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INVESTIGATIONS ON THE MIGRATION MODE METHOD (MMM) FOR REACTOR CALCULATIONS

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ABSTRACT

Current calculation codes for reactor analysis are based on the multi-group method to evaluate energy distribution of neutron flux. Usually a two energy group diffusion equation is adopted. This choice is adequate for PWRs associated to cross sections libraries tabulated versus fuel exposure and other state parameters as moderator density, fuel temperature, boron concentration. An improvement of this approach is represented by the Migration Mode Method (MMM) by which the neutron spectrum is expanded in terms of base functions corresponding to the different modes of migration of the neutrons in the energy dimension. For a thermal reactor, three such functions may be easily identified: the Maxwellian distribution of the neutrons at thermal equilibrium with the moderator, the $1/E$ slowing down distribution (corrected to take into account resonance absorption effects) and the fission neutron spectrum. The (space-dependent) coefficients of the expansion are calculated by solving a differential equation which results having a structure similar to the one relevant to multi-group theory. The method can therefore be easily implemented adopting existing diffusion theory codes.
With the present work, some investigations on the MMM are described relevant to UO_2 fuelled PWR systems. Demonstrative results are given to validate the potentiality of the method.

Key Words: MMM method, base functions, PWR, few-group, neutron diffusion

1. INTRODUCTION

Current calculation codes for reactor analysis implement the multi-group method (MGM) to evaluate the energy distribution of neutron flux and a two-group diffusion model is largely adopted. In order to achieve accurate results the two-group model uses cross sections that conserve the reaction rates of fine multi-group calculations on an infinite medium condition. For this purpose the assembly cross sections are homogenized on the fine spectrum. Since the spectrum depends on the current conditions of the reactor, varying spatially, the cross sections depend on several parameters as moderator density, moderator temperature, fuel temperature,

hard absorbers densities. This dependency is treated by tabulating the cross sections versus these parameters. But local spectrum variations in the reactor can also be due to the proximity of very different fuel types or to the proximity of the reflector or to the insertion of a control rod in the neighboring assembly. This dependency, usually called the leakage effect, is not seen by the cross sections, which are homogenized on an infinity medium spectrum. Since the leakage effect takes importance with reload strategies as MOX/UO_2, AREVA NP has developed an innovative method (as an alternative of the multi-group method), the Migration Mode Method (MMM) [1,2], to implicitly represent the local spectrum variations produced by local conditions and leakage effects. In order to further investigate on the benefits of this method, AREVA NP has recently launched a research action involving collaborations with universities. The benefits searched with this method are:

- increase accuracy with the same number of unknown variables (2 energy groups versus 2 base functions),
- reduce the computation burden (CPU time and memory size) associated to the interpolation in the cross section tables,
- take into account for the leakage effect, neglected by the current methods.

The Migration Mode Method (MMM) is based on the Minimum Weighted Residual (MWR) methodology, in particular on the so called 'Subdomain Collocation Approach' [3]. The MMM approach intends to extend the MWR method, so far widely used in the space domain, to the neutron energy dimension. The aim of this paper is to show the ability of MMM to implicitly represent feed-backs due to variations in conditions as moderator density or boron concentration, etc. After a short description of the theory, applications of the method to a number of test cases relevant to UO_2 fuelled PWR are illustrated, making use of an interface module, connecting the cell code for the determination of the homogenized multi-group cross sections with the diffusion, few-group code.

2. THE METHOD

2.1. Theoretical Background

Let us write the reference governing equation of the neutron flux $\phi(\mathbf{r},E)$:

$$D\nabla^2\phi - \Sigma_t\phi + \int_o^\infty \Sigma_s(E'\to E)\phi(E')\mathrm{d}E' + \frac{1}{k_{eff}}\chi(E)\int_o^\infty \nu\Sigma_f(E')\phi(E')\mathrm{d}E' = 0 \qquad (1)$$

Define M base functions $q_1(\mathbf{r},E)$, ..., $q_M(\mathbf{r},E)$ and let us assume that the flux is approximated by

$$\tilde{\phi}(\mathbf{r},E) = \sum_{m=1}^{M} a_m(\mathbf{r})q_m(\mathbf{r},E) \qquad (2)$$

where $a_m(\mathbf{r})$ are space-dependent coefficients to be determined. Replacing (2) into (1), gives

$$\sum_{m=1}^{M} q_m D\nabla^2 a_m - \sum_{m=1}^{M} \Sigma_t q_m a_m + \sum_{m=1}^{M} \int_o^\infty q_m(E') a_m \Sigma_s(E' \to E) \mathrm{d}E' + \frac{\chi(E)}{k_{eff}} \sum_{m=1}^{M} \int_o^\infty \nu\Sigma_f(E') q_m(E') a_m \mathrm{d}E' = 0 \tag{3}$$

Let us define weight (or test) functions u_ℓ $(\ell = 0,1,2,...,M)$. Values u_ℓ are equal to unity within a given energy range and zero otherwise. The ranges within which they are defined may overlap. The base functions q_m may be arbitrarily normalized. In order that the governing equations approximate a standard group form, we shall assume that

$$\int_0^\infty u_\ell(E) q_\ell(E) \mathrm{d}E = 1 \tag{4}$$

Multiplying Eq. (3) by u_ℓ and integrating we obtain

$$\sum_{m=1}^{M} D_m^\ell \nabla^2 a_m - \sum_{m=1}^{M} \Sigma_{t,m}^\ell a_m + \sum_{m=1}^{M} a_m \Sigma_{s,m}^{\to\ell} + \frac{\chi^\ell}{k_{eff}} \sum_{m=1}^{M} \nu\Sigma_{f,m} a_m = 0 \tag{5}$$

having defined the generic quantity X_m^ℓ as:

$$X_m^\ell = \int_0^\infty u_\ell(E) X(E) q_m(E) \mathrm{d}E \tag{6}$$

where $X(E)$ is a cross section $\Sigma(E)$ or the diffusion coefficient $D(E)$ and

$$\Sigma_{s,m}^{\to\ell} = \int_o^\infty u_\ell(E) dE \sum_{m=1}^{M} \int_o^\infty q_m(E') \Sigma_s(E' \to E) \mathrm{d}E' \ ,$$

$$\chi^\ell = \int_0^\infty u_\ell \chi(E) \mathrm{d}E \ . \tag{7}$$

Assuming a fine group approximation and considering that the in- and out-scattering terms within the same energy range cancel each other, we obtain

$$\sum_{m=1}^{M} D_m^\ell \nabla^2 a_m - \sum_{m=1}^{M} \hat{\Sigma}_{t,m}^\ell a_m + \sum_{m=1}^{M} a_m \hat{\Sigma}_{s,m}^{\to\ell} + \frac{\chi^\ell}{k_{eff}} \sum_{m=1}^{M} \nu\Sigma_{f,m} a_m = 0 \tag{8}$$

where $\hat{\Sigma}_{s,m}^\ell = \sum_{g\in\ell} \sum_{g'\notin\ell} \Sigma_s^{g\to g'} q_{m,g}$, $\hat{\Sigma}_{s,m}^{\to\ell} = \sum_{g\in\ell} \sum_{g'\notin\ell} \Sigma_s^{g'\to g} q_{m,g'}$ and $\hat{\Sigma}_{t,m}^\ell = \Sigma_{a,m}^\ell + \hat{\Sigma}_{s,m}^\ell$.

2.2. Leakage Term

The leakage term $\sum_{m=1}^{M} D_m^{\ell} \nabla^2 a_m$, in cases in which more than one base function appears within the energy range where u_ℓ is defined, poses a problem if, as a reference, a standard diffusion equation is considered in which only diagonal leakage terms are allowed. A possible approach to solve this problem is described here. Consider the case of two base functions. The governing equations will be:

$$D_1^1 \nabla^2 a_1 + D_2^1 \nabla^2 a_2 - \hat{\Sigma}_{t,1}^1 a_1 - \hat{\Sigma}_{t,2}^1 a_2 + a_1 \hat{\Sigma}_{s,1}^{\to 1} + a_2 \hat{\Sigma}_{s,2}^{\to 1} + \frac{\chi^1}{k_{eff}} \sum_{m=1}^{M} \nu\Sigma_{f,m} a_m = 0 \tag{9}$$

and

$$D_1^2 \nabla^2 a_1 + D_2^2 \nabla^2 a_2 - \hat{\Sigma}_{t,1}^2 a_1 - \hat{\Sigma}_{t,2}^2 a_2 + a_1 \hat{\Sigma}_{s,1}^{\to 2} + a_2 \hat{\Sigma}_{s,2}^{\to 2} + \frac{\chi^2}{k_{eff}} \sum_{m=1}^{M} \nu\Sigma_{f,m} a_m = 0 \tag{10}$$

Multiplying Eq. (9) by D_2^2 and Eq. (10) by D_2^1, subtracting and setting

$$\overline{X}_i^1 = (D_2^2 X_i^1 - D_2^1 X_i^2), \quad (i = 1,2) \tag{11}$$

X_i^g representing a generic coefficient, we obtain the standard form

$$\overline{D}_1^1 \nabla^2 a_1 - (\overline{\Sigma}_{t,1}^1 a_1 + \overline{\Sigma}_{t,2}^1 a_2) + (\overline{\Sigma}_{s,1}^{\to 1} a_1 + \overline{\Sigma}_{s,2}^{\to 1} a_2) + \overline{\chi}^1 \sum_{m=1}^{M} \nu\Sigma_{f,m} a_m = 0 \tag{12}$$

Setting

$$\overline{X}_i^2 = (D_1^1 X_i^2 - D_1^2 X_i^1), \quad (i = 1,2) \tag{13}$$

Eq. (10) may be likewise written in the standard form

$$\overline{D}_2^2 \nabla^2 a_2 - (\overline{\Sigma}_{t,1}^2 a_1 + \overline{\Sigma}_{t,2}^2 a_2) + (\overline{\Sigma}_{s,1}^{\to 2} a_1 + \overline{\Sigma}_{s,2}^{\to 2} a_2) + \overline{\chi}^2 \sum_{m=1}^{M} \nu\Sigma_{f,m} a_m = 0 \tag{14}$$

A general solution may be envisaged adopting recurrently the above procedure. At the end, a generic ℓ'th equation will be of the form

$$D_\ell^\ell \nabla^2 a_\ell - \sum_{m=1}^{M} \overline{\Sigma}_{t,m}^{\ell} a_m + \sum_{m=1}^{M} a_m \overline{\Sigma}_{s,m}^{\to \ell} + \frac{\overline{\chi}^\ell}{k_{eff}} \sum_{m=1}^{M} \nu\Sigma_{f,m} a_m = 0 \tag{15}$$

2.3. The Multiplication Factor

In order to determine the multiplication factor, let us integrate over space and sum Eq. (15) for $\ell = 1,...,M$. It results, recalling that the in- and out-scattering compensate each other and then cancel out, the expression of the multiplication factor

$$k_{eff} = \frac{\sum_{\ell=1}^{M} \int_{vol} \mathrm{d}\mathbf{r}\, \overline{\chi}^{\ell} \sum_{m=1}^{M} \nu\Sigma_{f,m} a_m}{\sum_{\ell=1}^{M} \int_{vol} \mathrm{d}\mathbf{r} \sum_{m=1}^{M} \overline{\Sigma}_{a,m}^{\ell} a_m - \sum_{\ell=1}^{M} \int_{vol} \mathrm{d}\mathbf{r} D_{\ell}^{\ell} \nabla^2 a_{\ell}} \tag{16}$$

2.4. Base Functions

The base functions may be selected basing on physical laws governing the neutron density during the slowing down of neutrons from fission to thermal energies. They have been defined in order to represent the main behaviors in the migration of the neutrons in the energy domain:

- the thermal behavior (having the major magnitude below 0.625 eV), governed by the Maxwell distribution (which is assumed as a base function);
- the fast/epithermal behavior governed by a 1/*E* law in a slightly absorber medium;
- the fission emission behavior.

The maxwellian distribution is taken into account for the thermal behavior in all cases considered

$$\Phi_{\mathrm{M}}(E) = E \exp(-\frac{E}{kT_{\mathrm{n}}}) \tag{17}$$

where k is Boltzmann's constant and T_{n} is the neutron temperature.

The base function groups considered in the fast/intermediate/epithermal flux are described in the following subsections.

2.4.1. Base functions form 1

For the fast/intermediate/epithermal behavior the following slowing-down distribution has been considered:

$$\Phi_{\mathrm{SD}}(E) = \frac{\widetilde{\chi}^{\theta}(E) p^{\alpha}(E)}{E^{\beta} \Sigma_t^{\gamma}(E)} \tag{18}$$

where $\widetilde{\chi}(E) = \int_E^{\infty} \chi(E')\mathrm{d}E'$ is the cumulative fission spectrum, p is a coefficient of a form similar to the expression of the resonance escape probability, i.e. (defining $\Sigma_a = \Sigma_c + \Sigma_f$):

$$p(E) = \exp\left(-\int_E^{\infty} \frac{\Sigma_a}{\overline{\xi}\Sigma_s} \frac{\mathrm{d}E'}{E'} \right) \tag{19}$$

The fission spectrum and the slowing down distribution could be also considered as two independent base functions or, alternatively, coupled into a single base function, as follows:

$$\Phi_{\mathrm{SD}}(E) = \frac{\widetilde{\chi}^{\theta}(E) p^{\alpha}(E)}{E^{\beta} \Sigma_t^{\gamma}(E)} + f_{eM} \chi(E) \tag{20}$$

where $\alpha, \beta, \gamma, \theta$ and f_{eM} are parameters subject to possible adjustment (tuning).

2.4.2. Base functions form 2

Rather than Eq. (20), the following slowing down distribution is considered:

$$\Phi_{\mathrm{SD}}(E) = \frac{E\mathrm{e}^{\frac{b_1}{\sqrt{E}}}}{(E+E_{\mathrm{SD}})^{b_2}} + f_{eM}\chi(E) \qquad (21)$$

Parameters b_1, b_2 and f_{eM} are subject to possible adjustment (tuning).

3. RESULTS

Several tests have been performed with the interface module MMM and the code DEMO solving the diffusion equation with the multi-group (2 up to 99 groups) and migration mode methods.

3.1. The MMM Interface Module architecture

The MMM Interface Module is written in FORTRAN 90 and is composed of a number of sub-modules organized in a tree structure. The most important sub-modules are:

1. *the reading sub-module*: it allows to read from different input files the data specified as the energy ranges, the cross-sections, the base functions parameters, the test functions.
2. *the base functions sub-module*: this sub-module could read the base functions from an input file (numerical formulation) or built them according to the user defined input;
3. *the coefficients calculation sub-module*: this sub-module calculates the elements of the matrix (H) governing the MMM function a;
4. *the writing sub-module*: this sub-module writes on different files the matrix coefficients as "modified cross-sections".

The MMM Interface Module has been coupled to the DEMO code, with the aim to identifying the optimum sets of base functions for UO_2 fuelled PWR systems, performing both zero- and mono-dimensional calculations in diffusion theory.

3.2. Zero Dimensional Calculations

The tests performed with the MMM Interface Module consist of calculations of neutron flux, fission rate and k_{inf} value, for a UO_2 fuel composition in the following cases:

0. reference conditions;
1. fuel temperature variation;
2. moderator temperature variation;
3. moderator density variation;
4. boron concentration variation;

The purpose of these calculations was to verify the ability of MMM to implicitly represent feed-backs due to variations of conditions as moderator density or boron concentration, etc. Indeed, in a few-group approach (FGA), feed-backs could be taken into account only using parameterized tables of cross sections versus moderator density, fuel temperature, boron concentration, etc., because changing the conditions changes the spectrum on which the cross sections are homogenized. In this context, MMM simplifies cross section tables.

The MMM microscopic and macroscopic equivalent cross sections, as defined in section 2.1, have been computed in the same reference condition (case 0) using 99-group fine cross sections (calculated by APOLLO2 code [4]) in computing the integrals over the energy range. The MMM macroscopic cross sections in the perturbed conditions (cases 1, 2 and 3) have been computed using the microscopic cross sections computed in the reference condition. The cross sections have been corrected versus fuel temperature in order to take into account the Doppler effect. A few-group approach (two groups) has been used in order to compare with MMM. The two-group calculations have been performed using cross sections homogenized in the reference spectrum. This choice, which does not correspond to the procedure used in neutronic design, has been done only with the purpose to quantify the benefit of MMM to need less spectral calculations compared to FGA. In the comparisons this approach will be indicated by FGA. The results obtained are shown in Figures 1, 2, 3, and 4, which show the flux per unit lethargy versus energy and in Tables I, II, III and IV. It can be seen that MMM approximates with good accuracy the reference k_{inf} whereas FGA does not, unless parameterized cross sections tables or a spectral correction model are used. The reference MMM equivalent cross sections have been also used in the space dependent calculations (section 3.3).

3.3. Space Dependent Calculations for PWR Systems

Test cases have been carried out in mono-dimensional geometry (slab), considering an optimum set of two base functions, identified with the zero dimension tests. They have the same geometry consisting of 5 UO_2 slabs of 10 cm at different:

1. fuel temperature: 303°K, 553°K, 773°K, 873°K and 973°K;
2. moderator density: 0.55 g/cm^3, 0.60 g/cm^3, 0.65 g/cm^3, 0.70 g/cm^3 and 0.75 g/cm^3;
3. boron concentration: 0 ppm, 130 ppm, 260 ppm, 520 ppm and 779 ppm;

Comparisons have been done with two-group calculations performed with two different approaches: cross sections homogenized in the infinite medium spectrum relative to

- the reference condition (indicated by the symbol FGA);
- the condition of the slab (indicated by the symbol MGM).

The results obtained are shown in Figures 5, 6, and 7 and in Table V. It can be seen that the power distribution computed with MMM (forms 1 and 2) matches with good accuracy the 99-group calculation. The two-group calculations show good results but with the use of cross sections homogenized in the infinite medium spectrum relative to the condition of the slab (indicated by the symbol MGM). This point is well showed in the calculation with varying moderator density where the importance of using a good homogenization spectrum appears (Fig. 6). The results in terms of k_{eff} show a better evaluation of MMM with respect to two-group calculations (either FGA or MGM). This behavior is in part due to the fact that the cross sections of the two-group calculations have been collapsed from 99 to 2 groups using the infinite medium spectrum and the leakage in the 1D core configuration changes the spectrum. It must be noticed that using a fundamental mode spectrum for the collapse from 99 to 2 groups the discrepancy affecting the two-group calculations would be reduced. MMM result is closer to the reference because it is, in some extent, to be considered as a few-group method that self-homogenizes the cross sections in the right spectrum.

4. CONCLUSIONS

From this study it appears that the MMM methodology is a valid alternative to the standard few-group approach for the analysis of PWR systems. Thanks to its ability to implicitly represent the spectrum variations due to local conditions variations it needs data files (containing the MMM cross sections) less cumbersome with respect to the ones used with few-group calculations. Moreover leakage effects due to environment configurations as the ones encountered in reload strategies as MOX/UO_2, not seen by classical few-group calculations (unless an environment spectral correction is applied), can be taken into account by the method. The benefits of the method are therefore not only in computation time (less interpolation time in the cross sections tables) but also in accuracy. In order to implement MMM in an industrial code AREVA NP is pursuing the development effort which is oriented in the improvement of the base functions in order that they fit at best the local spectrum in a wide range of operational conditions.

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Table I. UO_2 0D – Fuel temperature effect: k_{inf} values (reference Tf = 873°K)

Tf (°K)	303	553	773	873	973
99 groups	1.31675	1.30553	1.29739	1.29405	1.29097
FGA 2 groups	1.31596	1.30505	1.29722	1.29405	1.29113
MMM 2 BF form 1	1.31648	1.30548	1.29740	1.29405	1.29094
MMM 2 BF form 2	1.31628	1.30520	1.29727	1.29405	1.29108

Table II. UO_2 0D – Moderator temperature effect: k_{inf} values (reference Tm = 503°K)

Tm (°K)	303	403	503	553	603
99 groups	1.29391	1.29389	1.29350	1.29325	1.29299
FGA 2 groups	1.29351	1.29351	1.29350	1.29350	1.29350
MMM 2 BF form 1	1.29462	1.29413	1.29350	1.29321	1.29293
MMM 2 BF form 2	1.29466	1.29415	1.29350	1.29321	1.29293

Table III. UO_2 0D – Moderator density effect: k_{inf} values (reference Dm = 0.65 g/cm^3)

Dm (g/cm^3)	0.55	0.60	0.65	0.70	0.75
99 groups	1.25169	1.26196	1.26981	1.27564	1.27975
FGA 2 groups	1.28981	1.28083	1.26981	1.25720	1.24335
MMM 2 BF form 1	1.25503	1.26344	1.26981	1.27446	1.27764
MMM 2 BF form 2	1.25323	1.26264	1.26981	1.27510	1.27879

Table IV. UO_2 0D – Boron concentration effect: k_{inf} values (reference CB = 260 ppm)

CB (ppm)	0	130	260	520	779
99 groups	1.36296	1.34559	1.32872	1.29639	1.26582
FGA 2 groups	1.36208	1.34516	1.32872	1.29716	1.26726
MMM 2 BF form 1	1.36293	1.34558	1.32872	1.29640	1.26583
MMM 2 BF form 2	1.36315	1.34568	1.32872	1.29621	1.26547

Table V. UO_2 1D with 5 regions with local varying conditions: k_{eff} values

Varying condition	Fuel Temperature	Moderator Density	Boron Concentration
99 groups	1.23021	1.18039	1.26475
MGM 2 groups	1.23224	1.17850	1.26703
FGA 2 groups	1.23139	1.21381	1.26366
MMM 2 BF form 1	1.23038	1.18168	1.26502
MMM 2 BF form 2	1.23054	1.18254	1.26493

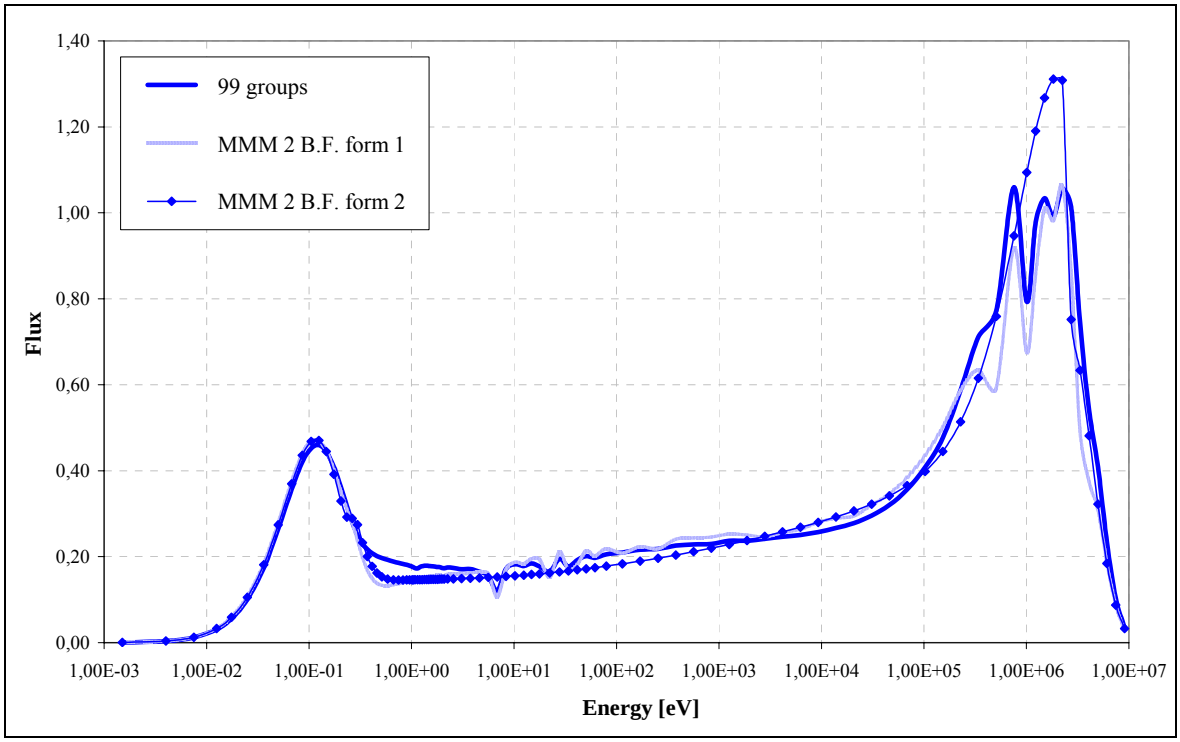

Figure 1. UO_2 0D– fuel temperature effect: reference flux and MMM 2BF fluxes.

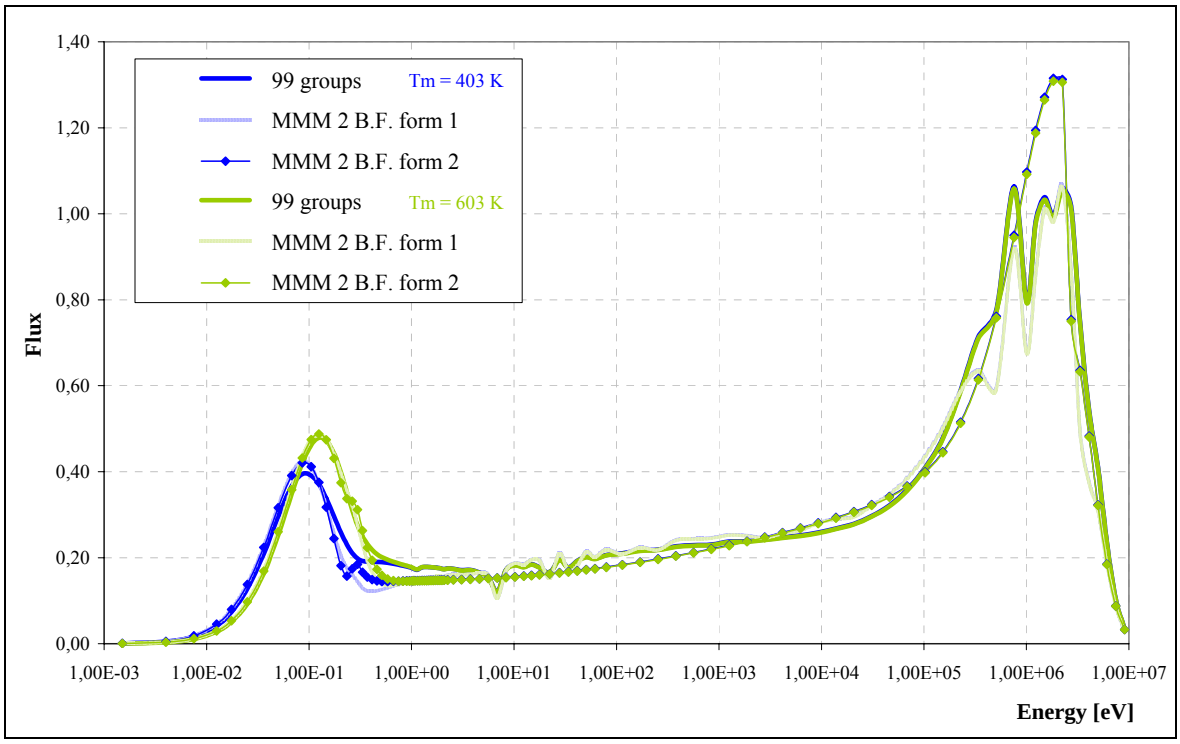

Figure 2. UO_2 0D–Moderator temperature effect: reference and MMM 2BF fluxes.

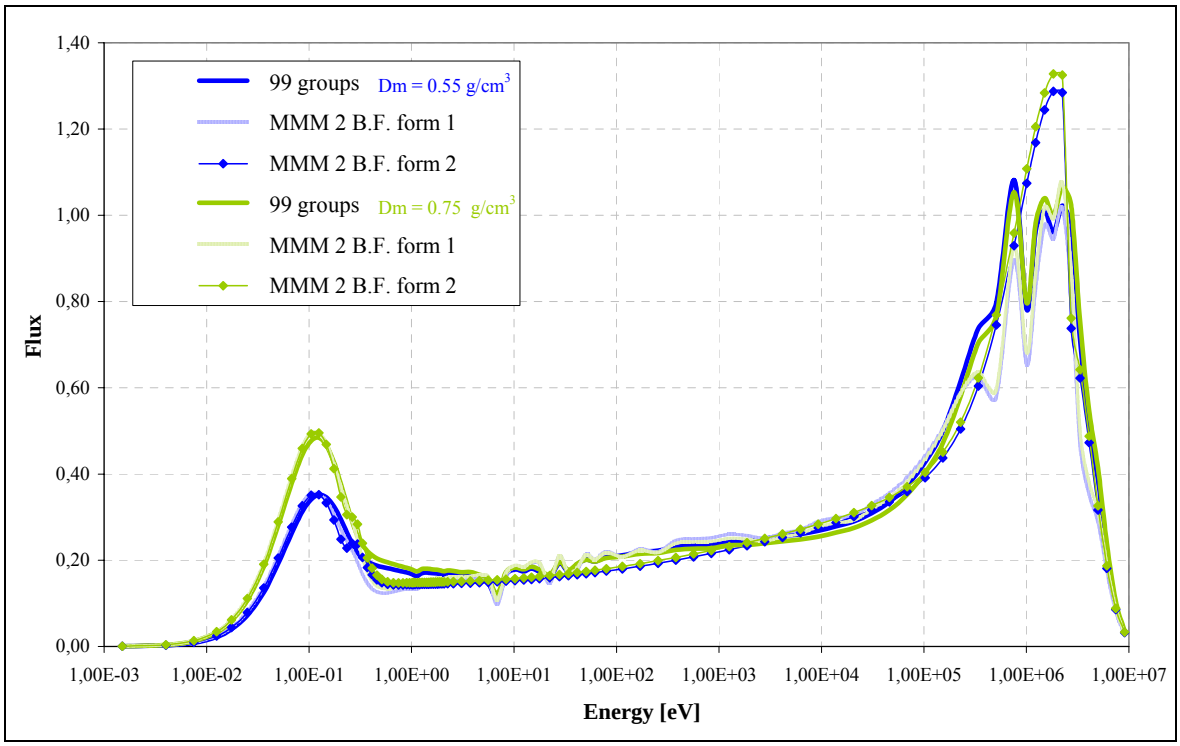

Figure 3. UO_2 0D– Moderator density effect: reference flux and MMM 2BF fluxes.

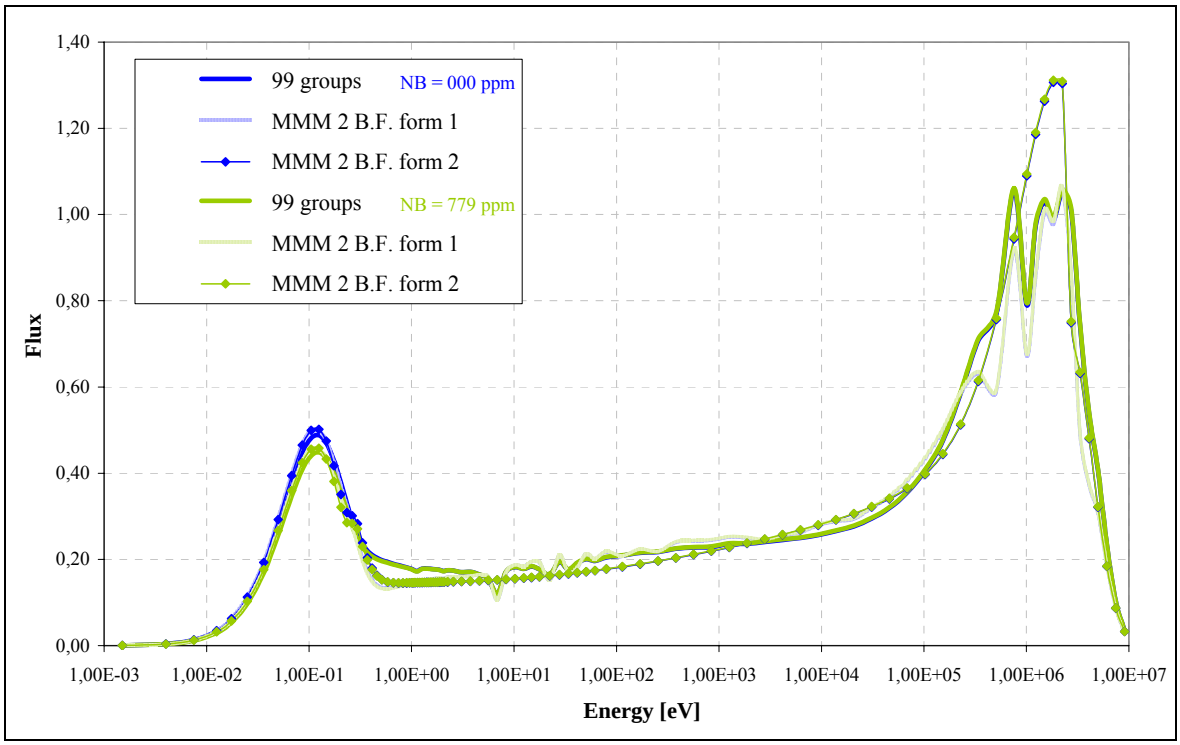

Figure 4. UO_2 0D– Boron concentration effect: reference flux and MMM 2BF flux.

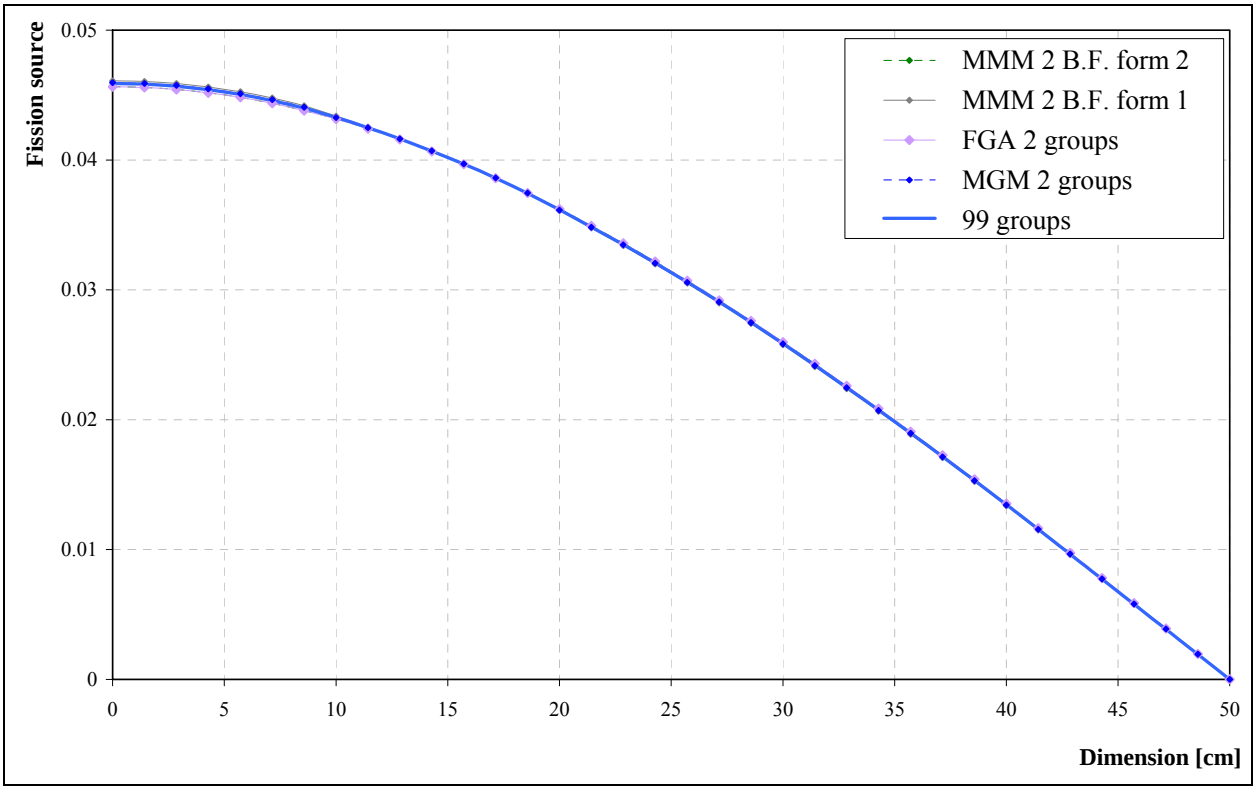

Figure 5. UO_2 1D: 5 slabs at different fuel temperature.

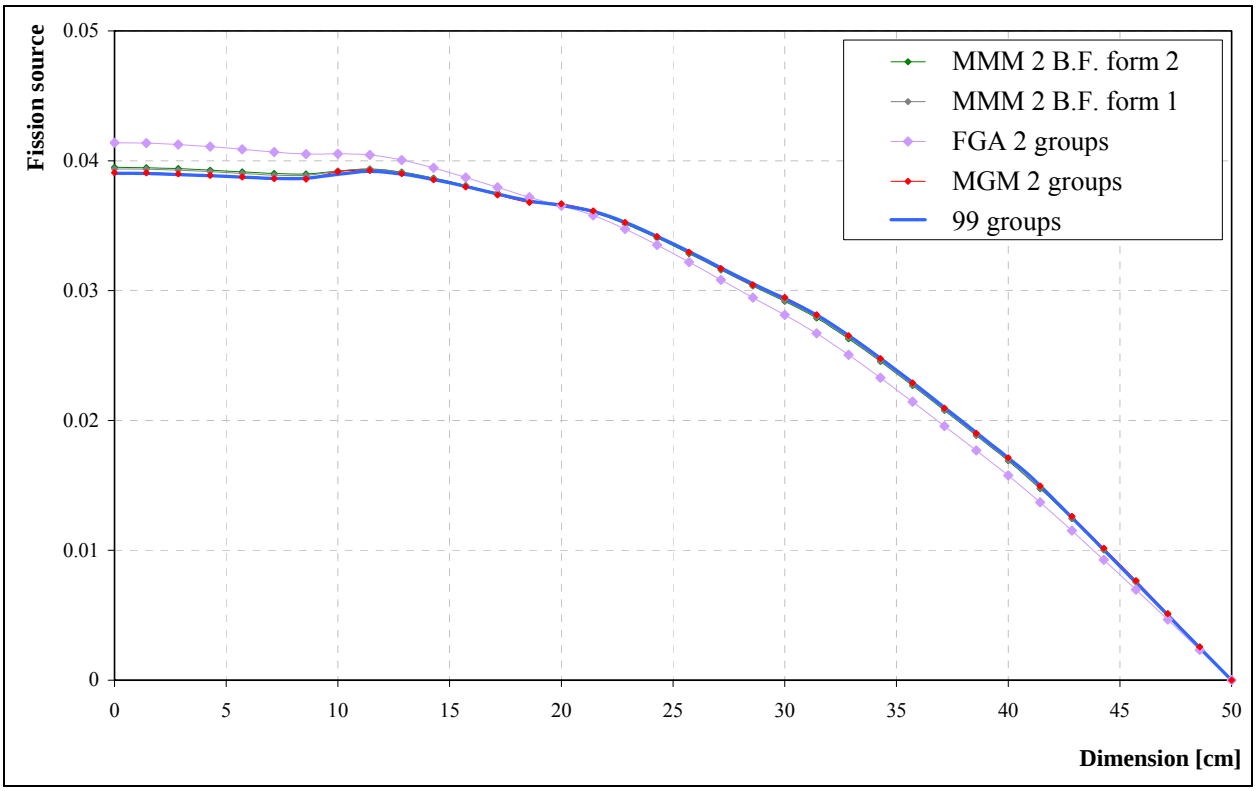

Figure 6. UO_2 1D: 5 slabs at different moderator density.

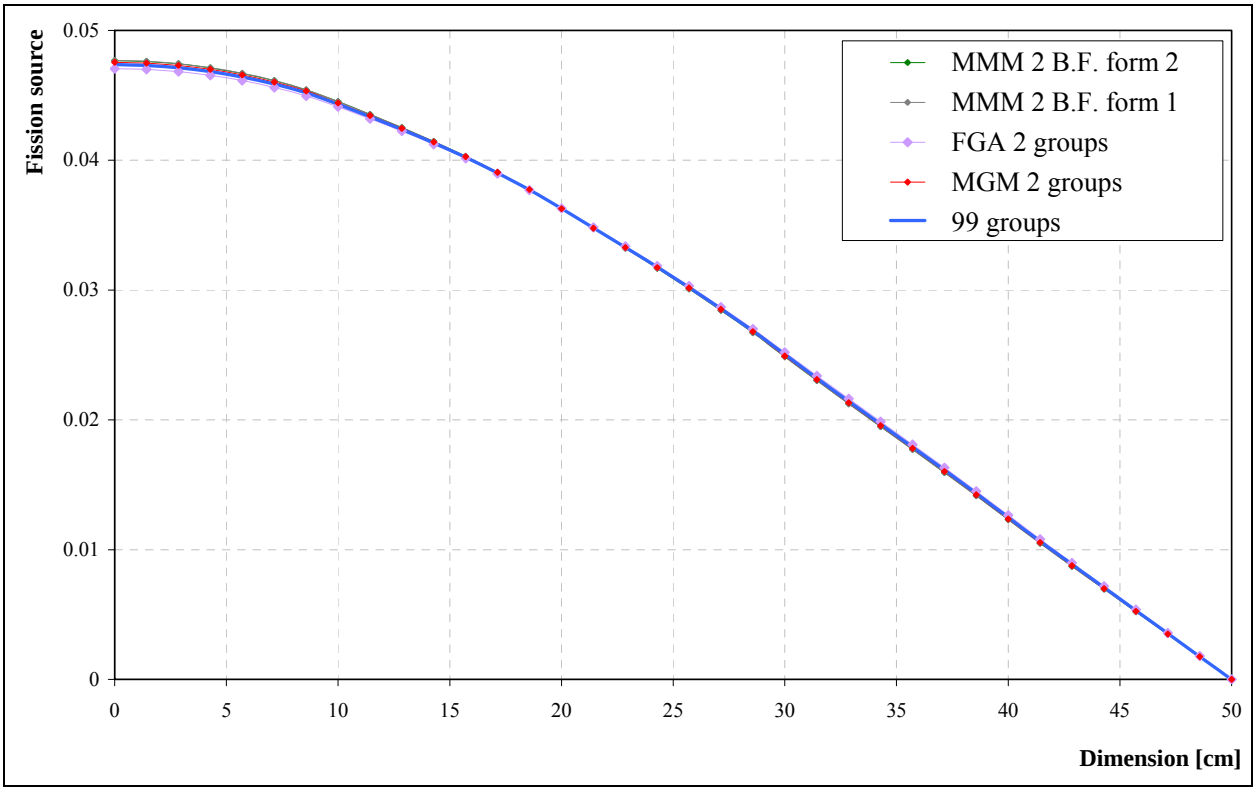

Figure 7. UO_2 1D: 5 slabs at different boron concentration.