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Stochastic Geometry in PRIZMA Code

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ABSTRACT

The paper describes a method used to simulate radiation transport through random media – randomly placed grains in a matrix material. The method models the medium consequently from one grain crossed by particle trajectory to another. Like in the Limited Chord Length Sampling (LCLS) method, particles in grains are tracked in the actual grain geometry, but unlike LCLS, the medium is modeled using only Matrix Chord Length Sampling (MCLS) from the exponential distribution and it is not necessary to know the grain chord length distribution. This helped us extend the method to media with randomly oriented arbitrarily shaped convex grains. Other extensions include multicomponent media – grains of several sorts, and polydisperse media – grains of different sizes. Sort and size distributions of crossed grains were obtained and an algorithm was developed for sampling grain orientations and positions. Special consideration was given to medium modeling at the boundary of the stochastic region.
The method was implemented in the universal 3D Monte Carlo code PRIZMA. The paper provides calculated results for a model problem where we determine volume fractions of modeled components crossed by particle trajectories. It also demonstrates the use of biased sampling techniques implemented in PRIZMA for solving a problem of deep penetration in model random media. Described are calculations for the spectral response of a capacitor dose detector whose anode was modeled with account for its stochastic structure.

Keywords: random disperse medium, particle transport, Monte Carlo method, code PRIZMA

1. INTRODUCTION

Monte Carlo simulation of radiation transport in random media is the question of the day for many applications in modern science and engineering (nuclear physics, meteorology, reactor physics and others). This complicated problem is often solved in a simplified setup for a binary random medium defined as random grains of one material in another (a matrix material).

In solving the equations of particle transport in a random medium, sought are linear functionals of the solution averaged over the ensemble of random medium realizations. The benchmark method to calculate ensemble-averaged values implies multiple modeling of random medium realizations in the entire volume and the random estimation of functionals from one or several trajectories for each realization. However, the implementation of the method for arbitrary random media in Monte Carlo codes causes algorithmic difficulties and requires huge computational resources.

One of the approximated approaches suggests modeling realizations not in the entire volume of the random medium, but on particle trajectories only. This can be justified by the fact that the straight trajectory in the random medium consists of alternating segments of random lengths – matrix chords and grain chords. Knowing chord length distributions for each component and assuming their independence, we can model medium realizations on straight trajectories [1]. In the 3D random medium, the matrix chord length defines the distance in matrix to the nearest grain along the particle path and the grain chord length defines the distance to its interface.

Methods used in the approach differ mainly in medium modeling after particle scattering. In the Chord Length Sampling (CLS) method proposed by Zimmerman and Adams [2], the matrix material and grains are modeled with the same algorithms: the new chord length from the collision point in the direction of the scattered particle is samples from the same distribution as it is done when the particle crosses into the component. This approach is valid for Markovian statistics. In random disperse media, the matrix chord length distribution can roughly be assumed Markovian, but the grain chord length distribution cannot.

In this context, Donovan, Danon and Sutton [3, 4] proposed a Limited Chord Length Sampling (LCLS) method for 2D and 3D random media with grains of one size (disks and spheres, respectively). Modeling in the matrix is done in the same manner as in CLS, but modeling in grains differs: the grain chord length is only sampled when a particle crosses into the grain for the first time and is used for sampling coordinates of the grain center. The particles scattered in the grains are tracked in the actual grain geometry which increases accuracy.

A method similar to LCLS was developed by Malyshkin [5] for multicomponent polydisperse random media with spherical grains, i.e., random media with several sorts of grains of different sizes. In grains, particles are tracked in the actual grain geometry, but unlike LCLS, the grain chord length distribution is not necessary to sample coordinates of grain centers. So, the medium is modeled using only the matrix chord length distribution and the method can thus be called Matrix Chord Length Sampling (MCLS).

The paper describes MCLS extension to multicomponent polydisperse random media with randomly oriented, arbitrarily shaped convex grains and its implementation on the universal 3D Monte Carlo code PRIZMA.

2. MULTICOMPONENT POLYDISPERSE MEDIA WITH RANDOMLY ORIENTED CONVEX GRAINS

2.1. Medium Model, Basic Assumptions and Modeling Algorithm

Generally, an (n+1)-component random medium consists of n sorts of randomly oriented non-overlapping grains (components 1,..,n) whose distribution is statistically uniform in a 3D region called stochastic, and a matrix material (component 0) filling the rest of the region. Grains of sort k, k=1,...,n, are similar convex bodies with a specified probability distribution function (pdf) $g_k(\delta)$ of the similarity factor δ – the ratio of a grain length $R_k(\delta)$ to a minimum grain length $R_k^{\min}$: $R_k(\delta) = R_k^{\min}\delta$, $1 \le \delta \le \delta_k^{\max}$. For spherical grains, R is their radius.

Each component is characterized by its composition and volume fraction α_k. It is assumed that the ensemble averaged volume fraction of component *k* is equal to α_k everywhere in the region including its boundary. It then becomes clear that some grains are cut by the boundary (their outer part is neglected). This is the difference from [3, 4] where all grains are strictly inside the region and therefore the volume fraction of grains near the boundary is smaller and that of the matrix is larger than in the region on the average.

In the MCLS method, the random medium is modeled consequently from one grain crossed by a particle to another including matrix chord length sampling and sampling of parameters for the nearest grain – its sort, size, orientation and position. The chord length l of the matrix material is assumed to obey the exponential distribution density

$$p(t) = \lambda e^{-\lambda t} \ , \ t \geq 0 \, . \tag{1}$$

As in [6], λ is calculated from

$$\frac{\overline{l}}{\overline{l} + \overline{h}} = \alpha_0 \tag{2}$$

with account that $\lambda = \frac{1}{\overline{l}}$; then

$$\lambda = \frac{(1-\alpha_0)}{\overline{h}\alpha_0} \, , \tag{3}$$

where $\overline{l}$ and $\overline{h}$ are the mean chord lengths of the matrix and grains of all sorts, respectively, and α_0 is the matrix volume fraction.

The exponential distribution is exact for the Boolean model [7] of a random medium where grain centers produce the point Poisson process and grains hence may overlap. For a model with non-overlapping grains, the exponential distribution is an approximation.

By the Markovian property, Eq. (1) describes not only the distribution of distances between grains, but also the distribution of distances from an arbitrary matrix point to the nearest grain. The matrix chord length is sampled from Eq. (1) when a particle crosses into the stochastic region from the matrix, after birth or scattering in the matrix, and after escape from the grain. If the free path length of a particle is greater than the matrix chord length, the particle crosses into the nearest grain.

It is assumed that characteristics of the nearest grain are independent of the matrix chord length and their joint distribution is

$$f(\vec{u}_c, C, \delta) = \sum_{k=1}^{n} q_k f_k(\delta) f_k^*(\vec{u}_c, C \mid \delta) \, , \tag{4}$$

where q_k is the probability that the nearest grains is of sort *k*,

$f_k(\delta)$ is the pdf of the similarity factor δ for the nearest grain of sort *k*, and

$f_k^*(\vec{u}_c, C \mid \delta)$ is the conditional pdf of coordinates $\vec{u}_c$ of the center and the rotation matrix *C* of the nearest grain of sort *k* for known δ.

The algorithm of modeling the nearest grain in accord with Eq. (4) is as follows:

- Sample sort *k* of the grain from the probabilities q_k;
- Sample the similarity factor δ of sort *k* from the pdf $f_k(\delta)$;
- Recalculate the size of the grain with respect to δ;
- Sample grain orientation and position.

After a particle crosses into the grain, it is modeled in the actual grain geometry until it is killed or escapes from the grain.

2.2. Mean Chord, Sort and Size Distribution for the Nearest Grain

The surface area $S_k(\delta)$ and the volume $V_k(\delta)$ of a grain of sort *k* can apparently be expressed in terms of $S_k^{\min}$ and $V_k^{\min}$ which are known for the minimal grain:

$$S_k(\delta) = S_k^{\min}\delta^2, \qquad V_k(\delta) = V_k^{\min}\delta^3 . \tag{5}$$

The mean chord $h_k(\delta)$ of a convex grain of sort *k* is [8]

$$h_k(\delta) = \frac{4V_k(\delta)}{S_k(\delta)} . \tag{6}$$

In a random medium, the probability that a random straight line (trajectory) crosses the grain is proportional to its surface area; hence, the distribution density $f_k(\delta)$ of the similarity factor δ of the nearest grain of sort *k* is

$$f_k(\delta) = \frac{S_k(\delta)g_k(\delta)}{\overline{S}_k} . \tag{7}$$

Averaging $h_k(\delta)$ with $f_k(\delta)$ and using Eq. (5) yield the mean chord length $\overline{h}_k$ of grains of sort *k* on the random line:

$$\overline{h}_k = \frac{4\overline{V}_k}{\overline{S}_k} = h_k^{\min}\frac{\int\delta^3 g_k(\delta)d\delta}{\int\delta^2 g_k(\delta)d\delta}, \tag{8}$$

where $\overline{V}_k$ and $\overline{S}_k$ are mean volume and mean surface area of grains of sort *k*, and

$h_k^{\min}$ is the mean chord length of the minimal grain of sort *k*.

The probability q_k that the nearest grain is of sort *k* is

$$q_k = \frac{N_k^L}{N^L}, \tag{9}$$

where $N_k^L = \frac{\alpha_k}{\bar{h}_k}$ is the average number of grains of sort *k* per unit length, $N^L = \sum_{k=1}^{n} N_k^L$.
Averaging $\bar{h}_k$ with the probabilities q_k gives the mean chord length $\bar{h}$ of all grains, which is present in Eq. (3):

$$\bar{h} = \frac{1-\alpha_0}{N^L}. \tag{10}$$

2.3. Sampling Orientation and Position of the Nearest Grain

Particles are tracked in an absolute frame of coordinates XYZ. The geometry of the minimal grain of each sort is defined in its relative frame $X^oY^oZ^o$ whose origin is arbitrary taken to be the center of the grain. Its translation $\vec{u}_B$ and rotation matrix *B* relative to XYZ are known. The position of the nearest grain is defined through translation and rotation of $X^oY^oZ^o$ which gives a new frame $X^{o'}Y^{o'}Z^{o'}$ for each grain. Let $\vec{u}_A$ stand for the random translation of the origin and *A* stand for the random rotation matrix of $X^{o'}Y^{o'}Z^{o'}$ relative to $X^oY^oZ^o$. The conversion from XYZ to $X^{o'}Y^{o'}Z^{o'}$ is made in two steps: first from XYZ to $X^oY^oZ^o$ and then from $X^oY^oZ^o$ to $X^{o'}Y^{o'}Z^{o'}$. Hence, the origin translation $\vec{u}_C$ and the rotation matrix *C* of $X^{o'}Y^{o'}Z^{o'}$ relative to XYZ, which are present in Eq. (4), have the form:

$$\vec{u}_C = \vec{u}_B + B^{-1}\vec{u}_A, \tag{11}$$

$$C = AB. \tag{12}$$

Let $\vec{r}_0$ and $\vec{\Omega}_0$ be absolute coordinates and direction of a particle, and *l* be a distance to the nearest grain of known sort and size (see Fig. 1).

The algorithm of sampling $\vec{u}_C$ and *C* for the nearest grain is based on the fact that the point where a particle crosses into a randomly oriented convex grain is uniformly distributed over the grain surface and the cosine angle μ between the particle direction and the normal to the surface at the point of crossing obeys Lambert distribution: $p(\mu) = 2\mu$, $0 < \mu \le 1$. The algorithm randomly selects a point $\vec{r}_1^{\,o}$ on the grain surface and a direction $\vec{\Omega}_1^{o}$ in $X^oY^oZ^o$ and then, using rotation and translation, brings them into the point $\vec{r}_3$ where the particle crosses the grain and the direction $\vec{\Omega}_0$, respectively.

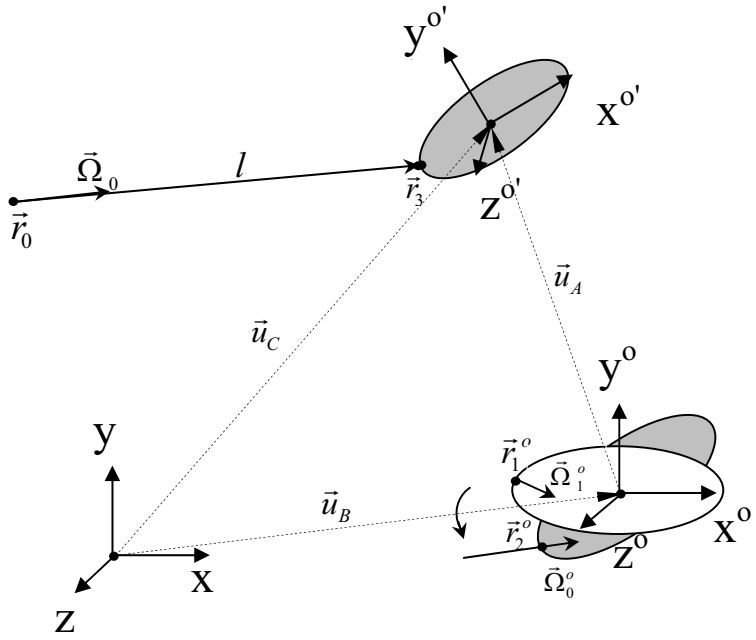

Figure 1. Conversion from $\vec{r}_1^{\,o}$, $\vec{\Omega}_1^o$ to $\vec{r}_3$, $\vec{\Omega}_0$.

The algorithm to sample orientation and position of the nearest grain is as follows:

- Sample coordinates $\vec{r}_1^{\,o}$ in $X^oY^oZ^o$ uniformly over the surface area of the grain;
- Sample $\vec{\Omega}_1^o$ at $\vec{r}_1^{\,o}$ from Lambert distribution;
- Convert $\vec{\Omega}_0$ to $X^oY^oZ^o$: $\vec{\Omega}_0^o = B\vec{\Omega}_0$;
- Compute the rotation matrix A^T converting $\vec{\Omega}_1^o$ to $\vec{\Omega}_0^o$: $\vec{\Omega}_0^o = A^T\vec{\Omega}_1^o$ (A^T is a transpose of A); compute A;
- Compute coordinates of $\vec{r}_2^{\,o}$ in which $\vec{r}_1^{\,o}$ is converted through rotation: $\vec{r}_2^{\,o} = A^T\vec{r}_1^{\,o}$;
- Compute the absolute coordinates of $\vec{r}_3$ – the point where the particle crosses into the grain: $\vec{r}_3 = \vec{r}_0 + \vec{\Omega}_0 l$;
- Convert the absolute coordinates $\vec{r}_3$ to the relative $\vec{r}_3^{\,o}$: $\vec{r}_3^{\,o} = B(\vec{r}_3 - \vec{u}_B)$;
- Compute the translation $\vec{u}_A$ of the grain to the current position relative to its initial position: $\vec{u}_A = \vec{r}_3^{\,o} - \vec{r}_2^{\,o}$;
- Compute coordinates $\vec{u}_C$ of the center and the rotation matrix *C* of the grain using Eqs. (11) and (12).

For spherical grains, we only sample coordinates of their centers with a simpler algorithm. We define coordinates X′Y′Z′ whose origin coincides with the particle coordinates $\vec{r}_0$ and the X′ axis coincides with the particle direction $\vec{\Omega}_0$. In X′Y′Z′, the coordinates y_c', z_c' of the center of the

grain are sampled uniformly over the area of grain projection onto the plane Y′Z′ after which the coordinate x_c' is computed uniquely. Then the coordinates x_c', y_c', z_c' of the grain center are converted to the absolute coordinates XYZ.

2.4. Modeling at the Stochastic Region Boundary

When a particle crosses into the stochastic region, the point $\vec{r}_0$ where it crosses the boundary is in the matrix material with probability α_0 and in a grain with probability (1-α_0). In the first case, we model the nearest grain in the particle direction and in the second case (see Fig. 2) the particle is tracked in the grain and the part of the grain that is outside the stochastic region is neglected.

The distribution of grain characteristics at a random point including $\vec{r}_0$ differs from that of the nearest grain in the particle direction despite that its general form Eq. (4) remains unchanged. In this case, the probability q_k that the boundary grain is of sort *k* is proportional to the volume fraction α_k:

$$q_k = \frac{\alpha_k}{\sum_{k=1}^{n} \alpha_k}, \tag{13}$$

and the distribution density $f_k(\delta)$ of the similarity factor on the boundary is

$$f_k(\delta) = \frac{V_k(\delta) g_k(\delta)}{\bar{V}_k}. \tag{14}$$

The algorithm to sample orientation and position of a boundary grain is as follows:

- Sample relative coordinates $\vec{r}_1^{\,o}$ uniformly over grain volume;
- Sample a random grain rotation [8] – coefficients of *A* and compute the transposed A^T;
- Compute coordinates of $\vec{r}_2^{\,o}$ in which $\vec{r}_1^{\,o}$ is converted through rotation with A^T: $\vec{r}_2^{\,o} = A^T \vec{r}_1^{\,o}$;
- Convert the absolute coordinates of $\vec{r}_0$ to the relative $\vec{r}_0^{\,o}$: $\vec{r}_0^{\,o} = B(\vec{r}_0 - \vec{u}_B)$;
- Compute the translation $\vec{u}_A$ of the grain into the current position relative to its initial position $\vec{u}_A = \vec{r}_0^{\,o} - \vec{r}_2^{\,o}$;
- Compute coordinates $\vec{u}_C$ of the center and rotation matrix *C* of the grain using Eqs.(11) and (12).

For spherical grains, we only sample the absolute coordinates $\vec{u}_C$ of the grain center uniformly over the volume of a sphere which is centered at $\vec{r}_0$ and whose radius equals the radius of the grain.

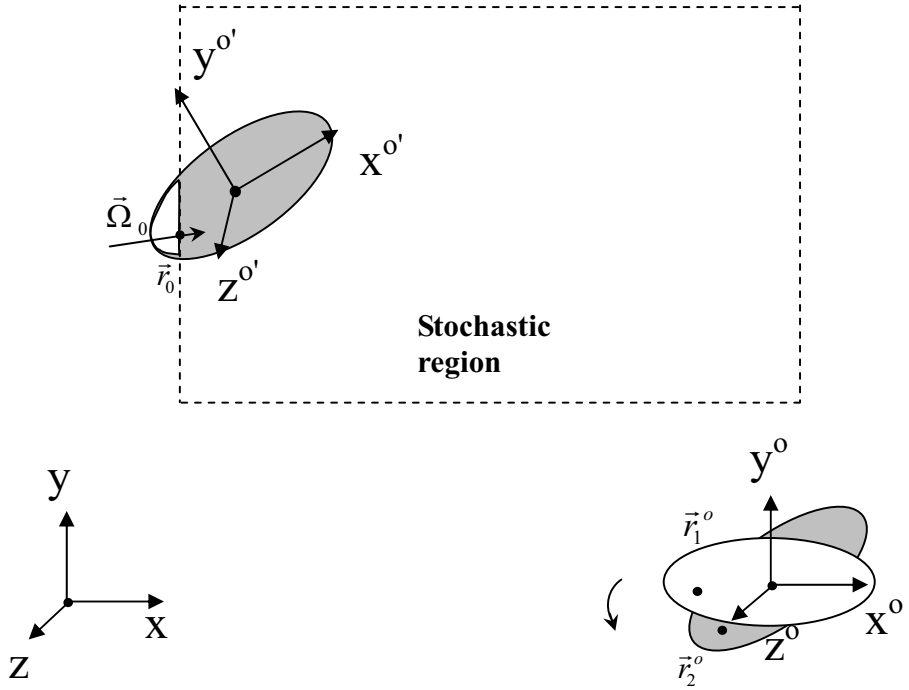

Figure 2. Crossing into the stochastic region through a grain.

The particle is tracked in the actual geometry of the grain until it is killed or escapes from the grain. If a particle escapes from the stochastic region through a grain, the outer part of the grain is neglected as it is done when a particle crosses into the region.

3. IMPLEMENTATION IN PRIZMA

For many years RFNC-VNIITF scientists have developed a universal code PRIZMA [9] for solving linear continuous-energy radiation transport problems by the Monte Carlo method. The code simulates separate and coupled transport of neutrons, photons, electrons, positrons and ions. One-, two- and three-dimensional geometries and lattices are used. The code provides wide capabilities to describe radiation sources and estimated results. Biased sampling techniques are well developed and used for a wide class of problems.

In the mid 80-s, an approach was developed to track particles in a stochastic geometry that described random disperse media with spherical grains [5]. In 2002, the approach was extended to random non-spherical grains and implemented for all types of particles as a standard capability. Medium modeling and particle tracking implemented fully correspond to what is discussed in Section 2. Random media can be modeled in 1-, 2- and 3-dimensional geometrical cells whose number is arbitrary. The number of grain sorts is also arbitrary.

The grains are convex bodies of revolution; the contour of their plane section contains segments of straight lines, circles and ellipses. They are homogeneous or heterogeneous in material.

Spherical grains may consist of several layers of different materials and non-spherical ones may contain arbitrary cells filled with different materials. The similarity factor distribution density is defined by a piecewise-constant, piecewise-linear or discrete function; for layered spherical grains, the distribution may be assigned to all layers or to one of them, the thickness of the others remaining unchanged.

Results in the stochastic region are estimated separately for each component. They include particle density, particle flux, number of specified interactions, absorbed energy etc. It is possible to predict particle or energy fluxes (currents) across the boundary of the stochastic region and across the surface of grains. All results can be calculated in space, direction, energy and time intervals, and space and direction distributed results in grains can be defined in the coordinate frame of the stochastic region or in the coordinate frame of the grain.

The range of problems PRIZMA can solve with account for the stochastic nature of materials is rather wide: penetration of radiations through shielding layers of composite materials; estimation of absorbed energy and other results in components of a disperse medium [10]; prediction of the effective neutron multiplication in critical systems with disperse fuel [11] and others.

3.1. MCLS Testing: Estimation of Modeled Component Fraction

To estimate the volume fraction of components modeled along particle trajectories, we do an additional calculation for a specific random medium: only isotropic scattering in all components, the total cross section in the components is unity, component densities may differ. We estimate the time integrated flux of particles over component volumes and its fraction in a component gives an estimate to the volume fraction of the component.

As an example, Table I provides calculated integral fluxes (cm per source particle) for direct Φ_k^0 and total Φ_k radiations (k=0,1,2,3) in components of a specific random medium – normal incidence on a 10-cm-thick slab containing spherical (sort 1) and cylindrical (sorts 2 and 3) grains of different sizes. σ is relative standard deviation in percent.

Table I. Estimated volume fractions for three sorts of grains of different sizes

k	Medium components	Minimal grain size	α_k	Φ^0_k (σ, %)	$\Phi_k^0 \Big/ \sum_{i=0}^{3} \Phi_i^0$	Φ_k (σ, %)	$\Phi_k \Big/ \sum_{i=0}^{3} \Phi_i$
0	**Matrix**		0.8	0.798964 (0.09)	0.800005	18.9843 (0.14)	0.799994
1	**Grain - cylinder**	Rc_{min} = 0.001 Lc_{min} = 0.167	0.05	0.049907 (0.11)	0.049972	1.18655 (0.14)	0.050001
2	**Grain - sphere**	R_{min} = 0.005	0.1	0.099921 (0.11)	0.100051	2.37312 (0.14)	0.100003
3	**Grain - cylinder**	Rc_{min} = 0.016 Lc_{min} = 0.000651	0.05	0.049906 (0.11)	0.049971	1.18657 (0.14)	0.050002

The similarity factor for all grain sorts is uniformly distributed in (1,10). The sizes of minimal grains of one volume are provided in the table (R_{min} is the radius of spherical grains and Rc_{min} and Lc_{min} are cylinder radius and length, respectively). The volume fractions α_k of the components are presented in the table.

The modeled component fractions (columns 6 and 8) are seen to excellently agree with the defined values α_k (column 4). A great number of calculations have demonstrated the same agreement for any sizes of the stochastic cell and grains independently of their shape, sort and volume fraction, and for any relations between grain sizes and mean free paths of particles in the components which allows adequate modeling of the random medium both in tracking of neutrons and photons and in tracking of electrons, positrons and ions.

3.2. A Model Deep Penetration Problem

A number of calculations were done to predict the time integrated flux of particles at different distances (to 100 optical thicknesses) from a point isotropic source in an infinite 2-component model random medium. Only isotropic scattering was defined in the matrix and only absorption in grains; the total cross section was unity in both components; the volume fraction of grains was 0.1. We considered one variant of the medium with spherical grains of radius R=0.4 cm and five variants of the medium with cylindrical grains of the same volume. Cylindrical radius Rc=0.35 and length Lc=0.7 corresponded to a square in the axial section of the cylinder; the other values of Rc and Lc were obtained by dividing and multiplying the radius by 2 and 4. The appropriate homogeneous medium (atomic mix) was considered. The calculations were performed with a technique developed by Y.Z. Kandiev [12] for deep penetration problems. Calculated results are presented in Table II in the units of particles per source particle.

Table II. Integral particle flux across surfaces of spheres of radius r, centered at the source point in the model random medium (sizes in cm)

r	Homogeneous medium	Spherical grains	Cylindrical Grains				
	R=0	R=0.4	Rc=0.0875 Lc=11.2	Rc=0.175 Lc=2.8	Rc=0.35 Lc=0.7	Rc=0.7 Lc=0.175	Rc=1.4 Lc=0.04375
10	1.32e-01 (0.12)	2.37e-01 (0.66)	1.65e-01 (0.72)	2.01e-01 (1.05)	2.32e-01 (1.57)	2.01e-01 (1.16)	1.58e-01 (0.77)
20	1.38e-03 (0.15)	4.21e-03 (0.71)	2.13e-03 (0.80)	3.09e-03 (1.15)	4.12e-03 (1.70)	3.01e-03 (1.13)	1.93e-03 (0.89)
50	4.91e-10 (0.19)	7.45e-09 (0.76)	1.41e-09 (0.95)	3.39e-09 (1.28)	6.84e-09 (1.81)	3.26e-09 (1.27)	1.12e-09 (1.03)
100	3.83e-21 (0.37)	8.34e-19 (0.78)	3.08e-20 (1.17)	1.69e-19 (1.39)	7.19e-19 (1.84)	1.64e-19 (1.39)	1.95e-20 (1.37)

It is seen that the fluxes for cylindrical square-section grains are almost identical to those for spherical grains which seem quite natural. When the cylindrical radius becomes smaller or

larger, the flux reduces because of the larger surface area of grains and, hence, the mean chord length of grains crossed by particles decreases (see Eq. (6)). This results in a proportionally reduced mean chord length of the matrix and the number of grains crossed by particles increases. In this sense, the media becomes closer to homogeneous and, accordingly, the passed flux reduces.

3.3. Dose Detector Calculation

The recent PRIZMA calculations were done in order to evaluate the spectral response of a capacitor dose detector used to measure gamma doses to 1000 roentgen and higher from research reactors [13]. The working medium (electrolytic capacitor) of the detector has a sophisticated structure because of the stochastic structure of its anode which consists of many ($\sim 4\times10^{6}$) compressed oxidized Ta grains from 10 to 60μ in size, with the electrolyte (38% aqueous solution of H_2SO_4) in between (see Fig. 3). Thin layers ($\sim 2.4\times10^{-2}$μ) of Ta_2O_5 on grain surfaces act as dielectric for the capacitor. On average, the grain size is ~ 24μ; their volume fraction is ~ 0.524.

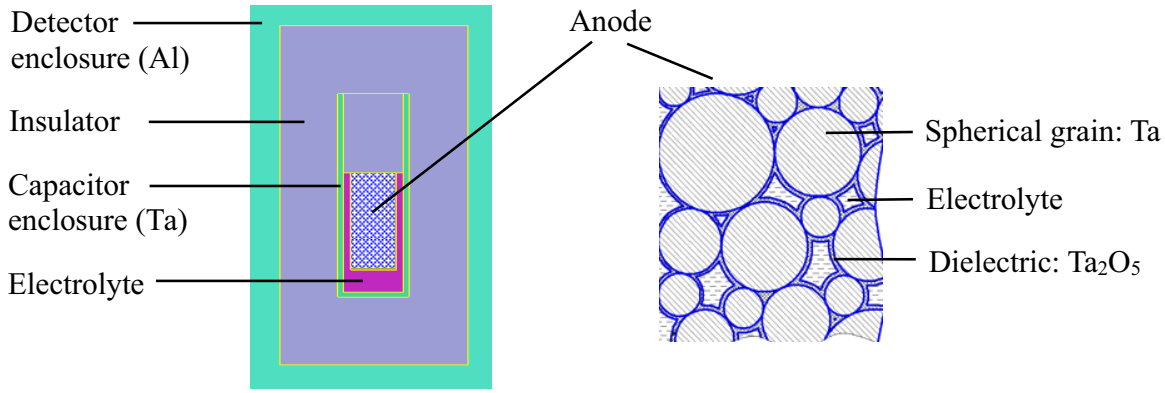

Figure 3. A cross sectional view of the detector and the anode structure.

It is impossible to measure the spectral response in the entire range of energies because of the limited number of adequately intense monoenergetic gamma sources. However, with certain assumptions, the spectral response can be calculated in terms of energy absorbed in dielectric.

To evaluate the spectral response of the detector to gamma radiation of energy *E*, we calculated doses deposited in the dielectric from gammas, secondary electrons and positrons with account for the stochastic structure of the anode. The energy of the source was varied from 0.1 to 7 MeV.

The size distribution of Ta grains for a constant thickness of the dielectric on the grain surface was described by a truncated normal distribution of the grain diameter D between 10 and 60μ with a maximum at $D_m = 24$μ for three values of σ: 4, 8 and 20 (see Fig. 4).

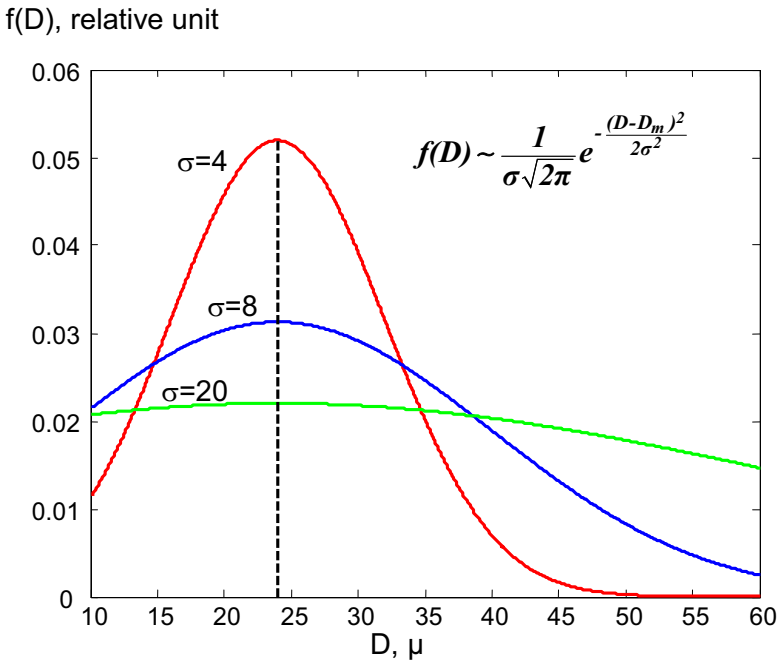

Figure 4. Three normal distributions of grain diameters.

In addition to these three variants, we made calculations for media with grains of constant sizes which were equal to size minimum (D = 10μ) and maximum (D = 60μ), and to a value corresponding to D_m (D = 24μ).

Calculated results show the total energy absorbed in the dielectric to be strongly dependent on grain size. The main reason was that the mass of the dielectric on the grain surface differed (it is proportional to the surface area, not to volume), while the volume fraction of grains was same in all variants. However, if change from absorbed energy to spectral response $F(E)$ in the relative form:

$$F(E) = \frac{K(E)}{K(E_0)},$$

where $K(E)$ [1/R] is sensitivity to gammas of energy E and E_0 = 1.25 MeV is the energy of ^{60}Co, insignificant differences will only remain in a narrow energy range about E = 0.2 MeV. A good agreement is seen between calculated and experimental data for a ^{137}Cs source of energy E=0.661 MeV, for which experiment gives *F(E)* = 1.68 (see Fig. 5).

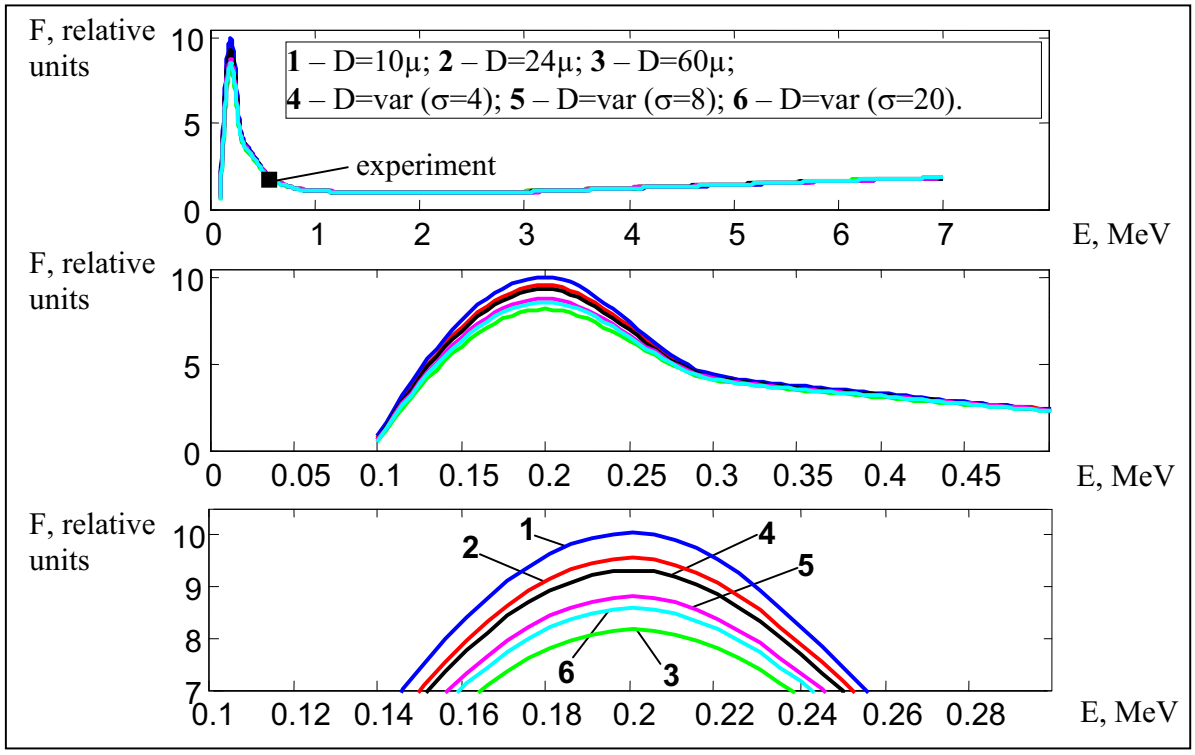

Figure 5. Spectral response of the dose detector.

The dependence of $F(E)$ on grain size at low energies of source gammas is related to the fact that the path of secondary electrons at these energies becomes comparable with grain size. Electrons are generated mainly in Ta grains and, therefore, the larger the grain (3 in Fig.5), the lower the energy carried out the grain by electrons. This reduces dose in the dielectric on the grain surface and, accordingly, detector's sensitivity.

4. CONCLUSION

Combining capabilities provided for the description of stochastic geometries with the wide capabilities of PRIZMA makes it possible to use the code for solving different problems of radiation transport through stochastic multicomponent polydisperse media with rather arbitrarily shaped grains. Our calculations of the dose deposited in the dielectric material of a capacitor dose detector made for the first time in a realistic setup with account for the stochastic structure of the anode, helped us calculate the spectral response of the detector in a wide range of energies. Though the developed MCLS method, as well as the LCLS method, is an approximation, but it is a very good, and even sufficient approximation for most radiation transport problem. In cases where the absolute values of calculated quantities raise doubts, the method can successfully be used in perturbation problems to evaluate how the variation of medium parameters influences characteristics of interest.

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