. Integral quantities are the eigen value

k_{eff} and the mean transient power $P(t)$. Local quantities gather in the Table I are based on the power distribution F(x,y,z).

Table I. Local quantities used for analysis

Radial form factor	$F_{xy} = \max\left[\frac{1}{\Delta z}\int_{\Delta z} F(x,y,z)dz\right]$
Axial form factor	$F_z = \max\left[\frac{1}{\Delta x \Delta y}\int_{\Delta z \Delta y} F(x,y,z)dxdy\right]$
Hot spot factor	$F_q = \max[F(x,y,z)]$

The Table II gives the mains characteristics of the performing tests. One diffusion calculation with transport correction has been conducted for testing the consistence between solvers (cases 1 and 2 comparison). The diffusion coefficient D for each energy group g has the following expression:

$$D^g = \frac{1}{3\sigma_{tr}^g} \tag{29}$$

where σ_{tr}^g is the transport section combination of the total section σ_{total}^g and the 1 order scattering section σ_{s1}^g:

$$\sigma_{tr}^g = \sigma_{total}^g - \sigma_{s1}^g \tag{30}$$

Table II. Numerical tests

Case	Solver	Number of energy groups (G)	Flux development (N)	Scattering sections development (n)
1	Diffusion	2	1	1
2	Even Simplified PN transport	2	1	1
3		2	3	1
4		2	3	3
5		2	5	3
6		2	5	5
7		8	3	1
8		16	3	1

4.2 Static analysis

First of all, a static analysis has been performed. RIA configuration is studied before and after the “ejection”. Results are given in the Table III.

Table III. results in static configuration

Case	Before "ejection"				After "ejection"				$r_0 = 10^5 \ln \frac{k_{eff}^{after}}{k_{eff}^{before}}$ [pcm]
	k_{eff}	F_{xy}	F_z	F_q	k_{eff}	F_{xy}	F_z	F_q	
1	0.50804	1.			0.51209	1.75	1.69	3.74	794
2					0.51211	1.74	1.69	3.72	798
3					0.51228	1.79	1.71	3.88	831
4					0.51230	1.80	1.71	3.90	835
5					0.51231	1.81	1.71	3.91	837
6									
7					0.51123	1.51	1.53	2.9	626
8					0.51112	1.48	1.50	2.78	604

Response before "ejection"
We note that responses before the "ejection" are identical for all cases. This result is due to the application of the same medium in the whole design and the use of reflexion conditions that induce a homogeneous power distribution. This conf iguration allows beginning the transient with the same response that is useful for neutronic comparisons.

Response after "ejection"
Results from cases 1 and 2 show that diffusion solver with transport correction and transport SP1-P1 solver are equivalent as expected. Furthermore, a transport effect appears with N increase. A numerical convergence is reached for $(G,N,n) = (8,3,1)$ (case 7).

4.3 Dynamic analysis

RIA is simulated by considering case 1 (diffusion solver), cases 3 and 7 (SPN transport solver). The fraction of delayed neutrons β is homogeneous in the whole core and equal to 521 pcm that conduct to superprompt-critical transients ($\rho_0 > \beta$). The calculation lasts 0.4s with a time step of 10^{-3} s.
Figure 3 show the evolution of the normalized mean power during the transient. We observed previously that transport solver induces modifications of the inserted reactivity (ρ_0) and the power distribution (F_q as illustration). These modifications have an impact on the power pulse that illustrate the analytical expression of the maximum of power P_{max} obtained by a point kinetic approach:

$$P_{max} \propto \frac{(r_0 - b)^2}{w_D a_D} \tag{31}$$

In this expression, α_D and ω_D are respectively the Doppler coefficient and the associated weight. ω_D is correlated to the deformation of the power distribution: the more the power distribution is heterogeneous, the more the Doppler weight is high. Thus, transport solver impact on P_{max} through ρ_0 and ω_D.

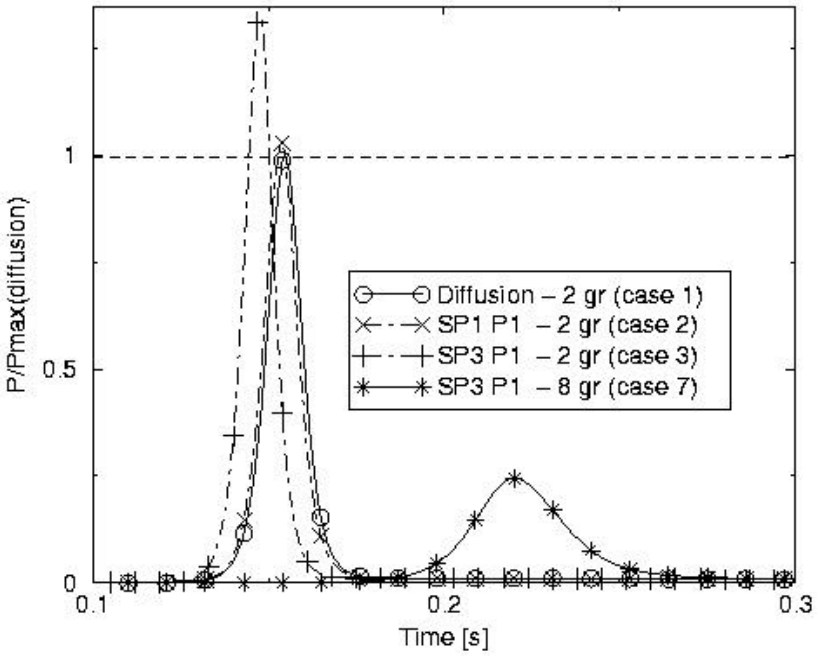


Figure 3. Mean power evolution versus time corresponding to cases 1, 3 and 7

Effect of the time dependency on the odd components

In the present RIA configuration, the take in account of odd component time dependence has a slightly effect on the kinetic behavior. The figure 4 shows the "odd" effect by considering the case 2 configuration (SP1-P1 – 2 groups of energy). We observe a decrease of the maximum of power reached during the RIA (-3%) induced by a modification of the power grade (integral effect). Investigations are carried out to find some configurations that clearly underline this effect.

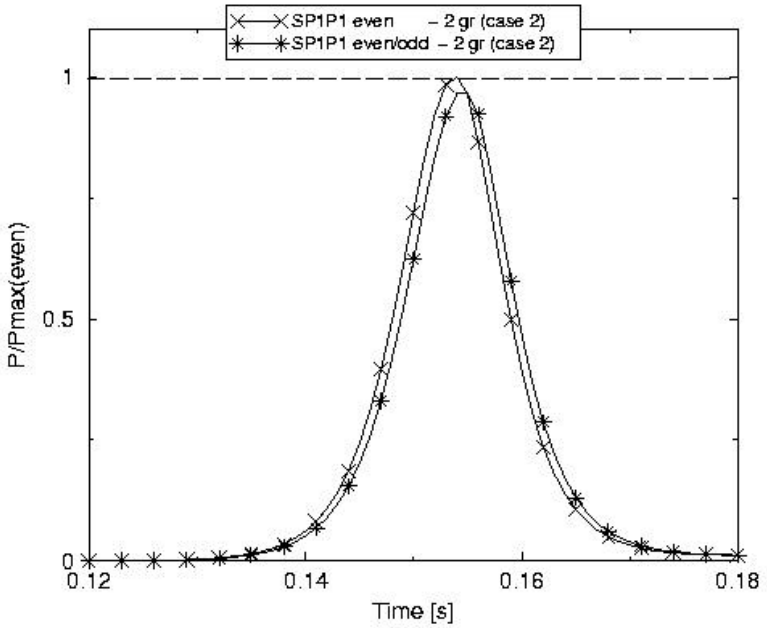


Figure 4. "odd" effect on the evolution on the mean power during a RIA

5 CONCLUSION

This paper presents one of the last evolution in the Cronos Code which allows to perform 3D-kinetic calculations in the context of the simplified PN transport theory. This implementation has been carried out in the solver Minos based on the mixed dual finite element method. The solver has been successfully tested in an RIA configuration. Numerical tests show all the benefit to use simplified transport approximation in the kinetic equations compared to a diffusion approximation. The impact of the time dependency on the odd components of the flux has been quantified. We didn't observe significant effects, this is probably due to the configuration used for this analysis. In the future other configurations where the transport effects are more significant will be performed.

ACKNOWLEDGMENTS

Numerical applications were realized in the framework of a collaboration with IRSN (Institut de Radioprotection et de Sûreté Nucléaire)

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