Joint International Topical Meeting on Mathematics & Computation and Supercomputing in Nuclear Applications (M&C + SNA 2007)
Monterey, California, April 15-19, 2007, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2007)

CALCULATION OF THE PULSE HEIGHT DISTRIBUTION INDUCED BY FAST NEUTRONS IN A SCINTILLATION DETECTOR

Andreas Enqvist, Imre Pázsit
Chalmers University of Technology
Department of Nuclear Engineering
412 96 Göteborg, Sweden
andreas@nephy.calmers.se; imre@nephy.calmers.se

Sara Pozzi
Oak Ridge National Laboratory
Oak Ridge TN 37831-6010, USA
pozzisa@ornl.org

ABSTRACT

The probability distribution of light pulses produced in an organic scintillator by impinging fast neutrons is calculated. It is assumed that the slowing down takes place on a mixture of hydrogen and carbon atoms, and that the light produced is a non-linear function of the energy transferred to the H atoms and a linear function of the transferred energy to C atoms in each individual collision. The light pulses from all collisions of one neutron add up. Due to the non-linear relationship between energy transfer and light production, neutrons with the same total transferred energy (after several collisions) but with a different collision history generate different amounts of light. The pulse height distribution of n-times collided neutrons is calculated for the few first values, i.e. $n = 1 - 4$. The results are compared with Monte-Carlo simulations and an excellent agreement is observed. Due to the limitations in transferred energy, the results for collisions with only carbon are of less interest, hence the main emphasis is put on the light generation in collisions with H atoms. The treatment for slowing down in a mixture of H and C atoms is also outlined and solved analytically.

Key Words: light pulse distribution, slowing down, scintillation detectors

1. BACKGROUND

One possible method of detecting nuclear materials in nonproliferation and homeland security applications is neutron detection by organic scintillators, in both liquid and plastic form. Neutron detection in this type of detectors occurs by multiple scatterings on hydrogen (H) and carbon (C), the main constituents of the scintillator. During slowing down of the neutrons, scintillation light is produced. Although a light quantum is generated at each individual energy transfer by the individual neutron collisions, only the total light intensity by a single neutron can be detected, because the individual collisions cannot be resolved. However, the different pulses by the different neutrons can be distinguished, and the light intensity distribution of a flux of incoming detectors can be experimentally determined. Because such a distribution will depend on the energy of the incoming neutron, it is possible to unfold an unknown energy spectrum from the pulse height distribution of the scintillation light.

Calculating the pulse height distribution is a relatively intricate task. Even with the same transferred total energy, due to the non-linear relationship between energy transfer in single collisions and the induced light, the total induced light intensity per neutron will be a random process. Then, the energy transfer of neutrons during slowing down also follows particular statistical laws.

To obtain some insight into the statistics of the light generation process, we will study the distribution of total light intensity produced by neutrons collided once, twice, etc. For a given detector, by accounting for the scattering and absorption cross sections, all such neutron histories can be summed up to obtain the total distribution of neutrons that either escaped the detector or slowed down sufficiently so that they cannot produce any more light. In practice, due to the large leakage probability in detectors of typical sizes, it is sufficient to study the light transfer for neutrons that collided up to four times.

A derivation of the light distribution of the n-times collided neutrons is the purpose of the present paper. A general formula is derived for an arbitrary number of collisions. In the quantitative analysis, the result for the n-times collided neutrons is given as an irreducible n-fold convolution, hence the calculations for increasing n become increasingly difficult. We calculated numerically these distributions up to $n = 4$. The results were compared to Monte-Carlo calculations, and an excellent qualitative agreement was obtained. The Monte-Carlo simulations were made by the code MCNP-PoliMi, which can be easily used to compute such distributions.

2. THEORY

The light intensity L for a given energy transfer T in a single collision of a neutron with hydrogen is given as

$$L = aT^2 + bT \equiv L_h(T) \qquad (1)$$

with $a = 0.035$ and $b = 0.141$ (in MeVee for transferred energy in MeV). The distribution of T will depend on the actual number of the neutron collisions and the energy transfer on the individual collisions. It is the physics of the neutron slowing down and the distribution of the transferred energy, as known from the theory of neutron slowing down, which determines the statistics of the generated light pulse amplitudes.

The distribution T can be easily calculated from the energy distribution of n-times collided neutrons as follows. Assume first slowing down on hydrogen in an infinite homogeneous system. Then the distribution of the energy transferred in one collision by a neutron having an initial energy E_0 is equal to

$$p(T, E_0)dT = \frac{dT}{E_0}. \qquad (2)$$

Likewise, the post-collision energy distribution in a scattering on a nucleus other than that of hydrogen will have the form

$$p_\alpha(T, E_0)dT = \frac{dT}{E_0(1-\alpha)}, \qquad (3)$$

where α is the standard notation for the parameter used in calculating the maximum energy transfer:

$$\alpha = \left(\frac{A-1}{A+1}\right)^2 . \tag{4}$$

For carbon α is about 0.716, leading to a restricted maximum transferrable energy, while for hydrogen α is zero.

As is known, the distribution of the transferred energy is independent of the amount of transferred energy. Given Eqns (1) and (2) it is possible to calculate the distribution $f_{nX}(L, E_0)$, i.e. the total light generated in n collisions by a single neutron on element X. The light intensities generated in the single collision add up to a total intensity. To obtain the distribution of $f_{1X}(L, E_0)$ it is sufficient to find the distribution of the function of a random variable with a known distribution. Using the relationship between the distribution of a random variable x and of another variable, given as a single-valued function of x in the form $y = g(x)$, the distribution of y, $f_y(y)$, is given in terms of the distribution of x, i.e, $f_x(x)$, as

$$f_y(y) = \frac{f_x(x)}{|g'(x)|} \tag{5}$$

where now $x = g^{-1}(y)$, i.e, it is expressed with y [1]. Then from (1) and (2) it follows that for collisions on hydrogen

$$f_{1h}(L, E_0) = \frac{1}{E_0\sqrt{b^2 + 4aL}} \tag{6}$$

for L lying between 0 and $L_{\max,h} = aE_0^2 + bE_0$, and zero outside, because the maximum transferred energy is E_0, and also because due to the monotonically increasing character of $L_h(T)$ with a positive second derivative, the maximum light for a given energy transfer is achieved in a single collision. For a given light intensity L in (6), the transferred energy (which will be needed later) is given as

$$T_h(L) = \frac{\sqrt{b^2 + 4aL} - b}{2a}. \tag{7}$$

From here the distribution of multiply collided neutrons can be calculated by convolution-type integrals. For the distribution after two collisions, $f_{2h}(L, E_0)$, one has:

$$\begin{aligned} f_{2h}(L, E_0) &= \int_0^L f_{1h}(L - l, E_0 - T_h(l)) f_{1h}(l, E_0) dl = \\ &= \frac{1}{E_0} \int_0^L \frac{1}{(E_0 - T_h(l))\sqrt{b^2 + 4a(L-l)}} \frac{1}{\sqrt{b^2 + 4al}} dl \end{aligned} \tag{8}$$

where

$$T_h(l) = \frac{1}{2a}\left\{\sqrt{b^2 + 4al} - b\right\}. \tag{9}$$

This procedure can be continued, i.e,

$$f_{3h}(L,E_0) =$$
$$= \int_0^L dl_1 \int_0^{L-l_1} dl_2 f_{1h}\left\{L-(l_1+l_2), E_0-(T_h(l_1)+T_h(l_2))\right\} f_{1h}(l_2, E_0-T_h(l_1)) f_{1h}(l_1,E_0) =$$
$$= \frac{1}{E_0}\int_0^L dl_1 \int_0^{L-l_1} dl_2 \frac{1}{(E_0-T_h(l_1)-T_h(l_2))\sqrt{b^2+4a(L-l_1-l_2)}} \times$$
$$\times \frac{1}{(E_0-T_h(l_1))\sqrt{b^2+4al_2}} \frac{1}{\sqrt{b^2+4al_1}} \tag{10}$$

with

$$T_h(l_i) = \frac{1}{2a}\left\{\sqrt{b^2+4al_i}-b\right\}; \qquad i=1,2 \tag{11}$$

and so on.

If we express the integral for $f_{nh}(L,E_0)$ as

$$f_{nh}(L,E_0) = \underbrace{\int_0^L dl_1 \int_0^{L-l_1} dl_2 \cdots \int_0^{L-(l_1+l_2+\cdots+l_{n-2})} dl_{n-1}}_{\int_{n-1}} I_n, \tag{12}$$

then the next order in the distribution can be formed as:

$$f_{(n+1)h}(L,E_0) =$$
$$\int_{n-1} \int_0^{L-(l_1+l_2+\cdots+l_{n-1})} dl_n f_{1h}\left\{L-(l_1+l_2+\ldots+l_n), E_0-[T_h(l_1)+\cdots+T_h(l_n)]\right\} I_n. \tag{13}$$

This does not imply any simple recursive way of calculating high order distributions, but it makes it easier to verify the correctness of higher order expressions when compared to lower order expressions.

The graphs in Fig. 1 show a comparison with Monte Carlo-simulations performed with the code MCNP-PoliMi [2] and show a good agreement. The quadratic dependence for the light produced in hydrogen scattering has also been investigated slightly where we show the distribution for a stronger quadratic dependence with $a = 0.085$ in Fig. 2. This does not change the characteristics of the pulse distribution, only the value for L_{max} is slightly higher. Taking a very small moves the pulse height distribution towards becoming a mirror of the energy distribution, i.e. of the distribution of the total energy transfer of neutrons slowing down, since the produced light would then just be proportional to the total transferred energy.

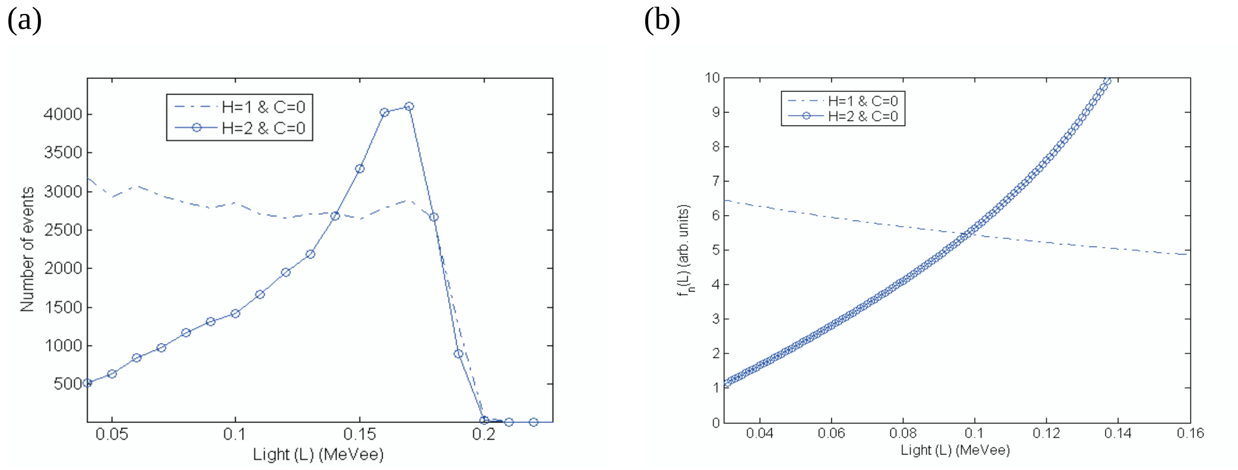

Figure 1. Monte Carlo simulation vs analytical solution for light output. (a) Monte Carlo simulation of light output for 1-MeV incident neutrons in the case of one scattering on hydrogen and two scatterings on hydrogen. (b) Analytical solution of the light output for the case of one scattering on hydrogen and two scatterings on hydrogen (1-MeV incident neutrons)

2.1 Carbon Moderation

In the case of slowing down on carbon we need to take into account that not all of the energy can be transferred in a single collision with carbon, as opposed to hydrogen. The maximum amount of energy that can be transferred is given by

$$T_{max,c} = (1-\alpha)\, E_0. \tag{14}$$

From this we have a restriction in the maximum amount of light created from one one scattering. The generated light output for scattering on carbon is very small and for this reason we use the following relationship:

$$L_c(T) = c\, T, \tag{15}$$

with c=0.02 MeVee/MeV. This gives the maximum amount of light generated in one collision as:

$$L_{max,c} = L(T_{max,c}) = c\,(1-\alpha)\, E_0. \tag{16}$$

The total maximum transferred energy will increase with the number of scatterings as $T_{max,c} = (1-\alpha^n)E_0$ for n collisions on carbon. If, on the other hand the neutron has one or several scatterings on hydrogen in its history, the maximum energy transfer is the same as the initial energy of the neutron. The light distribution from one scattering on carbon is thus given by:

$$f_{1c}(L, E_0) = \begin{cases} \dfrac{1}{c\,(1-\alpha)\, E_0} & \text{for} \quad 0 \leq L \leq L_{max,c}; \\ 0 & \text{for} \quad L \geq L_{max,c}. \end{cases} \tag{17}$$

To make the calculations easier we use the Heaviside unit step function, θ, to write the light distribution for one collision as:

$$f_{1c}(L, E_0) = \frac{\theta(L_{max,c} - L)}{c\,(1-\alpha)\, E_0}. \tag{18}$$

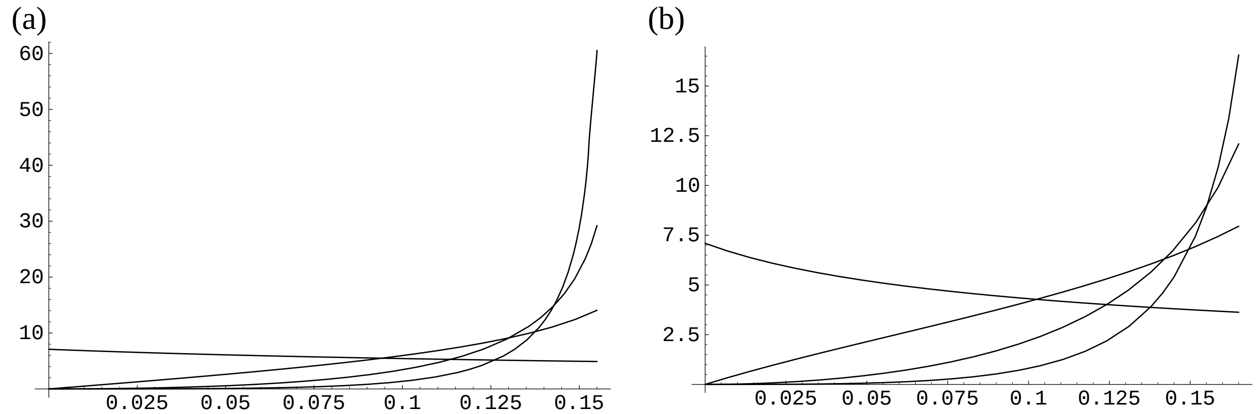

Figure 2. Analytical solution for the distributions up to four collisions on hydrogen. (a) Regular quadratic model with the parameters $a = 0.035$, $b = 0.141$. (b) Plot of the distribution for a case with stronger quadratic dependence between the transferred energy and the produced light with $a = 0.085$, $b = 0.141$.

Double scattering on carbon then results in the expression:

$$f_{2c}(L,E_0) = \int_0^L f_{1c}(L-l_1, E_0 - T_c(l_1)) f_{1c}(l_1, E_0) dl_1 = \\ = \frac{1}{c^2\,(1-\alpha)^2\,E_0} \int_0^L \frac{\theta(c\,(1-\alpha)\,E_0 - L)\cdot\theta(c\,(1-\alpha)\,(E_0 - T_c(l_1)) - (L-l_1))}{(E_0 - T_c(l_1))} dl_1 \tag{19}$$

where

$$T_c(l_1) = \frac{l_1}{c}. \tag{20}$$

Higher orders of f_{nc} are defined as convolution integrals in the same way as for hydrogen. Due to the simplified relation between transferred energy and light produced, the expressions simplify somewhat, but the fact that we have functions defined over different intervals, the Heaviside functions will lengthen the expressions somewhat.

In Fig. 3 one can see the pulse height distribution for pure carbon-moderation within the scintillating detector. More complicated behavior will occur when one observes the non-linear light production of hydrogen, or alternatively when considering neutrons that collide with both types of atoms.

2.2 Carbon-Hydrogen Moderation

Just as the neutrons can suffer one or more collisions on either hydrogen or carbon, they could also have collision histories containing both hydrogen and carbon scattering.

One can find the pulse distributions for histories containing both hydrogen and carbon by using the definitions of the distribution functions for hydrogen and carbon, $f_{1h}(L, E_0)$, $f_{1c}(L, E_0)$

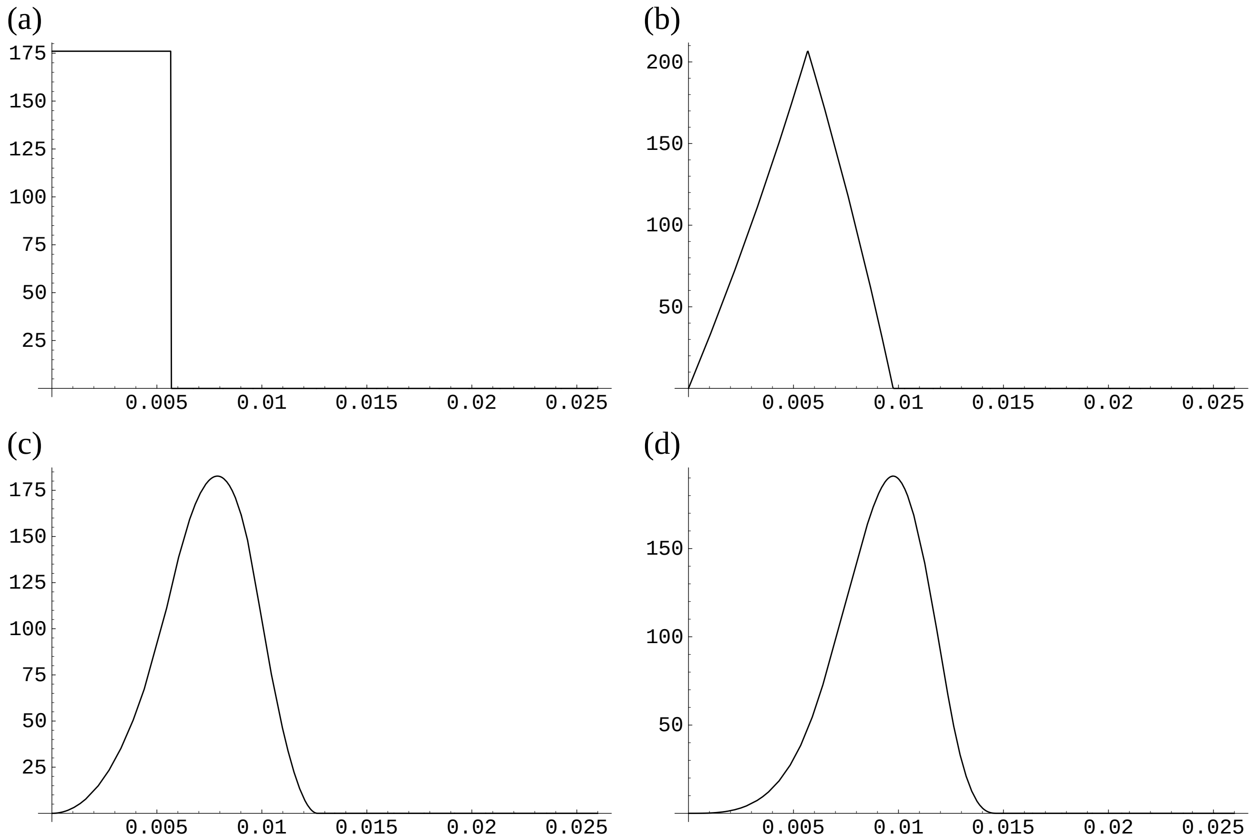

Figure 3. The light distribution from one to four scatterings on carbon resembles the distribution of energy transfer of the scattered neutron since for carbon a linear dependence between the transferred energy and the light produced is used.

respectively. The expression for the light generated from a neutron that collides first with hydrogen and thereafter with carbon will read as

$$\begin{aligned} f_{hc}(L,E_0) &= \int_0^L f_{1c}(L-l_1, E_0 - T_h(l_1)) f_{1h}(l_1, E_0) dl_1 = \\ &= \frac{1}{E_0} \int_0^L \frac{\theta(c\,(1-\alpha)\,(E_0 - T_h(l_1)) - (L-l_1))}{c\,(1-\alpha)} \frac{1}{\sqrt{b^2 + 4al_1}} dl_1. \end{aligned} \tag{21}$$

Alternatively one could use the following formula for the reverse process: colliding first with carbon and thereafter with hydrogen:

$$\begin{aligned} f_{ch}(L,E_0) &= \int_0^L f_{1h}(L-l_1, E_0 - T_c(l_1)) f_{1c}(l_1, E_0) dl_1 = \\ &= \int_0^L \frac{1}{E_0 - T_c(l1)\sqrt{b^2 + 4a(L-l_1)}} \cdot \frac{\theta(c\,(1-\alpha)\,E_0 - l_1)}{c\,(1-\alpha)\,E_0} dl_1. \end{aligned} \tag{22}$$

One can extend the formulas to histories of three collisions such as a hydrogen-hydrogen-carbon sequence of collisions.

$$f_{hcc}(L, E_0) = \int_0^L dl_1 \int_0^{L-l_1} dl_2\, f_{1c}\left\{L-(l_1+l_2), E_0-(T_h(l_1)+T_c(l_2))\right\} f_{1c}(l_2, E_0-T_h(l_1)) f_{1h}(l_1, E_0). \tag{23}$$

Using these formulas one can observe the effect of having between zero and two scatterings on carbon, and for example one scattering on hydrogen (Fig. 4). To find what the total combined

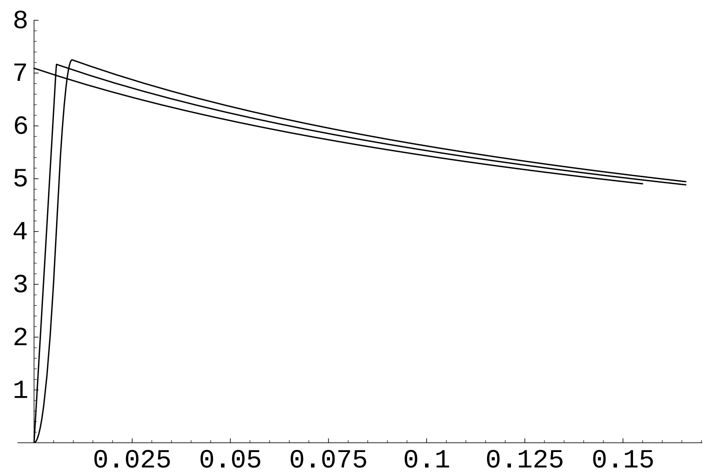

Figure 4. The pulse height distribution for neutrons that undergo one collision with hydrogen and 0, 1 or 2 collisions with carbon. As can be observed, the extra collisions on hydrogen makes it more unlikely to have very low amounts of light produced, while at the same time slightly shifting the rest of the distribution upwards to maintain normalization.

pulse height distribution looks like one needs to use the cross sections associated with different collisions, and use them as weighting factors.

3. CONCLUSIONS

Analytical expressions were derived for the distribution of light pulses generated by neutrons in a detector containing a mixture of H and C atoms. The formulas are totally resolved in the collision number and sequence of the neutrons on these two types of atoms. For quantitative work, but even for getting a qualitative picture, the resulting expressions need to be evaluated numerically. The formulas for the light distribution were evaluated by using the values $a = 0.035$ and $b = 0.141$ for light produced from energy transferred in collisions with hydrogen and $c = 0.02$. Some of the results were compared with Monte-Carlo simulations. These latter were obtained using a suitably modified version of MCNP-PoliMi. There is good qualitative agreement between the theory and the simulations. It is evident that collisions with carbon contribute little to the detector response. At the same time having many collisions on hydrogen compacts the light pulse distribution at a value corresponding to the maximum transferrable energy which in the case of hydrogen scattering is the same as the initial energy.

Finding the pulse height distributions for neutrons colliding on scintillator material opens up for using these data in combination with cross section data as weighting factors to calculate the total distribution, corresponding to a statistical average over all collisions. This way light distribution spectra can be calculated for a large number of initial neutron energies, or even spectrum of impinging neutrons, and one can use these distributions to elaborate algorithms to unfold unknown neutron energy spectra from the amplitude distribution of the light pulses.

ACKNOWLEDGEMENTS

The work of the Swedish authors was supported by the Swedish Nuclear Power Inspectorate. The work of the U.S. author was supported in part by the U.S. Department of Energy National Nuclear Security Administration Office of Non-proliferation Research Engineering NA-22. The Oak Ridge National Laboratory is managed and operated for the U. S. Department of Energy by UT-Battelle, LLC, under contract DE-AC05-00OR22725.

REFERENCES

[1] A. Papoulis: Probability, Random Variables and Stochastic Processes. 3rd Edition. McGraw Hill, New York (1991).

[2] S.A. Pozzi, E. Padovani, M. Marseguerra, “MCNP-PoliMi: a Monte-Carlo code for correlation measurements,” *Nucl. Instr. and Meth. A* **513** pp. 550 (2003).