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# RECONSTRUCTION OF THE LOCAL NEUTRON-PHYSICAL FUNCTIONALS IN SURFACE HARMONICS METHOD

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## ABSTRACT

Formulas used in surface harmonics method for reconstruction of the local neutron-physical functionals in three-stage 2D calculation of VVER core are described and verified. Set of technical tests and international benchmark for 2D VVER-1000 MOX core were used for verification.

*Key Words*: Surface harmonics method, reconstruction, local functionals, SUHAM-U code.

## 1. INTRODUCTION

In Surface Harmonics Method (SHM) equations [1], group vectors connected with different laws of neutron current incoming into the cell are the unknown values. Vectors of average neutron fluxes in cells or cell zones are not the unknown vectors in SHM equations. As result, procedure of reconstruction of local functionals including the average neutron fluxes in cells and cell zones gets complicated. In the case if calculation is carried out in three stages – cells, fuel assemblies (FA-s), reactor core – this procedure gets still more complicated. In so doing, it is necessary to calculate and keep reaction matrices both for cells and for FA-s and for each reaction type.

Another way for reconstruction of local functionals is possible too. In this way, only reaction matrices for average neutron fluxes in cell zones are calculated. In so doing, for calculation of any other functionals it is necessary to keep group cross-sections of isotopes and materials. Namely this way is used in SUHAM-U code [2] and described in this paper. It allows us to reduce essentially the quantity of kept information.

## 2. FORMULAS FOR RECONSTRUCTION OF LOCAL FUNCTIONALS IN THREE-STAGE CALCULATION OF THE 2D LAYER OF VVER BY SHM

In SHM, group function of neutron distribution in a reactor core is presented as a superposition of group trial matrices in FA-s.

$$\vec{\Phi}_R^N(w) = \sum_{i=1}^{I_{FA}} \sum_{n=0}^{N_{FA}} \hat{\phi}_{FA,i}^{(n)}(w) \vec{J}_{FA,i}^{(n)} \quad , \tag{1}$$

where $\hat{\phi}_{FA,i}^{(n)}(w)$ - trial matrices for FA-s, $I_{FA}$ – number of fuel assemblies, ($N_{FA}$+1) - number of trial matrices for each FA, $w = \{\vec{r}, \vec{\Omega}\}$, $\vec{J}_{FA,i}^{(n)}$ - vector of unknown group amplitudes. Group trial matrices for FA-s are presented as a superposition of group trial matrices in cells

$$\hat{\phi}_{FA,i}^{(n)}(w) = \sum_{k=1}^{K_c} \sum_{n'=0}^{N'_c} \hat{\phi}_{c,k,i}^{(n')}(w)\ \hat{J}_{c,k,i}^{(n,n')} \qquad , \tag{2}$$

where $\hat{\phi}_{c,k,i}^{(n')}(w)$ - trial matrices for cells, $K_c$ – number of cells in each FA, ($N'_c$+1) - number of trial matrices for each cell, $\vec{J}_{c,k,i,g'}^{(n',n)}$ - group vector of unknown amplitudes for cell trial matrix n′. As result group function of neutron distribution in a reactor core is presented in the form

$$\vec{\Phi}_R^N(w) = \sum_{i=1}^{I_{FA}} \sum_{n=0}^{N_{FA}} \sum_{k=1}^{K_c} \sum_{n'=0}^{N'_c} \hat{\phi}_{c,k,i}^{(n')}(w)\ \hat{J}_{c,k,i}^{(n,n')}\ \vec{J}_{FA,i}^{(n)} \qquad , \tag{3}$$

where $\hat{J}_{c,k,i}^{(n,n')} = (\vec{J}_{c,k,i,1}^{(n,n')}, \vec{J}_{c,k,i,2}^{(n,n')}, ..., \vec{J}_{c,k,i,G}^{(n,n')})$.
Write expression (3) in the form

$$\vec{\Phi}_R^N(w) = \sum_{i=1}^{I_{FA}} \sum_{k=1}^{K_c} \vec{\Phi}_{c,i,k}^N(w) \tag{4}$$

where

$$\vec{\Phi}_{c,i,k}^N(w) = \sum_{n=0}^{N_{FA}} \sum_{n'=0}^{N'_c} \hat{\phi}_{c,k,i}^{(n')}(w)\ \hat{J}_{c,k,i}^{(n,n')}\ \vec{J}_{FA,i}^{(n)} \tag{5}$$

$\vec{\Phi}_{c,i,k}^N(w)$ - group function of neutron distribution in cell k of FA i. Values of elements of matrices $\hat{J}_{c,k,i}^{(n,n')}$ are obtained when trial matrices $\hat{\phi}_{FA,i}^{(n)}(w)$ are calculated. Values of elements of vectors $\vec{J}_{FA,i}^{(n)}$ are obtained when coarse-mesh calculation is carried out.

Let's consider now formulas for group vectors of reactions in cells, in so doing we will only consider reactions which are symmetric on cell (symmetry means independence on azimuth angle – angle between vector $\vec{\rho}$ and axis **X**), in so doing n′=0. Let's multiply each group element of vector $\vec{\Phi}_{R,i,k}^N(w)$ by group cross-section of type 'x' - $\Sigma_x^g(\vec{r})$ and integrate result over $\{\vec{r}, \vec{\Omega}\}$. As result, we can write group vector of reaction of type 'x' in cell k of FA i in the form

$$\vec{F}_{k,i}^x = V_{cell} \hat{\vec{F}}_{c,k,i}^x \sum_{n=0}^{N_{FA}} \hat{J}_{c,k,i}^{(n,0)}\ \vec{J}_{FA,i}^{(n)} \qquad , \tag{6}$$

where $V_{cell}$ – volume of two-dimensional cell,

$$(\hat{\bar{F}}^{x}_{c,k,i})_{gg'} = \frac{1}{V_{cell}} \int_{V_{cell}} d\vec{r} \Sigma^{g}_{x}(\vec{r}) (\hat{\varphi}^{(0)}_{c,k,i}(\vec{r}))_{gg'}, \tag{7}$$

$$\hat{\varphi}^{(0)}_{c,k,i}(\vec{r}) = \frac{1}{4\pi} \int_{4\pi} d\vec{\Omega} \hat{\varphi}^{(0)}_{c,k,i}(\vec{r}, \vec{\Omega}) \tag{8}$$

Vectors $\vec{J}^{(n,0)}_{c,k,i,g'}$ can be expressed in terms of group vector of neutron levels [3] in cells $\vec{\Phi}^{(n)}_{c,k,i,g'}$. Just last vectors are unknown ones when trial vectors in FA $\vec{\varphi}^{(n)}_{FA,i,g'}(w)$ are calculated.

$$\vec{J}^{(n,0)}_{c,k,i,g'} = \frac{V_{cell}}{Ma_{cell}} \hat{\Sigma}_{c,k,i} \vec{\Phi}^{(n)}_{c,k,i,g'} \quad , \tag{9}$$

where $a_{cell}$ - length of one boundary side of cell, M – number of cell boundary sides. Let's write expression (9) in the matrix form

$$\hat{J}^{(n,0)}_{c,k,i} = \frac{V_{cell}}{Ma_{cell}} \hat{\Sigma}_{c,k,i} \hat{\Phi}^{(n)}_{c,k,i} \tag{10}$$

Substitution of (10) to expression (6) gives:

$$\vec{F}^{x}_{k,i} = V_{cell} \hat{\bar{F}}'^{x}_{c,k,i} \hat{\Sigma}_{c,k,i} \sum_{n=0}^{N_{FA}} \hat{\Phi}^{(n)}_{c,k,i} \vec{J}^{(n)}_{FA,i}, \tag{11}$$

where

$$(\hat{\bar{F}}'^{x}_{c,k,i})_{gg'} = \frac{1}{\Pi_{cell}} \int_{V'_{cell}} d\vec{r} \Sigma^{g}_{x}(\vec{r}) (\hat{\varphi}^{(0)}_{c,k,i}(\vec{r}))_{gg'} \tag{12}$$

Here $\Pi_{cell}$ - perimeter of cell. Let's rewrite (11) in the form

$$\vec{F}^{x}_{k,i} = V_{cell} \hat{\bar{F}}''^{x}_{c,k,i} \sum_{n=0}^{N_{FA}} \hat{\Phi}^{(n)}_{c,k,i} \vec{J}^{(n)}_{FA,i}, \tag{13}$$

where

$$\hat{\bar{F}}''^{x}_{c,k,i} = \hat{\bar{F}}'^{x}_{c,k,i} \hat{\Sigma}_{c,k,i} \tag{14}$$

Exactly such matrices $\hat{\bar{F}}''^{x}_{c,k,i}$ for cells are calculated in module RACIA of SUHAM-U code. Let $(N_{FA}+1) = 6$, i.e. only 6 trial matrices for each FA are used. Unknown vectors $\vec{J}^{(n)}_{FA,i}$ can be expressed in terms of group vectors $\vec{\Phi}_{FA,i}$, $\vec{X}^{(1)}_{FA,i}$, $\vec{X}^{(2)}_{FA,i}$ and $\vec{X}^{(3)}_{FA,i}$ [4]. Just lust vectors are

unknown vectors in coarse-mesh SHM equations with six trial matrices per each FA. According to [4] we can write

$$\begin{cases} \vec{J}^{(0)}_{FA,i} = \dfrac{V_{FA}}{MS_{FA,r}} \hat{\Sigma}_{FA,i} \vec{\Phi}_{FA,i} \\ \vec{J}^{(3)}_{FA,i} = \dfrac{2V_{FA}}{MS_{FA,r}} \hat{\Sigma}^{(1)}_{FA,i} \vec{X}^{(1)}_{FA,i} \\ \vec{J}^{(4)}_{FA,i} = \dfrac{2V_{FA}}{MS_{FA,r}} \hat{\Sigma}^{(1)}_{FA,i} \vec{X}^{(2)}_{FA,i} \\ \vec{J}^{(5)}_{FA,i} = \dfrac{V_{FA}}{MS_{FA,r}} \hat{\Sigma}^{(3)}_{FA,i} \vec{X}^{(3)}_{FA,i} \end{cases} \tag{15}$$

Matrices $\hat{\Sigma}_{FA,i}$, $\hat{\Sigma}^{(j)}_{FA,i}$ (j=1,3) are the functionals of trial matrices for FA-s and their combinations form the coefficients of coarse-mesh SHM equations. Special formulas are obtained for $\vec{J}^{(1)}_{FA,i}$, $\vec{J}^{(2)}_{FA,i}$:

$$\begin{cases} \vec{J}^{(1)}_{FA,i} = \hat{\Lambda}''_0 \vec{\Phi}_{FA,i} - \hat{\Lambda}''_1 \vec{X}^{(1)}_{FA,i} - \hat{\Lambda}''_2 \vec{X}^{(2)}_{FA,i} + \hat{\Lambda}''_2 \vec{X}^{(3)}_{FA,i} \\ \vec{J}^{(2)}_{FA,i} = \hat{\Lambda}''_4 \vec{\Phi}_{FA,i} - \hat{\Lambda}''_5 \vec{X}^{(1)}_{FA,i} - \hat{\Lambda}''_6 \vec{X}^{(2)}_{FA,i} + \hat{\Lambda}''_7 \vec{X}^{(3)}_{FA,i} \end{cases} \tag{16}$$

Here $\hat{\Lambda}''_j$ - different finite-difference operators, for example

$$\begin{cases} \hat{\Lambda}''_0 \vec{A}_k = \dfrac{2}{Mh} \sum_{i=1}^{M} \hat{H}_{ik} (\vec{A}_{j_i} - \vec{A}_k) \cos(\alpha_i) \\ \hat{\Lambda}''_2 \vec{A}_k = \dfrac{2}{Mh} \sum_{i=1}^{M} \hat{H}_{ik} (\vec{A}_{j_i} - \vec{A}_k) \cos(\alpha_i) \sin(2\alpha_i) \end{cases} \tag{17}$$

Substitution of (15), (16) to expression (13) gives the final expression for integral over cell volume group vector of reaction 'x' in cell k and fuel assembly i:

$$\begin{aligned} \vec{F}^{x}_{k,i} = V_{cell} \hat{\vec{F}}''^{x}_{c,k,i} [ & \frac{V_{FA}}{MS_{FA,r}} \hat{\Phi}^{(0)}_{c,k,i} \hat{\Sigma}_{FA,i} \vec{\Phi}_{FA,i} + \hat{\Phi}^{(1)}_{c,k,i} (\hat{\Lambda}''_0 \vec{\Phi}_{FA,i} - \hat{\Lambda}''_1 \vec{X}^{(1)}_{FA,i} - \hat{\Lambda}''_2 \vec{X}^{(2)}_{FA,i} + \hat{\Lambda}''_3 \vec{X}^{(3)}_{FA,i}) + \\ + \hat{\Phi}^{(2)}_{c,k,i} (\hat{\Lambda}''_4 \vec{\Phi}_{FA,i} - \hat{\Lambda}''_5 \vec{X}^{(1)}_{FA,i} - \hat{\Lambda}''_6 \vec{X}^{(2)}_{FA,i} + \hat{\Lambda}''_7 \vec{X}^{(3)}_{FA,i}) + & \frac{2V_{FA}}{MS_{FA,r}} \hat{\Phi}^{(3)}_{c,k,i} \hat{\Sigma}^{(1)}_{FA,i} \vec{X}^{(1)}_{FA,i} + \\ + \frac{2V_{FA}}{MS_{FA,r}} \hat{\Phi}^{(4)}_{c,k,i} \hat{\Sigma}^{(1)}_{FA,i} \vec{X}^{(2)}_{FA,i} + & \frac{V_{FA}}{MS_{FA,r}} \hat{\Phi}^{(5)}_{c,k,i} \hat{\Sigma}^{(3)}_{FA,i} \vec{X}^{(3)}_{FA,i} ] \end{aligned} \tag{18}$$

Analogous formulas may be written for functionals averaged on cell and on cell zones. It should be noted that expression (18) takes place for symmetrical functionals over cell (or cell zone)

volume and for three trial matrices for each cell and six trial matrices for each FA. Expressions for group vector of reaction with use, five, four, three trial matrices for each FA can be obtained from expression (18) when vectors $\vec{X}^{(3)}_{FA,i}$, $\vec{X}^{(2)}_{FA,i}$ and $\vec{X}^{(1)}_{FA,i}$ are equaled zero (one, two or three vectors correspondingly).

## 3. VERIFICATION OF FORMULAS FOR RECONSTRUCTION OF LOCAL FUNCTIONALS

Adduced formulas were realized in SUHAM-U code. Following verifications were carried out.

1. Seven identical fuel assemblies with uranium or MOX fuel.
2. Seven identical fuel assemblies consisting of 397 identical cells.
3. Seven fuel assemblies with two types of FA-s with uranium and MOX fuel.
4. International benchmark for 2D VVER-1000 MOX core [5].

### 3.1. Test N:1.

Verification by calculations of seven identical FA-s of VVER-1000 with uranium (U42G6) or MOX (Pu39G8) fuel and with zero group neutron currents on the external boundary were carried out. Both FA-s were described in [2]. Results of two-stage calculation of separate FA with SUHAM-U code were taken as a reference results (calculation N:0). FA $k_{inf}$, average neutron fluxes and fission reaction in fuel pins were compared. Three types of three-stage calculations with SUHAM-U code were carried out.

1. Calculation N:1: All FA-s were given as FA-s of one type.
2. Calculation N:2: All FA-s were given as FA-s of two types.
3. Calculation N:3: All FA-s were given as FA-s of seven types.

Tables I, II demonstrate calculational results for test N: 1. One can see that all results are practically identical.

**Table I. Values of $k_{inf}$ for test N:1**

| FA | Calc. N:0 | Calc. N:1, 2, 3 |
|---|---|---|
| U42G6 | 1.254797 | 1.254800 (2.4E-04 %) |
| Pu39G8 | 1.159544 | 1.159546 (1.7E-04 %) |

**Table II. Deviation in % of local functionals for test N:1 from calculation N:0**

| FA | SUHAM-U, Calc. N:1, 2, 3 | | | |
|---|---|---|---|---|
| | $\delta\overline{\Phi}_g^{\max}$ | $RMS(\delta\overline{\Phi}_g)$ | $\delta F^{\max}$ | $RMS(\delta F)$ |
| U42G6 | 0.0075 | 0.0022 | 0.0015 | 0.0002 |
| Pu39G8 | 0.0077 | 0.0024 | 0.0018 | 0.0004 |

### 3.2. Test N:2.

Object consisting of seven identical FA-s with uranium fuel consisting of 397 identical cells U42 [2] and with zero group neutron currents on the external boundary was calculated. Comparison was carried out with calculational results of separate cell by RACIA [6] module of SUHAM-U code with zero group neutron currents on the external cell boundary. Cell $k_{inf}$ value equals 1.3061396. Three-stage calculation of this object gives $k_{inf}$ value equaled cell $k_{inf}$ value with all significant figures. Besides, three-stage calculation reconstructs the group zone fluxes practically exactly. Maximum deviation doesn't exceed 2.7E-04 %.

### 3.3. Test N:3.

Objects consisting of seven FA-s with uranium (U42G6) and MOX (Pu39G8) fuel with zero group neutron currents on the external boundary were calculated. Fig. 1 shows the composition of calculated object. Two variants of object were calculated:

- Variant 1: FA Pu39G8 is in the centre of object
- Variant 2: FA U42G6 is in the centre of object

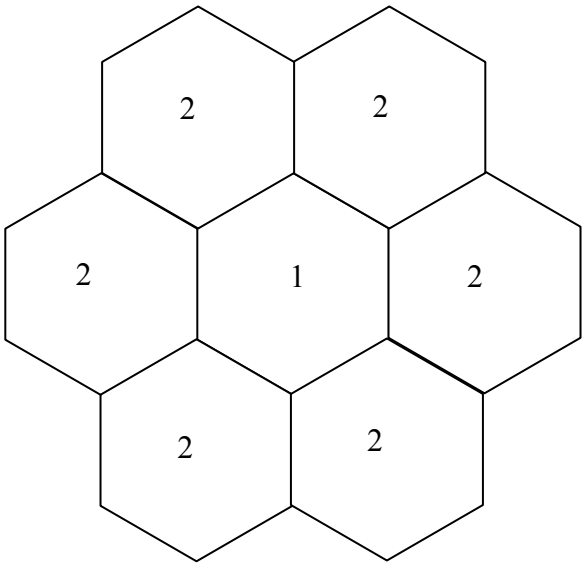


**Figure 1. Composition of calculated object in test N:3.**

Calculations were carried out with 3, 4, 5 and 6 trial matrices per each FA (calculations SUH-I, I=3,4,5,6). So long as calculations of this test with other codes don't exist, comparisons between calculations by SUHAM-U code with different trial matrices per each FA were carried out. Table III demonstrates the values of $k_{eff}$ in calculations SUH-I and deviations in % from values in calculations SUH-6. Table IV demonstrates assembly average fission reaction rates in calculations SUH-I and deviations in % from values in calculations SUH-6. Table V demonstrates maximum and root mean square deviations in % of fission reaction rates in fuel pins (Max(δF) and RMS(δF) correspondingly) from values in calculations SUH-6.

**Table III. Values of $k_{eff}$ for test N:3**

| Var. | SUH-3 | SUH-4 | SUH-5 | SUH-6 |
|---|---|---|---|---|
| **1** | 1.240799<br>0.03*) | 1.240653<br>0.02 | 1.240535<br>0.01 | 1.240371 |
| **2** | 1.171321<br>0.03 | 1.171195<br>0.02 | 1.171098<br>0.01 | 1.170960 |

*) Deviation in % from value in calculation SUH-6.

**Table IV. Assembly average fission reaction rates for test N:3**

| Variant | FA | SUH-3 | SUH-4 | SUH-5 | SUH-6 |
|---|---|---|---|---|---|
| **1** | U42G6,<br>2 row | 1.0287<br>0.41*) | 1.0305<br>0.59 | 1.0261<br>0.16 | 1.0245 |
| | U42G6<br>1 and 3 rows | 1.0287<br>0.41 | 1.0256<br>0.11 | 1.0261<br>0.16 | 1.0245 |
| | P39G8 | 0.8280<br>-2.94 | 0.8366<br>-1.93 | 0.8433<br>-1.15 | 0.8531 |
| **2** | U42G6 | 1.1937<br>2.64 | 1.1829<br>1.971 | 1.1747<br>1.01 | 1.1630 |
| | P39G8<br>2 row | 0.9677<br>-0.52 | 0.9666<br>-0.64 | 0.9709<br>-0.20 | 0.9728 |
| | P39G8<br>1 and 3 rows | 0.9677<br>-0.52 | 0.9710<br>-0.19 | 0.9709<br>-0.20 | 0.9728 |

*) Deviation in % from value in calculation SUH-6.

**Table V. Deviations in % of fission reaction rates in fuel pins (Max(δF) and RMS(δF)) from values in calculation SUH-6 for test N:3**

| Variant | Value | FA | SUH-3 | SUH-4 | SUH-5 |
|---|---|---|---|---|---|
| 1 | Max(δF) | U42G6, 1 and 3 rows | 9.1 | 7.5 | 3.0 |
| | | U42G6, 2 row | 8.9 | 2.8 | 3.1 |
| | | P39G8 | 9.4 | 2.8 | 3.1 |
| | RMS(δF) | U42G6, 1 and 3 rows | 2.0 | 1.8 | 0.9 |
| | | U42G6, 2 row | 2.0 | 0.9 | 0.9 |
| | | P39G8 | 2.1 | 0.9 | 0.9 |
| 2 | Max(δF) | U42G6 | 8.2 | 3.3 | 2.9 |
| | | P39G8, 1 and 3 rows | 8.1 | 7.9 | 2.9 |
| | | P39G8, 2 row | 7.9 | 3.3 | 2.9 |
| | RMS(δF) | U42G6 | 2.1 | 1.1 | 1.1 |
| | | P39G8, 1 and 3 rows | 2.1 | 2.0 | 1.1 |
| | | P39G8, 2 row | 2.1 | 1.1 | 1.1 |

One can see the good convergence of results to the values in calculations SUH-6 both for assembly functionals and for pin ones.

### 3.4. Test N:4.

The international benchmark for 2D VVER-1000 MOX core [5] was calculated in this test. The benchmark model consists of a full-size 2-D VVER-1000 core with heterogeneous 30% MOX fuel loading. The core consists of fresh and burned FA-s of the following types: graded UOX with U-Gd burnable absorber rods (BA) and graded, profiled MOX FA with U-Gd BA rods. Twenty eight assemblies in 60° rotation angles are considered. The system has an infinite axial dimension and vacuum condition on the side surface. A pattern of core is shown in Fig. 2.
The following designations are used in Fig. 2: N is the number of the assembly (N=1,…,28), type of assembly: type=1 for UOX and type=2 for MOX FA, numbers at the bottom indicate the burn-up value in MWd/kg. More detail information for benchmark including patterns of FA-s and isotopic compositions for all materials see in work [5]. Only state 1 without absorber rods was considered here. Results of benchmark are: $k_{eff}$, assembly average fission reaction rates for 28 assemblies and pin average fission reaction rates within assemblies N:3, N:21 and N:27 (see Fig. 2).

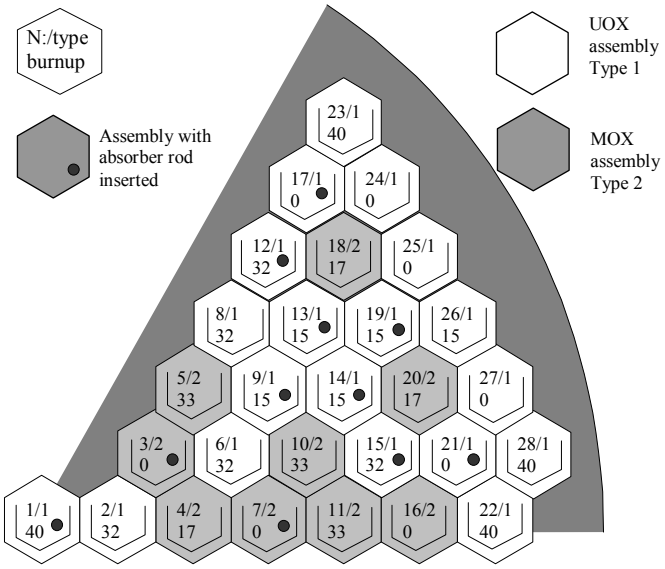


**Figure 2. Pattern of the VVER core with 30% MOX-fuel loading**

Table VI demonstrates values of $k_{eff}$ calculated by SUHAM-U code with different number of trial function per each FA and comparison with MCNP and MCU Monte-Carlo calculations. One can see that value of $k_{eff}$ in calculation SUH-6 lies between values calculated by MCNP and MCU codes.

**Table VI. Values of $k_{eff}$ for test N:4**

| SUH-3 | SUH-4 | SUH-5 | SUH-6 | MCNP | MCU |
|---|---|---|---|---|---|
| 1.03906 | 1.03803 | 1.03717 | 1.03661 | 1.03770 | 1.03341 |
| 0.13*) | 0.03*) | -0.05 | -0.11 | | |
| 0.55**) | 0.45**) | 0.36 | 0.31 | | |

*) Deviation in % from MCNP calculation; **) Deviation in % from MCU calculation

Table VII demonstrates assembly average fission reaction rates for 28 assemblies in calculation SUH-6 and deviations of corresponding values in calculations SUH-I from values calculated by MCNP and MCU codes.

**Table VII. Comparison of assembly average fission reaction rates in calculations SUH-I with values calculated by MCNP and MCU codes for test N:4**

| N | Fission rate | $\delta F_{MCNP,I}$ $\delta F_{MCU,I}$ % | | | |
|---|---|---|---|---|---|
| | SUH-6 | SUH-3 | SUH-4 | SUH-5 | SUH-6 |
| **1** | 0.778 | -17.3, -16.1 | 0.3, 1.7 | 6.7, 8.2 | 1.8, 3.3 |
| **2** | 0.934 | -15.9, -15.2 | -1.3, -0.4 | 2.5, 3.4 | 0.6, 1.5 |
| **3** | 1.226 | -12.0, -11.9 | 1.2, 1.3 | 4.1, 4.2 | 0.0, 0.1 |
| **4** | 1.091 | -13.9, -13.7 | -0.8, -0.5 | 0.4, 0.6 | -0.8, -0.5 |
| **5** | 0.932 | -14.5, -13.8 | -4.0, -3.3 | -0.2, 0.5 | -0.9, -0.1 |
| **6** | 0.994 | -12.5, -12.0 | -0.8, -0.3 | 1.0, 1.5 | 0.0, 0.5 |
| **7** | 1.183 | -7.6, -7.9 | 0.8, 0.6 | 3.1, 2.8 | 0.3, 0.0 |
| **8** | 0.999 | -8.8, -8.6 | -2.1, -1.9 | 0.0, 0.2 | 0.0, 0.2 |
| **9** | 1.297 | -6.0, -6.2 | 0.6, 0.4 | 1.7, 1.5 | 0.1, -0.2 |
| **10** | 0.916 | -10.5, -10.3 | -3.7, -3.5 | -0.8, -0.5 | -0.7, -0.4 |
| **11** | 0.86 | -8.1, -7.9 | -3.2, -3.0 | 0.1, 0.3 | -0.5, -0.2 |
| **12** | 1.004 | -3.6, -3.5 | -0.5, -0.4 | -1.0, -0.9 | -0.5, -0.4 |
| **13** | 1.384 | -1.5, -1.8 | -0.4, -0.6 | -1.0, -1.3 | -0.4, -0.6 |
| **14** | 1.354 | -1.6, -1.8 | -0.7, -0.8 | -1.1, -1.2 | -0.5, -0.7 |
| **15** | 0.973 | -3.5, -3.8 | -2.1, -2.4 | -0.7, -1.0 | -0.4, -0.7 |
| **16** | 1.147 | 3.6, 2.7 | -1.6, -2.5 | -2.6, -3.4 | -1.1, -2.0 |
| **17** | 1.201 | 11.2, 11.1 | 4.4, 4.3 | -0.2, -0.2 | 0.0, -0.1 |
| **18** | 1.137 | 2.3, 1.7 | -0.4, -1.0 | -3.0, -3.6 | -1.1, -1.7 |
| **19** | 1.321 | 5.1, 4.5 | 1.1, 0.6 | -1.1, -1.6 | 0.2, -0.4 |
| **20** | 1.08 | 1.6, 1.4 | -4.1, -4.4 | -3.1, -3.4 | -1.5, -1.7 |
| **21** | 1.162 | 10.7, 11.1 | 0.8, 1.1 | -0.4, -0.1 | -0.3, 0.1 |
| **22** | 0.559 | 4.1, 4.2 | -4.1, -3.9 | -2.1, -1.9 | -1.2, -1.1 |
| **23** | 0.36 | 12.3, 12.9 | -3.8, -3.3 | -3.8, -3.3 | -1.6, -1.1 |
| **24** | 0.888 | 22.9, 22.9 | 3.9, 3.9 | 1.4, 1.4 | 2.3, 2.3 |
| **25** | 1.014 | 22.4, 22.6 | 7.9, 8.2 | 2.7, 3.0 | 3.2, 3.4 |
| **26** | 0.843 | 18.0, 17.7 | 3.5, 3.3 | 1.5, 1.2 | 2.7, 2.4 |
| **27** | 0.822 | 21.6, 21.7 | 4.8, 5.0 | 1.0, 1.1 | 1.9, 2.0 |
| **28** | 0.347 | 11.6, 12.3 | -2.0, -1.4 | -3.7, -3.1 | -1.7, -1.1 |
| **Max** | | 22.9, 22.9 | 7.9, 8.2 | 6.7, 8.2 | 3.2, 3.4 |
| **RMS** | | 12.0, 11.9 | 3.0, 2.9 | 2.4, 2.6 | 1.3, 1.4 |

The following designations are used in Table VII: F – assembly average fission reaction rate,

$$\delta F_{L,I} = 100 * (1 - F_L / F_{SUH-I}), \text{ here L=\{MCNP, MCU\}, I=3,4,5,6.} \quad (19)$$

Fig. 3 shows the deviations of assembly average fission reaction rates for 28 assemblies in calculation SUH-6 from values calculated by MCNP and MCU codes.

One can see from Table VII and Fig. 3:

- Convergence of results in calculations SUH-I to corresponding values calculated by MCNP and MCU codes.
- Deviation of assembly average fission reaction rates in calculation SUH-6 from corresponding values calculated by MCNP and MCU codes doesn't exceed 3.5 %

Table VIII shows comparison calculations of pin average fission reaction rates within assemblies N:3, N:21 and N:27, namely maximum and root mean square deviations in % ($Max(\delta F_{MCNP,I})$ and $RMS(\delta F_{MCNP,I})$, I=SUH-3,…, SUH-6). Comparison was carried out only with MCNP results. One can see:

- Convergence of results in calculations SUH-I to corresponding values calculated by MCNP code.
- For calculations SUH-6, $Max(\delta F_{MCNP,SUH-6})$ doesn't exceed 4.7 % and $RMS(\delta F_{MCNP, SUH-6})$ doesn't exceed 1.6 % for all considered FA-s.

It should be noted that there is very big changing the pin average fission reaction rate for FA N: 27: from 0.45 till 1.58, that it is very hard test.

Fig. 4 shows the deviations of pin average fission reaction rates within assemblies N:27 in calculation SUH-6 from values calculated by MCNP and MCU codes for diagonal direction from bottom left pin to top right pin. These pins include the pins with maximum deviations.

## 3. CONCLUSIONS

Formulas used in SHM for reconstruction of local neutron-physical functionals in three-stage 2D calculation of VVER core have been described and verified. Comparisons were carried out both for technical tests and for international 2D VVER MOX core computational benchmark. It was shown:

- Convergence of SUH-I calculations to SUH-6 calculation.
- Convergence of SUH-I calculations to MCNP and MCU calculations both for assembly and pin functionals.
- Only SUH-6 calculations give enough accuracy in comparison with Monte-Carlo calculations both for assembly and pin functionals.
- Deviations of SUH-6 calculation from Monte-Carlo calculations for assembly average fission reaction rates don't exceed: 3.4 % in $Max(\delta F_{L,SUH-6})$ and 1.4 % in $RMS(\delta F_{L, SUH-6})$.
- Deviations of SUH-6 calculations from Monte-Carlo calculations for pin average fission reaction rates don't exceed: 4.7 % in $Max(\delta F_{L,SUH-6})$ and 1.6 % in $RMS(\delta F_{L, SUH-6})$.

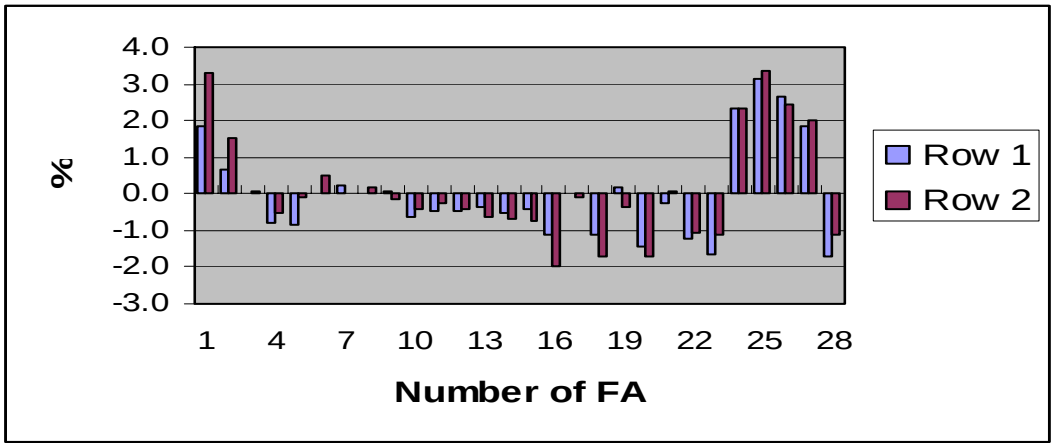


**Figure 3. Deviations of assembly average fission reaction rates in calculation SUH-6 from values calculated by MCNP (row 1) and MCU (row 2) codes for test N:4.**

**Table VIII. Comparison calculations of pin average fission reaction rates within assemblies N:3, N:21 and N:27 for test N:4**

| Value | FA N:3, % | FA N:21, % | FA N:27, % |
|---|---|---|---|
| $Max(\delta F_{MCNP,SUH-3})$ | 6.1 | 11.8 | 11.4 |
| $RMS(\delta F_{MCNP,SUH-3})$ | 1.8 | 3.5 | 4.7 |
| $Max(\delta F_{MCNP,SUH-4})$ | 5.1 | 6.3 | 12.0 |
| $RMS(\delta F_{MCNP,SUH-4})$ | 1.5 | 1.9 | 3.4 |
| $Max(\delta F_{MCNP,SUH-5})$ | 4.6 | 5.3 | 7.8 |
| $RMS(\delta F_{MCNP,SUH-5})$ | 1.3 | 1.8 | 2.6 |
| $Max(\delta F_{MCNP,SUH-6})$ | 4.2 | 4.4 | 4.7 |
| $RMS(\delta F_{MCNP,SUH-6})$ | 1.2 | 1.1 | 1.6 |

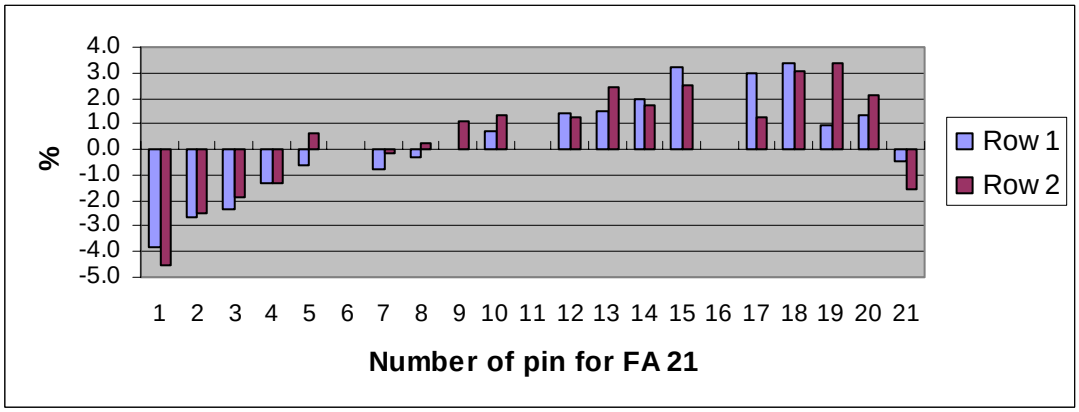


**Figure 4. Deviations of pin average fission reaction rates within assembly N:27 in calculation SUH-6 from values calculated by MCNP (row 1) and MCU (row 2) codes for diagonal direction from bottom left pin to top right pin for test N:4.**

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