Joint International Topical Meeting on Mathematics & Computation and Supercomputing in Nuclear Applications (M&C + SNA 2007)
Monterey, California, April 15-19, 2007, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2007)

A NEW STRATEGY FOR OPTIMAL FUEL CORE LOADING PATTERN DESIGN IN PWR NUCLEAR POWER REACTORS

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ABSTRACT

Most of the strategies yet implemented to optimize fuel loading pattern design in the nuclear power reactors, are based on maximizing the core effective multiplication factor (Keff) in order to extract maximum energy and to reduce the local power peaking factor (Pq) from a set value. However, a new optimization criterion would be of interest in order to maximize the burn-up of the plutonium content (Pu) in the fuel assemblies. In this research we have developed a new strategy based on multi objective optimization, to optimize fuel core loading pattern for a VVER-1000 reactor. In this approach we simultaneously have reduced the Pq, maximized Keff and also minimized the plutonium content in the fuel at the end of cycle. This strategy has been implemented via exact calculation cycle using WIMS and CITATION calculation codes. We use the genetic algorithm to find the optimum fuel loading pattern. Simulation results Show that this strategy can reduce the plutonium content of the fuel at EOC while maintaining a limit on the core power peaking and multiplication factor.

Key Words: Nuclear reactor core, Fuel loading pattern design, Genetic algorithm, Multi objective optimization

1. INTRODUCTION

The general in-core fuel management problem for a PWR consists of determining the fuel reloading policy for each cycle that minimizes the local power peaking factor (P_q) and/or maximizes the effective multiplication factor (K_{eff}). When the fresh and retained fuel assemblies are loaded, the P_q and the K_{eff} must be considered. The value of P_q must be kept lower than a predetermined value to maintain fuel integrity, and K_{eff} must be maximized under these constraints to extract the maximum energy. However, a new optimization criterion could be of interest, aiming maximum burn up of the plutonium content in nuclear fuel assemblies, i.e, minimization of remaining plutonium (Pu) in spent fuel at the end of cycle (EOC), which lead to the production of more energy. In this research, we developed a new strategy for optimal fuel core loading pattern of a VVER-1000 reactor, based on multi-objective optimization: lowering the P_q, maximization of the K_{eff} and minimization of Pu in fuels at EOC. This strategy has been implemented via exact calculations of fuel burn up during the equilibrium cycle of a typical VVER reactor using WIMS and CITATION calculation codes [1,2].

To find a fuel core loading pattern which resolves these limitations, we used genetic algorithms (GAs). GAs are a computational paradigm that has been emerging in recent years. This method is effective for solving difficult multi objective optimization problems such as functional and combinational optimizations.

Simulation results show that this strategy can reduce the remaining *Pu* of the fuels at EOC conditions while considering limitations on core power peaking and multiplication factor.

2. CONVENTIONAL FUEL LOADING PATTERN OPTIMIZATION STRATEGY IN NUCLEAR POWER PLANTS

In pressurized water reactors, the fuel reloading problem has significant meaning in terms of both safety and economics. The fuel integrity is severely challenged if the Pq is too high. Thus, the Pq must be kept lower than a predetermined value. On the other hand, from the economic point of view, the core Keff, and consequently the cycle burn up may be maximized under a given number of fresh fuel assemblies.

An optimal loading pattern is defined as a pattern in which the Pq is lower than a predetermined value during one cycle while the Keff is maximized to extract the maximum energy. In some cases, minimization of the Pq may be regarded as objective function (to satisfy safety criteria), regardless of its effect on Keff.

Local power peaking in nuclear reactors is a complex 3-dimensional phenomenon, resulting from different reactor parameters (fuel loading, power level, temperature distribution, position of control rod groups, fuel burn up, spatial Xe oscillations…). To simplify this phenomenon, local power peaking is usually divided into two components as:

$$P_q = P_r \times P_z \times P_e \tag{1}$$

Where Pr and Pz are the radial and axial power peaking and Pe is an uncertainty factor. Radial power peaking is usually flattened via an optimal fuel loading/reloading pattern, once at the beginning of cycle (BOC). Axial power peaking which is continuously changing by perturbations created by control rod groups manoeuvres, power level changes and Xe oscillation, is not subject to flattening at BOC, but is usually kept under a certain value during reactor operation, via the reactor core controller. Therefore, in PWRs, minimization of Pq at BOC is reduced to minimization of Pr. The permissible amount of Pr in these reactors is 1.32[3].

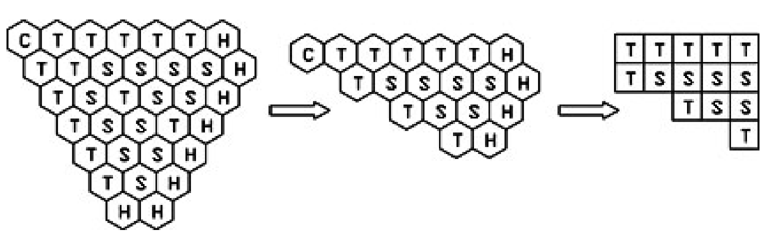


Figure 1. One-sixth and One-twelfth view of a VVER core

3. NEW STRATEGY FOR CORE LOADING PATTERN DESIGN

Other optimization objectives such as maximization of burnup (BU) at EOC conditions could be of interest in some fuel management strategies. However, another criterion which can be considered in the fuel reloading design is increasing the amount of plutonium burn up in fuel assemblies. In this research, we aim at finding a coreloading pattern to satisfy following three objectives in reactor core:

- *Maximization of* K_{eff}
- *Minimization of PU*
- *Limitation on* P_r

We calculates K_{eff}, and P_r for each fuel core pattern using WIMS and CITATION calculation codes at BOC conditions. In order to calculate the amount of remaining plutonium at EOC, we need fuel burn up calculations. We did the burn up calculations via exact numerical calculations of fuel burn up while considering power distribution in the core. In this method cycles` duration is divided into equal time intervals and power distribution is considered invariable during each interval. Obviously, shorter intervals will lead to more accurate results.

To calculate the remaining plutonium in EOC conditions we have implement following steps:

Step 1: Calculation of multi-group neutronic charactristics (D, $\sum_a$, $\upsilon\sum_f$ and $\sum_r$) for fuel assemblies at BOC conditions in absence of boric acid using WIMS calculation code.
Step 2: Calculation of K_{eff} using neutronic charactristics and CITATION core calculation code.
Step 3: Estimation of require boric acid concentration for criticality.
Step 4: Repetition of steps 1 to 3 until K_{eff} converges to one.
Step 5: Calculation of power distribution in the core and power generated in each fuel assemblies.
Step 6: Calculation of neutronic charactristics of each fuel assemblies for the next time interval based on burn up calculations during the previous time interval.
Step 7: Repetition of steps 2 to 6 as many times as needed during the cycle.

In this way remaining plutonium concentration at EOC in all of the fuel assemblies is calculable.

4. REACTOR CORE MODEL

A Russian PWR, the VVER-1000, was chosen as the reference reactor to demonstrate the applicability of the reload core design method. The VVER-1000 core consists of 163 hexagonal fuel assemblies producing 3000 MWth at full power. In practice, loading pattern design is carried out on a slice of one- twelfth or one-sixth of the core. We have used the one-twelfth symmetry. The cells in the slice are divided into two classes: six-symmetric and twelve-symmetric cells, except for the centre cell. Fig.1 shows a graphical representation of a loading pattern. All assemblies are uniquely labelled according to the symmetry information: six-symmetry assemblies are labelled with a prefix ‘S’; twelfth-symmetry assemblies are labelled with a prefix

‘T’and the high enrichment fuel assemblies labelled with prefix ‘H’ ; the centre assembly is labelled with ‘C’.

We used CITATION, a 2-dimensional core calculation code with a coarse mesh, for calculation of the Pr, Keff and Pu. Using a coarse mesh sharply reduces the algorithm execution time while the accuracy of calculation of the average power distribution of fuel assemblies, is not so affected.

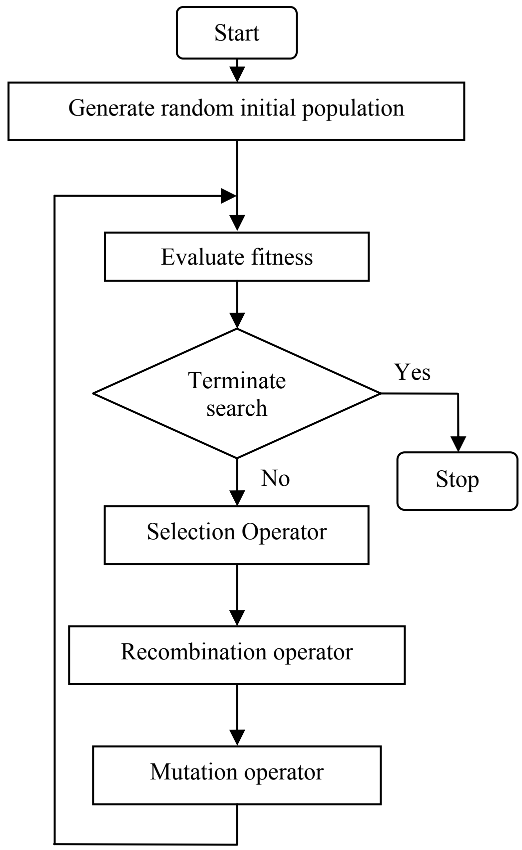


Figure2. Diagram of genetic

5. PRINCIPLES OF GENETIC ALGORITHMS

Genetic algorithms [4,9,10], or GAs, are stochastic search techniques based on the mechanism of natural selection and reproduction. They search from a population of points, typically producing new populations known as *generations*, by probabilistically selecting pairs of individuals for mating, recombining the genetic material (genotypes) of these pairs to produce new offspring, and then subjecting these offspring to random modifications known as mutations.

GAs for optimization were formally introduced in 1970`s by Holland [9]. The continuing performance improvements of computational systems have made them attractive for some types of optimization. Many optimization methods move from a single point in the decision space to the next using some transition rule to determine the next point. This point-to-point method is dangerous because it is a perfect prescription for locating false peaks in many peaked search spaces. By contrast, GAs work from a rich database of points, simultaneously climbing many peaks in parallel. Thus, the probability of finding a false peak is reduced over methods that go point-to-point. Therefore, GAs are less susceptible to getting stuck at local optima than conventional search methods.

In GAs, the term *chromosome* typically refers to a candidate solution to a problem, often encoded as a string of bits or numbers. Each chromosome can be thought of as a point in the search space of candidate solutions. The GAs process populations of chromosomes, successively replacing one such population with another. The GAs require a fitness function that assigns a score to each chromosome in the current population. The fitness of a chromosome depends on how well that chromosome solves the problem at hand [7]. After an initial population of chromosomes is randomly generated, the algorithm (see Fig. 2) evolves the population through the three operators: namely, *selection, recombination* and *mutation*, all based on random sampling.

-*Selection operator*: This operator selects individuals (chromosomes) in the population for reproduction. The goodness of each individual depends on its fitness. Fitness may be determined by an objective function. The fitter the chromosome, the more times it is likely to be selected to be reproduced.

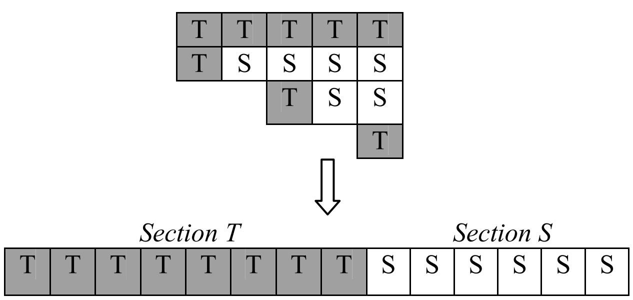


Figure3. Equal chromosome for fuel core loading pattern with one-sixth and one-twelfth

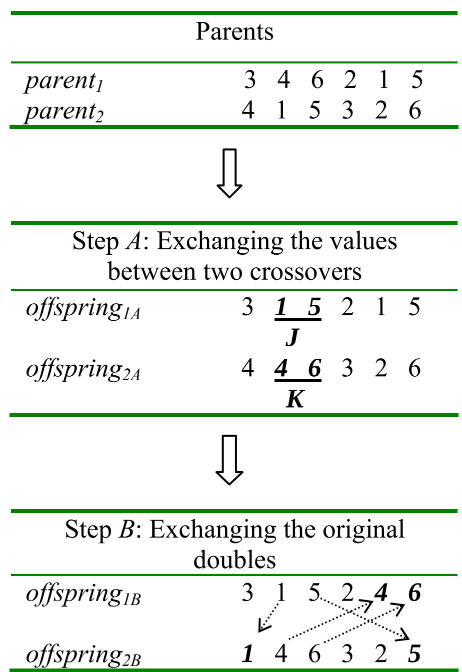


Figure4. An example of PMX method for two chromosomes recombination

-*Recombination operator*: This operator creates one or more offspring from the parents selected in the pairing process. Two individuals are chosen from the population using the selection operator. This operator randomly chooses a crossover site along the strings and exchanges the subsequences before and after that crossover site between the two individuals to create two offspring.

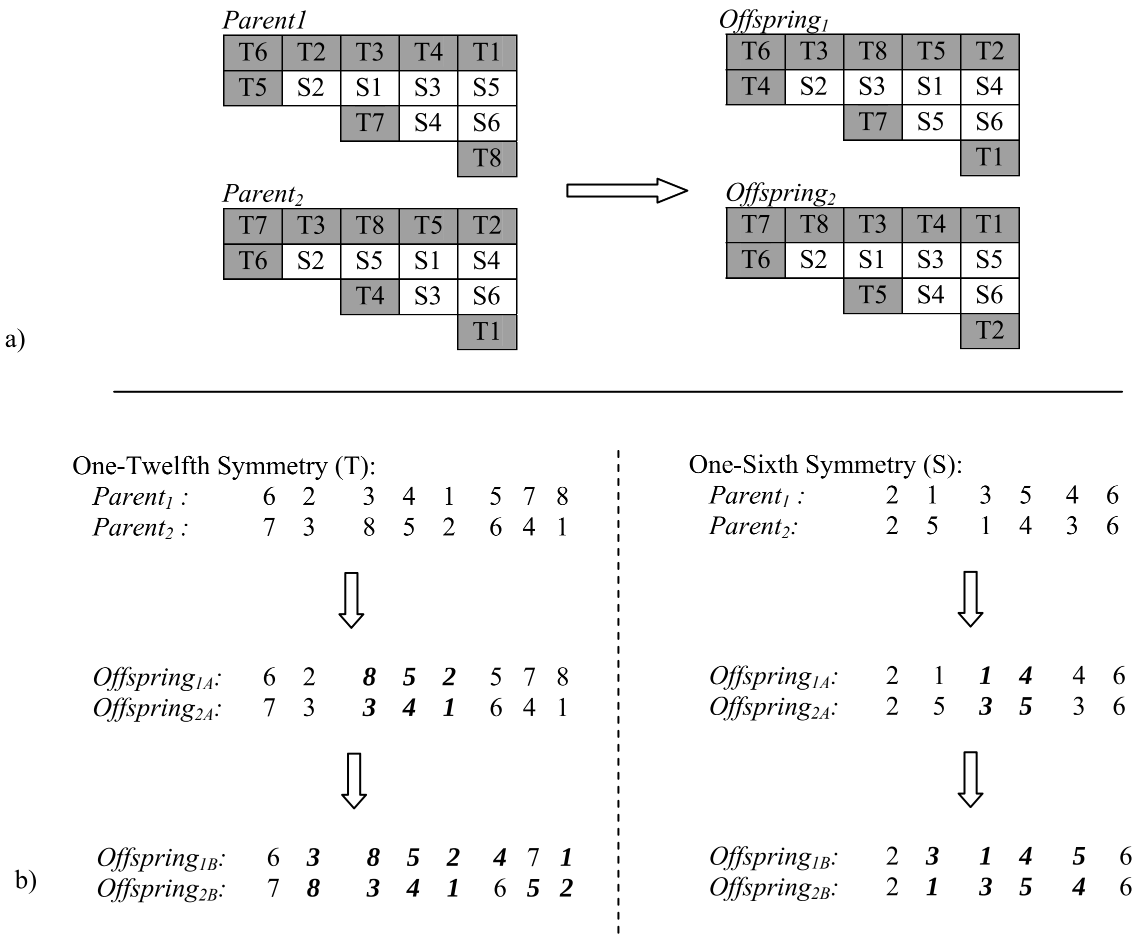


Figure 5. Example of recombination operator using PMX method on one-sixth and one-twelfth symmetries fuel assemblies: a) Real core loading pattern. b) Equivalent chromosomes with genes string.

6. USE OF GENETIC ALGORITHM IN LOADING PATTERN PROBLEM

We have used genetic algorithms to implement our approach on a typical reactor core [3,7]. During the optimization, each fuel assemblies should be exchanged with its own symetries. We implemented GAs only on one-twelfth and one-sixth symetries fuel assemblies while high enriched fuel assemblies are exchanged randomly.

Table I. Charactristics of the fuel assemblies in equlibrium cycle and number of different fuel assemblies in different sections of One-sixth of core

Fuel Assembly Type	Average Enrichment, ^{235}U (%weight)	Year of Burn-up	Burnable Absorber	Boron Concentration in Burnable Absorber (gr/cm^3)
36B20 0	3.62	0 (Fresh)	Type 1	0.02
36 1	3.62	1	-	-
36 2	3.62	2	-	-
40B36 0	4.02	0 (Fresh)	Type 2	0.036
40 0	4.02	0 (Fresh)	-	-
40 1	4.02	1	-	-
40 2	4.02	2	-	-
40 3	4.02	3	-	-

Solving a genetic problem starts with definition of chromosome as a combination of problem variables whose optimal amount must be found. In this research, we represent each core loading pattern as a chromosome in which fuel assemblies are as genes (see Fig. 3). Each chromosome contain eight genes in section 'T'and six genes in section 'S' while the total gene number, $N_{gene} = 8 + 6 = 14$ genes.

Now we need to define genetic algorithm operators: selection, recombination and mutation in fuel core loading problem and also scoring the chromosomes.

6.1. Selection Operator

The number of initial population (N_{ipop}) is too large to undergo the journey through the iterative steps of the genetic algorithm. Therefore Firstly, the N_{ipop} fitness function and associated chromosomes are ranked from highest rank to lowest. Then, only the best $N_{pop} < N_{ipop}$ members of the population are kept for each generations, while the others are discarded.

We have used $N_{ipop} = 32$ while, only $N_{pop} = 16$ of these chromosomes are kept for the next generation. In every generation, $N_{good} = 8$ chromosomes are placed in a mating pool for reproducing and $N_{bad} = 8$ are discarded. The first generation is selected randomly but in the next generations, the chromosomes are ordered based on their level of fitness and half of them which are fitter, are selected.

Using this operator, two chromosomes are selected to produce two offspring. There are different methods for pairing including: random pairing, rank weighting, cost weighting, tournament selection and pairing from top to bottom. We have used pairing from top to bottom. In this method, we start at the top of the list and pairs the each subsequent chromosome at a time until the top N_{good} chromosomes are selected for mating. Thus the algorithm pairs *chromosomes*$_{2i-1}$ with *chromosomes*$_{2i}$ for $i = 1, 2, \ldots$ [4]. In our study we would pair chromosomes *1&2*, *3&4*, *5&6* and *7&8*, respectively.

6.2. Recombination Operator

Using this operator, two selected chromosomes as mother and father exchange information similar to biological reproduction. This stage implement through different methods. We used partialy matched crossover method (PMX) which is applied separately on one-sixth and one-twelfth symmetry fuel assemblies [4,5,6]. This method explained by an example for two chromosomes with 6 genes in Fig. 4. This method includes two steps as follows:

Step1: Two crossover points are chosen randomly, then exchanging the genes of the parents between these points. If crossover points are between gene1 and gene2, and gene3 and gene4, then string K from parent$_2$ is switched with string J from parent$_1$. All values exchanged between parents are shown in bold type.

Step2: So far we still have the problem of having doubles of some integers and none of others. The switched strings, J and K, remain untouched throughout the rest of the procedure. The original doubles in *offspring*$_{2A}$ are exchanged for the original doubles in *offspring*$_{1A}$ (the original 4 in *offspring*$_{2A}$ exchanged with the original 1 in *offspring*$_{1A}$, and the original 6 in *offspring*$_{2A}$ with the original 5 in *offspring*$_{1A}$) to obtain the final solution.

To implement PMX method to the core loading pattern chromosomes in Fig. 3, we have used PMX to one-sixth and one-twelfth symmetry sections separately (Fig. 5).

6.3. Mutation Operator

In order to implement mutation we select two genes randomly in‘T’or‘S’sections and transpose them in own section. Mutations points are randomly selected between Npop× Ngene genes.We do not allow mutations on the best chromosome in each generation. We used mutation rate μ=0.042, means that 4.2 percent of total Npop× Ngene genes substitude in each generation.

6.4. Scoring the Chromosomes

Fuel core loading pattern design problem is a multi-objective optimization problem and it is difficult to define a suitable mathematical fitness function for it. In order to determine a suitable fitness function, we compare each *chromosome*$_i$ with all *chromosomes*$_j$ in each generation and use a scoring function as follows:

Where:

$$
\begin{aligned}
&- \ \textit{Scoring function}_i = a_i+b_i+c_i+10\times(1.32-P_{r\text{-}i})\\
&- \ \textit{Scoring function}_j = a_j+b_j+c_j+10\times(1.32-P_{r\text{-}j})
\end{aligned}
\qquad (2)
$$

$$
\begin{aligned}
&- \ \text{If} \quad (K_{eff\text{-}i} > K_{eff\text{-}j}) \quad \text{Then} \quad (a_i = 1 \text{ and } a_j = 0)\\
&- \ \text{If} \quad (P_{r\text{-}i} > P_{r\text{-}j}) \quad \text{Then} \quad (b_i = 0 \text{ and } b_j = 1)\\
&- \ \text{If} \quad (Pu_i > Pu_j) \quad \text{Then} \quad (c_i = 0 \text{ and } c_j = 1)
\end{aligned}
\qquad (3)
$$

Fig. 6 shows scoring approach for two chromosomes. In this method the considered parameters (K_{eff}, P_r and Pu) for each chromosome extracted by CITATION and WIMS core calculation codes as illustrated in Section 3. This task is repeated $N_{pop} \times (N_{pop}\text{-}1)$ times in each generation. In this way all the chromosomes are ranked based on their scores, and half of them will be selected as N_{good}.

$Chromosome_i \rightarrow [K_{eff\text{-}i}=1.198765,\ P_{r\text{-}i}=1.289806,\ Pu_i=0.039871]$
$Chromosome_j \rightarrow [K_{eff\text{-}j}=1.210956,\ P_{r\text{-}j}=1.320987,\ Pu_j= 0.041980]$

$Scoring\ Function_i = 0 +1 + 1 + [10\times(1.32-1.289806)] = 2.150970$
$Scoring\ Function_j = 1 + 0 + 0 + [10\times(1.32-1.320987)] = 0.995065$

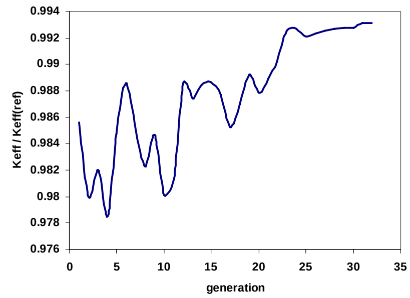
Figure 6. An example for calculating of scoring function between two chromosomes i and j

7. CASE STUDY

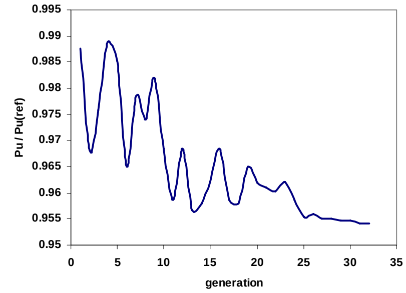
In previous sections, we presented the GAs algorithm for loading pattern design problem and our new approach on minimizing the remained Pu at EOC. We designed a fuel management code using Visual Basic.Net program language for implementing this strategy. This program calls WIMS and CITATION calculation codes for calculating K_{eff}, P_r and Pu for each fuel core loading pattern in a typical VVER-1000 nuclear reactor.

We have considered a VVER-1000 reactor core in equilibrium cycle. Table I shows detailed enrichment and burn-up of the fuel assemblies. The aim of this case is to implement our approach on equilibrium cycle, while high enrichment fuel assemblies are installed on the core periphery. Each fuel cycle of the reactor is considered 300-days which is divided to three equal time intervals (100-days) for burn-up calculations (see section3, step 6).

Simulation results show that after 32 generations, remained plutonium at EOC has been reduced 4.60% as compared to the reference equilibrium cycle [12], while K_{eff} has been decreased 0.69% and P_r reduced 0.31%. Table II and Figure 7 show result found by genetic algorithm and the evolutions of the K_{eff}, P_r and Pu versus the genarations, respectively. Figure 8 compares the one-sixth core loading pattern design calculated by our approach to minimize the remained plutonium at EOC with the reference cycle pattern.



a)



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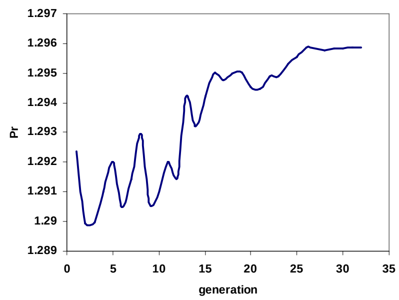

c)

Figure 7. Evolutions of the effective multiplication factor (K_{eff}), power peaking factor (P_r) and remained plutonium concentration at EOC (*Pu)* of the core versus generations: a) Relative K_{eff} of the core compare to reference equilibrium cycle loading pattern. b) Relative *Pu* compare to reference equilibrium cycle loading pattern. c) Exact P_r of the core

Table II. The best result found by genetic algorithm

Pu (Kg / Ton)	K_{eff}	P_r	
143.253654	1.299938	1.199302	Referernce equilibrium fuel loading pattern
136.66401	1.29587	1.191209	Optimal result after 32 generation using our strategy
-4.60%	-0.31 %	-0.69 %	Percent of change

8. DISCUSSION AND CONCLUSIONS

When attempting to optimize performance of a complex engineering system such as nuclear reactor, we are always faced within a problem of achieving several optimizing parameters, some of which may conflict. In reality, it is imposible to optimize all these parameters simultaneously and thus any “optimal” solution must inevitably represent some sort of compromise in which a degradation performance in at least one of the desired parameters must be tolerated. In this case, no unique “optimal” solutions exist but a set of “non-inferior” solutions, which may satisfy the safety constraints of a given reactor [11]. This research presents an innovative strategy on fuel core loading pattern design in PWR nuclear reactors aims maximization of the K_{eff} and minimization of the *Pu* while limiting the P_r lower than a predetermined value. Primary results show that this strategy has the ability to present a loading pattern with -4.60% lower level of remained plutonium at EOC conditions while retain K_{eff} and P_r parameters close to reference core loading pattern. Decreasing the time intervals will improve the results but increase the calculation time. As shown in Figure 8, the aim of decreasing the remained plutonium at EOC will result to a core loading with 0.0069 $^{\Delta k}/_{k}$ in K_{eff} which is equal to 10 days decrease in cycle length. This research is a first effort to takle the problem. In practice other constraints may be considered. Adding a new constraint to the optimal fuel core loading will affects the amount of the other core parameters.

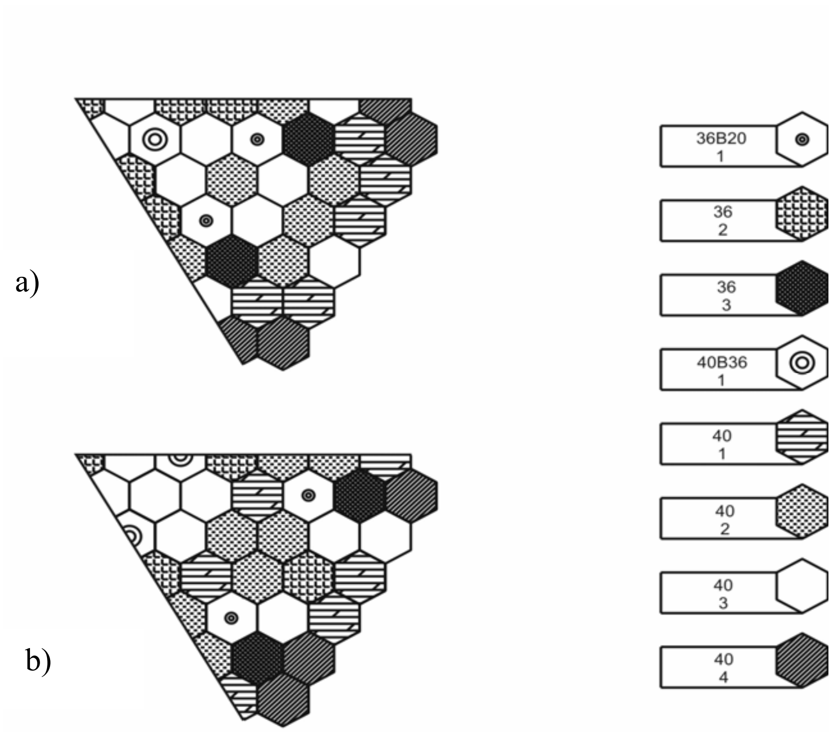

Figure 8. Comparing the one-sixth core loading pattern design calculated by our approach to minimize the remained plutonium at EOC conditions with the reference cycle pattern: a) reference core loading pattern. b) proposed core loading pattern. The right column shows different fuel assemblies and year of burnup.

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