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ON SOME FEATURES OF SPATIAL NEUTRON KINETICS FOR MULTIPLYING SYSTEMS

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ABSTRACT

The paper considers some physical aspects of the neutron space kinetics of critical and source-driven subcritical systems. The possibility of introducing indicators of the spatial nature of neutronic transients is investigated. It is shown theoretically and then proved by numerical examples that the separation of the eigenvalues of the mathematical operator defining the problem can be taken as a good indicator of the importance of space effects in time dependent conditions. To get a good physical insight into the phenomena, paradigmatically simple configurations are considered and, whenever possible, a fully analytical approach is used. The results presented evidence the limits of applicability of classic simplified models for transient analyses, such as for instance point kinetics. In a second part, the paper considers the problem of the choice of the weighting function to be used either for the generation of the kinetic coefficients of point-like models or for the exploitation of quasi-static procedures, analyzing comparatively the effect of different options on the results of transient calculations.

Key Words: Nuclear Reactor Physics, Neutron Kinetics, Space Transients

1. INTRODUCTION

An important problem in applications of reactor physics concerns the determination of the role of neutronic space and energy effects in time-dependent situations. The presence of such effects often limits the applicability of simplified models, such as point kinetics, in the kinetic studies of neutron multiplying systems. In recent times, new problems have been introduced in the physics of accelerator-driven systems. These systems are obviously subcritical and the neutron distribution is heavily dominated by the presence of the source. New physical features can come into play and their simulation may require a completely new approach, with respect to the standard approach adopted to study transient behaviors in reactors departing from an initially critical state [1]. Therefore, models and methods might need to be extended and adapted [2]. For instance, point kinetics for close-to-critical systems is generated using as a reference system a critical reactor in a steady-state configuration and the kinetic parameters are generated utilizing the critical adjoint as a weighting function for the projection operation. Consequently, the point kinetic model is interpreted as describing the evolution of the component of the fundamental eigenfunction of the critical problem. This interpretation is no longer possible for source-driven systems. However, since the system is strongly dominated by the source, the neutron distribution is somewhat more *rigid* than for a system evolving from a critical state, and thus in many situations space and spectrum distortions can become less important. The construction of an appropriate point model requires obviously to refer to a source-driven system. Furthermore, the problem of the choice of the most suitable weighting function comes into play. It is worth to investigate the effect of such a choice for different transient situations. Similar considerations as above apply also in a quasi-static framework.

It looks very important to have indicators of the nature of transients that can be expected when a perturbation is introduced into a nuclear reactor, both starting from a steady-state critical condition or a subcritical source-driven situation. Important works have been performed to analyze the spatial distortion effects with respect to simplified point or two-point models [3][4].

From a mathematical point of view, the eigenvalue spectrum is greatly informative of the intimate nature of the physico-mathematical problem [5]. It has been long well-known that from a purely mathematical point of view the separation between the first two eigenvalues of the operator of a physical model is important in characterizing the system response. This feature is used for the characterization and clarification of many aspects of the neutronic behavior of nuclear reactors [6]. The eigenvalue separation has been experimentally measured in loosely coupled cores and related to spatial effect, such as flux tilts [7][8][9]. The concept has been extensively used for various applications, e.g. to foresee the appearance of spatial effects coming forward in strongly decoupled structures with large optical dimensions in steady-state and transient conditions [10],[11], to investigate the coupling coefficients between zones of the system [12][13] and for the interpretation of noise measurements[14].

It is worth-while to try to investigate in a systematic manner the importance of spatial effects in neutron kinetics. The present paper considers some simplified paradigmatic geometrical and material configurations. Its aim is mainly theoretical; being finalized to get a full insight into the physical phenomena, analytical procedures are employed whenever possible. This approach has proved fruitful also in recent quite general discussions of the basic physics of subcritical reactors [1] and can enlighten the physical features of the problem better than numerical solutions needed when treating realistic configurations. For materially homogeneous systems a well-assessed technique involving a proper eigenfunction expansion of the unknown solution is employed [15]. In the whole paper the diffusion theory model is always assumed.

2. THE BASIC EQUATIONS OF THE MODEL AND THE ANALYTICAL SOLUTION

Let us consider the one-group diffusion model for slab one-dimensional systems including fission delayed emissions and, in the most general case, an external source:

$$\begin{cases} \dfrac{1}{v}\dfrac{\partial \Phi(x,t)}{\partial t} = D\dfrac{\partial^2 \Phi(x,t)}{\partial x^2} - \Sigma_a \Phi(x,t) + (1-\beta)\nu\Sigma_f \Phi(x,t) + \lambda C(x,t) + S(x,t), \\ \dfrac{\partial C(x,t)}{\partial t} = -\lambda C(x,t) + \beta\nu\Sigma_f \Phi(x,t). \end{cases} \tag{1}$$

The one-family assumption for delayed neutrons is not limiting the generality of the conclusions drawn and it is only taken here for formal simplicity. The system is assumed materially homogeneous in its final state. Boundary and initial conditions need to be specified as follows:

$$\begin{array}{l} \Phi(0, t>0) = \Phi(H, t>0) = 0, \\ \Phi(x, t=0) = \Phi_0(x), \\ C(x, t=0) = C_0(x). \end{array} \tag{2}$$

The use of different boundary conditions for the neutron flux is obviously possible without any additional problem.
An exact analytical solution can be obtained making use of the spatial eigenfunctions $\varphi_n(x)$ of the following Helmholtz eigenproblem associated to eigenvalue B_n^2:

$$\begin{array}{l} \dfrac{d^2\varphi_n(x)}{dx^2} = -B_n^2 \varphi_n(x), \\ \varphi_n(0) = \varphi_n(H) = 0. \end{array} \tag{3}$$

The above eigenfunctions constitute a complete orthogonal set and can be normalized. They are mathematically the most suitable basis to treat the problem (1). Hence, the spatial dependences of the solution and of the source are expanded at each time in terms of these eigenfunctions, introducing the infinitely many (unknown) components of the flux $a_n(t)$ and of the delayed neutron precursor concentrations $c_n(t)$, together with the (known) components of the source $s_n(t)$, as:

$$\begin{array}{ll} \Phi(x,t) = \displaystyle\sum_{n=0}^{\infty} a_n(t)\varphi_n(x), & C(x,t) = \displaystyle\sum_{n=0}^{\infty} c_n(t)\varphi_n(x), \\ S(x,t) = \displaystyle\sum_{n=0}^{\infty} s_n(t)\varphi_n(x). & \end{array} \tag{4}$$

The source components can be expressed by a spatial inner product as:

$$s_n(t) = \int_0^H dx S(x,t)\varphi_n(x) \equiv \left(\varphi_n, S(t)\right). \tag{5}$$

Also the initial state for neutrons and precursor can be expanded in terms of the eigenfunctions as:

$$\begin{aligned} \Phi_0(x) &= \sum_{n=0}^{\infty} a_{n0}\varphi_n(x) \equiv \sum_{n=0}^{\infty} (\varphi_n, \Phi_0)\, \varphi_n(x), \\ C_0(x) &= \sum_{n=0}^{\infty} c_{n0}\varphi_n(x) \equiv \sum_{n=0}^{\infty} (\varphi_n, C_0)\, \varphi_n(x). \end{aligned} \tag{6}$$

The expansions (4) are now introduced into the basic equations (1) and then a projection of the equations of the system on each eigenfunction is taken and use is made of the orthogonality property, thus decoupling the spatial modes of the system. It is useful to simplify the mathematics using a matrix notation, and introducing the vectors including the unknown components of the solution for each eigenfunction and the source:

$$|x_n(t)\rangle = \left| \begin{array}{c} a_n(t) \\ c_n(t) \end{array} \right\rangle, \qquad |s_n(t)\rangle = \left| \begin{array}{c} s_n(t) \\ 0 \end{array} \right\rangle, \tag{7}$$

The unknown vector obeys the following system of first-order differential equations:

$$\frac{d}{dt}\,|x_n(t)\rangle = M_n\,|x_n(t)\rangle + |s_n(t)\rangle\,, \tag{8}$$

where

$$M_n = \left(\begin{array}{cc} v\left[(1-\beta)\,\nu\Sigma_f - DB_n^2\right] & v\lambda \\ \beta\nu\Sigma_f & -\lambda \end{array} \right). \tag{9}$$

The initial conditions are given through Eq. (6). The problem (8) can be solved using the direct and adjoint eigenvectors of M_n, namely the solutions of the following eigenproblems:

$$\begin{aligned} M_n\,|u_n\rangle &= \omega_n\,|u_n\rangle \\ \langle u_n|\,M_n &= \omega_n\,\langle u_n| \end{aligned} \tag{10}$$

where the two (real) time-eigenvalues are found through the algebraic inhour equation:

$$\det M_n = 0, \tag{11}$$

which can be analytically solved. It is easily found that:

$$\begin{aligned} |u_n\rangle &= \left| \begin{array}{c} 1 \\ \dfrac{\beta\nu\Sigma_f}{\omega_n+\lambda} \end{array} \right\rangle, \\ \langle u_n| &= \left\langle \begin{array}{cc} 1 & \dfrac{v\lambda}{\omega_n+\lambda} \end{array} \right|, \end{aligned} \tag{12}$$

and, therefore, the full closed form solution takes the following form:

$$|x_n(t)\rangle = \sum_{j=1}^{2} \frac{1}{\left\langle u_n^{(j)} \middle| u_n^{(j)} \right\rangle} \left[\left\langle u_n^{(j)} \right. |x_n(0)\rangle \, e^{\omega_n^{(j)} t} + \int_0^t dt' \left\langle u_n^{(j)} | s_n(t') \right\rangle e^{\omega_n^{(j)}(t-t')} \right] \left| u_n^{(j)} \right\rangle \equiv$$
$$\sum_{j=1}^{2} \left[b_{n0}^{(j)} e^{\omega_n^{(j)} t} + \int_0^t dt' \sigma_n^{(j)}(t') e^{\omega_n^{(j)}(t-t')} \right] \left| u_n^{(j)} \right\rangle . \tag{13}$$

It is seen that the component of the solution along each spatial eigenfunction evolves as a superposition of two exponential, whose time-constants are solutions of the proper inhour equation (11).
At last, the flux solution can be expressed taking the first components of the vectors appearing in Eq. (13), as:

$$\Phi(x,t) = \sum_{n=0}^{\infty} \left\{ \sum_{j=1}^{2} \left(1 + \frac{\beta v \Sigma_f v \lambda}{(\omega_n^{(j)} + \lambda)^2} \right)^{-1} \left[(\varphi_n, \Phi_0) \, e^{\omega_n^{(j)} t} + \frac{v\lambda}{\omega_n^{(j)} + \lambda} (\varphi_n, C_0) \, e^{\omega_n^{(j)} t} + \right. \right.$$
$$\left. \left. \int_0^t dt' \, (\varphi_n, S(t')) \, e^{\omega_n^{(j)}(t-t')} \right] \right\} \varphi_n(x). \tag{14}$$

The above equation is the basis for the physical investigation that is following, for which different physical cases are considered.

2.1. Initially critical reactor in absence of delayed emissions

No source injection is considered for initially critical reactors. In the degenerate case of no delayed emissions ($\beta = 0$), the second sum appearing in Eq. (14) reduces to one term only. The explicit expression for the time-eigenvalues follows:

$$\omega_n = v \left[\nu \Sigma_f - \left(D B_n^2 + \Sigma_a \right) \right] . \tag{15}$$

(dropping the unnecessary upper index). Hence the full exact solution reads:

$$\Phi(x,t) = \sum_{n=0}^{\infty} a_{n0} e^{\omega_n t} \varphi_n(x). \tag{16}$$

It is useful for the subsequent discussion of the results to define the *asymptotic* portion of the solution Φ_{asy}, which is also the most persisting component in the case of a final subcritical system, as:

$$\Phi_{asy}(x,t) = a_{00} e^{\omega_0 t} \varphi_0(x), \tag{17}$$

and the relative difference between the full solution and its asymptotic part with respect to the complete solution, as:

$$\mathcal{R}_{asy} = \left| \frac{\Phi - \Phi_{asy}}{\Phi_{asy}} \right| .$$

It is straightforward to obtain from Eq. (16) that, for sufficiently large values of time, the ratio $\mathcal{R}_{asy}$ is dominated by a pure exponential behavior:

$$\mathcal{R}_{asy} \rightarrow e^{(\omega_1 - \omega_0)t}, \tag{18}$$

which shows that the evolution for $\mathcal{R}_{asy}$ is regulated by the difference between the first two time eigenvalues of the system.

The ratio $\mathcal{R}_{asy}$ is a suitable parameter to measure the *spatiality* of the transient, i.e. the deformation of the neutron flux shape from the initial state to the shape of the asymptotic evolution. In the present problem this space feature is connected to the persistence of higher-order harmonics in determining the spatial distortion of the solution with respect to the initial state in its evolution towards the fundamental Helmholtz eigenfunction of the critical system. A small space distortion of the neutron distribution implies a point-like behavior of the system.

2.2. Initially critical reactor with delayed emissions

When delayed emissions are accounted for, the contribution of the fundamental eigenfunction takes the time-dependent form:

$$\Phi_{fund}(x,t) = \left(b_{00}^{(1)} e^{\omega_0^{(1)} t} + b_{00}^{(2)} e^{\omega_0^{(2)} t} \right) \varphi_0(x). \tag{19}$$

To obtain the asymptotic flux, further simplifications can be introduced noticing that for each space eigenfunction:

$$\left| \omega_n^{(1)} \right| \ll \left| \omega_n^{(2)} \right| , \tag{20}$$

and $\omega_n^{(2)} < 0$, hence:

$$\Phi_{asy}(x,t) = b_{00}^{(1)} e^{\omega_0^{(1)} t} \varphi_0(x). \tag{21}$$

Consequently, also in this case, $\mathcal{R}_{asy}$ can again be given a simple expression:

$$\mathcal{R}_{asy} \rightarrow e^{(\omega_1^{(1)} - \omega_0^{(1)})t}. \tag{22}$$

2.3. Subcritical system

For the study of subcritical systems an external source must be accounted for, thus including the

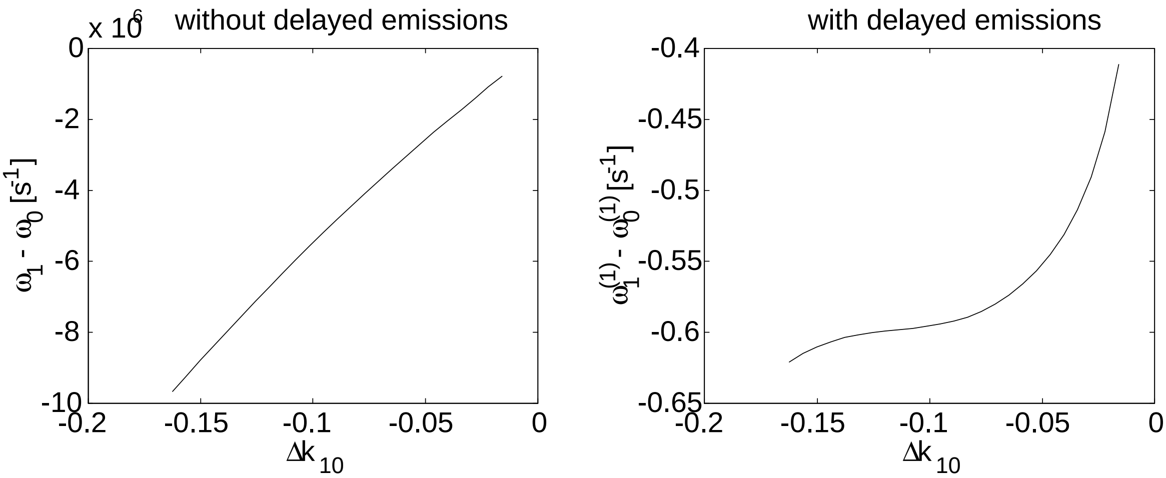

Figure 1. Relation between the separation of the first static multiplication eigenvalues and the separation of the first time eigenvalues.

convolution term appearing in Eq. (13). If the source components $\sigma_n^{(j)}$ are assumed to be constant in time, one obtains:

$$\int_0^t dt' \sigma_n^{(j)}(t') e^{\omega_n^{(j)}(t-t')} = -\frac{\sigma_n^{(j)}}{\omega_n^{(j)}} \left(1 - e^{\omega_n^{(j)} t}\right). \tag{23}$$

2.3.1. No delayed neutron case

For subcritical systems two different situations need to be considered. In fact, the final system can be either supercritical and time-diverging or subcritical. In the former case the ratio $\mathcal{R}_{asy}$ shows the same time evolution as for the initially critical reactor considered above. In the latter case, the solution is dominated by the presence of the source, and the asymptotic behavior takes the form:

$$\Phi_{asy}(x,t) = -\sum_{n=0}^{\infty} \frac{\sigma_n}{\omega_n} \varphi_n(x), \tag{24}$$

thus explicitly evidencing the fact that the steady-state condition includes contribution from all space eigenfunctions, excited by the source spatial distribution.
However, during the transient towards the asymptotic flux also contributions coming from the fundamental eigenfunction need to be taken into account. Being the system subcritical, these contributions are to die out. Hence, a *dominant* flux $\Phi_{\mathcal{D}}$ can be defined according to the following sum:

$$\Phi_{\mathcal{D}}(x,t) = \left(a_{00} + \frac{\sigma_0}{\omega_0}\right) e^{\omega_0 t} \varphi_0(x) + \Phi_{asy}(x,t). \tag{25}$$

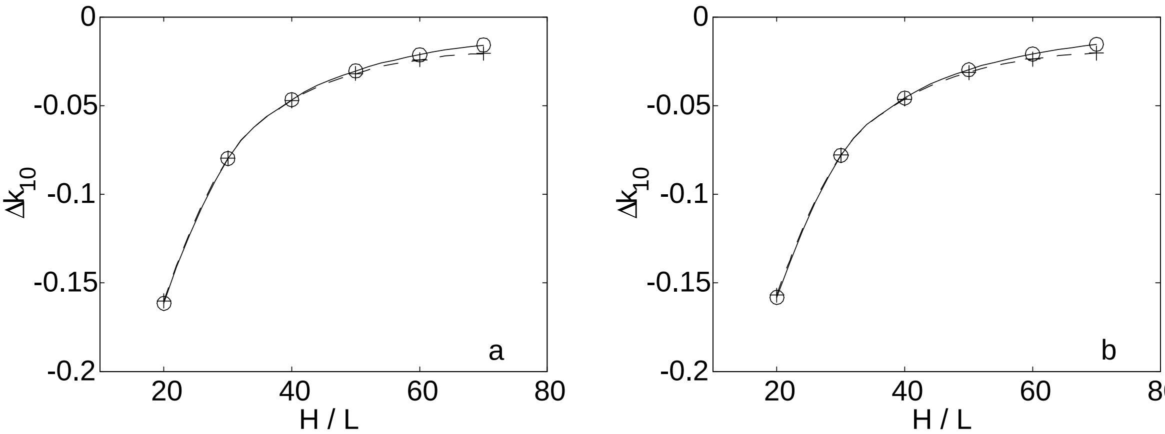

Figure 2. Eigenvalue separation for critical and subcritical systems. The circles indicate results for the homogeneous system and crosses for a system with an absorber that introduces a change in k of -500 pcm. Graph (a) on the left refers to the case in which the homogeneous system is critical, while graph (b) refers to the case in which the homogeneous system is subcritical ($k = 0.98$).

The dominant flux is a suitable physical quantity to describe the spatiality of the transient towards the asymptotic situation described by Eq. (24). Therefore, a new ratio $\mathcal{R}_{\mathcal{D}}$ can be defined according to the following relationship:

$$\mathcal{R}_{\mathcal{D}} = \left| \frac{\Phi - \Phi_{\mathcal{D}}}{\Phi_{\mathcal{D}}} \right| . \tag{26}$$

After some obvious simplifications, one obtains:

$$\mathcal{R}_{\mathcal{D}} \rightarrow \left| \frac{\left(a_{00} + \frac{\sigma_0}{\omega_0} \right) e^{\omega_0 t} \varphi_1(x)}{\Phi_{\mathcal{D}}} \right| . \tag{27}$$

2.3.2. Case with delayed neutrons

The portion of the flux in response to the component of the source according to the fundamental eigenfunction yields the steady state asymptotic behavior, as:

$$\Phi_{asy}(x,t) = \sum_{n=0}^{\infty} \left(\frac{\sigma_n^{(1)}}{\omega_n^{(1)}} + \frac{\sigma_n^{(2)}}{\omega_n^{(2)}} \right) \varphi_n(x), \tag{28}$$

while the dominant behavior in the transient is:

$$\Phi_{\mathcal{D}}(x,t) = \sum_{j=1}^{2} \left(b_{00}^{(j)} + \frac{\sigma_0^{(j)}}{\omega_0^{(j)}} \right) e^{\omega_0^{(j)} t} \varphi_0(x) + \Phi_{asy}(x,t). \tag{29}$$

Once the simplifications are carried out, the following expression can be written:

$$\mathcal{R}_{\mathcal{D}} \rightarrow \left| \frac{\left(b_{10}^{(1)} + \frac{\sigma_1^{(1)}}{\omega_1^{(1)}} \right) e^{\omega_1^{(1)} t} \varphi_1(x)}{\Phi_{\mathcal{D}}} \right| . \tag{30}$$

3. RESULTS AND DISCUSSION

In the above analytical discussion a strong connection between the time-eigenvalue separation and the evolution of the space transient is evidenced. The separation of these eigenvalues is also isomonotonically connected to the separation between the first two static multiplication eigenvalues, the first of which is the classic (fundamental) multiplication constant. This fact is clearly demonstrated by Fig. 1, where the difference between the first two time eigenvalues is plotted against the separation between the first two static multiplication constants Δk_{10}, for both a situation with no delayed neutrons and with delayed neutrons.

The following set of results is obtained with reference to a slab reactor in which the value of the multiplication eigenvalue is maintained with the presence of a symmetrically located absorber (referred to as *control device* in the following), the thickness of which is assumed to be $4.10^{-2}H$. The presence of the control device establishes the required value of the multiplication constant and determines an initial flux distribution which differs from the fundamental mode of the final system. Hence, the removal of the control device produces a transient situation, caused by both the change in reactivity and the initial condition. With this physical situation, the above analytical solution can be easily applied, since the final system is homogeneous. The subcritical systems considered are injected by an external neutron source, symmetrically located in a thin slab within the system, whose thickness is $2.10^{-2}H$. The material data are typical of a Myrrha system [16], recently proposed as an ADS configuration.

Figure 2 reports the effect of the dimensions of the system in terms of diffusion lengths on the eigenvalue separation Δk_{10}. It is clearly seen how the separation is increasing by decreasing the physical dimensions of the system, no matter if initially critical or subcritical and irrespectively of the presence of the control device.

As it has been shown in the theory above, the ratios $\mathcal{R}_{asy}$ and $\mathcal{R}_{\mathcal{D}}$ yield a meaningful indication of the spatial nature of the transient and are directly connected to the eigenvalue separation. Figure 3 reports the time evolution of such ratios for transients in a critical and in two subcritical systems with different initial effective multiplication constants. The spatial characteristics of the transient is studied by introducing the relative norm of the difference between the full solution and the reference steady-state flux distribution

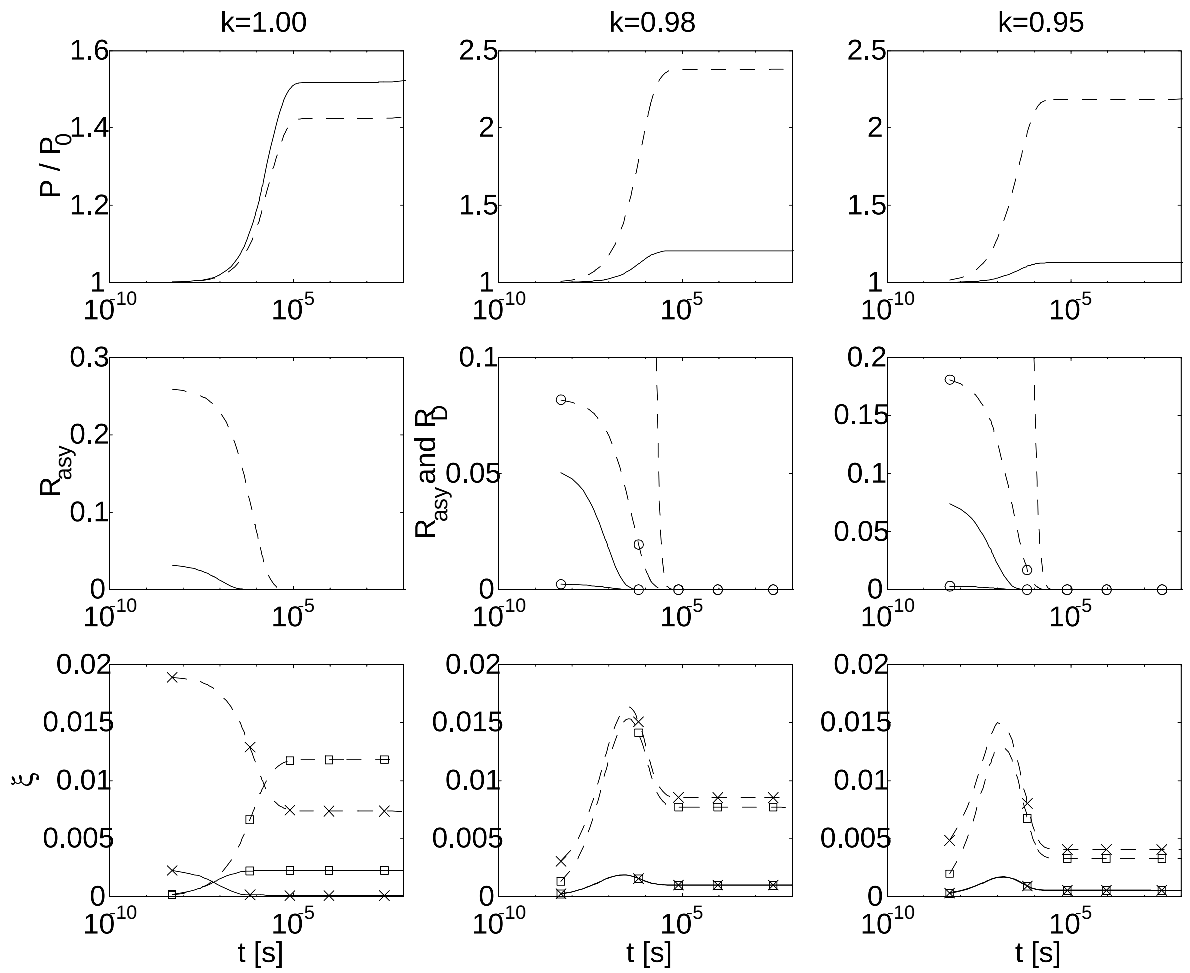

Figure 3. Spatiality of transients in critical and subcritical systems, for a fixed reactivity insertion of 500 pcm. The power evolution for the initially critical case is diverging and such behavior appears at longer times than shown in the graph. Solid line refers to a small system ($H = 20L$) and broken line to a large system ($H = 70L$). Crosses ($\times$) results for ξ are produced using as reference Ψ the final flux distribution, while squares ($\square$) using the initial state. For subcritical systems circles ($\circ$) indicate the ratio $\mathcal{R}_{asy}$.

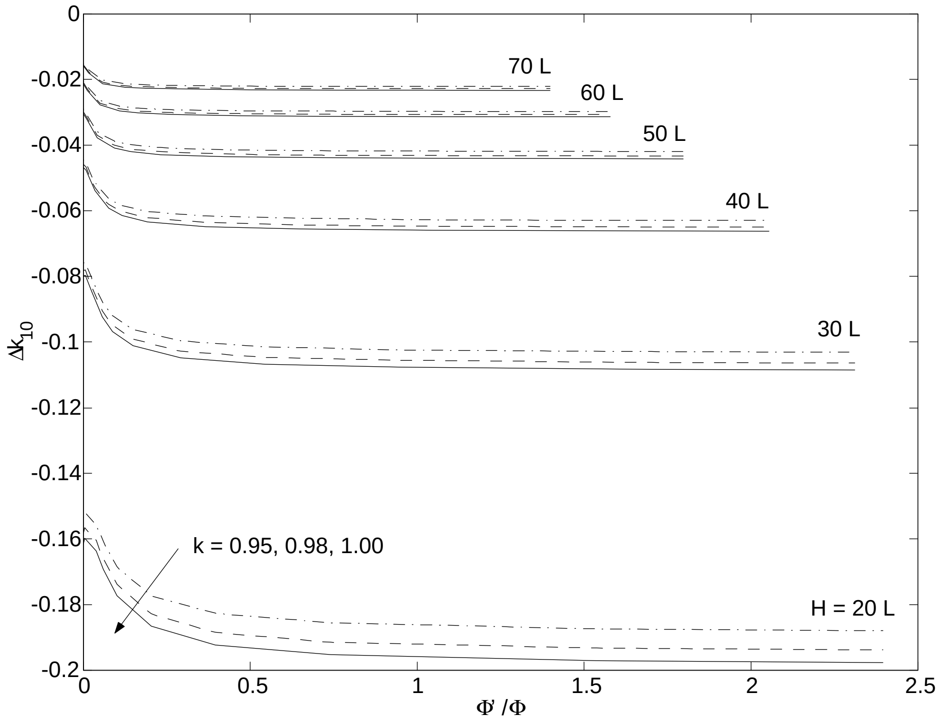

Figure 4. Eigenvalue separation for different values of the greyness of the control device and of the physical dimensions of the system.

(spatiality parameter ξ), as

$$\xi \equiv \frac{1}{\langle \Phi \rangle} \sqrt{\left\langle \left(\Phi - \frac{\langle \Phi \rangle}{\langle \Psi \rangle} \Psi \right)^2 \right\rangle}, \tag{31}$$

where $\langle \quad \rangle$ denote space integration and Ψ is the reference steady-state flux distribution. The reactivity introduction for all transients is 0.3458 β. From the graphs the following considerations can be made:
1. The response to a perturbation in a critical reactor is spatially more significant in a large system, i.e. $\mathcal{R}_{asy}$ is larger and takes longer to reduce to 0, and thus the contribution of higher order harmonics is more persistent;
2. The evolutions of both $\mathcal{R}_{asy}$ and $\mathcal{R}_{\mathcal{D}}$ for subcritical systems show that the importance of higher-order harmonics increases with increasing subcriticality, as the systems are more *source-dominated*;
3. The comparison of initially critical and subcritical systems shows that the spatial feature of the transients is larger in systems departing from criticality; therefore, one can expect that the point model may have obvious limitations of applicability in these situations.
In order to analyze the appearance of spatially important phenomena, the eigenvalue separation is studied by varying the neutronic spatial decoupling by means of a change of the greyness of the control device; the effect can be measured by the ratio between the flux derivative and the flux itself at the boundary of the

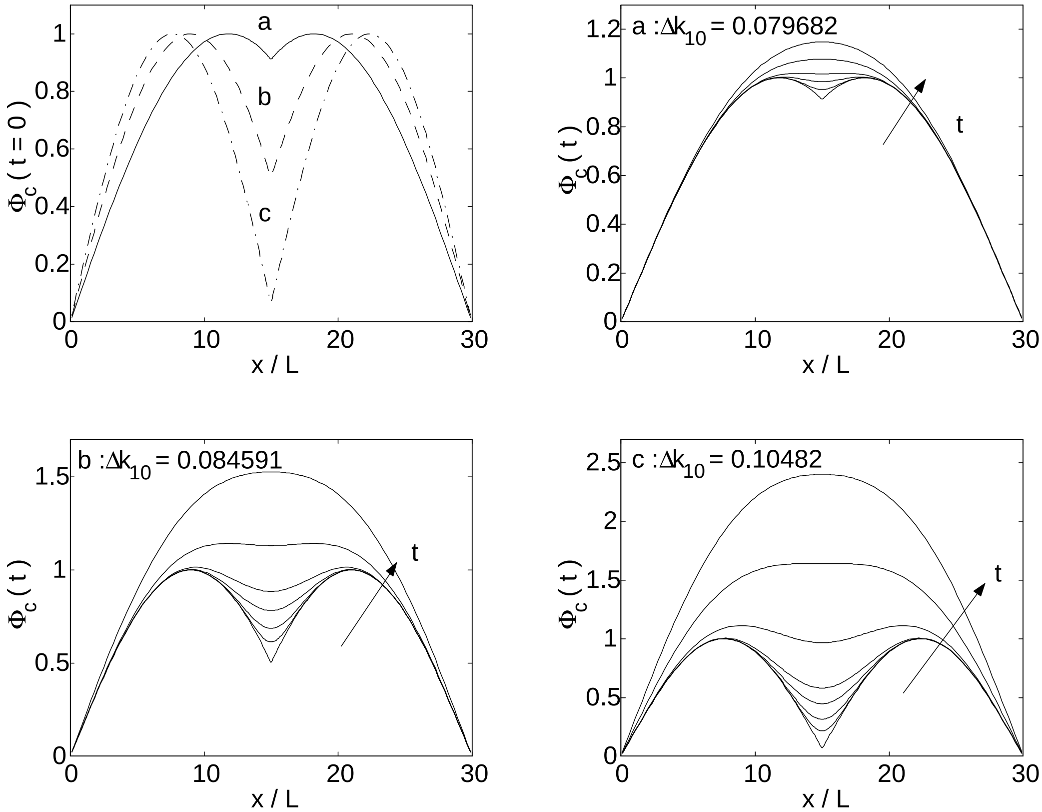

Figure 5. Spatial evolution in initially critical systems with different eigenvalue separations. The top left graph reports the initial flux distributions for three values of the greyness of the control device.

device, Φ'/Φ. Figure 4 illustrates the behavior of the eigenvalue separation against such parameter. The following considerations are in order:

1. For a fixed dimension of the system, the eigenvalue separation increases as k increases, thus the kinetic spatiality of the system is decreasing;

2. As already observed, the separation decreases as the physical dimensions increase, thus increasing the spatiality of the kinetic behavior; this effect is by far dominating with respect to the effect due to the multiplication eigenvalue and the greyness of the control device; for large systems the difference induced by different subcriticalities is almost irrelevant.

The spatial behavior of the flux for transients in an initially critical system are plotted in Fig. 5.

A source transient is now considered in a subcritical system. To better represent the physical situation involving a spallation neutron source with a strong spectral effect, a two group calculation is performed in this case. In Fig. 6 the spatiality parameter ξ is plotted separately for the two energy groups and for two different system dimensions, implying the same subcriticality level $\Delta k/k = (1 - 1/k) = -1.4114\beta$. Also in this case, the validity of a point-like model is reduced by increasing the geometrical dimensions of the structure.

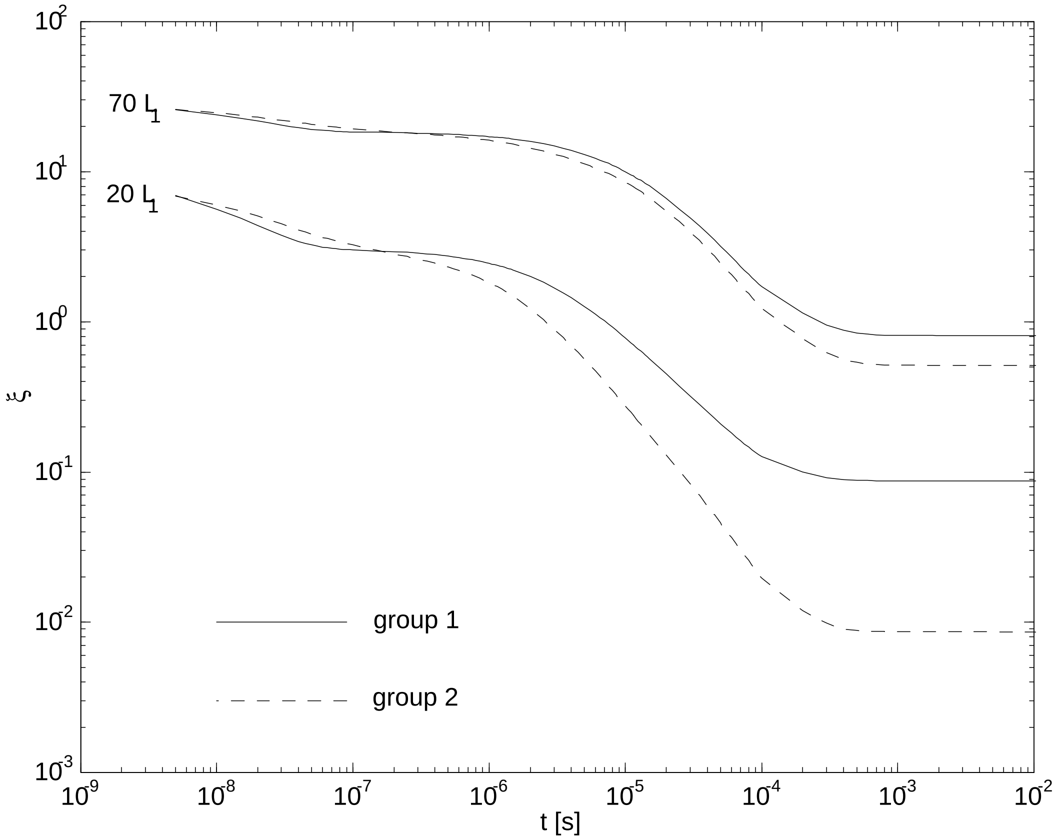

Figure 6. Evolution of the spatiality parameter ξ for two subcritical systems having $\Delta k/k = -1.4114\beta$. The reference Ψ is the final flux distribution.

4. ON THE CHOICE OF THE WEIGHTING FUNCTION

When constructing the point model from an initially critical reactor, there is no ambiguity in choosing as weighting function for generating the kinetic parameters the critical adjoint. The same consideration applies also for quasi-static procedures, in which this option has always proved to yield satisfactory results. However, when considering transients in source-driven systems, the definition of the adjoint problem requires either the introduction of source, which is somewhat arbitrary, or, alternatively, of a fictitious critical system with the introduction of an eigenvalue, such as the usual effective multiplication constant. The choice of the weighting function can affect the accuracy of the transient predictions and the effectiveness of the quasi-static procedure, although in the latter case the convergence of the method is always assured whatever the choice of the weighting function may be.
The intent of the results we are presenting in this section is to evidence the problem, rather then to propose solutions. In past investigations concerning quasi-static calculations of three-dimensional systems, a difference was observed if the weighting function was chosen according to either the critical adjoint or the adjoint driven by the fission cross-section as a source [2].
Figures 7 and 8 compare results for the power evolutions obtained by the point kinetic model derived

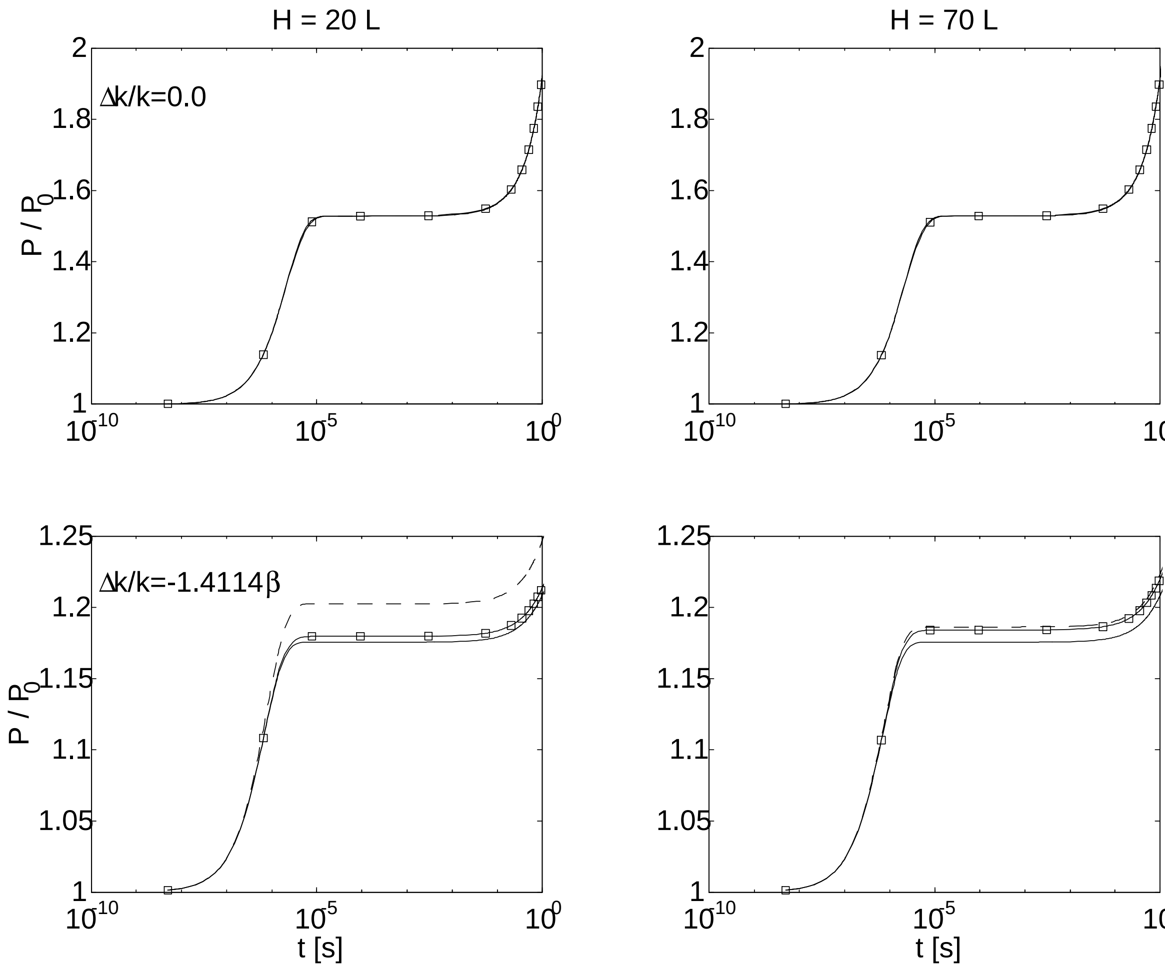

Figure 7. Comparison of the power evolutions calculated using different weighting functions, in response to a homogeneous perturbation, for systems with different multiplication constants. Top curves: initial critical system; bottom curves: initial subcritical system, $k = 0.98$. Squares: results of the reference full space calculation; broken line: constant adjoint; solid line: critical adjoint; dot-point line: source-driven adjoint.

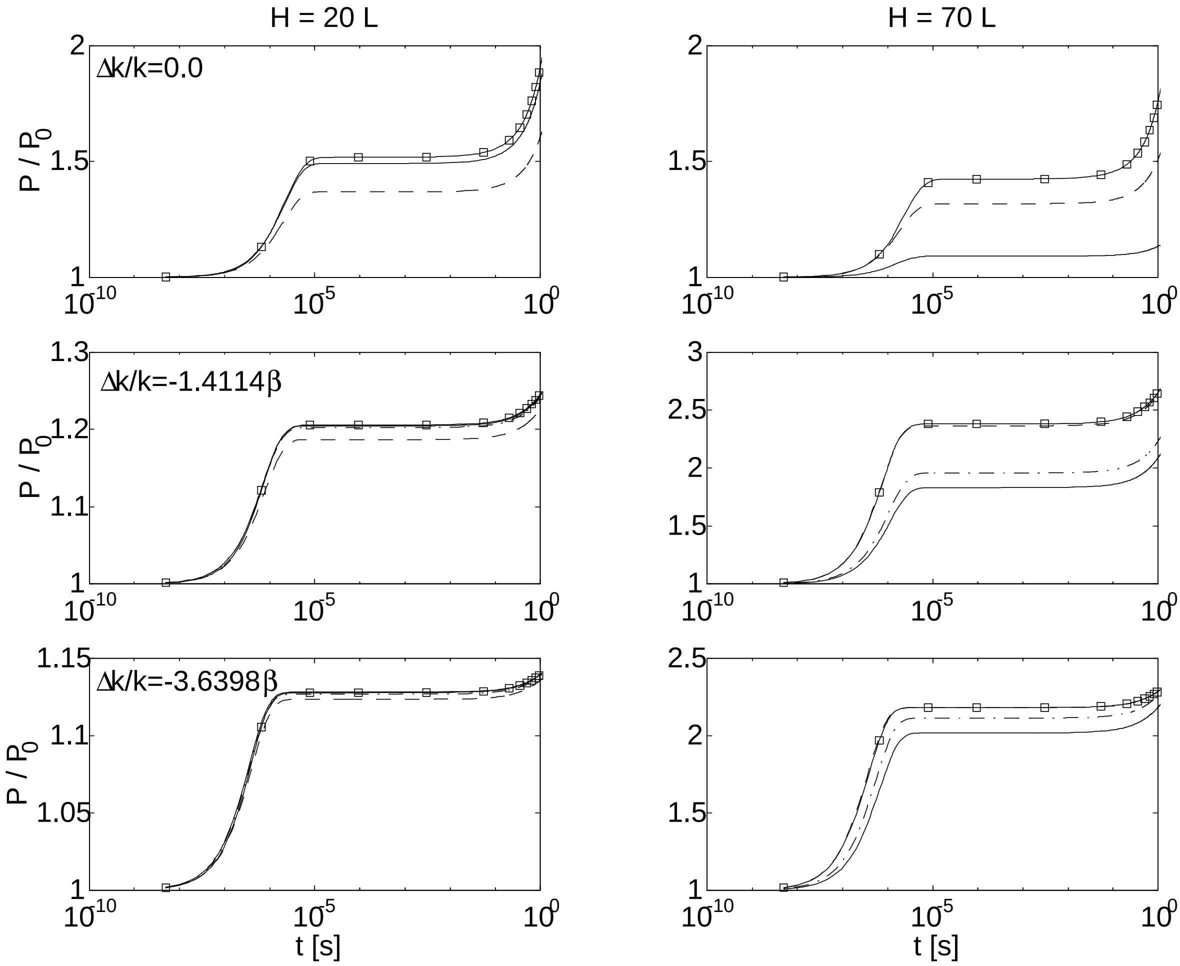

Figure 8. Comparison of the power evolutions calculated using different weighting functions in response to a localized perturbation. Squares: results of the reference full space calculation; broken line: constant adjoint; solid line: critical adjoint; dot-point line: source-driven adjoint.

using different weighting functions, namely, a constant, the critical adjoint and, for subcritical systems, a fission cross section source-driven adjoint. Transients are started by a perturbation implying a change in the multiplication constant of 0.3458 β. Figure 7 refers to a homogeneous perturbation, while Fig. 8 to a localized perturbation caused by the removal of the control device. The line with squares plots the results of the reference full space calculation. In both figures, the broken line, solid line and dot-point line refer to the choice of the constant, the critical adjoint and the source-driven adjoint, respectively.
Figure 9 illustrates the effect of different choices of the weighting function in quasi-static schemes. The transient is the same as the one analyzed in Fig. 8 for $H = 70L$ and for a subcriticality level of -1.4114β. As can be seen, all quasi-static calculations converge to the full space results. Rather amazingly, for this transient the choice of a spatially flat weighting function performs better than the other choices. This is due to the particular perturbation inserted, which involves a removal of an initial control device from a region close to the external source, hence characterized by a high neutron importance. If one uses either the source-driven or the critical importance, the dynamic effect of the cross section change is initially underestimated with respect to the situation following the perturbation. When the shape is updated, the method is numerically trying to correct, with overcompensations which cause power overshoots and then oscillations as appear in the graphs of the. The effect can be seen also from the evolution of the reactivity, which changes according to the shape updates. The reactivities computed with the three different choices of the weighting function converge to different steady-state values, according to the corresponding values of the effective source. These results seem to evidence that no unique recipe can be generally given as far as the choice of the weighting function for quasi-statics is concerned, since the accuracy of the results is highly dependent on the physical characteristics of the transient being calculated.

5. CONCLUSIONS

The work presented concerns some basic aspects in the neutron kinetics of multiplying systems. The scope of the analysis is to put in evidence the role of spatial effects in different physical configurations. The extent of the spatial nature of a transient is strictly connected to the separation of the time-eigenvalue of the system. This characteristic is thus informative on the limits of applicability of simplified models such as point kinetics. Some attention is given to the new problems arising in source-driven systems. Some considerations are given to the open problem of the choice of the proper adjoint function to be used for the generation of the kinetic parameters in subcritical systems. The results presented show remarkable effects; however, no solution to the problem is proposed and the question remains open for discussion at the Conference.

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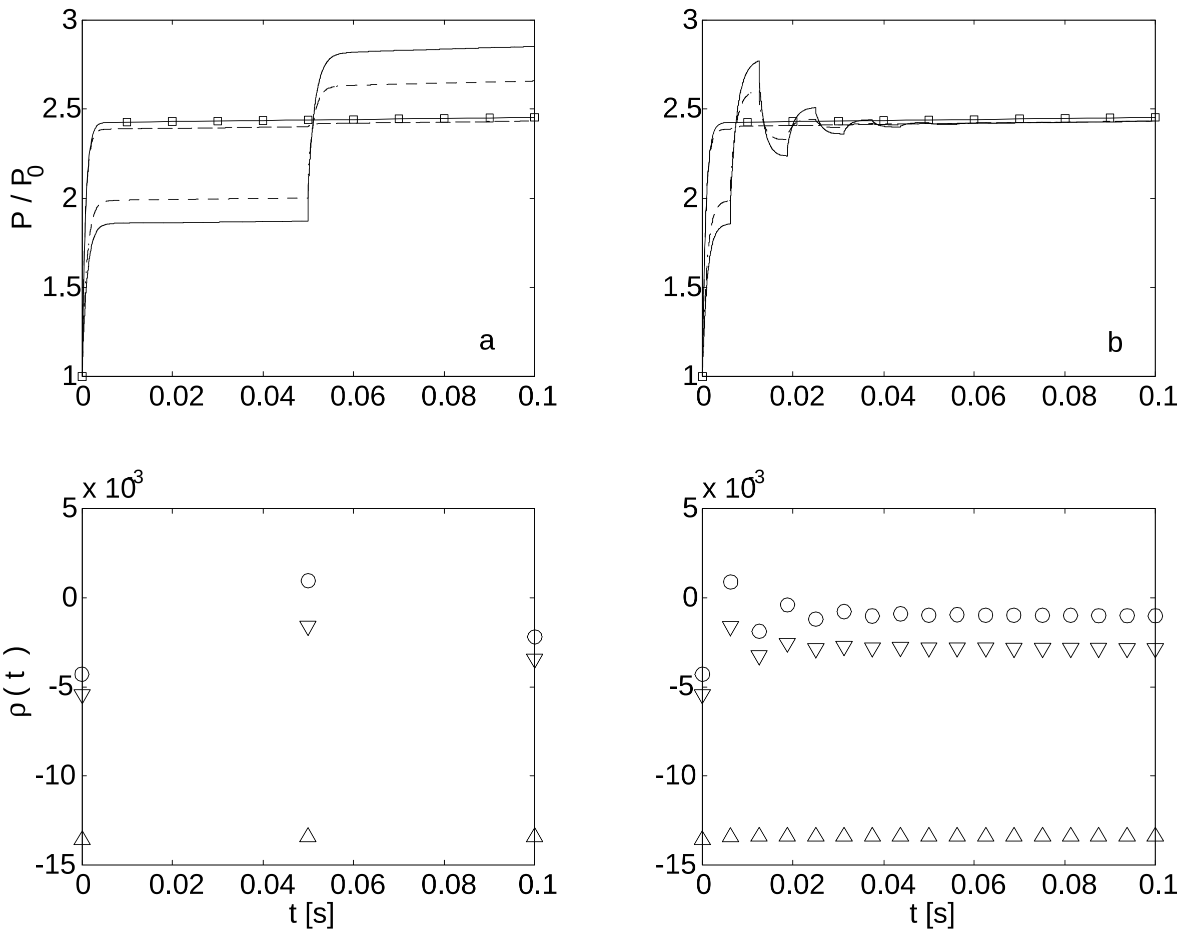

Figure 9. Power evolution in the quasi-static calculations for the transient of Fig. 8 with $H = 70L$ and $\Delta k/k = -1.4114\beta$, compared to the full spatial results. Only one shape update is used for graphs a), while 15 shape recalculations are performed in case b). Curves at the bottom report evolutions of the reactivity; $\triangledown$: source adjoint; $\triangle$: constant; $\circ$: critical adjoint.

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