Joint International Topical Meeting on Mathematics & Computation and Supercomputing in Nuclear Applications (M&C + SNA 2007)
Monterey, California, April 15-19, 2007, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2007)

OPTIMIZATION PROCEDURE FOR RADIAL BWR FUEL LATTICE DESIGN USING GENETIC ALGORITHMS

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ABSTRACT

An optimization procedure based on the Genetic Algorithms method was developed for the optimization of enrichment and gadolinia distributions in boiling water reactor fuel lattices. The optimization process was linked to the HELIOS code to evaluate and qualify all the investigated solutions. The goal is to search for the optimal fuel lattice distribution, which has the lowest average enrichment between two defined values, and the minimum local power peaking factor; which satisfies an average gadolinia concentration target and a k-infinite multiplication factor, also target. The weighting factors, used to give the relative importance of the evaluation parameters in the objective function, take two different values depending on the values of the evaluation parameters. This strategy helped the process to rapidly guide the search toward configurations which satisfy all the constraints. The genetic algorithm uses one crossover point, and two offspring. The main contribution of this work is the development of an efficient procedure for BWR fuel lattice design, using a powerful algorithm and an objective function which saves computing time, because it does not require lattice burnup calculations. The results show the good performance of this procedure; fuel distributions were found with low enrichment, low radial power peaking factor and good reactivity performance during the lattice life.

Key Words: BWR, Fuel Lattice Design, Genetic Algorithms

1. INTRODUCTION

In Mexico there is one nuclear power plant with two Boiling Water Reactors (BWR). Unit 1 is operating since 1990 and unit 2 since 1995. Today both reactors have 2027 MW of thermal power and are operating with cycles of 18 months. The reactor's core has 444 fuel assemblies (FA) and 109 control rods. Some of these fuel assemblies are fresh and others have one, two or more cycles of operation. The FA is an array of 10x10 fuel pins, using 6 or 7 axial sections, containing different characteristics related with burnable poisons, water zones, void zones and ^{235}U enrichment. An axial section of FA is called a radial fuel lattice and it uses approximately ten different pin compositions in the 10x10 array. The present study considers predefined lattice geometry, with fixed fuel rod size, pitch and water zones. Only the distribution of enrichment and gadolinia content in the fuel pins within the lattice are investigated. The radial fuel lattice design

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is focused on finding the adequate distribution to obtain very good lattice neutronic characteristics, when a window of values for the average lattice enrichment, and a target average lattice gadolinia concentration, are previously defined. To obtain this “optimal” design we need to solve a huge combinatorial problem and we propose to use Genetic Algorithms.

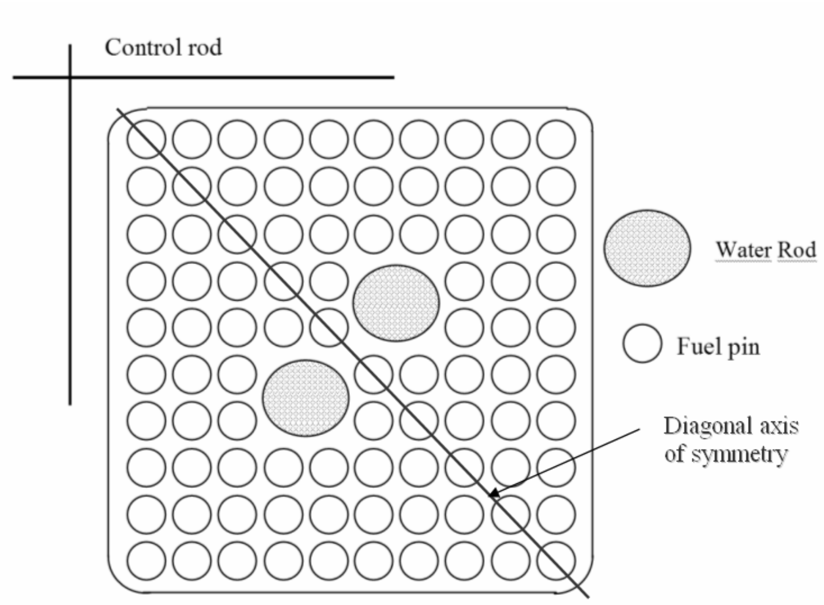

Figure 1. Radial representation of a fuel lattice

This large combinatorial problem has been treated on several works, for example; the method of approximation programming [1] gives quite satisfactory results producing feasible candidate designs comparable to those elaborated by an expert engineer. A methodology that combines the response matrix method with non-linear programming techniques [2] was applied to search for an optimal pin enrichment distribution that gives the best approximation to a prescribed power distribution in a two-dimensional FA. Another application to the optimization of MOX enrichment distributions in typical light water reactor assemblies [3] used a simplex method-based algorithm. Maldonado [4] developed a Simulated Annealing based multi-objective and multi-constraint pin-by-pin lattice optimization engine. Tabu Search optimization method has also been used successfully [5]. A work related to the burnable poison placement optimization has also been performed in the Pressurized Water Reactor (PWR) side [6]. Recently, fuel rod optimization for the coupled bundle-core design was developed, using the simulated annealing method to produce fresh bundle designs, and the linear superposition method, based upon sensitivity coefficients, to approximate core parameters. [7].

There are also several applications of GA, to solve different problems on nuclear fuel management tasks, which have proposed good ideas in order to implement, with high efficacy, this optimization method. GAs were applied to the reload pattern optimization for BWRs [8], GAs were also applied to the axial FA design of BWRs [9]. GAs were also used for in-core fuel management in PWRs [10]. Recently, GAs were also implemented as optimization method for in-core loading optimization in the VVER, which is the Russian version of the Pressurized Water Reactor [11].

2. METHODOLOGY DESCRIPTION

2.1. Genetic Algorithms description

Genetic Algorithms [12] combines mathematical analysis with random search to build artificial systems having properties similar to natural systems according to the Darwinian mechanisms of evolution. GAs is a highly parallel mathematical algorithm that transforms a set (population) of individual mathematical objects (chains of chromosomes); each of them associated to a fitness criterion, into a new population (next generation) using genetic operations. The GAs method has very good characteristics for solving complex combinatorial problems.

2.2. Representation of a candidate solution

In order to investigate the performance of different fuel distributions, a candidate solution **x** is represented by a bi-dimensional array indicating the fuel composition (with different ^{235}U enrichment and gadolinia (Gd_2O_3) concentration) located at each pin position in the lattice.

2.3. Mathematical model of the objective function

The objective function was formulated to find the solution ***x*** with:

1st. The minimum average lattice enrichment E(**x**) between E_{lower} and E_{upper}
2nd. An average lattice gadolinia concentration G(**x**) equal to the target G_{target}
3rd. An infinite k-multiplication factor at 0 MWd/t, $k_{inf,0}$(**x**) equal to the target $k_{inf,0,target}$
4th. The minimum local power peaking factor at 0 MWd/t, PPF_0(**x**)

The average lattice enrichment E(**x**) and the average gadolinia concentration G(**x**), are obtained considering the fuel composition located in each pin position using Equation (1&2), where *E(j)* is the enrichment of the fuel pin located in position *j* in the lattice with *92* total fuel pin positions.

$$E(x) = \sum_{j=1}^{92} \frac{E(j)}{92} \qquad G(x) = \sum_{j=1}^{92} \frac{G(j)}{92} \qquad (1\&2)$$

The values for $k_{inf,0}$(**x**) and PPF_0(**x**) are obtained modeling the lattice with the neutronic simulator HELIOS [13]; in which the fuel pins, the water regions, the channel and the control rod are explicitly represented in two-dimensions. In HELIOS, this lattice can be represented using half diagonal symmetry.

Because the lattice reactivity is proportional to the enrichment, the first constraint allows the minimization of the enrichment, between values that guaranty a desired reactivity behavior. To satisfy the first constrain, each lattice design generated by the algorithm, which no satisfy Equation (3), must be discarded and other lattice must be generated. This constraint allows the elimination of the lattices with very low or very high enrichment, before their neutronic evaluation.

$$E_{lower} < \mathrm{E}(\mathbf{x}) < E_{upper} \tag{3}$$

To satisfy the second constrain, Equation (4) must be minimized.

$$\Delta\mathrm{G}(\mathbf{x}) = \mathrm{abs}\,(\mathrm{G}(\mathbf{x}) - G_{target}) \tag{4}$$

To satisfy the third constrain, the quadratic deviation between $\mathrm{k}_{\mathrm{inf},0}(\mathbf{x})$ and $k_{inf,0,target}$, Equation (5), must be minimized.

$$\mathrm{D_k}(\mathbf{x}) = (\,\mathrm{k}_{\mathrm{inf},0}(\mathbf{x}) - k_{inf,0,target})^2 \tag{5}$$

The forth constrain is accomplished when $\mathrm{PPF_0}(\mathbf{x})$ is minimized.

Then the objective is to minimize the function $\mathrm{F}(\mathbf{x})$ presented in Equation (6).

$$\mathrm{F}(\mathbf{x}) = \mathrm{C} + \mathrm{w_E}\cdot\mathrm{E}(\mathbf{x}) + \mathrm{w_G}\cdot\Delta\mathrm{G}(\mathbf{x}) + \mathrm{w_P}\cdot(\mathrm{PPF_0}(\mathbf{x}) - PPF_{max}) + \mathrm{w_k}\cdot\mathrm{D_k}(\mathbf{x}) \tag{6}$$

Here w_i indicates the relative importance that the decision-maker attaches to objective/constraint *i*.

3. APPLICATION TO THE STUDY CASE

The optimization process was developed for a 10X10 fuel pin array with two water zones and diagonal symmetry. The fuel used is uranium dioxide (UO_2) and some fuel pins are mixed with gadolinia (Gd_2O_3) as burnable poison. The lattice performance is evaluated using the objective function that was previously presented.

3.1 Fuel compositions to accommodate in the lattice

To calculate the objective function, the FA is simulated using the HELIOS code, in which the fuel pins, the water regions, the channel and the control rod are explicitly represented in two-dimensions. Ten different pin enrichment and gadolinia concentrations (see Table 1) were used in the lattice. Six fuel compositions do not have gadolinia.

Table 1. Enrichment and gadolinia concentrations in fuel compositions.

Composition name	**2**	**2.8**	**3.6**	**4.4**	**3.95**	**4.9**	**3.95g5**	**4.4g5**	**4.4g4**	**4.4g2**
U_{235} %w	2.0	2.8	3.6	4.4	3.95	4.9	3.95	4.4	4.4	4.4
Gadolinia %w	0	0	0	0	0	0	5	5	4	2

3.2 Heuristic accommodation rules

To simplify the searching process, three heuristic accommodation rules were used:

a. The lowest enriched fuel (UO_2-1) is only used at the corner positions: (0,0), (0,9) and (9,9) shown in Figure 2.
b. Fuels containing gadolinia cannot be placed in the lattice's edge;
c. Water region position is fixed.

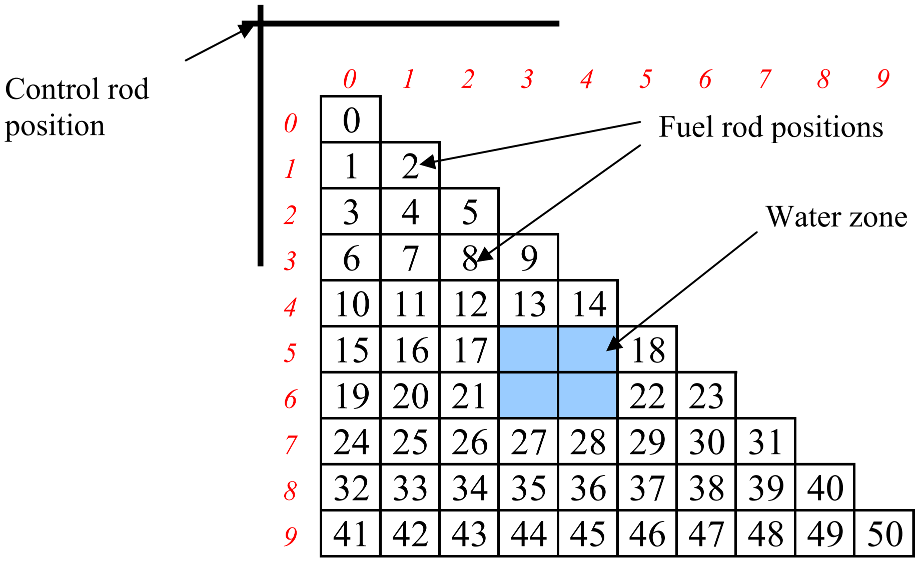

Figure 2. Schematic representation of the fuel lattice.

Taking into account the available fuel compositions and the heuristic accommodation rules, there are 16 pins that could have 5 different fuel compositions, and there are 32 pins that could have 9 different fuel compositions. The resulting number of total combinations is calculated as:

$$5^{16} \text{ x } 9^{32} \approx 5\text{x}10^{41}$$

Consequently the search must be well conducted to save computing time without loss of accuracy.

3.3 Heuristic composition rules

As it was mentioned in the introduction of this paper, in order to save computing time, burnup calculations were avoided, but the average enrichment and gadolinia in the lattice were used as constraints, to ensure that k-infinite was optimized inside a window of values. Then the following two heuristic rules were implemented:

a) Each individual must have an average enrichment between E_{lower} and E_{upper}.

b) Each individual must have a Gadolinia concentration between G_{lower} and G_{upper}

When an individual, generated by the genetic algorithm, does not satisfy these conditions, it is not evaluated and other one is created.

3.4 Genetic Algorithm specifications

a. The first population of individuals is randomly generated by placing the 10 available compositions in the lattice positions. This is done until the number of individuals, representing the first population, is obtained.
b. The proportional roulette technique was used as selection operator, where the individuals with better qualifications have higher probability of being selected as parents to be crossed.
c. The selection of individuals that survive from one generation to the next is elitist.
d. The crossover operator creates new individuals by the combination of parts from two individuals called parents, using one crossover point, chosen randomly. Furthermore, once two individuals are chosen as parents, they can not be chosen again in the same generation, and the offspring can not have the same quality than its two parents.
e. In the mutation operator, the position and the replacing composition are randomly selected; the mutated individual can not have the same quality than any other individual.
f. The population of individuals does not contain identical individuals
g. The crossover and mutation operations are repeated until the precedent conditions are satisfied.

3.5 Specific data for the application case

In order to fix the values for the limits and targets used in the OF, a reference lattice was selected. This is a fuel lattice which was used in the fresh fuel bundle of the cycle 10 core of the Unit 1 of the Laguna Verde nuclear power plant. All HELIOS calculations were done at 40 % voids, a fuel temperature of 793 ºK, and a moderator temperature of 560 ºK. Table 2 shows the Gd_{target}, the k_{inf_target} and the PPF_{max} imposed as constraints at 0 MWd/T. The limits E_{lower}, E_{upper}, G_{lower} and G_{upper} are also shown in this table.

Table 2. Pre-established data.

Variable	Value	Variable	Value	Variable	Value
G_{upper} (%)	0.82	E_{upper} (%)	4.15	PPF_{max}	1.435
G_{target} (%)	0.8152	E_{good} (%)	4.1	$k_{inf,0,target}$	1.02996
G_{lower} (%)	0.80	E_{lower} (%)	3.87		

The weighting factors used in Equation 6 are the same utilized in a previous work [14]. They were determined by the evaluation of hundreds of solutions and the values were adjusted in order to obtain an adequate response on the OF. These weights depend on the lattice evaluation parameters; when these values are far from the target values, the weights are higher than when they are close to the targets, because OF must be more penalized and, remember that the main goal is to minimize OF in Equation 6. The selected values are presented in Table 3. In this application E_{good} is the average enrichment of the reference lattice.

Table 3. Weighting factors.

Factor	Value	
	$E(\mathbf{x}) < E_{good}$	$E(\mathbf{x}) > E_{good}$
w_E	2	3
	$G(\mathbf{x}) < G_{target}$	$G(\mathbf{x}) > G_{target}$
w_G	15	20
	$PPF_0(\mathbf{x}) < PPF_{max}$	$PPF_0(\mathbf{x}) > PPF_{max}$
w_P	100	300
	$k_{inf,0}(\mathbf{x}) < k_{inf,0,target}$	$k_{inf,0}(\mathbf{x}) > k_{inf,0,target}$
w_k	13000	12000

4. RESULTS

The optimization process was executed, and sensitivity studies were done, changing some of the parameters of the genetic algorithm as the Population Size (PS), the Number of Generations (NG), the Crossover Probability (CP), and the Mutation Probability (MP). Only four optimization cases are presented in this paper; Table 4 shows the evaluation parameters for the *best* solution obtained at the end of the optimization. The values for PS/NG and CP/MP used in each case are also indicated in the first and second rows. For example, Case A used PS equal 20, NG equal 200, CP equal 50 and MP equal 20. Case A differs from Case B because the last one used 10% of MP compared with 20% in the first one; and Case A had better results. Case C differs from Case A because the second one used a PS of 20 individuals and Case C used 30; Case C had better results than Case A, but it required much more evaluations. Case D differs from Case C because Case D used a half CP value, and this action does not help to improve the process; Case D is the worst case and it used 6000 evaluations. In conclusion a good set of CP and MP is 50 and 20. Results could be improved when PS is increased, but 20 individuals are good enough.

Case A was selected to show the performance of the optimization algorithm, because it obtained very good results; 4000 evaluations were done and the *best* solution was found at the evaluation 3620. Figure 3 shows the PPF_0 and $k_{inf,0}$ as function of the generation number, for all the evaluated individuals during the optimization process of Case A. It is verified that PPF_0 was minimized and k-infinite converged to the target value. Figure 4 shows the *Objective Function* and the lattice *Enrichment* as function of the generation number, for the *best* solution, in each generation, during the optimization process of Case A. Figure 5 shows the pin enrichment and gadolinia distribution in the final best lattice of Case A.

Table 4. Results for the *best* solutions at the end of the optimization processes.

Variable	Case A	Case B	Case C	Case D	Reference
PS/NG	20/200	20/200	30/200	30/200	
CP/MP	50/20	50/10	50/20	25/20	
OF	53.33	57.86	53.06	67.74	
Enrichment (%)	4.0832	4.0636	4.0766	4.0973	4.1056
Gadolinia (%)	0.815	0.815	0.815	0.815	0.815
$k_{inf,0}$	1.0358	1.0372	1.0368	1.0489	1.0300
PPF_0	1.382	1.426	1.378	1.453	1.435
Number of evaluations	3640	3780	5910	6000	

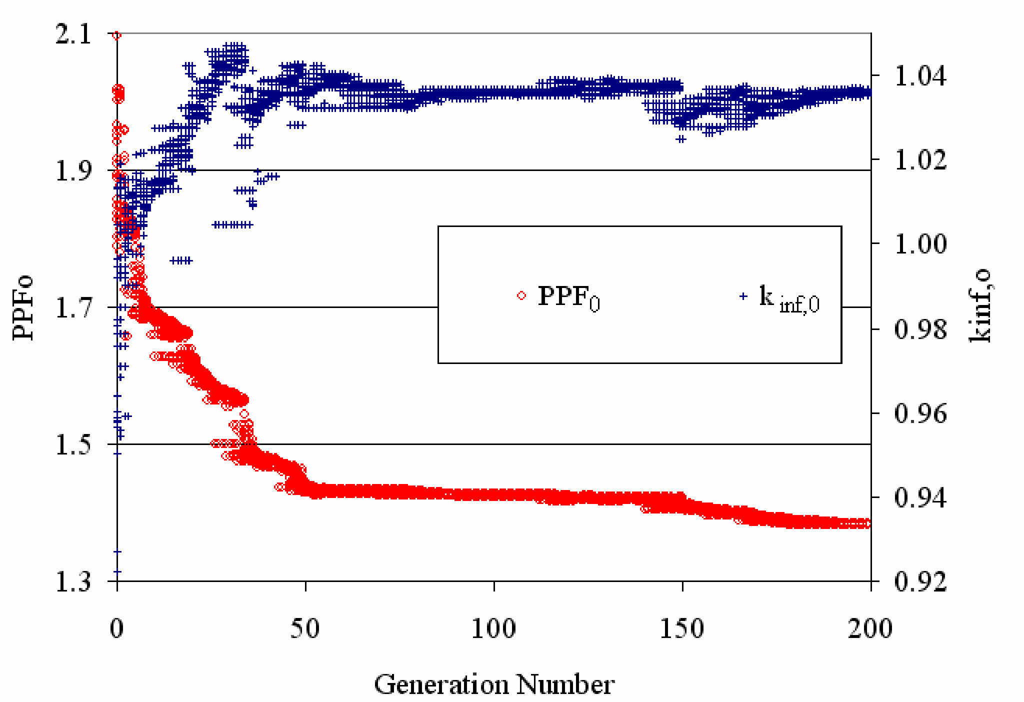

Figure 3. PPF_0 and $k_{inf,0}$ as function of generation number for all the evaluated individuals during the optimization process of Case A.

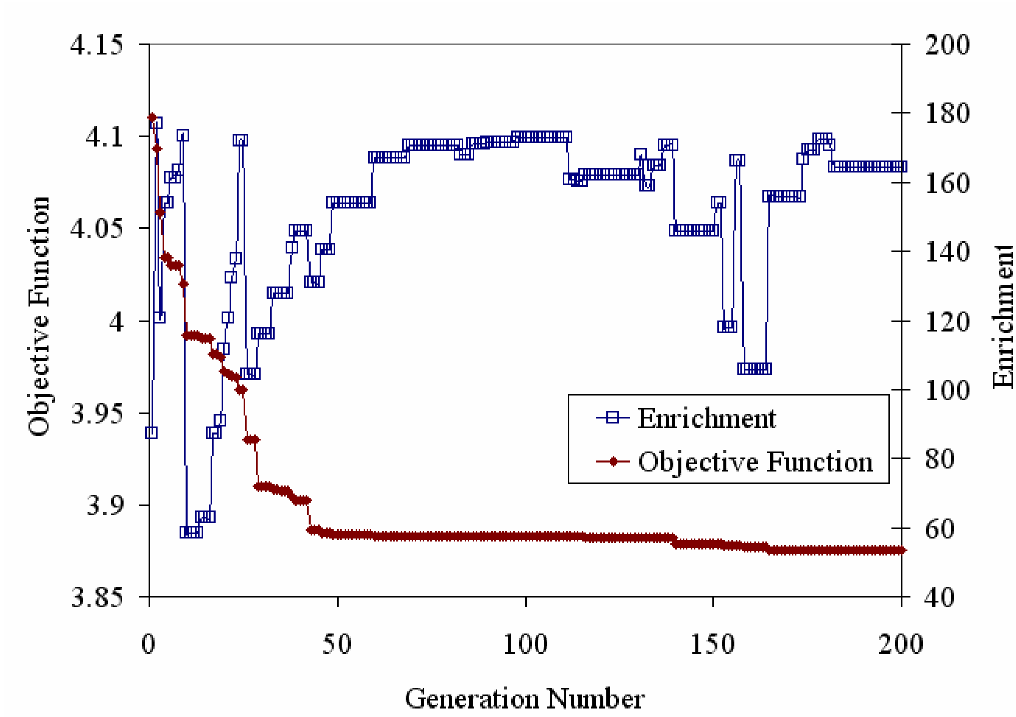

Figure 4. Objective Function and Enrichment as function of the generation number for the best solution during the optimization process of Case A.

	0	1	2	3	4	5	6	7	8	9
0	2									
1	2.8	3.6								
2	3.6	4.9	4.4							
3	3.95	4.4	3.95	3.95g5						
4	4.4	3.6	3.95g5	3.95	3.6					
5	4.4	3.95	4.4g4			2.8				
6	4.4	3.95g5	3.95g5			4.4	3.6			
7	3.6	4.9	4.9	3.95	4.9	4.9	4.4g4	4.9		
8	2.8	3.95	4.4	3.6	3.95g5	4.9	4.9	4.4g5	4.4g4	
9	2	2.8	3.6	4.4	4.9	4.9	4.9	4.9	4.4	2

Figure 5. Pin enrichment and gadolinia distribution in the final *best* lattice of Case A.

In order to verify that BOC calculations were good enough as to ensure that the designed lattices perform well in terms of reactivity and PPF versus burnup, the twenty different individuals (enrichment-gadolinia distributions) of the generation number 200 (the last population), were simulated with HELIOS until 60,000 MWd/t, and the PPF and k-infinite as function of burnup were graphed. Figure 6 and 7 show these functions and they proved that *k-infinite* is comparable to the reference lattice and that PPF always decreases with burnup.

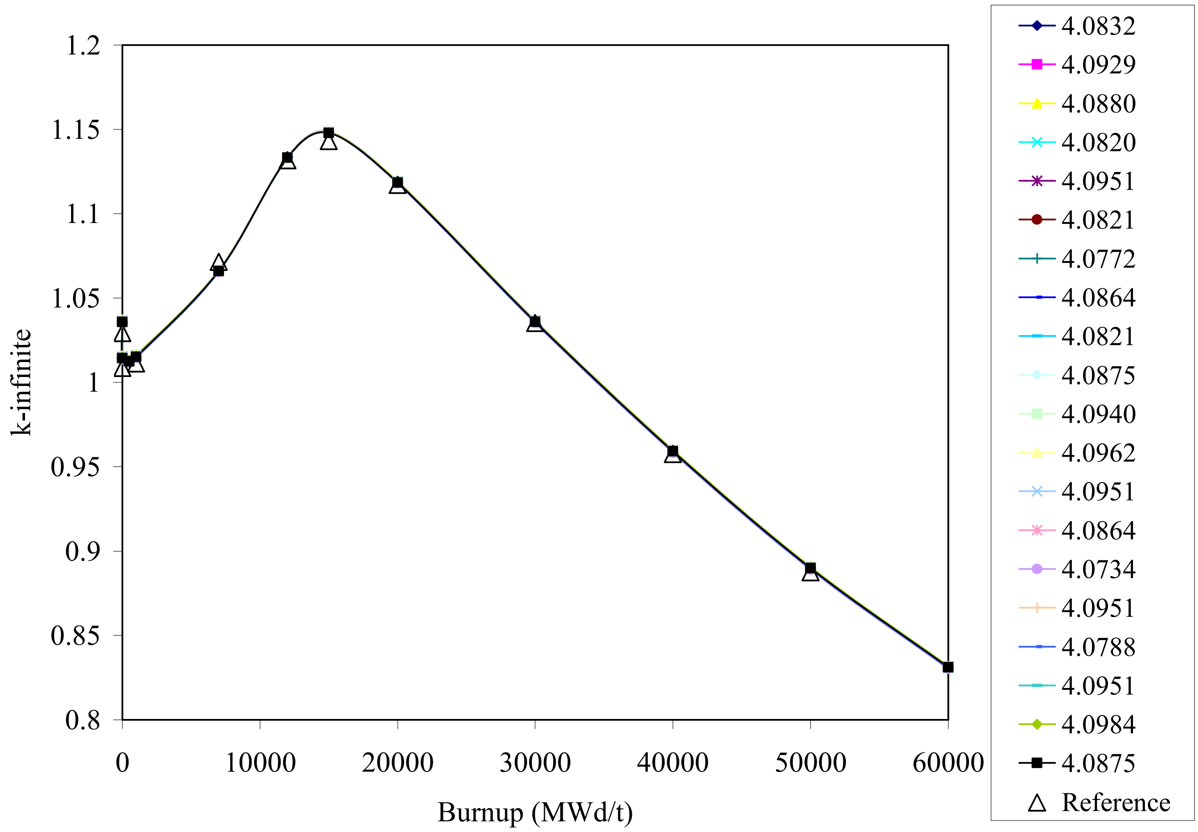

Figure 6. k-infinite vs Burnup for the 20 individuals of the generation 200 of the optimization process of Case A.

5. CONCLUSIONS

Genetic algorithms method was applied to optimize the fuel enrichment and gadolinia distribution in a BWR fuel lattice. It was not required to execute burnup lattice calculations to assure an adequate reactivity performance during fuel exposure, because the average enrichment reduction was limited to values that ensure the lattice reactivity. The use of weights, depending on the lattice parameters values, helped to rapidly guide the search of enrichment and gadolonia distributions toward configurations which satisfy all the constraints.

As general result, an effective optimization procedure for BWR fuel lattice design using Genetic Algorithms was developed. On this procedure several strategies for reducing computing time were implemented without penalizing the accuracy of the results. The system's coding and architecture is so flexible that the objective function is very easy to change.

This methodology can be applied with few modifications to the design and optimization of fuel lattices for the Advanced Boiling Water reactor and the Evolutionary simplified Boiling Water Reactor. Also it is possible to parallelize the optimization process and to replace the HELIOS code by other lattice's neutronic simulator or by a trained neural network.

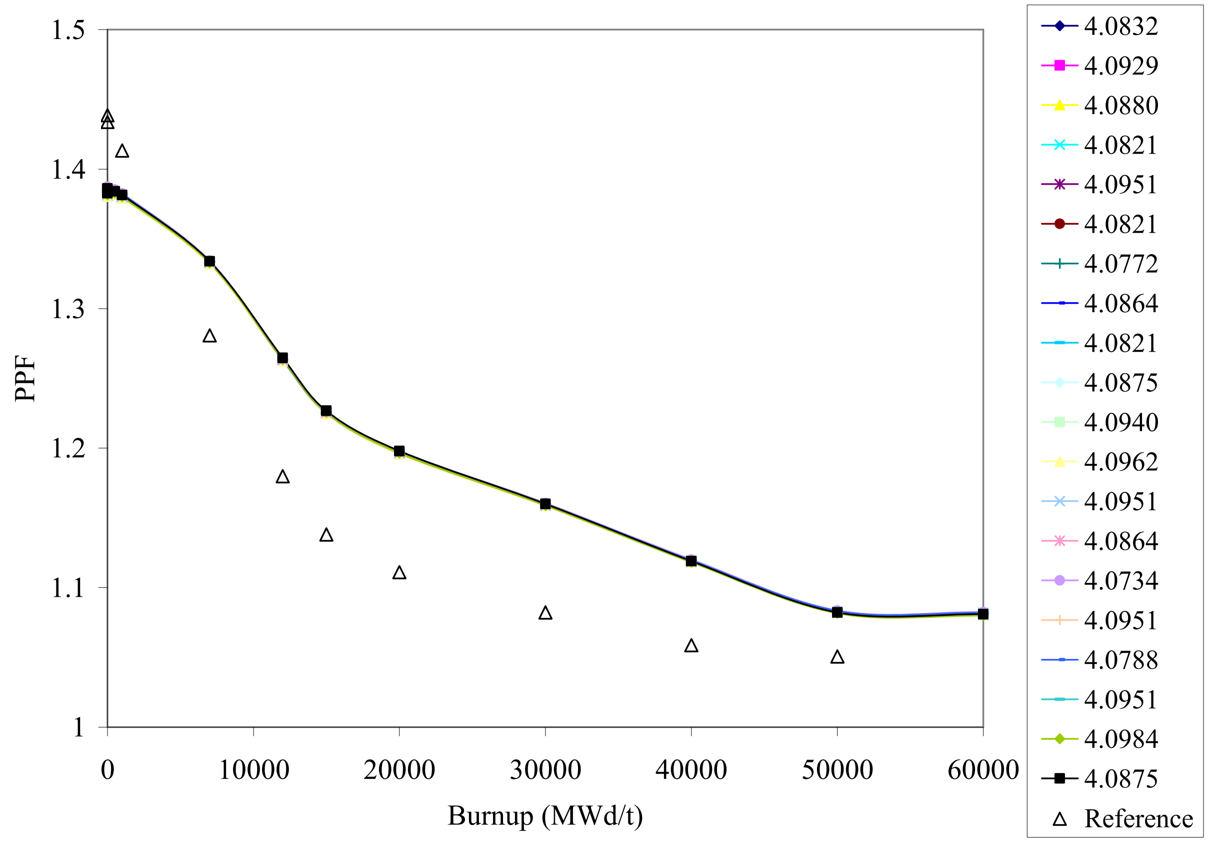

Figure 7. PPF vs Burnup for the 20 individuals of the generation 200 of the optimization process of Case A.

ACKNOWLEDGMENTS

The authors acknowledge the support given by CONACyT through the research project: NSF-2004-C01-9.

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