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# POINT GENETICS: A NEW CONCEPT TO ASSESS DYNAMIC BEHAVIOR IN ACCELERATOR DRIVEN SYSTEMS

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## ABSTRACT

In this paper we derive a coupled set of equations called the point *genetic* equations, which describe the behavior of neutron generations. For an ADS, the point genetic equations provide a more detailed and more accurate description of the behavior of system, than the conventional point kinetic equations, which are in fact a special case of the point genetic equations. The main reason to propose this set of equations is its use in coupled neutronic and thermal hydraulic transient analysis. The equations also provide basic insight in the dynamic behavior of an ADS.
We define the kinetic parameters for the point genetic equations and consider the entire fission chain to characterize the integral behavior of the system. In doing this we derive integral kinetic parameters for a system driven by an external source. We also describe the behavior of detectors in terms of the point genetic equations. We then consider the asymptotic behavior of the point genetic equations and make two different approximations for kinetic parameters that can be used in the conventional point kinetic equations.
The calculation of the kinetic parameters for the point genetic equations requires additional work in the near future and, hence, we cannot make a comparison yet with other reference calculational routes. Currently we use the point genetic equations in fictive, but realistic, situations and show the nature and use of these equations. We compare the results with the two point kinetic approximations. We also make simulations of the behavior in situations specific for an ADS, like source variations and typical changes in reactivity.
We conclude that the point genetic equations, with a limited number of generations, are a useful set of equations that describe the integral behavior of an ADS in far more and greater detail than any point kinetic equation can do. The proposed set of equations is relatively easy and quickly solved numerically and can be used for coupled neutronic and thermal hydraulic transient analysis.

*Key Words*: Point Genetics, Point Kinetics, Accelerator Driven Systems, Dynamic Behavior and Transient Analysis

## 1. INTRODUCTION

The point kinetic equations are often applied for critical reactors, especially in thermal hydraulic transient analyses. For accelerator driven systems the point kinetic equations are less suited to describe the behavior of the system, because the external neutron source introduces a source neutron specific spatial and time dependent behavior. In this paper we propose a new set of so-called point *genetic* equations, which are able to describe the behavior of an ADS with greater accuracy and in more detail, still without having to resort to a full-scale 3D space-time kinetic approach.
The main reason to propose this set of equations is that it may reduce the calculational time for transient analysis, while maintaining accuracy. It also provides basic insight in the dynamic behavior of an ADS.

The point genetic equations consist of a coupled set of equations. Each set describes the time-dependent behavior of a specific neutron generation present in the system. The first neutron generation is created by the external source; the last set describes the fundamental mode or fundamental generation. The combined generations, or combined set of equations, describe the integral behavior of the subcritical system as induced by the external source.
Each set of the point genetic equations depends on generation specific kinetic parameters (i.e. a multiplication factor, a delayed neutron fraction and a lifetime) that may be different from generation to generation. We consider several specific characteristics of the point genetic equations, including the fission chain created by the external neutron source. The fission chain allows us to derive integral kinetic parameters that describe the entire fission chain, including the individual links in the chain. In doing this we derive a combined, or integral, multiplication factor and an effective delayed neutron fraction that are characteristic for the response of the system. These integral kinetic parameters are, in some sense, similar to the usual point kinetic parameters. And we will show that the point kinetic equations are a special case of the point genetic equations.

The calculation of the kinetic parameters for each generation in a realistic system is feasible, but will require some additional effort in the near future. In this paper we will apply the theory to a fictive case with realistic kinetic parameters to show the nature and use of the point genetic equations. More specific, we will compare the results with the predictions by conventional point kinetic equation(s), we will study the behavior following specific reactivity insertions and we will consider the role of delayed neutrons. The results show that the point genetic equations are very useful in describing the integral behavior of an ADS, making them suitable for application in coupled neutronic and thermal hydraulic transient analysis.

# 2 Point Genetics

## 2.1 Neutron generations

Neutron generations offer a simplified conceptual view of neutronics [1][5]. A generation is simply a collection of neutrons, represented by a certain distribution in phase space. When a neutron is absorbed, it terminates the generation. Absorptions that lead to a fission event create neutrons that start their 'life' in the next generation. In this way a sequential chain of fission sources is created in the system. Neutrons from the external source are the first generation. See Figure 1 for a schematic drawing of generations and fission sources.

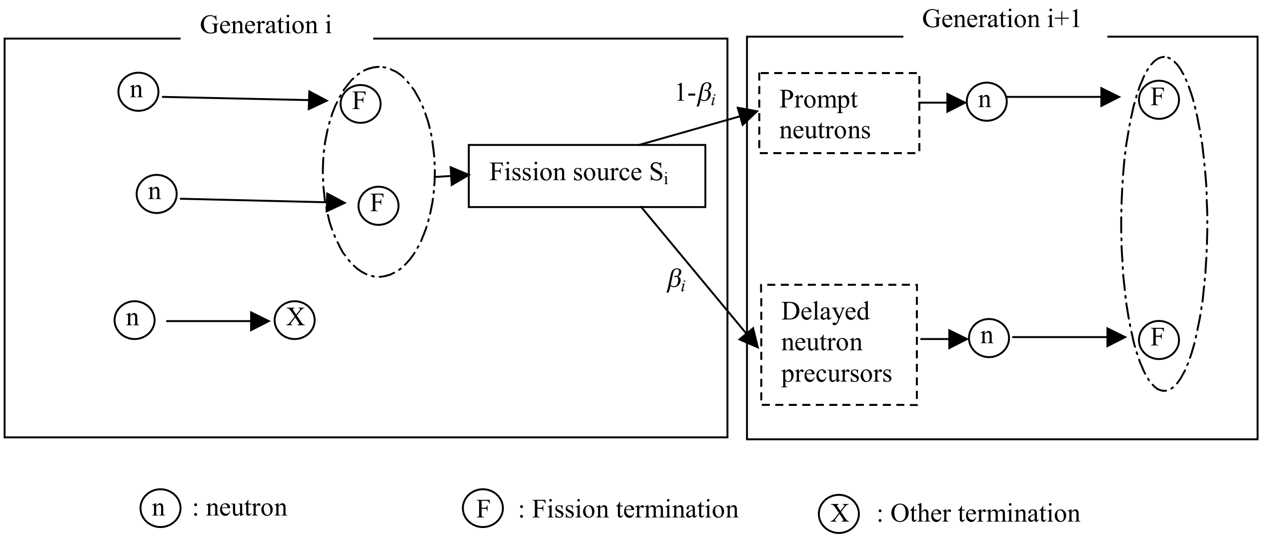


**Figure 1. Schematic drawing of two generations**

Assigning characteristic kinetic parameters to a generation automatically means that they are averaged over phase space, including time, which makes it somewhat strange to consider the time evolution of generations. However, the neutrons belonging to a particular generation are typically created and terminated within a few (prompt) neutron lifetimes, and we assume that the kinetic parameters are averaged over this time interval. The kinetic parameters may change on time scales much larger than the prompt neutron lifetime. See Appendix A.1 for a more elaborate discussion on this subject.

In the usual view of generations no difference is made between delayed neutrons and prompt neutrons. They are in the same generation, although they are released significantly later than prompt neutrons and may also behave differently due to the lower initial kinetic energy. In the point genetic equations we wish to catch this specific behavior of delayed neutrons and therefore we introduce sub-generations that contain either prompt neutrons or groups of delayed neutrons (and precursors). Note however, that both contribute to the same fission source, the starting point of the next generation.

The point genetic equations can be constructed with precursor decay constants that depend on the specific generation (characterized by $\lambda_{ij}$ : *i* generation, *j* precursor group), however, to simplify the equations somewhat, we will only use one group of delayed neutrons ($\lambda$).

## 2.2 Point genetic equations

The flux $\varphi_i(\mathbf{r}, \Omega, E, t)$[cm$^{-2}$s$^{-1}$] is associated with a specific generation *i* and is the sum of the fluxes $\varphi_i^P(\mathbf{r},\Omega,E,t)$ and $\varphi_i^D(\mathbf{r},\Omega,E,t)$, respectively representing the prompt and the delayed neutron sub-generation. The evolution of these fluxes is governed by the Boltzmann equation[1][3], without the term for the production of neutrons by fission, because this term is a source ($f$ [cm$^{-3}$]) that produces neutrons for the next generation.

The flux of each sub generation is described by:

$$\begin{aligned}
&\frac{1}{\mathrm{v}}\frac{\partial \varphi_i^{\mathrm{X}}(\mathbf{r},\Omega,E,t)}{\partial t} = -\mathbf{E}\varphi_i^{\mathrm{X}} - \mathbf{A}\varphi_i^{\mathrm{X}} + s_i^{\mathrm{X}}(\mathbf{r},\Omega,E,t) && \mathrm{x} = \{P, D\} \\
&f_i(\mathbf{r},t) = \nu\Sigma_F\left(\varphi_i^P + \varphi_i^D\right) && \text{: Fission source for generation } i+1 \\
&\frac{\partial c_i}{\partial t}(\mathbf{r},t) = -\lambda c_i + \beta_{i-1} f_{i-1} && \text{: Precursors} \\
&s_i^{\mathrm{P}} = (1-\beta_{i-1}) f_{i-1} && \text{: Prompt neutron production} \\
&s_i^{\mathrm{D}} = \lambda c_i && \text{: Delayed neutron production}
\end{aligned} \qquad (1)$$

With **E** the escape, or leakage, operator and **A** the absorption operator, which also includes scattering and (n,xn). A single group precursors $c_i$[cm$^{-3}$] has been introduced to model the delayed neutrons, with $\lambda$[s$^{-1}$] the characteristic time scale associated to the decay of the precursor of each generation and $\beta_i(t)$ the delayed neutron fraction.

The source for the first generation is given by:

$$f_0 = s_{\mathrm{External}}, \quad c_1 = 0, \quad \beta_0 = 0,$$

with which we account for the fact that the external source neutrons (first generation) are not accompanied by delayed neutrons.

The above set of equations are integrated over position, energy and direction and the various parameters, averaged over the entire system, are introduced:

$$\langle \ldots \rangle_V \equiv \iiint \ldots d\mathbf{r}\,dE\,d\Omega,$$

$$N_i^{\mathrm{X}}(t) = \left\langle \frac{\varphi_i^{\mathrm{X}}}{\mathrm{v}} \right\rangle_V, \quad C_i(t) = \langle c_i \rangle_V, \quad \mathrm{S}_i^{\mathrm{X}}(t) = \langle s_i^{\mathrm{X}} \rangle_V, \quad \mathrm{F}_i(t) = \langle f_i \rangle_V,$$

$$k_i^{\mathrm{X}}(t) = \frac{\langle \nu\Sigma_F \varphi_i^{\mathrm{X}} \rangle_V}{\langle (\mathbf{E}+\mathbf{A})\varphi_i^{\mathrm{X}} \rangle_V}, \quad \mathrm{l}_i^{\mathrm{X}}(t) = \frac{N_i^{\mathrm{X}}}{\langle (\mathbf{E}+\mathbf{A})\varphi_i^{\mathrm{X}} \rangle_V},$$

with $N_i^{\mathrm{x}}$ the number of neutrons in a certain generation, $S_i^{\mathrm{x}}$ the number of neutrons produced per unit of time, with ‘x’ for either the prompt or the delayed neutrons. Furthermore, we have introduced $C_i$ the number of precursors and $F_i$ the number of fissions.

In applying the point genetic equations we will usually assume that the system is completely empty at $t=0$: $N_i^x(0)=0$. If the system is not empty at t=0, the initial conditions can be determined by decomposing the initial flux in the flux for (sub-) generations and average ($<\ldots>_V$) over the system to obtain $N_i^x(0)$.

A coupled system of equations, the so-called point genetic equations, is now obtained:

$$\begin{aligned}
&\frac{dN_i^{\mathrm{X}}}{dt} = -\frac{N_i^{\mathrm{X}}}{l_i^{\mathrm{X}}} + S_i^{\mathrm{X}}(t) && \mathrm{x} = \{P, D\} \\
&F_i(t) = \frac{k_i^{\mathrm{P}} N_i^{\mathrm{P}}}{l_i^{\mathrm{P}}} + \frac{k_i^{\mathrm{D}} N_i^{\mathrm{D}}}{l_i^{\mathrm{D}}} && \text{Fission source} \\
&\frac{dC_i}{dt} = -\lambda C_i + \beta_{i-1} F_{i-1} && \text{Precursors} \\
&S_i^{\mathrm{P}} = (1-\beta_{i-1}) F_{i-1} && \text{Prompt neutron production rate} \\
&S_i^{\mathrm{D}} = \lambda C_i && \text{Delayed neutron production rate}
\end{aligned} \tag{2}$$

Instructive examples of the behavior of this set of equations are presented in Section 3.1.

## 2.3 The fission chain

An important characteristic of a subcritical system is its chain of multiplication [5] and we are interested how this chain is unfolding in the point genetic equations. We start by considering each link in the chain and determine the multiplication characteristic to each generation.
To obtain the total number of fissions, we integrate over time by introducing the shorthand notation:

$$\langle \ldots \rangle = \int_{-\infty}^{\infty} \ldots dt\,,$$

and find, with the help of (13) (Appendix A.2.2), that:

$$\begin{aligned}
&\langle N_i^X \rangle = l_i^X \langle S_i^X \rangle, \quad \langle F_i \rangle = \frac{k_i^P}{l_i^P} \langle N_i^P \rangle + \frac{k_i^D}{l_i^D} \langle N_i^D \rangle, \\
&\langle C_i \rangle = \beta_{i-1} \frac{\langle F_{i-1} \rangle}{\lambda}, \quad \langle S_i^P \rangle = (1-\beta_{i-1}) \langle F_{i-1} \rangle, \qquad \langle S_i^D \rangle = \lambda \langle C_i \rangle.
\end{aligned} \tag{3}$$

The fission source produced by one generation depends on the fission source produced by the preceding generation:

$$\begin{aligned}
\langle F_i \rangle \quad &= k_i^P \langle S_i^P \rangle + k_i^D \langle S_i^D \rangle \\
&= k_i^P (1-\beta_{i-1}) \langle F_{i-1} \rangle + k_i^D \beta_{i-1} \langle F_{i-1} \rangle \\
&\equiv k_i^{\mathrm{av}} \langle F_{i-1} \rangle \equiv k_i^{\mathrm{av}} (1-\beta_i^{\mathrm{av}}) \langle F_{i-1} \rangle + k_i^{\mathrm{av}} \beta_i^{\mathrm{av}} \langle F_{i-1} \rangle \\
\Rightarrow \quad & k_i^{\mathrm{av}} = k_i^P (1-\beta_{i-1}) + k_i^D \beta_{i-1} \\
\Rightarrow \quad & \beta_i^{\mathrm{av}} = k_i^D \beta_{i-1} / k_i^{\mathrm{av}}.
\end{aligned} \tag{4}$$

with $k_i^{\mathrm{av}}$ the average multiplication of a generation and $\beta_i^{\mathrm{av}}$ the average delayed neutron fraction of a generation. These kinetic parameters are the point genetic analogies of the well-known 'effective' kinetic parameters. For a system in fundamental mode (1 generation) the expression for the delayed neutron fraction equals the expression derived by Spriggs et al.[6] for the effective delayed neutron fraction.

With each link in the fission chain characterized, the total number of fissions produced as a result of the external source is:

$$\langle F_{\text{Total}} \rangle = \sum_{i=1} \langle F_i \rangle \ = \langle S_{\text{External}} \rangle \sum_{i=1} \prod_{j=1}^{i} k_i^{\text{av}} \ .$$

Sometimes the term on the right is written :

$$M_s = \sum_i \prod_{j=1}^{i} k_i^{\text{av}} \equiv \frac{k_s}{1-k_s}, \tag{5}$$

with $M_s$ the source multiplication and $k_s$ the source multiplication factor, which is a constant that characterises the multiplication of the system. See Appendix A.3 for a discussion on $k_i^{\text{av}}$, $k_s$ and $k_{\text{eff}}$.

The total number of fissions that is produced by delayed neutrons is:

$$\left\langle F_{\substack{\text{Total}\\ \text{Delayed}}} \right\rangle = \sum_i k_i^{\text{av}} \beta_i^{\text{av}} \langle F_{i-1} \rangle = \langle S_{\text{External}} \rangle \sum_i k_i^{\text{av}} \beta_i^{\text{av}} \prod_{j=1}^{i-1} k_j^{\text{av}},$$

with which a delayed source fraction $\beta_s$ can be introduced:

$$\begin{aligned}
\langle F_{\text{Total}} \rangle \quad &= M_s(1-\beta_s)\langle S_{\text{External}} \rangle + M_s \beta_s \langle S_{\text{External}} \rangle, \\
M_s \quad &= \sum_i \prod_{n=1}^{i} k_i^{\text{av}}, \\
\beta_s \quad &= \frac{\sum_i \beta_i^{\text{av}} \prod_{j=1}^{i} k_j^{\text{av}}}{\sum_i \prod_{j=1}^{i} k_j^{\text{av}}}.
\end{aligned} \tag{6}$$

Using the point genetic equations, we have now described the entire fission chain, including the individual links, with only a few kinetic parameters, namely $k_i^{\text{av}}$,$\beta_i^{\text{av}}$, $k_s$ and $\beta_s$.

## 2.4 Other characteristics

### 2.4.1 Average lifetime

In considering the fission chain we were concerned with the total number of fission neutrons, but not with the time these neutrons spend in the system. Using equation (3), we integrate the number of neutrons in generation i over time:

$$\begin{aligned}
\langle N_i \rangle \quad &= \langle N_i^P \rangle + \langle N_i^D \rangle = l_i^P \langle S_i^P \rangle + l_i^D \langle S_i^D \rangle \\
&= \left(l_i^P (1-\beta_{i-1}) + l_i^D \beta_{i-1}\right) \langle F_{i-1} \rangle \\
&\equiv l_i^{\text{av}} \langle F_{i-1} \rangle,
\end{aligned}$$

with $<N_i>$ a quantity that is related to the neutron fluence, but is not the same. Summing over all generations, $<N_{Total}>$ scales with the strength of the external source:

$$\langle N_{\text{Total}} \rangle = \sum_i \langle N_i \rangle = \langle S_{\text{External}} \rangle \left( l_1^{\text{av}} + \sum_{i=2} l_i^{\text{av}} \prod_{j=1}^{i-1} k_j^{\text{av}} \right) .$$

The average lifetime $l_s$ can now be introduced by writing:

$$\begin{aligned} & l_s + \sum_{i=2} l_s \prod_{j=1}^{i-1} k_s = l_1^{\text{av}} + \sum_{i=2} l_i^{\text{av}} \prod_{j=1}^{i-1} k_j^{\text{av}} \\ \Rightarrow\ & l_s = (1-k_s)\left( l_1^{\text{av}} + \sum_{i=2} l_i^{\text{av}} \prod_{j=1}^{i-1} k_j^{\text{av}} \right). \end{aligned} \tag{7}$$

#### 2.4.2 Detectors

The response of a detector is formally something like:

$$D(t) = \sum_i \int .. \int \mathbf{R}(....)\left(\varphi_i^P + \varphi_i^D\right) d.... ,$$

with $D(t)$ the response and $R$ the response operator. The integration is performed over variables like energy and position, but can be very complex.
For the generations this can be rewritten as:

$$\begin{aligned} D(t) &= \sum_i r_i^P(t) N_i^P(t) + r_i^D(t) N_i^D(t), \\ \Rightarrow \qquad r_i^X(t) &= \frac{\int .. \int \mathbf{R}(....)\varphi_i^X d....}{N_i^X}, \end{aligned}$$

with $r_i^{\text{X}}$ the specific response to each sub-generation. We will usually assume that $r_i^{\text{X}}$ does not depend on time (Appendix A.1). The major value of this expression is that each generation may give a different contribution to the response, for instance by a difference in the energy or spatial distributions.
Examples of specific generation-dependent detector responses are shown in Section 3.1.

### 2.5 Limit cases and approximations

#### 2.5.1 Termination of the infinite chain of generations

An important characteristic of either a critical or a subcritical system is the so-called fundamental mode. The distributions characterizing the fundamental mode are such that the kinetic parameters do not depend on the specific generation. The reason for this is that the flux associated with different generations does not change 'shape', although it still may change in amplitude.

Any neutron generation will eventually evolve into a generation that is equivalent to the fundamental mode: after a sufficiently large number of generations, say $i \geq i_F$, the shape of the associated flux is equivalent to the fundamental mode. The recursive chain in the point genetic equations can then be terminated at $i=i_F$, provided the source term for the fundamental generation is replaced by:

$$S_F{}^X = S_{i_F-1}{}^X + S_{i_F}{}^X \ .$$

The evolution of the fundamental sub-generations is then given by

$$\frac{dN_F{}^X}{dt} = -\frac{N_F{}^X}{l_F{}^X} + S_F{}^X \ . \tag{8}$$

This expression is useful, because it terminates the infinite chain of generations and allows numerical procedures to find solutions of the point genetic equations. It can also be used to derive point kinetic equations.

### 2.5.2 Point kinetic approximations for an ADS

The point genetic equations reduce to the point kinetic equations if only one generation is considered. The point genetic equations offer a wider range of applicability than the point kinetic equations at the cost of a more complex calculation of the kinetic parameters. Moreover, several different choices are possible for the parameters used in a point kinetic equation for an ADS. The general formulation of the point kinetic equations for a source driven system is:

$$\begin{aligned} \frac{dN}{dt} &= \frac{k(1-\beta)-1}{l} N + \lambda C + mS, \\ \frac{dC}{dt} &= -\lambda C + \frac{k\beta}{l} N, \end{aligned} \tag{9}$$

with yet unspecified kinetic parameters $k,\beta,l$. An additional parameter $m$ is introduced, which scales the source and that will prove to be useful for some choices of kinetic parameters.
The kinetic parameters for an ADS can be chosen differently depending on the situation that is considered. The point genetic equations have a much broader range of validity than any point kinetic approach. The two approximations of the kinetic parameters, labeled 'eff' and 'src' are introduced in Table 1.

**Table 1. Kinetic parameters and source scaling (*m*) for two different point kinetic approximations of an ADS.**

| Approximation | $k$ | $l$ | $\beta$ | $m$ |
|---|---|---|---|---|
| 'eff' | $k_{eff}$ | $l_{eff}$ | $\beta_{eff}$ | $\Phi^*$ |
| 'src' | $k_s$ | $l_s$ | $\beta_s$ | 1 |

The 'eff' approximation takes the kinetic parameters from the fundamental mode and is accurate in describing the long-term behavior (α-decay) when the source is off and the behavior is dominated by the fundamental mode. Without choosing *m* as the source importance[2,4], these kinetic parameters do not conserve the total multiplication of the source (see Appendix A.3). Comparing with the point genetic equations, the scaling of the source essentially assumes that all generations before the fundamental mode contribute instantaneously. The time a neutron spends in the system is therefore not conserved, although the total number of neutrons that is produced by the source is conserved.

The 'src' approximation takes the kinetic parameters from the *integral* behavior of the system in response to the source (Equations (5),(6) and (7)). This approximation is accurate on longer time scales when the source is on. It does not conserve the long-term behavior when the source is off. No problems arise with the time a neutron spends in the system, nor with the multiplication of the source, since the choice of parameters conserves the integral behavior. An important drawback of this approximation is that the multiplication factor $k_s$ cannot be assigned to a supercritical system.

Both approximations have a specific range of validity and it will be instructive to compare both with the point genetic equations in a number of different situations (see Section 3.2 and 3.3).

## 3 Results

The calculation of the kinetic parameters for the point genetic equations is not straightforward and will require some additional effort in the near future. Currently we cannot compare the point genetic parameters with a realistic reference calculation. However, we will apply the theory using realistic, but fictive, kinetic parameters and show that the point genetic equations may be very useful in describing the behavior of an ADS.
The theory is applied to different instructive situations, and it is then compared to predictions by the conventional point kinetic equations with the approximations (Table 1 and Equation (9)) that describe different aspects of the behavior of an ADS. We finalize by considering situations typical to ADS.

Unless specified otherwise, the point genetic equations are used with $i_F$=4 and with the following basic set of kinetic parameters for the prompt neutrons:

$$[k_1^P, k_2^P, k_3^P, k_4^P] = [1.7, 0.99, 0.97, 0.96],$$
$$[l_1^P, l_2^P, l_3^P, l_4^P] = [1\cdot 10^{-6}, 2\cdot 10^{-6}, 3\cdot 10^{-6}, 3\cdot 10^{-6}].$$

The multiplication associated to the first generation is very high, indicating that the neutrons produced by the external source are efficient in producing fissions. Subsequent generations show a decreased multiplication. The presence of a reflector, which generally increases the lifetime, is modeled by a shorter lifetime for the first generation, since this generation is influenced less by the reflector.

For the delayed neutrons we usually make the unlikely assumption that they have the same kinetic parameters as the prompt neutrons. Moreover, to increase the influence of the delayed neutrons on shorter time scales, we assume that:

$$[\beta_1, \beta_2, \beta_3, \beta_4] = [0.1, 0.1, 0.1, 0.1],$$
$$[\lambda_1, \lambda_2, \lambda_3, \lambda_4] = [3\cdot 10^4, 3\cdot 10^4, 3\cdot 10^4, 3\cdot 10^4].$$

This behavior of the delayed neutron is somewhat unrealistic, but it simplifies the calculations and the comparison.

Several different types of the behavior of the source are studied. In all cases the source is scaled such that the number of neutrons produced in the time interval considered is of the order one.

When plotting the source (usually plotted together with the fission rate) the source is scaled such that the maximum of the source corresponds to the maximum of the fission rate as predicted by the point genetic equations.

### 3.1 Example of behavior of generations and detectors

In Figure 2 the number of neutrons of the different generations is shown for a square source of neutrons activated at t=0 and has a duration of $5 \cdot 10^{-6}$ s. Delayed neutrons are not taken into account. The figure shows that the first generation of neutrons quickly builds up and reaches an asymptotic behavior, just around the time that the source is shut off. Subsequent generations respond much slower to the change in the source strength. The fundamental mode dominates after roughly $2 \cdot 10^{-5}$ s.

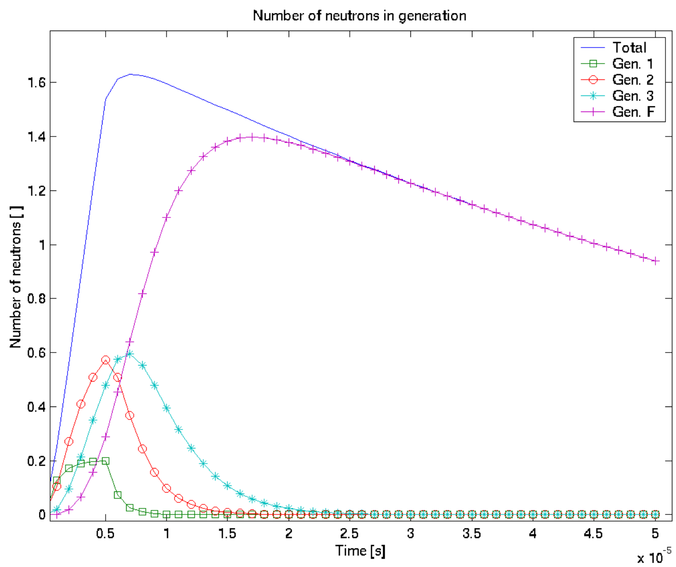


**Figure 2. The change of the number of neutrons in the different generations to a pulse of neutrons. The source is active in the time interval [0-5·10$^{-6}$ ] s. Delayed neutrons are not taken into account.**

The point genetic equations are also able to describe the response of detectors, taking into account each generation (Section 2.4.2). We consider two different detectors. The so-called fuel detector is located in the fictive fuel region close to the target. It is influenced strongly by the first generations. Subsequent generations influence this detector less and less, because the neutrons tend to spread out spatially over the system. The reflector detector is located in a fictive reflector region, far from the target. This detector mainly sees neutron from the later generations, since the neutrons in the first generations remain close to the target region. Both types of detector behavior can be modeled by choosing the generation dependent detector response as:

$$[r_1^{\text{Fuel}}, r_2^{\text{Fuel}}, r_3^{\text{Fuel}}, r_4^{\text{Fuel}}] = [1.2, 1.0, 0.5, 0.5]$$
$$[r_1^{\text{Reflector}}, r_2^{\text{Reflector}}, r_3^{\text{Reflector}}, r_4^{\text{Reflector}}] = [0.1, 0.2, 0.3, 0.3]$$

The response of both detectors, shown in Figure 3, differs strongly both from each other and from the behavior of the total neutron population:

- Fuel detector: a quick increase following the source, until a sharp peak occurs when the source is shut off. This is followed by rapid decrease that smoothly approaches the asymptotic behavior.
- Reflector: a slow and more smooth increase that reaches a maximum value at a moment that is delayed with respect to the moment at which the source was shut-off. The peak is almost instantly followed by the asymptotic behavior.

This behavior predicted by the point genetic equations is certainly not unrealistic, in fact, it strongly resembles the experimental trends observed in the European 5th Framework program MUSE[7][8]. This is by no means a final conclusion, since we have used fictive kinetic and detector parameters, but it gives confidence to see that the point genetic equation are able to describe these characteristics, whereas conventional point kinetic approximations are not.

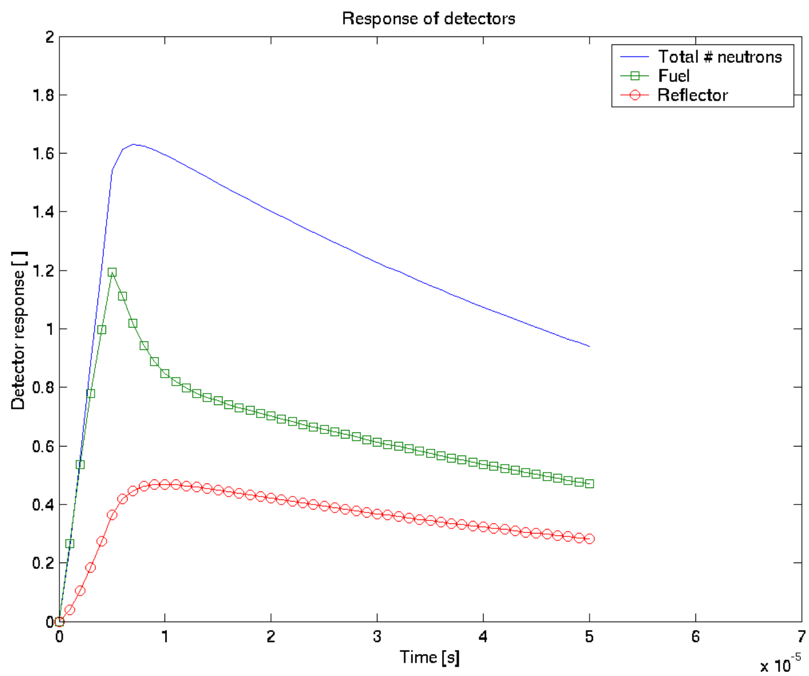


**Figure 3. Response of two different detectors, one located in a fictive fuel region near the target and one in a fictive reflector region. The source is constant in the interval [0-5·10$^{-6}$ ] s.**

## 3.2 Comparison of point kinetic equations

In Section 2.5.2 we discussed two point kinetic approximations (Table1 and Equation (9)): firstly the approximation named “eff”, which uses the usual ‘effective’ kinetic parameters with a source scaled by the source importance, and secondly the approximation named “src”, which uses the integral kinetic parameters for an external source (Equations (5),(6) and (7)).

As discussed in Section 2.5.2, both approximations are able to describe a certain aspect of the behavior of an ADS and in this section we compare both approximations with the predictions of the point genetic equations.
With the choice of kinetic parameters, we have restricted ourselves to realistic situations. It is easy to show that both approximations break down in unrealistic situations. An example of such a situation is a system for which $\beta_{eff}$=0, but $\beta_s$≠0. We consider a square source of neutrons with a duration of $1\cdot10^{-3}$s and include delayed neutrons. The fission rate [$s^{-1}$] and the total number of neutrons in the system are shown in Figure 4 for the point genetic equations and both kinetic approximations.
The results show that the 'eff' approximation predicts the fission rate really well, but fails to predict the total number of neutrons in the system. The 'src' approximation shows a larger deviation and is only accurate for longer time scales with the source on. The 'src' approximation fails on short time scales and whenever the source is off.

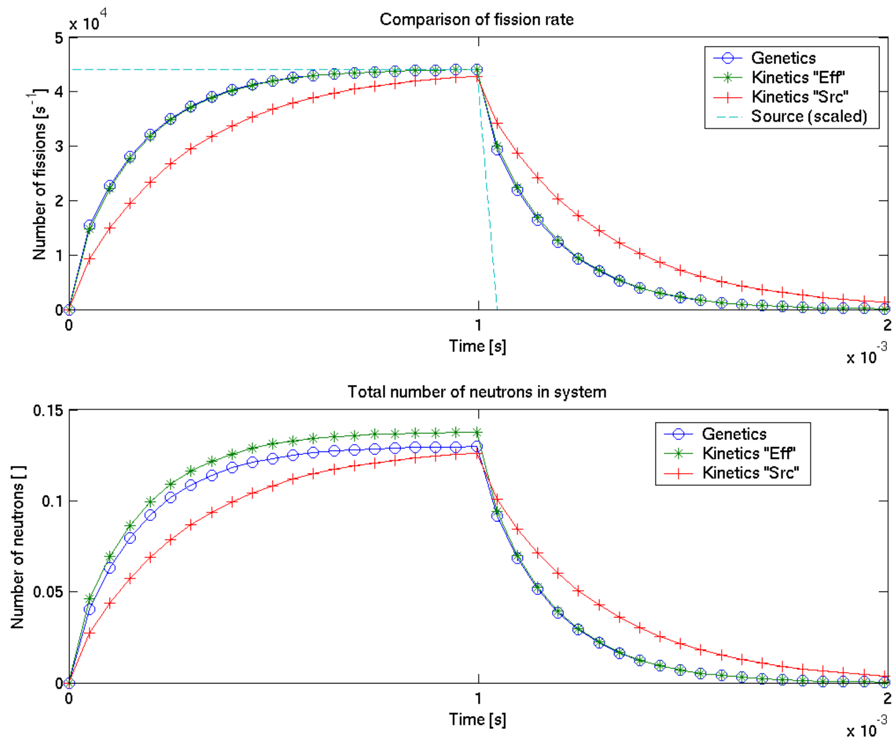


**Figure 4. Fission rate (above) and the number of neutrons (below) predicted by the point genetic equations and compared to the two point kinetic equations with approximated kinetic parameters. The constant source of neutrons is shut off after $1\cdot10^{-3}$ s.**

### 3.3 Application to typical situations for an ADS

We now apply the point genetic equations to a number of different situations that are typical to an ADS system. The delayed neutrons have been included for all simulations and we include the results for the two kinetic approximations for comparison. Note that the results for the genetic equations and the kinetic "eff" approximation are usually difficult to distinguish.

#### 3.3.1 Complex source variations

We consider two kinds of behavior of the external source that are typical for an ADS: a source with periodic variations following a (co)sine and a source for which the strength is instantly doubled. The results are shown in Figure 5.

For the periodic source the fissions rate follows the source variations with a delay of the order of $1{\cdot}10^{-4}$ s (about 100 times the prompt neutron lifetime). The fission rate reaches a stable average value after $1{\cdot}10^{-3}$ s. It is interesting to note that the difference with kinetics "eff" approximations are larger then observed before. This is caused by the strong time dependency of the source.

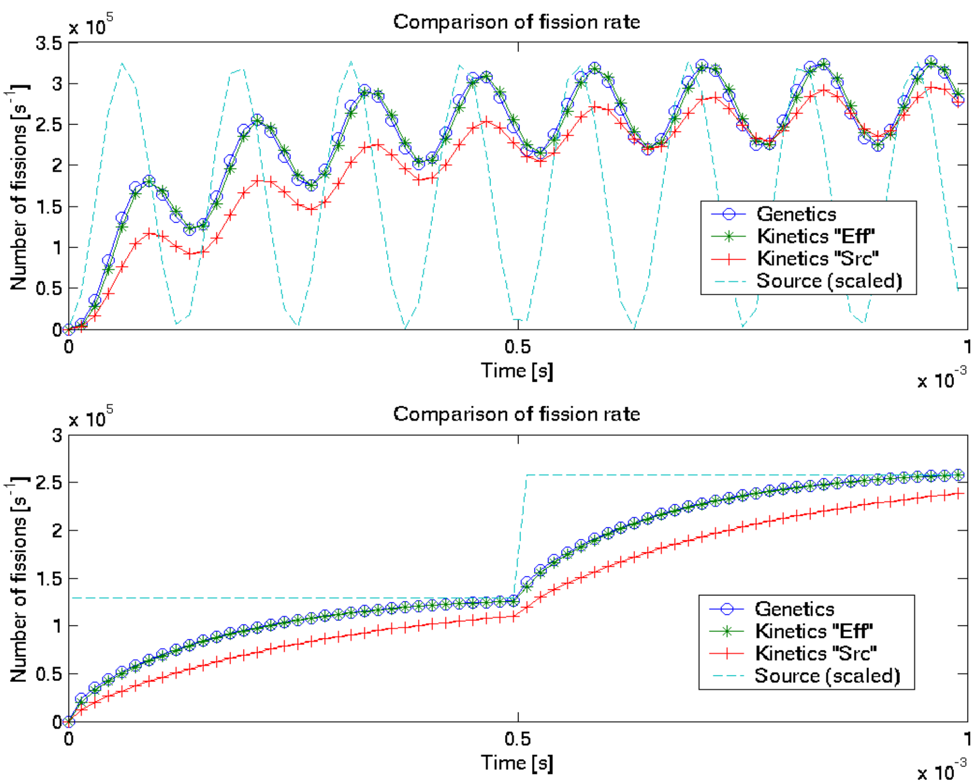


**Figure 5. The fission rate for two different kinds of behavior of the source. Above a source that varies periodically according to a cosine with period $1.25{\cdot}10^{-4}$ s. Below: a constant source the strength of which is increased by a factor 2 at $t=5{\cdot}10^{-4}$ s.**

### 3.3.2 Changes in multiplication

An interesting and important incident situation is one in which the reactivity, or multiplication, of the system changes. The point genetic equations consist of a number of multiplication factors each of which can be changed independently, which allows the point genetic equations to simulate specific 'ADS' incidents in a greater detail.
The source is constant and we consider two different changes of the multiplication:

1. The spallation neutrons suddenly become less efficient in producing fissions ($k_1$ decreases from 1.7 to 1. 1 at $t=5\cdot10^{-4}$ s), by for instance a loss of the target material, and
2. The multiplication of the fundamental mode is changed ($k_{eff}$ increases from 0.96 to 1.01 at $t=5\cdot10^{-4}$ s) by a change in the configuration (e.g. withdrawal of control rods).

The results for both situations are shown in Figure 6. The point genetic equations and kinetic "eff" approximations agree reasonably well. For the second situation the results are not shown, because the kinetic "src" approximations are not valid for a supercritical system.

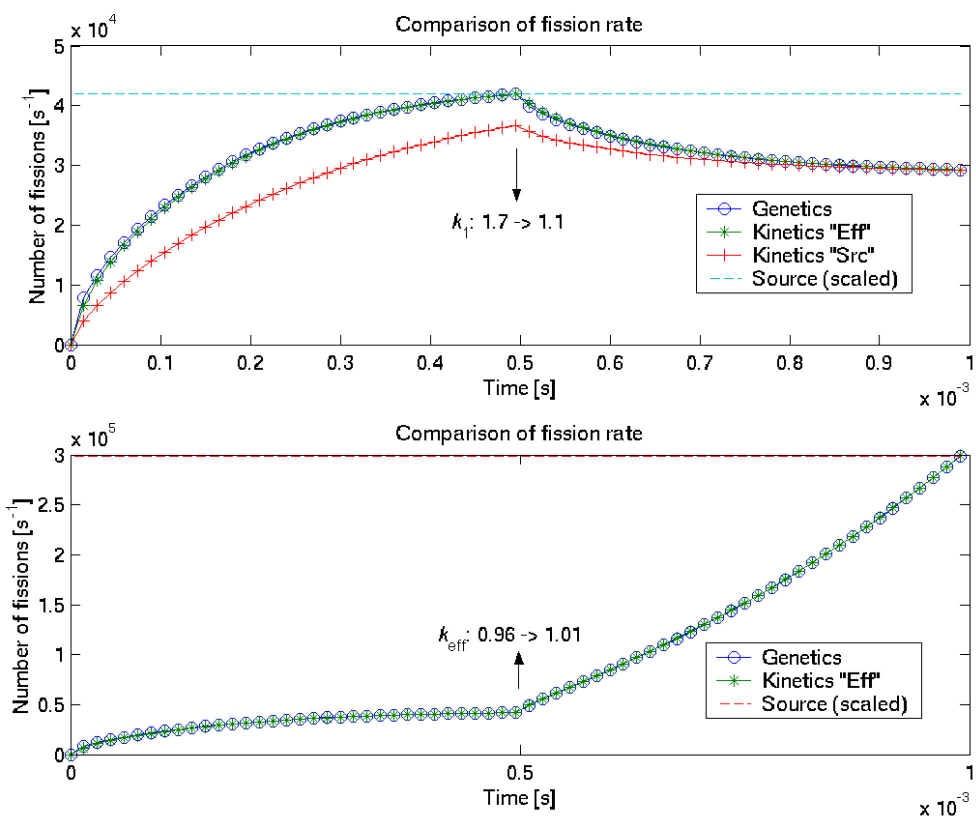


**Figure 6. Two situations in which the multiplication is changed at $t=5\cdot10^{-4}$ s. Above: a decrease of the multiplication in the first generation. Below: an increase in the effective multiplication factor.**

## 4. Discussion

With the neutron generation view as background, a coupled set of equations has been derived, which describe the average behavior of the number of neutrons and the fission source produced by each individual generation. Together these so-called point genetic equations describe the whole system in an integral sense. A set of kinetic parameters is representative for each generation. These may differ strongly from each other for a typical ADS, thus showing the added value of the point genetic equations, when compared to the kinetic equation, which is characterized by a single set of kinetic parameters.

The behavior of the system, especially the fission chain, can be characterized when the point genetic equations are averaged over time. In doing this, specific kinetic parameters $k_s$ and $\beta_s$ are obtained, which characterize the entire fission chain. Other characteristics of the system, like the neutron fluence and the response of detectors can also be described by the point genetic equations.

In principle the chain of generations created by the external source is infinite, but this chain can be written in a closed form, by assuming that for some generation the subsequent generations behave as the fundamental mode. This termination of the fission link can be used to derive the usual point kinetic equation without an external source, provided we have appropriate initial conditions.
A general formulation of the point kinetic equations, including an external source, can be constructed using different approximations. In this paper we considered two different approximations. The first approximation ("eff") states that the system behaves like the fundamental mode, which requires a scaling of the source by the source importance. A second approximation ("src") requires that the time-integrated behavior be conserved. Both approximations result in inaccuracies that have been investigated with simulations.

The calculation of the kinetic parameters used in the point genetic equations, which represent the behavior of each generation, is feasible but will require some additional effort in the near future. However, by choosing realistic kinetic parameters, we demonstrated the use and the value of the point genetic equations, when compared to the two kinetic approximations.
More specifically, we looked at the behavior of each neutron generation after a short pulse of source neutrons. For this source we also investigated the response of two detectors: one in a fuel region near the target and one in a reflector region far away from the target were considered. As expected, the response of both detectors is different and characteristic for their location in the system.
The two different point kinetic approximations were compared with the point genetic equations in a number of different situations, including some typical situations for an ADS. The kinetic "eff" approximation performs best when considering the total number of fissions. It is accurate in almost all situations, but shows minor deviations on time scales of the order of 10 to 100 times the prompt neutron lifetime. However, the kinetic "eff" approximation is unable to describe the total number of neutrons in the system accurately. The kinetic "src" approximation does conserve the time-integrated behavior of both the number of fissions and the number of neutrons, but only on long time scales. It is unable to describe the behavior on smaller time scales. More

important, the kinetic “src” approximation cannot be used for super critical systems, which makes its use questionable for transient analysis.

Finalizing, we feel that the point genetic equations, with a limited number of generations, are a useful set of equations that describe the integral behavior of an ADS in far more and greater detail than any point kinetic equation can do. The proposed set of equations is relatively easy and quick solved numerically and can be used for coupled neutronic and thermal hydraulic transient analysis.

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## APPENDIX A

### A.1 Time dependent behavior

Probably the most stringent assumption used to derive the point genetic equations is that the kinetic parameters do not depend on time within small time scales. This is also an assumption necessary to derive the point kinetic equations. Here we will discuss this assumption for the multiplication factor and generalize the discussion for other kinetic parameters and for the parameters describing the behavior of detectors.

The multiplication factor given by:

$$k(t) = \frac{\langle \nu \Sigma_F \varphi \rangle_V}{\langle (\mathbf{E} + \mathbf{A}) \varphi \rangle_V},$$

depends mainly on the ‘shape’ of the flux and this shape may change strongly and quickly in a number of different situations. It is safe to say that these changes of shape are inherent to an ADS, and we therefore have to discuss them.

The strongest variation in the shape of the flux is observed when all neutrons start their life in a generation at the same moment. This is an unlikely situation, even for the neutrons in the first generation that are produced by the external source. Nevertheless, for this δ(t) source, we find, with the help of (11), that in less than 5 neutron lifetimes only 1% of the initial population remains. Hence the time scale for the change of the shape of the flux is small compared to other time scales involved and we can take this effect into account by an appropriate averaging:

$$\bar{k}(t) = \frac{\int_0^L \langle \nu\Sigma_F \varphi \rangle_V \, dt'}{\int_0^L \langle (\mathbf{E}+\mathbf{A})\varphi \rangle_V \, dt'},$$

with $L$ a sufficient number of neutron life times. The averaging is not performed for *k(t)*, but for the numerator and denominator separately, because it is derived from a balance equation[5]. In most realistic situations the moment in time at which neutrons start in a certain generation shows a variation much larger than the neutron lifetime. The averaging is therefore acceptable.

Variations that occur on time scales larger than '*L*' can be handled by the point genetic equations and we therefore assume that the averaging has been performed in all cases.

## A.2 Solutions of the point genetic equations

### A.2.1 Formal solution

To obtain formal solutions of the point genetic equations, we assume that the kinetic parameters are constant. The point genetic equations can be rewritten in the general form:

$$\frac{d}{dt}\mathbf{N} = \mathbf{AN} + \mathbf{S} \ ,$$

with **A** a matrix and **S** a vector. The formal solution is:

$$\mathbf{N}(t) = \mathbf{Q}^{-1}\left[ \exp(\mathbf{D}(\boldsymbol{\omega} t)) \int_{-\infty}^{t} \exp(\mathbf{D}(-\boldsymbol{\omega} t'))\mathbf{Q} \ \mathbf{S}(t')\mathbf{Q}^{-1} dt' \right]\mathbf{Q} \ ,$$

with a matrix **Q** exists such that:

$$\mathbf{QAQ}^{-1} = \mathbf{D}(\boldsymbol{\omega}) \, ,$$

with **D**(..) a diagonal matrix with the entries of the vector **ω**. Both **Q** and **ω** can be determined by straightforward mathematics.
Another formal solution is found by direct integration of the differential equations:

$$N_i^{\mathrm{X}}(t) = \exp(t/l_i^{\mathrm{X}}) \int_{-\infty}^{t} \exp(-t'/l_i^{\mathrm{X}}) S_i^{\mathrm{X}}(t') dt' = \int_{-\infty}^{t} \exp([t-t']/l_i^{\mathrm{X}}) S_i^{\mathrm{X}}(t') dt'. \qquad (10)$$

### A.2.2 Convolution

Convolution is a well-known property and can also be used for the point genetic equations. From equation [A.1], we find that the solution for $S_i^{\mathrm{X}}(t) = \delta(t)$ is given by:

$$N_i^{\delta X}(t) = \theta(t)\exp(-t/l_i^{\mathrm{X}}) \, , \qquad (11)$$

with $\theta(t)$ the step function (($\theta(t)=0$, $t<0$ and $\theta(t)=1$, $t\geq 0$).

This solution is present in the formal solution (14), which may be rewritten to obtain a convolution integral:

$$N_i^{X}(t) = \int_{-\infty}^{\infty} S_i^{X}(t') N_i^{\delta X}(t'-t)dt' = \int_{-\infty}^{t} S_i^{X}(t') N_i^{\delta X}(t'-t)dt' \ . \tag{12}$$

This expression is useful, because it essentially removes all time dependency introduced by the source. By integrating over time, it is easy to show that:

$$\int_{-\infty}^{\infty} N_i^{X}(t)dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_i^{X}(t') N_i^{\delta X}(t'-t)dt'dt = l_i^X \int_{-\infty}^{\infty} S_i^{X}(t)dt \ , \tag{13}$$

which is used to determine the characteristics of the system when averaged over time.

### A.3 Relation with effective multiplication [5]

In a system in fundamental mode each generation behaves the same, and we have $k_F^{av}=k_{\text{eff}}$. For a subcritical system in fundamental mode equation (5) yields: $k_s=k_{\text{eff}}$. In general the neutrons in a subcritical system driven by an external source do not follow (entirely) the fundamental mode and there is a difference between $k_s$ and $k_{\text{eff}}$. This difference is expressed by the so-called source importance Φ*[2][4]. In terms of the total multiplication, given by:

$$M = \frac{k}{1-k} \ ,$$

the source importance is given by:

$$\Phi^* = \frac{M_s}{M_{\text{eff}}} \ .$$

The source importance equals the number of fissions produced by a neutron from the external source relative to the number of fissions produced by a 'source' neutron in the fundamental mode.