Joint International Topical Meeting on Mathematics & Computation and Supercomputing in Nuclear Applications (M&C + SNA 2007)
Monterey, California, April 15-19, 2007, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2007)

AN ALGORITHM FOR EFFICIENT COMPUTATION OF MULTI-LAYER NEUTRON REFLECTION AND TRANSMISSION CONDITIONS

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ABSTRACT

Presented here is an algorithm for efficient computation of multi-layer neutron reflection and transmission conditions in matrix form. These conditions have been derived from the nodal equations of our hybrid spectral diamond-spectral Green's function method for one-speed slab-geometry discrete ordinates neutron multiplication eigenvalue problems, and are aimed to replace without spatial truncation error a multi-layered boundary region in spectral nodal reactor computations. In contrast to the independent layer-by-layer algorithm considered thus far in our computations, the algorithm here is based on an inductive approach developed by the present author for deriving neutron reflection and transmission conditions for a boundary region with an arbitrary number of arbitrarily thick layers. With this new algorithm, we increased the computational efficiency of our spectral diamond-spectral Green's function method for solving neutron multiplication eigenvalue problems with multi-layered boundary regions. We present comparative results for an axial reactor core model to illustrate the increased efficiency of our spectral diamond-spectral Green's function method, and we close this article with concluding remarks and directions for further research.

Key Words: Nuclear reactor, neutron multiplication, discrete ordinates, spectral nodal method, multi-layer neutron reflection and transmission, computational efficiency.

1. INTRODUCTION

In a recent article [1], we have derived mathematical conditions for neutron reflection and transmission for a multi-layered boundary region of a nuclear reactor core. These conditions have been derived from the nodal equations of our hybrid spectral diamond-spectral Green's function (SD-SGF) method for one-speed discrete ordinates (S_N) neutron multiplication eigenvalue problems, and are aimed to replace without spatial truncation error a multi-layer boundary region in the nodal reactor computations. A multi-layered boundary region is to be understood as a region consisting of two or more contiguous and disjoint uniform layers of nonactive (nonfissionable) material surrounding the active (fissionable) region of a nuclear reactor core, e.g. the baffle-reflector region of a pressurized water reactor core [2] and the rod follower-coolant channel- reflector-shielding region of a liquid metal fast breeder reactor core [2,3].

In this article, we present an algorithm for efficient computation of the above multi-layer neutron reflection and transmission conditions. In contrast to the independent layer-by-layer algorithm considered thus far in our computations [1], the algorithm here is based on an inductive approach developed by the present author for deriving neutron reflection and transmission conditions for a

boundary region with an arbitrary number of arbitrarily thick layers. By just replacing the algorithm considered thus far by this new algorithm, we were able to increase significantly the computational efficiency of our hybrid SD-SGF method for solving S_N neutron multiplication eigenvalue problems with multi-layered boundary regions.

The remaining sections of this article are organized as follows. In Section 2, we review the underlying theory and main results concerning our multi-layer neutron reflection and transmission conditions. In Section 3, we present an algorithm for efficiently computing multi-layer neutron reflection and transmission conditions in matrix form. In Section 4, we display comparative results for an axial reactor core model to illustrate the increased efficiency of our hybrid SD-SGF method for solving S_N neutron multiplication eigenvalue problems with multi-layered boundary regions. In Section 5, we conclude this article with a discussion and directions for further research.

2. MULTI-LAYER NEUTRON REFLECTION AND TRANSMISSION CONDITIONS

In this section, we review the basic theory and main results concerning our multi-layer neutron reflection and transmission conditions.

2.1. Problem Formulation and the Hybrid SD-SGF Method

The mathematical formulation of nuclear reactor core problems that we are concerned with in this article is the one-speed slab-geometry S_N equations for neutron multiplication eigenvalue problems [4]. These equations can be expressed in the form

$$\mu_m \frac{d}{dx}\psi^i_m(x)+\sigma_t(x)\psi^i_m(x)= \sum_{\ell=0}^{L}\frac{(2\ell+1)}{2}\sigma_\ell(x)P_\ell(\mu_m)\sum_{n=1}^{N}\omega_n P_\ell(\mu_n)\psi^i_n(x)+\frac{1}{2k_i}\nu\sigma_f(x)\sum_{n=1}^{N}\omega_n\psi^i_n(x)\,,\ m=1:N. \tag{1}$$

The quantity k_i is the ith eigenvalue, $\psi^j_m(x)$ is the associated eigenfunction defined for the directional cosine μ_m at position x on a multi-layer spatial domain, and the remaining notation is quite usual in the open literature. Equations (1) represent a differential eigenvalue problem whose dominant eigenvalue k_0 and associated nonnegative eigenfunction $\psi^0_m(x)$, $m = 1 : N$, are the neutron multiplication factor and a nonnegative function proportional to the angular flux of neutrons traveling in the cone of directions defined by the directional cosine μ_m at position x, respectively. Equations (1) are subject to homogeneous boundary conditions. The usual conditions are source-free vacuum applied on the outer boundary of the core domain and specular reflection to account for geometric and material symmetry [4]. We hereafter suppress the "0" notation and simply write k and $\psi_m(x)$ for the dominant eigenvalue and the corresponding eigenfunction, respectively.

The hybrid SD-SGF method is a numerical nodal method for the generation of the dominant numerical solution to the eigenvalue problem (1). It is a spatial discretization method having

three main ingredients. The first ingredient is the definition of a spatial grid on the core model (multi-layer) domain consisting of J uniform (constant cross sections) spatial cells. The second ingredient is the derivation of the standard zeroth-order neutron balance equations in each spatial cell, i.e.

$$\frac{\mu_m}{h_j}(\psi_{m,j+1/2}-\psi_{m,j-1/2})+\sigma_{t,j}\overline{\psi}_{m,j}= \sum_{\ell=0}^{L}\frac{(2\ell+1)}{2}\sigma_{\ell,j}P_\ell(\mu_m)\sum_{n=1}^{N}\omega_n P_\ell(\mu_n)\overline{\psi}_{n,j}+\frac{1}{2k}\nu\sigma_{f,j}\sum_{n=1}^{N}\omega_n\overline{\psi}_{n,j}\ ,\ j=1:J,\ m=1:N. \tag{2}$$

The quantity $\overline{\psi}_{m,j}$ is proportional to the jth cell-average angular flux of neutrons in direction μ_m, $m = 1 : N$, $\psi_{m,j\pm1/2}$ is proportional to the jth cell-edge angular fluxes on the right (+) and left (–) edges in direction μ_m, h_j is the thickness of the jth cell, and the remaining notation is apparent from the constant cross section assumption within the jth cell. The third ingredient is the derivation of two types of auxiliary equations. For cells in the active core, we consider the SD auxiliary equations

$$\overline{\psi}_{m,j}=\sum_{p=1}^{N}\gamma_{m,p}^{j}(\psi_{p,j-1/2}+\psi_{p,j+1/2}),m=1:N, \tag{3}$$

while for cells in the nonactive boundary regions we consider the SGF auxiliary equations

$$\overline{\psi}_{m,j}=\sum_{p=1}^{N/2}\theta_{m,p}^{j}\psi_{p,j-1/2}+\sum_{p=N/2+1}^{N}\theta_{m,p}^{j}\psi_{p,j+1/2}\ ,m=1:N. \tag{4}$$

Equations (2), (3), and (4) are the spatially discretized equations of the hybrid SD-SGF method for one-speed slab-geometry S_N multiplication eigenvalue problems with anisotropic scattering. These equations together with homogeneous S_N boundary conditions plus angular flux continuity conditions at cell interfaces form an algebraic eigenvalue problem whose dominant solution consists of numerical values for the dominant eigenvalue k and for the entries of the corresponding eigenvector $[\psi_{m,j\pm1/2}]^T$, $m = 1 : N$, $j = 1 : J$. For a detailed discussion of the advantages and limitations of the hybrid SD-SGF method for solving one-speed slab-geometry S_N multiplication eigenvalue problems with anisotropic scattering, the reader is referred to the basic Ref. [1].

2.2. Two-layer Neutron Reflection and Transmission Conditions

Let us consider a multislab model of a nuclear reactor core consisting of (R-2) contiguous and disjoint active layers and two nonactive boundary layers (R-1) and R (from left to right). Vacuum conditions apply on the right boundary of the core and specular reflection is assumed on the left boundary. Our aim here is to derive a real square matrix that represents the neutron reflecting power of the two boundary layers (R-1) and R. For then we impose one single cell on layer (R-1)

and one single cell on layer *R*. Since the rightmost cell of the spatial grid has index *J*, the indices for the cells on layers (*R*-1) and *R* are, respectively, (*J*-1) and *J*. Since layers (*R*-1) and *R* are nonactive, we consider the SGF auxiliary equations

$$\overline{\psi}_{m,J-1} = \sum_{p=1}^{N/2} \theta_{m,p}^{J-1} \psi_{p,J-3/2} + \sum_{p=N/2+1}^{N} \theta_{m,p}^{J-1} \psi_{p,J-1/2} \ , m = 1:N, \tag{5}$$

defined on the (*J*-1)st cell, and

$$\overline{\psi}_{m,J} = \sum_{p=1}^{N/2} \theta_{m,p}^{J} \psi_{p,J-1/2} + \sum_{p=N/2+1}^{N} \theta_{m,p}^{J} \psi_{p,J+1/2} \ , m = 1:N, \tag{6}$$

defined on the *J*th cell. Since source-free vacuum conditions apply on the right boundary of the core, $\psi_{p,J+1/2} = 0$ for all $p = N/2+1 : N$ [1], and the SGF auxiliary Eqs. (6) become

$$\overline{\psi}_{m,J} = \sum_{p=1}^{N/2} \theta_{m,p}^{J} \psi_{p,J-1/2} \ , m = 1:N. \tag{7}$$

Equations (2) for $j = J$ and (7) can be worked out [1] to yield the matrix equation

$$\vec{\psi}_{J-1/2}^{\,ref,R} = \underline{\underline{D}}^{J} \ \vec{\psi}_{J-1/2}^{\,in,R}, \tag{8}$$

where

$$\vec{\psi}_{J-1/2}^{\,ref,R} \equiv [\psi_{N/2+1,J-1/2}; \psi_{N/2+2,J-1/2}; \ldots; \psi_{N,J-1/2}]^{T} \tag{9}$$

is a column matrix whose entries are angular fluxes of neutrons that are reflected back by layer *R* at grid position *J*-1/2,

$$\vec{\psi}_{J-1/2}^{\,in,R} \equiv [\psi_{1,J-1/2}; \psi_{2,J-1/2}; \ldots; \psi_{N/2,J-1/2}]^{T} \tag{10}$$

is a column matrix whose entries are angular fluxes of neutrons that are incident upon layer *R* at grid position *J*-1/2, and $\underline{\underline{D}}^{J}$ is an (N/2)-dimensional real square matrix whose entries are

$$D_{m,p}^{J} = \frac{h_J}{|\mu_m|} \left[\sum_{\ell=0}^{L} \frac{(2\ell+1)}{2} \sigma_{\ell,J} P_{\ell}(\mu_m) \sum_{n=1}^{N} \omega_n P_{\ell}(\mu_n) \theta_{n,p}^{J} - \sigma_{t,J} \theta_{m,p}^{J} \right], \tag{11}$$

for $m = N/2+1 : N$ and $p = 1 : N/2$. We next substitute the SGF auxiliary Eqs. (5) into Eqs. (2) for $j = J$-1, we split the set of resulting equations into two subsets: one subset for $m = 1 : N/2$ and the other subset for $m = N/2+1 : N$, and the coming equations may be expressed in the matrix form

$$\vec{\psi}^{\,in,R}_{J-1/2} = \underline{\underline{E}}^{\,J-1}\,\vec{\psi}^{\,in,R-1}_{J-3/2} + \underline{\underline{D}}^{\,J-1}\,\vec{\psi}^{\,ref,R}_{J-1/2} \tag{12}$$

and

$$\vec{\psi}^{\,ref,R-1}_{J-3/2} = \underline{\underline{D}}^{\,J-1}\,\vec{\psi}^{\,in,R-1}_{J-3/2} + \underline{\underline{E}}^{\,J-1}\,\vec{\psi}^{\,ref,R}_{J-1/2}\,. \tag{13}$$

The quantities $\underline{\underline{E}}^{\,J-1}$ and $\underline{\underline{D}}^{\,J-1}$ are (N/2)-dimensional real square matrices with known entries

$$E^{J-1}_{m,p} = \delta_{mp} + \frac{h_{J-1}}{\mu_m}\left[\sum_{\ell=0}^{L}\frac{(2\ell+1)}{2}\sigma_{\ell,J-1}P_\ell(\mu_m)\sum_{n=1}^{N}\omega_n P_\ell(\mu_n)\theta^{J-1}_{n,p} - \sigma_{t,J-1}\theta^{J-1}_{m,p}\right],$$
$$m = 1 : N/2,\ p = 1 : N/2, \tag{14}$$

where δ_{mp} is the Kronecker delta function, and

$$D^{J-1}_{m,p} = \frac{h_{J-1}}{|\mu_m|}\left[\sum_{\ell=0}^{L}\frac{(2\ell+1)}{2}\sigma_{\ell,J-1}P_\ell(|\mu_m|)\sum_{n=1}^{N}\omega_n P_\ell(\mu_n)\theta^{J-1}_{n,p} - \sigma_{t,J-1}\theta^{J-1}_{m,p}\right],$$
$$m = N/2{+}1 : N,\ p = 1 : N/2. \tag{15}$$

Equations (8), (12) and (13) can be reformulated further [1] to yield

$$\vec{\psi}^{\,ref,R-1}_{J-3/2} = \underline{\underline{B}}^{\,R-1,R}\,\vec{\psi}^{\,in,R-1}_{J-3/2}\,, \tag{16}$$

where the real matrix

$$\underline{\underline{B}}^{\,R-1,R} = \underline{\underline{D}}^{\,J-1} + \underline{\underline{E}}^{\,J-1}\left(\underline{\underline{I}} - \underline{\underline{D}}^{\,J}\,\underline{\underline{D}}^{\,J-1}\right)^{-1}\underline{\underline{D}}^{\,J}\,\underline{\underline{E}}^{\,J-1} \tag{17}$$

represents the neutron reflecting power of the two boundary layers (R-1) and R. A similar result can be found for the boundary layers 1 and 2 on the left side of the reactor core model.

We now describe a two-layer (matrix) model to the diffuse transmission of neutrons that is free from spatial truncation error. We substitute Eq. (7) into Eq. (2) for $j = J$, we let m vary from 1 to N/2, and we reformulate to write the matrix equation

$$\vec{\psi}^{\,out}_{J+1/2} = \underline{\underline{E}}^{\,J}\,\vec{\psi}^{\,in,R}_{J-1/2}\,, \tag{18}$$

where $\underline{\underline{E}}^{\,J}$ is a (N/2)-dimensional real square matrix whose entries follow from a general expression analogous to (15), and

$$\vec{\psi}^{\,out}_{J+1/2} \equiv [\psi_{1,J+1/2}; \psi_{2,J+1/2}; \ldots; \psi_{N/2,J+1/2}]^T \tag{19}$$

is a column matrix whose entries are angular fluxes of neutrons that leak out from the nuclear reactor core. After some work with Eqs. (8), (13), and (18), we can obtain the matrix equation

$$\vec{\psi}^{\,out}_{J+1/2} = \underline{\underline{T}}^{R-1,R}\, \vec{\psi}^{\,in,R-1}_{J-3/2}, \tag{20}$$

where the real matrix

$$\underline{\underline{T}}^{R-1,R} = \underline{\underline{E}}^{J} \left(\underline{\underline{I}} - \underline{\underline{D}}^{J-1}\, \underline{\underline{D}}^{J} \right)^{-1} \underline{\underline{E}}^{J-1} \tag{21}$$

gives us an account of the diffuse transmission of neutrons through the nonactive layers (R-1) and R. A similar result can also be found for the boundary layers 1 and 2 on the left side of the reactor core model.

2.3. Extension to an Arbitrary Number of Layers

Suppose that we have found the matrices $\underline{\underline{B}}^{R-K+1,R}$ and $\underline{\underline{T}}^{R-K+1,R}$, which govern the equations

$$\vec{\psi}^{\,ref,R-K+1}_{J-K+1/2} = \underline{\underline{B}}^{R-K+1,R}\, \vec{\psi}^{\,in,R-K+1}_{J-K+1/2} \tag{22}$$

and

$$\vec{\psi}^{\,out}_{J+1/2} = \underline{\underline{T}}^{R-K+1,R}\, \vec{\psi}^{\,in,R-K+1}_{J-K+1/2}, \tag{23}$$

for the reflection and transmission of neutrons for K nonactive boundary layers, with $K > 1$. Our aim here is to determine the real matrices $\underline{\underline{B}}^{R-K,R}$ and $\underline{\underline{T}}^{R-K,R}$ representing the neutron reflecting power and transmittance of (K+1) nonactive boundary layers in terms of the real matrices for K nonactive boundary layers and of matrices $\underline{\underline{E}}^{J-K}$ and $\underline{\underline{D}}^{J-K}$ defined on the (K+1)st outermost layer.

We start with the derivation of $\underline{\underline{B}}^{R-K,R}$. For this task, we consider the balance Eqs. (2) and the SGF auxiliary Eqs. (4) for j = (J-K). We substitute Eqs. (4) into Eqs. (2), we split the resulting equations as we did before, and after some algebra we obtain the matrix equations

$$\vec{\psi}^{\,in,R-K+1}_{J-K+1/2} = \underline{\underline{E}}^{J-K}\, \vec{\psi}^{\,in,R-K}_{J-K-1/2} + \underline{\underline{D}}^{J-K}\, \vec{\psi}^{\,ref,R-K+1}_{J-K+1/2} \tag{24}$$

and

$$\vec{\psi}^{\,ref,R-K}_{J-K-1/2} = \underline{\underline{D}}^{J-K}\, \vec{\psi}^{\,in,R-K}_{J-K-1/2} + \underline{\underline{E}}^{J-K}\, \vec{\psi}^{\,ref,R-K+1}_{J-K+1/2}. \tag{25}$$

We now left multiply Eq. (24) by $\underline{\underline{B}}^{R-K+1,R}$, we substitute Eq. (22) into the resulting equation, and after some matrix algebra we arrive at

$$\vec{\psi}_{J-K+1/2}^{\,ref,R-K+1} = \left(\underline{\underline{I}} - \underline{\underline{B}}^{R-K+1,R}\,\underline{\underline{D}}^{J-K}\right)^{-1} \underline{\underline{B}}^{R-K+1,R}\,\underline{\underline{E}}^{J-K}\,\vec{\psi}_{J-K-1/2}^{\,in,R-K}. \tag{26}$$

We substitute Eq. (26) into Eq. (25), and we obtain S_N return conditions in the compact form

$$\vec{\psi}_{J-K-1/2}^{\,ref,R-K} = \underline{\underline{B}}^{R-K,R}\,\vec{\psi}_{J-K-1/2}^{\,in,R-K}, \tag{27}$$

where the real matrix $\underline{\underline{B}}^{R-K,R}$, which represents the neutron reflecting power of (K+1) nonactive boundary layers, is given by

$$\underline{\underline{B}}^{R-K,R} = \underline{\underline{D}}^{J-K} + \underline{\underline{E}}^{J-K}\left(\underline{\underline{I}} - \underline{\underline{B}}^{R-K+1,R}\,\underline{\underline{D}}^{J-K}\right)^{-1} \underline{\underline{B}}^{R-K+1,R}\,\underline{\underline{E}}^{J-K}. \tag{28}$$

The derivation steps for determining the real matrix $\underline{\underline{T}}^{R-K,R}$ are quite similar. We substitute Eq. (22) into Eq. (24) to take $\vec{\psi}_{J-K+1/2}^{\,ref,R-K+1}$ out, we reformulate the resulting equation, and we obtain

$$\vec{\psi}_{J-K+1/2}^{\,in,R-K+1} = \left(\underline{\underline{I}} - \underline{\underline{D}}^{J-K}\,\underline{\underline{B}}^{R-K+1,R}\right)^{-1} \underline{\underline{E}}^{J-K}\,\vec{\psi}_{J-K-1/2}^{\,in,R-K}. \tag{29}$$

We left multiply Eq. (29) by $\underline{\underline{T}}^{R-K+1,R}$, we substitute Eq. (23) into the resulting equation and we obtain S_N transmission conditions in the compact form

$$\vec{\psi}_{J+1/2}^{\,out} = \underline{\underline{T}}^{R-K,R}\,\vec{\psi}_{J-K-1/2}^{\,in,R-K}, \tag{30}$$

where the real matrix $\underline{\underline{T}}^{R-K,R}$ is given by

$$\underline{\underline{T}}^{R-K,R} = \underline{\underline{T}}^{R-K+1,R}\left(\underline{\underline{I}} - \underline{\underline{D}}^{J-K}\,\underline{\underline{B}}^{R-K+1,R}\right)^{-1} \underline{\underline{E}}^{J-K}. \tag{31}$$

Results (28) and (31) are inductive extensions of results (17) and (21), respectively, to an arbitrary number of nonactive boundary layers. Once we have obtained the real matrices $\underline{\underline{B}}^{R-K+1,R}$ and $\underline{\underline{T}}^{R-K+1,R}$ representing the neutron reflecting power and transmittance of K nonactive boundary layers, respectively, with $K \geq 1$, we are able to determine the real matrices $\underline{\underline{B}}^{R-K,R}$ and $\underline{\underline{T}}^{R-K,R}$ representing the neutron reflecting power and transmittance of (K+1) nonactive boundary layers. We note that our two-layer results (17) and (21) can be stated in terms of our more general results (28) and (31) if we define the single layer quantities $\underline{\underline{B}}^{R,R} \equiv \underline{\underline{D}}^{J}$ and $\underline{\underline{T}}^{R,R} \equiv \underline{\underline{E}}^{J}$. These matrices represent the neutron reflecting power and

transmittance of layer *R*, respectively. Analogous results can be found for an arbitrary number of nonactive layers on the left side of the reactor core model.

3. AN ALGORITHM BASED ON THE INDUCTIVE RESULTS (28) AND (31)

In this section, we present an algorithm for efficient computation of the multi-layer matrices that represent the neutron reflecting power (28) and transmittance (31) of a multi-layered boundary region with an arbitrary number (*NL*) of arbitrarily thick layers. We assume for convenience that the multi-layered boundary region is located on the right side of a nuclear reactor core model. The left side case is completely analogous.

The algorithm here is based on the inductive approach of Subsection 2.3. We begin by defining and computing the matrices $\underline{\underline{B}}^{updated}$ and $\underline{\underline{T}}^{updated}$ using expressions (11) and (14) for $j = J$, respectively. That is to say,

$$\underline{\underline{B}}^{updated} = \underline{\underline{B}}^{R,R} \text{ and } \underline{\underline{T}}^{updated} = \underline{\underline{T}}^{R,R}.$$

If $NL = 1$ (this is the one single layer case), then we are done with the matrix computations. Otherwise ($NL > 1$) we do the following:

i) We define the matrices

$\underline{\underline{B}}^{local}$, $\underline{\underline{T}}^{local}$, and $\underline{\underline{B}}^{available}$.

ii) For *K* varying from 1 to (*NL*–1):

ii.1) we make the change of variable $J \rightarrow R–K$ and $J–1 \rightarrow R–K$ in expressions (11) and (14), respectively;

ii.2) we use the resulting expressions to compute the matrices $\underline{\underline{B}}^{local} = \underline{\underline{B}}^{R-K,R-K}$ and $\underline{\underline{T}}^{local} = \underline{\underline{T}}^{R-K,R-K}$, respectively;

ii.3) we set $\underline{\underline{B}}^{available} = \underline{\underline{B}}^{updated}$, we make use of expressions (28) and (31) with appropriate changes in superscript notation to compute

$$\underline{\underline{B}}^{updated} = \underline{\underline{B}}^{local} + \underline{\underline{T}}^{local}\left(\underline{\underline{I}} - \underline{\underline{B}}^{available}\underline{\underline{B}}^{local}\right)^{-1}\underline{\underline{B}}^{available}\underline{\underline{T}}^{local}$$

and

$$\underline{\underline{T}}^{updated} = \underline{\underline{T}}^{updated}\left(\underline{\underline{I}} - \underline{\underline{B}}^{local}\,\underline{\underline{B}}^{available}\right)^{-1}\underline{\underline{T}}^{local},$$

and we proceed to a new K. After (NL–1) steps, we get $\underline{\underline{B}}^{updated} = \underline{\underline{B}}^{R-NL+1,R}$ and $\underline{\underline{T}}^{updated} = \underline{\underline{T}}^{R-NL+1,R}$.

We have written a computer version of this algorithm in standard FORTRAN to replace our former routine based on a (less efficient) independent layer-by-layer approach [1]. Input data are the order N of angular quadrature, the corresponding Gauss-Legendre points and weights, the transport properties of the current layer, and the corresponding constants $\theta_{r,m,p}$. Computer memory requirements for matrix storage are $8*N^2/2 = 4N^2$ bytes for the $N^2/2$ constants $\theta_{r,m,p}$ (we need only to store the constants $\theta_{r,m,p}$ for the current layer) and $8*5*N^2/4 = 10N^2$ bytes for the five (N/2 x N/2) real square matrices

$$\underline{\underline{B}}^{local},\ \underline{\underline{T}}^{local},\ \underline{\underline{B}}^{available},\ \underline{\underline{B}}^{updated}, \text{ and } \underline{\underline{T}}^{updated},$$

in a total of $14N^2$ bytes for double precision computations. We note that additional memory for matrix inversion in step ii.3 is not needed. This is because computer memory for the constants $\theta_{r,m,p}$ is released as soon as the local matrices $\underline{\underline{B}}^{local}$ and $\underline{\underline{T}}^{local}$ become available. We use this released memory for the auxiliary matrices involved in matrix inversion in step ii.3. Matrix inversion is carried out using Crout's method with implicit pivoting [6], while the remaining matrix operations (addition and multiplication) are quite trivial to implement.

4. NUMERICAL RESULTS

In this section, we perform a numerical experiment to illustrate that, by just replacing our former independent layer-by-layer algorithm by the algorithm of Section 3 to generate the multi-layer matrices

$$\underline{\underline{B}}^{R-NL+1,R} \text{ and } \underline{\underline{T}}^{R-NL+1,R},$$

we can increase the computational efficiency of our hybrid SD-SGF method for solving S_N neutron multiplication eigenvalue problems with multi-layered boundary regions. For this purpose, we consider an axial reactor core model consisting of three active layers followed by a nonactive baffle-reflector region (from left to right). Reflective conditions apply on the left boundary and source-free vacuum conditions apply on the right boundary of the core model. We assume that neutron scattering is isotropic in the active layers and linearly anisotropic in the baffle-reflector. The transport properties and thicknesses of all layers are provided in Ref. [1].

To illustrate the increased efficiency of our hybrid SD-SGF method, we perform the following numerical experiment:

i) we run our hybrid SD-SGF code to get S_8 dominant numerical results (k and thermal power distribution) for the above core model with the two-layered baffle-reflector region

replaced by the matrices $\underline{\underline{B}}^{R-1,R}$ and $\underline{\underline{T}}^{R-1,R}$ generated with our independent layer-by-layer algorithm;

ii) we run our hybrid SD-SGF code to get (the same) S_8 dominant numerical results with the two-layered baffle-reflector region replaced by the matrices $\underline{\underline{B}}^{R-1,R}$ and $\underline{\underline{T}}^{R-1,R}$ now generated with the algorithm of Section 3; and

iii) we compare computer memory location and execution (CPU) time for the runs in steps i and ii.

We stress that the only difference in steps i and ii is the algorithm (computer routine) used to generate the two-layer matrices. This is why the dominant numerical results generated in steps i and ii are the same. Since the hybrid SD-SGF method is free from spatial truncation error and no spatial approximation has been introduced along the derivation of the multi-layer matrices, the dominant numerical results generated in steps i and ii are exactly those obtained from the dominant analytical solution of the S_N Eqs. (1), aside of computational finite arithmetic considerations. The dominant numerical results for k and thermal power distribution can be found in Ref. [1].

In Table I, we provide computer memory location and execution (CPU) time for the runs in steps i and ii. We remark that our hybrid SD-SGF code was executed on a 32-bit IBM compatible PC (1.4 GHz Pentium 4 Intel processor, 256 Mbytes of RAM) running on a GNU/Linux platform.

Table I. Computer memory and execution (CPU) time.

	Memory (Kbyte)	**CPU (s)**
Step i	2.68	31.7
Step ii	2.19	23.4
% Reduction[a]	18.3	26.2

[a] % Reduction ≡ 100*(Step i – Step ii)/Step i.

It is apparent from the results in Table I that the algorithm of Section 3 (step ii) increased the computational efficiency of our hybrid SD-SGF code (note the percent reductions in both computer memory and execution time in the third row of results). This owes to a couple of reasons. First, our “brute force” independent layer-by-layer algorithm requires us to store the constants $\theta_{r,m,p}$ and the local matrices (11) and (14) for all nonactive layers of the boundary region, while the algorithm of Section 3 requires only the storage of the same quantities for the current (K) layer. Second, in our independent layer-by-layer algorithm, the multi-layer matrix $\underline{\underline{B}}^{R-NL+1,R}$ is generated, and only afterwards the multi-layer matrix $\underline{\underline{T}}^{R-NL+1,R}$ is determined. This

makes the independent layer-by-layer algorithm more time consuming than the algorithm of Section 3. This was so because the multi-layer matrix $\underline{\underline{B}}^{R-NL+1,R}$ provides us with the S_N return conditions needed for the dominant numerical solution of the S_N eigenvalue problem. The multi-layer matrix $\underline{\underline{T}}^{R-NL+1,R}$ is not used in the dominant numerical computations. It is rather a post-processing tool for computing the neutron leakage from the nuclear reactor core model [1].

At this point we proceed to Section 5, where we close this article with concluding remarks and we report ongoing research.

5. CONCLUDING REMARKS

We have presented an algorithm for efficiently computing the neutron reflection and transmission matrices that replace without spatial truncation error a nonactive multi-layered boundary region in one-speed slab-geometry S_N neutron multiplication eigenvalue problems. The algorithm here is based on an inductive approach developed by the present author for deriving neutron reflection and transmission conditions for a boundary region with an arbitrary number of arbitrarily thick layers. It is aimed to replace our former (and less efficient) independent layer-by-layer algorithm in order to increase the computational efficiency of our hybrid SD-SGF code for solving S_N multiplication eigenvalue problems with multi-layered boundary regions. To illustrate the computational merit of the algorithm presented here, we have provided computer memory location and execution time for the SD-SGF runs with either algorithm. All of these figures of merit were favorable to our new algorithm because of the reduced location of computer memory and the concurrent computation of reflection and transmission matrices.

We conclude by noting that the algorithm of Section 3 can be implemented “as is” in our multigroup SD-SGF code to generate multigroup multi-layer matrices for neutron reflection and transmission, and we are currently working on such implementation. We intend to give a detailed account of such implementation together with numerical results in a forthcoming article.

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