

ON APPLICATION OF QUASISTATIC AND POINT-KINETICS SCHEMES FOR SUBCRITICAL SYSTEMS WITH EXTERNAL NEUTRON SOURCE

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ABSTRACT

Quasistatic and point-kinetics schemes provide an efficient way to simulate transients in conventional nuclear reactors. They imply utilization of a weighting function that is used to compute integral reactor parameters such as reactivity. Currently, the applicability of these schemes and the choice of the weighting function for modeling transients in subcritical systems with external neutron source are widely discussed. In the paper the direct, quasistatic, and point kinetics methods are compared for particular transients in subcritical systems. The applicability of the quasistatic scheme with a conventional weighting function is demonstrated. It is shown that the point-kinetics method can be extended to improve its accuracy and applied efficiently in the subcritical case with the conventional weighting function under certain conditions.

Key Words: point-kinetics, quasistatic method, adjoint function, accelerator driven systems

1. INTRODUCTION

Space-time factorization schemes, e.g. quasistatic and point-kinetics techniques [1], provide an efficient way - compared to the direct method, i.e. a finite-difference (with respect to time) approximation to the time-dependent neutron transport equation – for simulation of transients in conventional “critical” reactors. That is why an obvious tendency exists: to extend the application of these schemes [2] for analyzing transients in subcritical systems with external neutron source as Accelerator Driven Systems (ADSs) [3]. These systems are of special interest for incinerating/transmuting Pu and minor actinides as they inherently offer a larger flexibility in design and operation and for coping with safety problems. Therefore the performance of traditional and development of new kinetics models for ADSs are widely discussed [4, 5].

Two kinetics models for simulating transients and hypothetical accidents in advanced reactor systems, in particular in ADSs, have been developed at FZK/IKET in cooperation with CEA Cadarache and implemented in the SIMMER [6] code and in the KIN3D [7] module of the VARIANT code [8]. These models and some possible extensions are presented in [5]. In particular, new correction-factor and two-shape schemes were suggested to improve the accuracy of point-kinetics and assess the performance of the quasistatic method.

In this paper the analyses of [5] are developed further. Performance of quasistatic and direct methods for a short-term (compared to the neutron lifetime) transient is compared and convergence of the quasistatic scheme for short-term and long-term transients is demonstrated that gives an additional evidence of the applicability of this scheme for ADS.

Examples are given to demonstrate possible inaccuracies of the point-kinetics approach. The two-shape technique - that may improve the accuracy of point-kinetics - is formulated in a more general way (compared to [5]). This formulation facilitates implementation of the technique in conventional (i.e. developed for critical reactors) reactor analysis codes.

2. MATHEMATICAL FORMULATION

The time-dependent multigroup neutron transport equation (in the following, only one family of delayed neutrons is considered for simplicity, x denotes a space-angular position,

$\Phi, Q, \nu, \chi_p, \chi_d$ are "multigroup" vectors $\chi = \{1 - \beta\} \chi_p + \beta \chi_d$; M and F are operators having their standard meaning [1]: neutron "leakage, absorption, and scattering" and "generation"):

$$\frac{1}{\nu} \frac{\partial \Phi(x,t)}{\partial t} + M(t)\Phi(x,t) = (1 - \beta) \chi_p F(t)\Phi(x,t) + \chi_d \lambda C(x,t) + Q(x,t), \quad (1)$$

$$\frac{\partial C(x,t)}{\partial t} = \beta F(t)\Phi(x,t) - \lambda C(x,t), \quad (2)$$

can be solved using different (with respect to time) solution schemes. The direct method, as it is implemented in KIN3D, employs a fully implicit finite-difference scheme to Eqs. (1, 2) that transforms the original equations into a sequence of steady-state-like problems. The improved quasistatic scheme, as it is implemented in SIMMER (in KIN3D it is quite similar), factorizes the flux as $\Phi(x,t) = N(t)\psi(x,t)$. The shape equations (time-dependence is omitted hereafter in this section for N, M, F and all point-kinetics parameters):

$$\frac{1}{\nu} \frac{\partial \psi(x,t)}{\partial t} + \frac{1}{\nu N} \frac{dN}{dt} \psi(x,t) + M\psi(x,t) = (1 - \beta) \chi_p F\psi(x,t) + \frac{1}{N} \chi_d \lambda C(x,t) + \frac{1}{N} Q(x,t), \quad (3)$$

$$\frac{\partial C(x,t)}{\partial t} = N\beta F\psi(x,t) - \lambda C(x,t), \quad (4)$$

are solved (by employing the implicit finite-difference scheme) relatively seldom (shape steps) compared to the more frequently solved point-kinetics equations for the amplitude:

$$\frac{dN}{dt} = \left(\frac{\rho - \beta_{eff}}{\Lambda} \right) N + \lambda c + q_{eff}, \quad (5)$$

$$\frac{\partial c}{\partial t} = \frac{\beta_{eff}}{\Lambda} N - \lambda c. \quad (6)$$

Usually the "critical" adjoint flux is employed as a weighting function W for computing the parameters (for their definition, see Eqs. (13 – 17), omit the p and d subscripts) of Eqs. (5, 6):

$$M^*(0)W(x) = \frac{1}{k}(\chi F(0))^* W(x). \quad (7)$$

This weighting function is “optimal” for a “critical” reactor: it reduces errors in reactivity calculations related to shape inaccuracies. Therefore it makes more effective iterative procedures employed in SIMMER and KIN3D to compute the shape-dependent parameters of Eqs. (5, 6) and recalculate the amplitude-dependent shape each new time step. As a consequence, the shape steps of the quasistatic scheme may be significantly larger (e.g. by a factor of 100, see the first case in [7]) compared to the time steps of the direct method to provide similarly accurate results. In ADSs a reaction-rate-independent “optimal” weighting function may not exist due to possible remarkable shape variations; however, accurate results can be obtained with different weightings options if the shape step is small enough [5]. By default, the function W defined by Eq. (7) is used in KIN3D and SIMMER for both critical and subcritical cases (and employed in this paper for all examples). This option gives accurate reactivity values when an initially subcritical system approaches criticality during the transient. In SIMMER, one may optionally take the “alpha” adjoint flux as a weighting function; for fast transients it gives an effect of employing the asymptotic (largest possible) neutron lifetime and therefore the most “inertial” flux amplitude.

In the following we consider short-term and long-term (compared to the neutron lifetime) transients. The mentioned extensions (two-shape) of the point-kinetics scheme may improve the accuracy only for long-term (or slow) transients and therefore are not studied for fast events.

3. DIRECT AND QUASISTATIC METHODS FOR A FAST TRANSIENT

To compare the performance of the quasistatic and direct techniques, we investigate a case for a simplified 2D XY model of a subcritical core (see Fig. 1) with fast neutron spectrum for which some experiments and calculations were performed in the past [9].

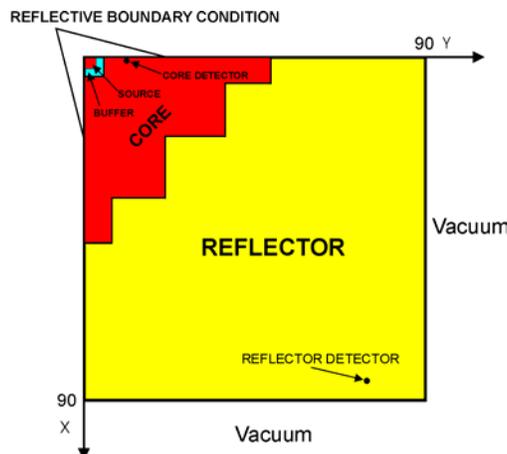


Figure 1. 2D XY model for fast transient simulations

We modeled core and reflector detector (see Fig. 1 for their location) rates to an almost prompt (within $0.1 \mu\text{s}$) switching-off the external (15 MeV neutrons) source, the reactivity being about

-4500 pcm, neutron generation time at steady-state about $0.65 \mu\text{s}$, β_{eff} about 325 pcm. The detector cross-sections were the fission cross-sections of U-235. The spatial kinetics effects in this case are related to the first term in Eq. (1) that takes into account the “flux inertia”, which is different at different spatial locations.

The quasistatic and direct methods were employed with shape (time) steps of 10, 1, and $0.1 \mu\text{s}$.

The results for the core and reflector detectors are shown in Figs. 2 and 3. In addition, point-kinetics curves (which are certainly similar for both detectors) are shown.

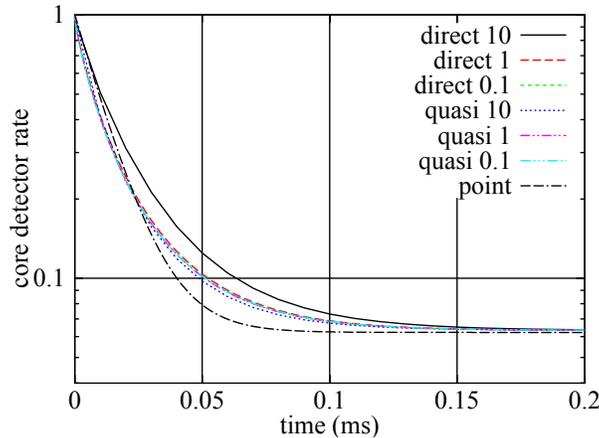


Figure 2. Source transient: relative core detector rate

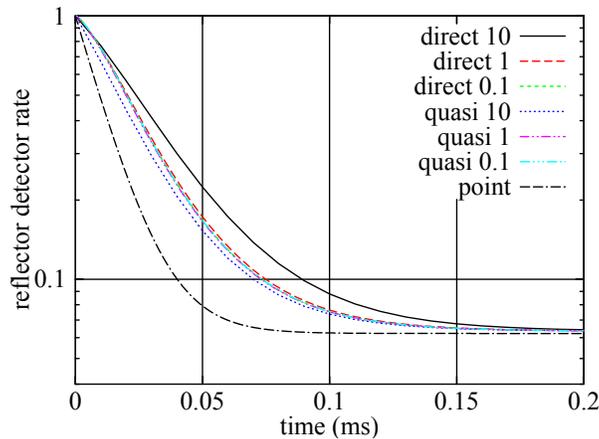


Figure 3. Source transient: relative reflector detector rate

The following conclusions can be drawn from Figs. 2 and 3.

- The strong spatial kinetics effects should be taken into account for this transient.
- The direct and quasistatic schemes do converge to the same limit.
- In the general case, the quasistatic scheme shows a better performance.
- In some particular cases (e.g. just after the switching off the source, see Fig. 3) the direct method may have advantages if the point-kinetics solution is too inaccurate (if the “alpha”

weighting were used, the point-kinetics amplitude near $t=0$ would be more accurate and the quasistatic scheme might perform better in this case).

It is clear that space-time factorization usually can be (at least) as accurate as the direct method provided that all time steps are sufficiently small and a “reasonable” (e.g. “critical” adjoint or “alpha” adjoint or flat) weighting function is used. To prove it, let us just consider a case when the amplitude is constant (that is usually a more poor approximation than a solution of the point-kinetics equation). Then the shape and flux equations are just the same.

4. QUASISTATIC AND POINT-KINETICS FOR A SLOW TRANSIENT

Fast transients are related to beam-trips (at a subcritical state) and may pose challenging calculation problems, but their relevance for accident analyses is often small (as we consider mainly the fast spectrum ADS): the energy deposition in a subcritical system may not usually change significantly during a very short time. Therefore, relatively slow (when in the subcritical domain) reactivity (i.e. related to material perturbations) transients are of main interest for safety analyses. We will investigate a transient of this type for an ADS model shown in Fig. 4. During the transient the flux shape may change significantly as reactivity varies (see Fig. 5).

For this model we simulated a transient caused by heating the fuel (U/Th) from 900K to 3300K and decreasing the coolant (lead) density by 5% within a time interval of 1s, β_{eff} being about 345 pcm. Two options were considered: an initially critical system and an ADS with initial reactivity of about -8%; the point-kinetics and the quasistatic schemes being employed.

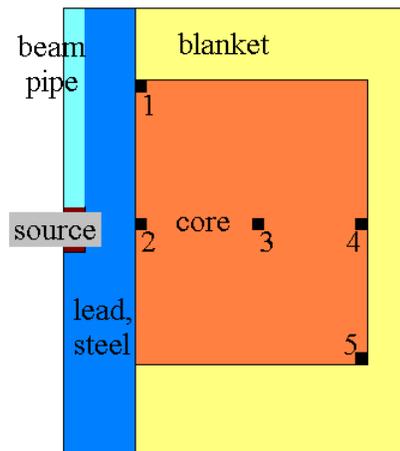


Figure 4. ADS model, locations of nodes 1 – 5 are shown

The results of the simulations for the critical case are shown in Fig. 6. The curves computed by the quasistatic schemes show the relative power variations (hereafter without taking into account decay heat) for the total reactor, and for nodes 2 and 5 (see Fig. 4 for their locations). In this case the spatial kinetics treatment affects mainly the transient reactivity; the power shape is almost

constant during the transient, therefore the total and nodal power traces are almost the same. Two point-kinetics cases are presented: (1) a standard approach, i.e. based on the reactivities computed by employing the first order perturbation theory (PT) and (2) an approach based on employing the exact PT, that means that the reactivity variations during the transient were multiplied by a certain value so that the exact value of reactivity (i.e. computed with the perturbed flux) at $t=1$ is met, this approach being (in this case) similar to the quasistatic option with a shape step of 1. One may see that the main problem here is to define the transient reactivity correctly. The spatial kinetics does it automatically; in the point-kinetics case corrections for transient reactivity should be employed.

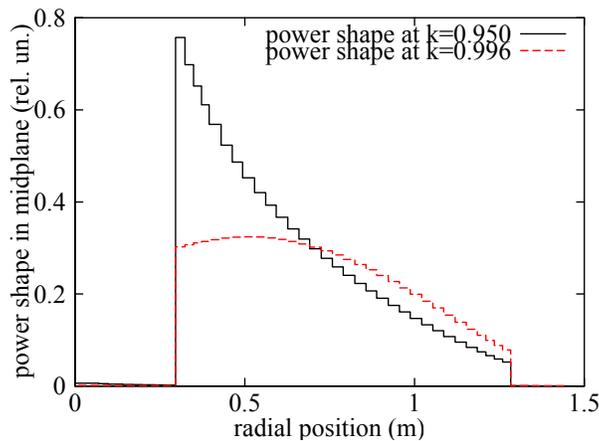


Figure 5. Power profile in the ADS at different subcriticality levels

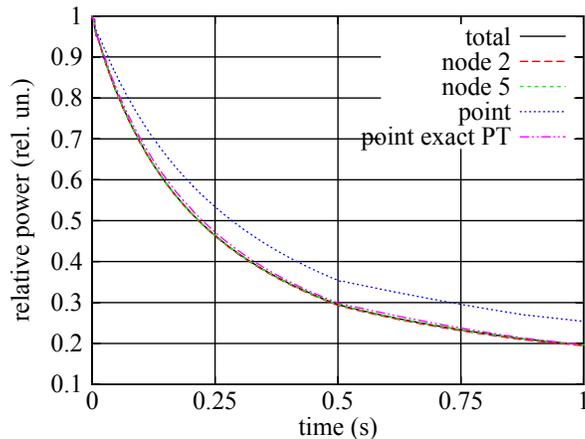


Figure 6. Quasistatic and point-kinetics: power traces in the initially critical reactor

In Fig. 7 results of similar computations for the subcritical case are shown. Due to significant shape changes related to reactivity variation (see also Fig. 5), exact PT reactivity corrections cannot bring the point-kinetics results sufficiently close to the accurate one's. The reason is that it is just impossible to approximate several different curves (total, nodes 2 and 5) by a single curve accurately.

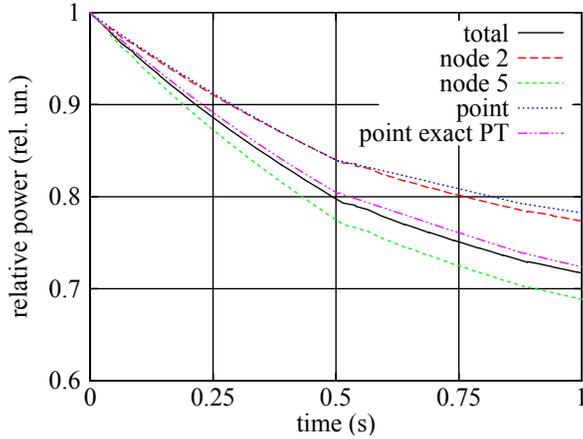


Figure 7. Quasistatic (small shape steps) and point-kinetics: power traces in the ADS

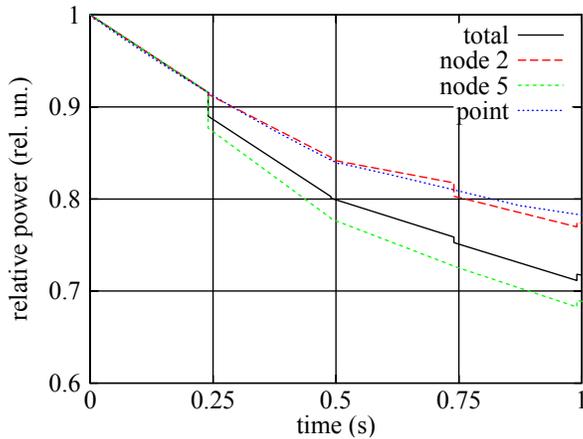


Figure 8. Quasistatic (large shape steps) and point-kinetics: power traces in the ADS

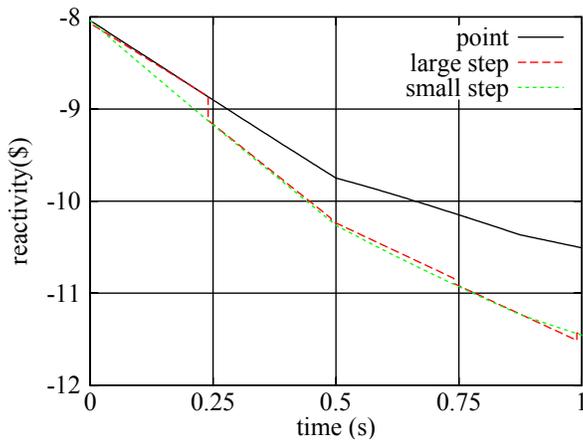


Figure 9. Quasistatic (large and small steps) and point-kinetics: reactivity in the ADS

In Fig. 8, the point and spatial kinetics results are shown for a rather coarse shape step of 0.25s. Due to employing the point-kinetics approach for computations within each time subinterval, steps in power traces are visible at the time points when the shape is recalculated. These steps and corresponding inaccuracies in power are certainly lower (thus invisible in Fig. 7) when the shape is recalculated more often. Comparison of reactivity curves related to the point-kinetics, coarse and fine time mesh quasistatic options is done in Fig. 9.

5. TWO-SHAPES MODEL FOR SLOW TRANSIENTS

The basis concept consists in splitting the flux into the “prompt” and “delayed” components,

$$\Phi(x, t) = \Phi_p(x, t) + \Phi_d(x, t), \quad (8)$$

so that Eq. (1) transforms into:

$$\frac{1}{v} \frac{\partial \Phi_p}{\partial t} + M(t) \Phi_p(x, t) = (1 - \beta) \chi_p F(t) \Phi_p(x, t) + Q(x, t), \quad (9)$$

$$\frac{1}{v} \frac{\partial \Phi_d}{\partial t} + M(t) \Phi_d(x, t) = (1 - \beta) \chi_p F(t) \Phi_d(x, t) + \chi_d \lambda C(x, t). \quad (10)$$

One may note that for a slow transient the first term of Eq. (9) can be neglected and therefore the prompt flux component can be computed reasonably accurately by a static code. On the other hand, Eq. (10) is similar to that of the critical reactor. Therefore, the delayed and “critical” flux shapes are quite close. This approximation may not be very accurate for a rather low k-eff value (e.g. 0.9), but then the contribution of the delayed flux component (to the total flux) is very low. Let us assume that W is defined by Eq. (7) and

$$\Phi_d(x, t) \approx N_d(t) \psi_d(x), \quad (11)$$

$$\Phi_p(x, t) = N_p(t) \psi_p(x, t), \quad (12)$$

$$F(t) = \langle W, \chi F \psi_d(x) \rangle = \langle W, \chi F \psi_p(x, t) \rangle, \quad (13)$$

$$\rho(t) = \frac{1}{F} \langle W, (\chi F - M) \psi_d(x) \rangle \approx \frac{1}{F} \langle W, (\chi F - M) \psi_p(x, t) \rangle, \quad (14)$$

$$\beta_{eff}(t) = \frac{1}{F} \langle W, \beta \chi_d F \psi_d(x) \rangle \approx \frac{1}{F} \langle W, \beta \chi_d F \psi_p(x, t) \rangle, \quad (15)$$

$$\Lambda(t) = \frac{1}{F} \langle W, \frac{1}{v} F \psi_d(x) \rangle \approx \frac{1}{F} \langle W, \frac{1}{v} F \psi_p(x, t) \rangle, \quad (16)$$

$$q_{eff}(t) = \frac{1}{F\Lambda} \langle W, Q \rangle. \quad (17)$$

With some additional assumptions (usual for the point-kinetics scheme) and assigning

$$N(t) = N_p(t) + N_d(t), \quad (18)$$

one may get Eqs. (5, 6) for the total amplitude and the following equation for the prompt one:

$$\frac{dN_p}{dt} = \left(\frac{\rho - \beta_{eff}}{\Lambda} \right) N_p + q_{eff}. \quad (19)$$

Then (see Eqs. (9 – 12))

$$\Phi(x, t) = N_p(t)\psi_p(x, \rho(t)) + (N(t) - N_p(t))\psi_d(x). \quad (20)$$

Eq. (20) gives rise to the two-shape scheme. At a subcritical state one may (by assuming $N(0)=1$ and taking into account Eqs. (5, 6) at $t=0$) approximate Eq. (19):

$$N_p(t) = \frac{\rho(0)q_{eff}(t)}{(\rho(t) - \beta_{eff})q_{eff}(0)}. \quad (21)$$

Under critical or overcritical conditions one may set $\psi = \psi_p = \psi_d$ in Eq. (20), therefore the conventional space-time factorization is employed, the prompt amplitude being not used.

In certain cases the prompt and delayed shapes employed in the two-shape scheme can be obtained by interpolation from a precomputed table. This table may include results of steady-state flux calculations for a quasi-critical (k-eff) and several subcritical (with external source) models (which may relate to possible reactor perturbed states):

$$M(0)\Phi_0(x) = \frac{1}{k} \chi F(0)\Phi_0(x), \quad (22)$$

$$M(\rho_i)\Phi_i(x, \rho_i) = \chi F(\rho_i)\Phi_i(x, \rho_i) + Q(x, 0). \quad (24)$$

which are normalized (for $i=0, 1, \dots, N$) as follows to meet Eq. (13):

$$\psi_i = c \frac{\Phi_i}{\langle W, \chi F \Phi_i \rangle}, \quad (25)$$

where the normalization constant c is chosen in such way that the “steady-state” shape is equal to the flux at $t=0$. The delayed shape (and the total flux shape at critical and overcritical states) is approximated by ψ_0 . For $\rho < 0$ the prompt shapes are computed from ψ_i as:

$$\psi_p(\rho_i) = \frac{\rho_i - \beta_{eff}}{\rho_i} \psi_i + \frac{\beta_{eff}}{\rho_i} \psi_0. \quad (26)$$

As ψ_d and $\psi_p(\rho_i)$ are computed, they can be used during a transient simulation to compute (by interpolation) the shapes employed in Eq. (20). It is clear that the choice of a weighting function is of minor importance for computing the prompt flux amplitude. On the other hand, “critical weighting” is important to compute the effective precursor concentrations and delayed flux amplitude properly (to understand it, one should take into account Eq. (14)).

Thus, an extension of a point-kinetics based code for ADS application can be done in a quite simple manner. In addition to the computed in the traditional manner - i.e. with the conventional “critical” direct (see Eq. (14)) and adjoint fluxes - table of reactivity coefficients and power shape, several calculations with external source at different subcritical states (see Eq. (24)) should be performed to get the corresponding power profiles. Then the power shapes should be processed (Eqs. (25, 26)) to form a table. During the transient Eq. (20) should be used instead of $\Phi(x,t) = N(t)\psi(x,t)$. The prompt shapes should be interpolated, on the basis of the precomputed table, the prompt amplitude can be calculated by either Eq. (19) or Eq. (21). If the reactivity is non-negative, the prompt shape is set equal to the delayed shape.

This technique was employed for the transient analyzed in the previous section. The steady-state power shape calculations were done for the k-eff problem and for two subcritical states (related to $t=0$ and $t=1$). The reactivity during the transient was “corrected” (i.e. computed by employing the exact PT mode). The obtained relative power traces are compared in Fig. 10 with spatial kinetics (quasistatic) one’s. One may note that the employment of two shapes improves the results significantly (compared to the traditional point-kinetics approach). Certainly the full agreement with spatial kinetics is not reached because of the approximations involved.

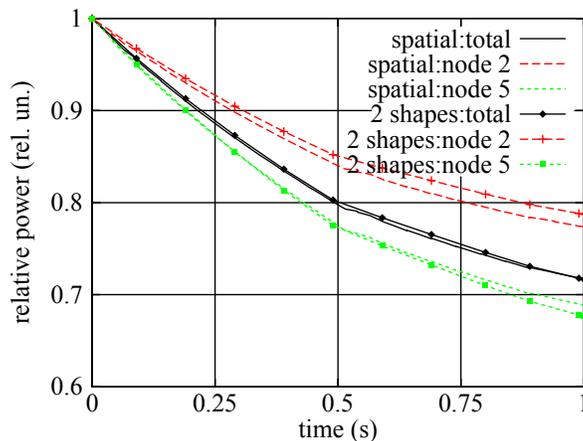


Figure 10. Spatial (quasistatic) and point (two-shape) kinetics power traces in the ADS

6. CONCLUSIONS

We have shown numerically that the quasistatic and direct kinetics options converge to the same limit for a short-term “subcritical” transient. The convergence of the quasistatic method is also demonstrated for a “long-term” reactivity transient. The performance of the quasistatic scheme is acceptable and the scheme is applicable in the general case for ADS analyses. However, the efficiency of the scheme for simulating fast transients in ADS may not be as favorable (compared to the direct method) as one may often observe in critical reactors.

The errors related to the point-kinetics method can be compensated in certain cases for a critical system by reactivity corrections (if the perturbed state is roughly known in advance). Similar reactivity corrections in the subcritical case may not be sufficient because the flux shape may change much stronger. To ameliorate the point-kinetics technique, a two-shape scheme is proposed in the paper. Though it is not exact (as the quasistatic method with a very small shape step), it may improve the point-kinetics accuracy. The proposed formulation makes easy extension of “critical” accident analysis codes by taking into consideration additional (to the “critical” shape) “subcritical” power (or detector rate) shapes and by modifying the way in which the power (or detector rate) should be computed from the point-kinetics amplitude and precomputed shapes.

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