Joint International Topical Meeting on Mathematics & Computation and Supercomputing in Nuclear Applications (M&C + SNA 2007)
Monterey, California, April 15-19, 2007, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2007)

TREATMENT OF THE CENTRAL SINGULARITY IN THE CONFORMAL MAPPING APPROACH TO THE ANALYTIC NODAL METHOD IN CYLINDRICAL GEOMETRY

Rian H. Prinsloo* **and Djordje I. Tomašević**
Necsa
Building P-1900, PO Box 582, Pretoria 0001, South Africa
rianp@necsa.co.za; djordje@necsa.co.za

ABSTRACT

The Analytic Nodal Method, as applied to full core neutronic diffusion calculations, has recently been extended to cylindrical geometry via the use of conformal mapping. The conformal mapping approach allowed the completion of the transverse integration procedure, which is essential within the Analytic Nodal Method. Although an investigation into the application of the method to the current design of the South African PBMR reactor proved successful, the approach does contain limitations not exposed by the PBMR design. These limitations reside within the nature of the conformal mapping function, which maps a cylindrical area element into a rectangular node, since this complex logarithmic function exhibits a singularity within the coordinate origin. The solution thus far imposed was the use of a small homogeneous cylindrical cut-out placed around the radial center of the system, with a zero net-current boundary condition at its edge. Furthermore, problems containing central inhomogeneity expose the weaknesses of the quadratic transverse leakage approximation in cylindrical geometry. In this paper, the method is applied to two severe neutronics problems. One of these is in the form of a "cylindrisized" version of the 2-group 3D IAEA PWR benchmark. Based on results, a series of extensions to the nodal method are developed in order to treat identified shortcomings. Potential solutions to this problem include an improved quadratic leakage model, a hybrid leakage approximation and a finite difference / nodal hybrid formalism.

Key Words: Analytic Nodal Method, Conformal Mapping, Transverse Leakage, reactor physics

1 Introduction

Nodal diffusion methods have been used extensively in nuclear reactor calculations, specifically for their performance advantage, but also for their superior accuracy. More specifically, the Analytic Nodal Method (ANM) [1], utilising the transverse integration principle, has been applied to numerous reactor problems with much success. Recently this method has been extended to cylindrical geometry via the use of conformal mapping [2][6], circumventing some of the mathematical difficulties experienced during the transverse integration process. The transverse integration method serves to produce auxiliary one-dimensional equations in all 3 directions. In the conformal mapping approach, a cylindrical (r, θ) node is mapped unto a Cartesian (u, v) node, and the transverse integration is completed in the locally mapped Cartesian coordinate system. The application of this method to the PBMR reactor design has been investigated [3] yielding encouraging results, but the design of PBMR does not pose significant challenges to the method, and therefore does not expose some of its shortcomings. These limitations reside within the nature of the conformal mapping function (which maps a cylindrical area element into a rectangular node), and how this complex Logarithmic mapping function affects the solution near the radial center of the

*Corresponding author

system. In order to quantify possible errors and propose solutions, the cylindrical nodal method is applied, in this paper, to two distinct problems.

The first represents an idealised cylindrical (r, θ) "stripe" problem which exhibits azimuthal inhomogeneity throughout the system (including the coordinate origin). The second is a realistic, neutronically difficult, 3D problem in the form of a cylindrisized version of the well-known 3D 2-group IAEA benchmark [4]. These models are used to evaluate and develop improvements and calculate performance advantage factors over finite-difference. Such improvements considered in this work include the following:

- Selection of the appropriate quadratic leakage approach for the axial leakage component in the one-dimensional radial equation. Within the conformal mapping approach, two options exist for the implementation of the radial quadratic leakage approximation, and specifically for the axial contribution to it. In the first option the three-node quadratic transverse leakage fit (generally implemented in transversely integrated nodal methods) is applied to transverse leakage in the original cylindrical coordinates, and in the second it is applied to transverse leakage in the mapped Cartesian coordinate system. These two approaches are developed and evaluated in this paper.
- A hybrid leakage approximation, making use of the flat approximation in selected directions near the center, and the quadratic approximation throughout the rest of the system. This approach aims to lesson the dependence on the quadratic leakage approximation in regions where the nodal aspect ratios are scewed. In this regard it is important to note that, due to the conformal mapping approach, a couple of extra radial nodal meshes are required near the radial center of the system. This could lead to skewed $(\triangle r : \triangle z)$ nodal aspect ratios in the center.
- The Logarithmic mapping function exhibits a singularity in the radial center of the system. The solution initially imposed was the use of a small homogeneous cylindrical cut-out placed in the center of the system, with a zero net current boundary condition at its edge. In this paper a hybrid finite difference / nodal solution scheme is proposed, utilizing the finite-difference form of the solution within the central cut-out and only in the (r, θ) plane. Such an approach would make use of the fact that the central cutout region is small (order of 1 cm for the sake of current implementation), and therefore does not require many finite-difference meshes to reproduce a reasonable result.

Before investigating some of these approaches and their necessity within the confines of the specified problems, some attention has to be given to the mathematical formulation of the conformal mapping solution method, and the definition of the transverse leakage terms which appear in the auxiliary one-dimensional equations.

2 Conformal mapping

In order to describe the flux distribution within a cylindrical reactor with given cross sections, we may begin by writing the 3-dimensional diffusion equation in cylindrical geometry.

$$-\bigtriangledown D^g(r,z,\theta)\bigtriangledown \Phi^g(r,z,\theta) + \sigma^{g,rem}(r,z,\theta)\Phi^g(r,z,\theta) - Q^g(r,z,\theta) = 0 \tag{1}$$

All symbols have their standard meaning in nuclear reactor analysis. This is the problem we need to solve in order to obtain the criticality level of the core, as well as the position and energy dependant distribution of the flux. If we divide the system into n zones and integrate the diffusion equation over each zone, we will obtain a set of coupled 3D balance equations over all nodes n:

$$\sum_{j=1}^{6} \overline{J}_{jn} S_{jn} - Q_n V_n + \sigma_n^{rem} \overline{\Phi}_n V_n = 0 \tag{2}$$

Before we can solve Eq. (2), we need a further relationship between node-averaged flux and side-averaged currents in each direction. The ANM requires that the 3D diffusion equation is transversely integrated in order to produce three one-dimensional differential equations, which are in turn solved to produce the required node-averaged flux to side-averaged current relationship in the r, z and θ directions respectively. Once these relationships are established, they are reinserted into Eq. (2) to complete the solution. Unfortunately, the traditional transverse integration process fails to produce a one dimensional equation in the θ direction [5]. In order to circumvent this difficulty, a conformal mapping technique was proposed to map the cylindrical (r, θ) nodes to rectangular (u, v) nodes, and then complete the transverse integration in the mapped Cartesian coordinate system. The details of this formulation have been previously published [2].

3 Potential nodal extensions

3.1 Quadratic leakage approaches

In this paper we aim to, amongst others, address various formulations of the transverse leakage term which appears in the mapped one-dimensional u- equation. We will not present the full one-dimensional $u-$equation here, but simply give the transverse leakage source term and discuss various approaches of treating it. The transverse leakage source term for node n in the $u-$direction has contributions from both mapped $v-$ and $z-$directions and may be written as

$$L_{n,u}(u) = L_{n,u}^{v}(u) + (r_n^- r_n^+) e^{2u} L_{n,u}^{z}(u) \tag{3}$$

with v and z components defined as

$$L_{n,u}^{v}(u) = \frac{J_v(u, \frac{h_v}{2}) + J_v(u, -\frac{h_v}{2})}{h_v} \tag{4}$$

and

$$L_{n,u}^{z}(u) = \frac{J_z(u, \frac{h_z}{2}) + J_z(u, -\frac{h_z}{2})}{h_z} \; . \tag{5}$$

Here $r_n^{\pm}$ denotes the left and right boundaries of node n and $(r_n^- r_n^+) e^{2u}$ represents the area mapping scale function which appears due to conformal mapping. The standard quadratic transverse leakage approximation requires that a three-node quadratic fit of average transverse leakages are performed. Within the conformal mapping approach, and with regard to the contribution from z-direction, we have two options. Fit the original coordinate system transverse leakage quantity with a quadratic polynomial, hence

$$(r_n^- r_n^+) e^{2u} L_{n,u}^z(u) = a + bu + cu^2 \tag{6}$$

or fit the mapped transverse leakage component with a quadratic polynomial as

$$L_{n,u}^z(u) = a + bu + cu^2 \tag{7}$$

In both these cases we would require that the node-averaged transverse leakage, in all three neighbouring nodes, is maintained. Hence that

$$\overline{L_{n,u}^z} = \frac{1}{\triangle u} \int_n (r_n^- r_n^+) e^{2u} L_{n,u}^z(u) du \tag{8}$$

is maintained in nodes n-1, n and $n+1$. These two approaches will be termed *original* and *mapped* quadratic transverse leakage, respectively. The *original* approach is analytically much simpler, but may prove inaccurate in some cases since the area mapping scale function is exponential. This exponential is steeper near the radial center of the system.

We will evaluate both these approaches numerically during the calculation of the cylindrisized IAEA 3D benchmark.

3.2 Hybrid flat / quadratic leakage

The usage of flat (constant) or quadratic leakage approximations within Cartesian nodal methods are well-known [1], and the use of quadratic "original" leakage within cylindrical geometry was demonstrated in [3]. The motivation behind proposing a hybrid of the quadratic and flat leakage approximations are summarised in the following arguments:

- As first implementation, a small cut-out was placed around the coordinate origin (1 cm) in order to exclude the central singularity in the mapping function [2]. At the edge of this cut-out a non-physical zero current boundary condition was placed and the cut-out zone flux was approximated as the azimuthal average of the neighbouring inner side-fluxes. This boundary condition potentially disrupts the true flux profile and may propagate some distance into the system. The propagation of this error is worsened by the use of the quadratic leakage approximation in the radial direction, since it utilises a three-node radial fit to reconstruct the one-dimensional radial leakage shape within the node. Since the conformal mapping approach requires a couple of extra inner meshes to limit the gradient of the exponential mapping function, a flat leakage approximation in the radial direction, in the first few radial meshes, will not be inaccurate. This statement will be evaluated numerically.
- The quadratic leakage approximation is known to break down for nodes with large aspect ratios. Near the center of the cylindrical system, the ratio of node sizes from r to z and θ to z may become large and lead to inaccuracies in the z-direction quadratic transverse leakage fit. Utilizing a flat leakage approximation in the z-direction, near the radial center, will therefore also be evaluated numerically.

These approaches will be applied to both the (r, θ) "Stripe" and the cylindrisized IAEA 3D benchmark problem.

3.3 Hybrid finite-difference / nodal solution

In order to improve upon the inner zero-current boundary condition placed at the edge of the 1 cm nodal cutout, a hybrid finite-difference / nodal solution scheme is proposed for the inner radial region. This approach will, in this paper, be developed for 2D (r,θ) geometry, and applied to the (r,θ) "Stripe" problem below. It will, in future work, be extended to 3D geometry by merging it with a nodal z-solution.

We begin by stating the balance equation in cylindrical geometry as derived in [6].

$$\left[\sum_{m=1}^{6} a_n^{nm} C_{mn} + \sigma_n^{rem}\right] \overline{\Phi}_n - \sum_{m=1}^{6} a_n^{nm} C_{nm} \overline{\Phi}_m = \sum_{m=1}^{6} a_n^{nm} \left[C_{mn} Z_{mn} - C_{nm} Z_{nm}\right] + \tag{9}$$

$$\frac{1}{k_{eff}} \chi^g \sum_{h=1}^{G} \upsilon^g \sigma_{f,n}^h \left(\overline{\Phi}_n\right) + \sum_{h=1,h\neq g}^{G} \sigma_{s0,n}^{h\rightarrow g} \left(\overline{\Phi}_n\right)$$

Here a_n^{nm} refers to the surface to volume ratios of node n at the interface with node m, C_{nm} denotes the nodal coupling coefficients and Z_{mn} denotes mesh edge source gradients ("tensorial" sources). All other symbols have their standard meaning. The above nodal balance equation may be reduced to produce a finite-difference solution by the following redefinitions within the chosen finite-difference part of the domain:

- Redefinition of nodal radial coupling coefficients as finite-difference mesh-edge coefficients (boundary conditions at system boundaries are maintained):

$$C_{mn}^{radial} = C_{nm}^{radial} = C_{FD,nm}^{radial} = \frac{2 \frac{D_n}{\triangle r_n} \frac{D_m}{\triangle r_m}}{\frac{D_n}{\triangle r_n} + \frac{D_m}{\triangle r_m}} \tag{10}$$

- Neglecting of contributions from the one-dimensional radial sources by setting the radial tensorial quantities to zero:

$$Z_{mn}^{radial} = Z_{nm}^{radial} = 0 \tag{11}$$

These simple adjustments will allow for the use of Eq. (9) for both finite-difference and nodal calculations in adjacent meshes. Note that the azimuthal coupling coefficients, as well as tensorial sources, will keep their nodal definitions in the finite-difference domain.

3.4 Adjustment to one-dimensional equations at nodal / finite-difference interface

The first non-finite-difference zone in the system (counting radially outward from the center) also requires some special attention. Adjustment to the nodal equations in this region relate mainly to the radial interface current between the nodal and finite-difference regions. In order to address these issues, a two-node problem is defined with node n representing the last finite-difference mesh and node m representing the first nodal mesh where the nodal solution is applied. The aim is to define a consistent expression for net current at the interface which satisfies the following continuity conditions for side-flux and net side-current respectively:

$$f_{nm} = f_{mn} \tag{12}$$

$$J_{mn}^{fd} = -\frac{1}{r_{nm}} J_{nm}^{nod} \tag{13}$$

Here we denote J_{mn}^{fd} as the interface current from the finite-difference domain, and J_{nm}^{nod} as the current from the nodal domain. The $\frac{1}{r_{nm}}$ term appears due to the conformal mapping approach. We have

$$J_{mn}^{fd} = \frac{2D_n}{\triangle r_n} \left(\Phi_n - f_{mn} \right) \tag{14}$$

as an expression for the finite-difference net current at the interface between node n and node m. The expression for J_{nj}^{nod} is somewhat more complicated, but assumes the standard form for side-averaged current from the Analytic Nodal Method (ANM) [6] . We apply the interface conditions (Eqs. (12) and (13)) to the respective side-averged net current expressions from both the finite-difference and nodal approaches and obtain as expression for the interface current

$$J_{mn}^{radial} = \widetilde{C}_{mn}^{radial}(\overline{\Phi}_n) - \widetilde{C}_{nm}^{radial}(\overline{\Phi}_m - Z_{nm}^{radial}) \tag{15}$$

Writing the current expression in this notation allows us to use the balance equation (9) by simply redefining the coupling coefficients in it.

4 Problem descriptions

4.1 (r, θ) "Stripe" problem

The first problem in this section can be described as a potential worst-case scenario for the conformal mapping based nodal method. The problem will test the mapping approach in multidimensional (r, θ) coordinates, addressing issues such as azimuthal inhomogeneity in the center, as well as the importance of quadratic transverse leakage within cylindrical geometry. The (r, θ) "striped" problem is illustrated in Fig. 1.

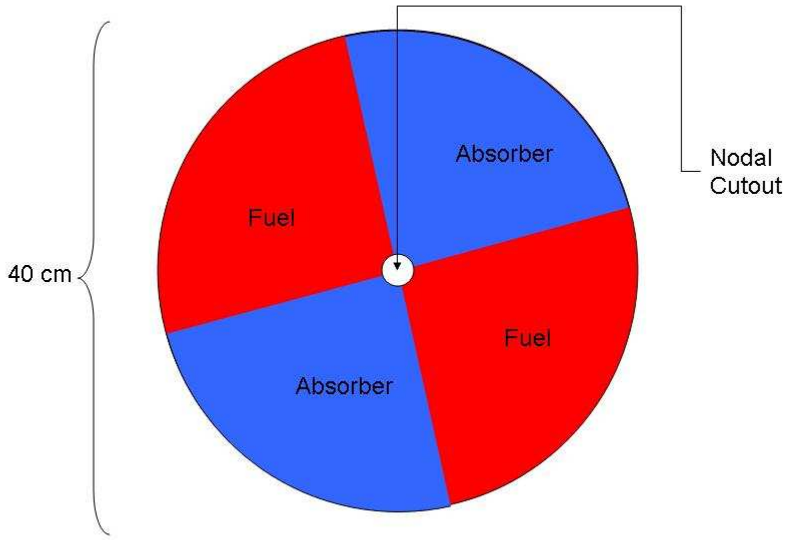

Figure 1. (r,θ) "Striped" problem geometric description

The problem has the following characteristics:

- Figure 1 contains azimuthally alternating fuel ($\nu\sigma_f$ = 0.3 cm^{-1}, σ_r=0.2 cm^{-1}, D=1 cm) and absorber (σ_r= 0.5 cm^{-1}, D=1 cm) regions.
- The problem contains azimuthal inhomogeneity throughout the system, including in the coordinate origin.

Nodal results are calculated with the cut-out plus five radial (sizes of 1, 3, 5, 5 and 5 cm), and twelve azimuthal meshes (30 degrees per node). Finite-difference reference results are 30 times refined in both directions, as compared to the nodal sizes.

5 IAEA cylindrical 3D benchmark

The chosen numerical problem addressed in this paper, is a "cylindrisized" version of the 3D IAEA PWR benchmark problem. The benchmark has been adapted to cylindrical coordinates by closely matching the volumes of fuel, control and reflector material, and hence preserving the nature of flux gradients in the problem. As basic illustration, Fig. 2 describes the material layout.

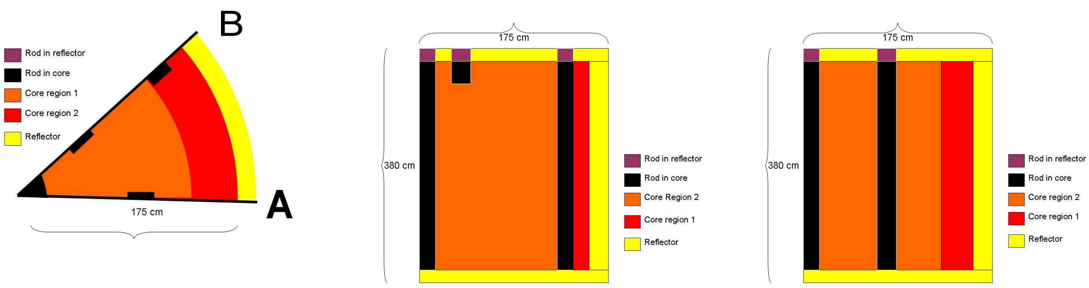

Figure 2. IAEA layouts given as (r,θ) cut, as well as (r, z) cuts at A and B

For a full illustration of the problem, the meshing scheme and material layout are described in Figs. 3 and 4 below. The (r, θ) planes below are presented as rectangular zones for simplification.

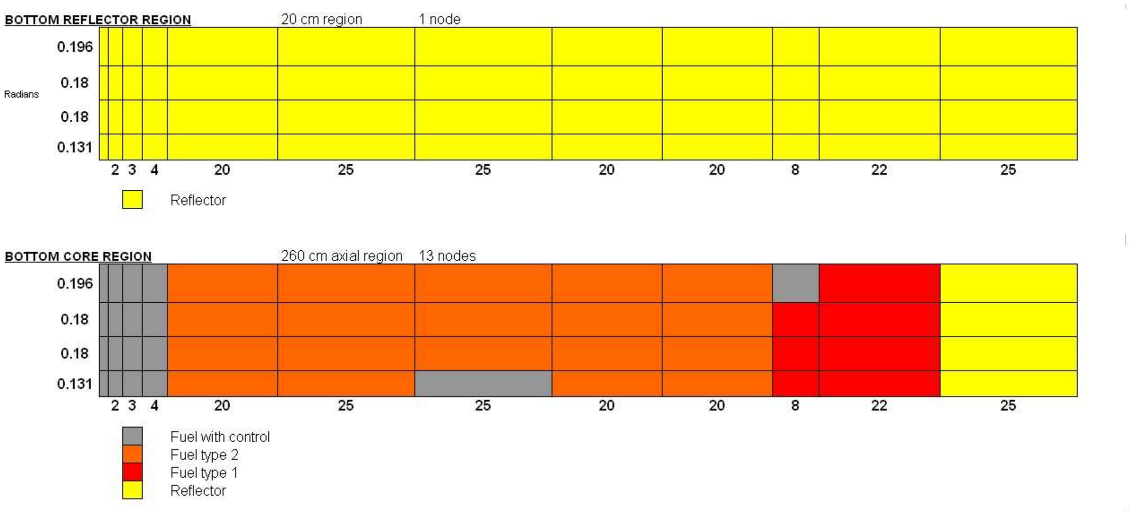

Figure 3. IAEA Nodal mesh - bottom reflector and bottom core regions

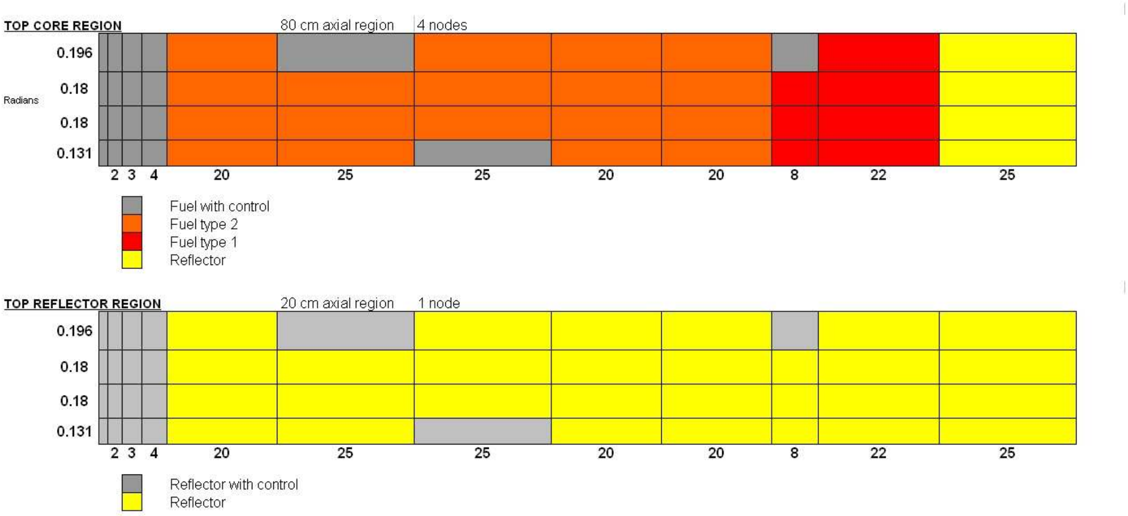

Figure 4. IAEA Nodal mesh - top core and top reflector regions

The nodal calculation is performed on the above mesh, and the reference finite-difference solution is obtained via extrapolation of 6, 8 and 12 times refined finite-difference solutions. The method of extrapolation is based on a quadratic polynomial fit over the three finite-difference mesh refinements (6,8 and 12 times). It is similar to that used in the original benchmark problem and is described in [4]. Five materials are utilized in this benchmark, and cross-sections are provided in this section. These cross-sections are specified in Table I below:

Table I. IAEA Material cross-sections

Region	G	$D(cm)$	σ^{rem}	$v\sigma^{fis}$	$\sigma_{1,2}^{scat}$
Reflector	1	2.0	0.04	0.0	0.04
	2	0.3	0.01	0.0	0.0
Fuel 1	1	1.5	0.03	0.0	0.02
	2	0.4	0.08	0.135	0.0
Fuel 2	1	1.5	0.03	0.0	0.02
	2	0.4	0.085	0.135	0.0
Control/Fuel	1	1.5	0.03	0.0	0.02
	2	0.4	0.13	0.135	0.0
Control/Ref	1	2.0	0.04	0.0	0.04
	2	0.3	0.055	0.0	0.0

6 Results and discussion

6.1 (r, θ) "Stripe" problem

Results for the 2D "stripe" problem, which was introduced in Section 4.1, are summarised in Table II. From the above mentioned nodal extensions, the "mixed" leakage, and "finite-difference / nodal hybrid" approaches are evaluated for this problem. Results are given for the finite-difference reference solution, a nodal solution with flat leakage approximation everywhere (Nodal - flat), a quadratic leakage approximation everywhere (Nodal - quad), the mixed leakage approximation (Nodal - mixed) and the finite-difference / nodal hybrid (Nodal - hybrid) cases.

Table II. "Stripe" problem results

Result	Eigenvalue	Inner 1 cm azimuthally-averaged flux
Finite-difference (reference)	1.21999	11.37
Nodal - flat	1.22975 (800 pcm)	12.16 (7%)
Nodal - quad	1.22167 (138 pcm)	13.03 (15%)
Nodal - mixed	1.22260 (137 pcm)	12.99 (14%)
Nodal - hybrid	1.22138 (113 pcm)	10.88 (4%)

The “stripe” problem is constructed to represent a potential worst case scenario for the cylindrical nodal method, due largerly to the scewed aspect ratios near the inhomogeneous center of the system, and its associated impact on the leakage approximation. This is substantiated by relatively poor performance of all the nodal approaches. Nevertheless, we may deduce from the table above that the Nodal / FD hybrid is the most accurate of all the nodal approaches and specifically so with regard to the estimation of the inner zone (1 cm) flux. The mesh used for this calculation is (10 $\times$ 0.1 cm FD meshes, followed by 1, 3, 5, 5 and 5 cm for the nodal meshes). The mixed leakage approximation, in this case, improves results only marginally. As a further indication of the gradients of this problem, Fig. 5 below compares average radial flux profiles through both the source and absorber azimuthal zones. Nodal values are given for the *Nodal-hybrid* case.

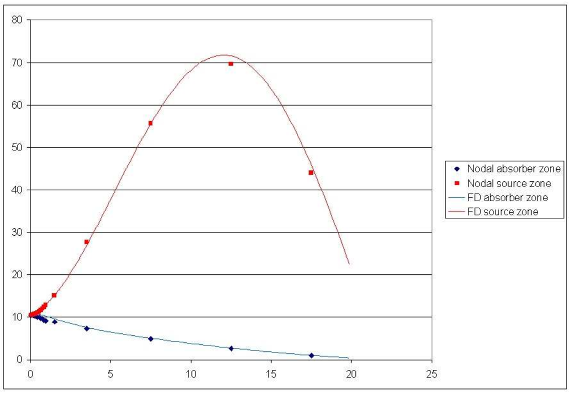

Figure 5. Nodal / FD hybrid vs reference finite-difference flux profiles - “Stripe” problem

We may deduce from this profile that the finite-difference / nodal hybrid approach allows for the correction of the inner zero-current boundary condition with a more realistic estimation of the net-current at the finite-difference / nodal interface. Furthermore, this approach allows for a detailed flux profile to be obtained arbitrarily close to the coordinate origin.

6.2 IAEA cylindrical 3D benchmark

Eigenvalue calculations were performed with both finite-difference and nodal approaches for the configuration specified in Section 5, and summary results are listed in Table III. The nodal calculation was performed upon a mesh with average axial- and radial sizes of approximately 20 cm (exact mesh specified in Section 5), and the finite-difference refinement factors upon this mesh are listed in Table III. The final two columns, named “RZ” and “Rθ”, list the maximum and the average power error from two perspectives. The “RZ”-column gives the (average and maximum) azimuthally-averaged (r, z) errors, and the “Rθ”-column gives the (average and maximum) axially-averaged (r, θ) errors. “Rθ” may be interpreted as the traditional assembly averaged powers for each axial channel. Initial nodal results, presented below, utilizes the “original” quadratic leakage approximation, along with the fixed 1 cm cut-out placed around the coordinate origin.

Table III. Eigenvalue results for 3D IAEA benchmark problem

Method (RxZxθ)	k_{eff}	Time	k_{eff} Error	RZ(Ave,Max)	Rθ(Ave,Max)
Extrapolated	1.02420	—	—	—	—
FD (12×12×12)	1.02410	34378 min	10 pcm	1% (3%)	1% (1.7%)
FD (8×8×8)	1.02399	6202 min	21 pcm	2% (6.2%)	2% (3.5%)
FD (6×6×6)	1.02388	2105 min	31 pcm	3% (9.4%)	3% (5.4%)
Nodal - original leakage	1.02424	10.2 min	4 pcm	0.5% (6.9%)	0.5% (2.8%)

The reference flux results were obtained via extrapolation using results from the (6×6×6), (8×8×8) and (12×12×12) finite-difference calculations as listed in Table III. An extrapolated solution was also obtained for the system eigenvalue (k_{eff}).

We may utilize Table III to estimate the performance advantage factor for the proposed nodal method over finite difference results. An estimation of the performance advantage factor over a finite difference calculation of equivalent accuracy (see Table III), would be in excess of 500. We further note from Table III, that the maximum nodal (r, z) power error is 6.9%, and maximum nodal (r, θ) error is 2.8%. These discrepancies are larger than what are typically expected from nodal methods and occur near the center line of the system, axially at the top. We proceed to apply both the "mapped" quadratic leakage, hybrid flat / quadratic methodologies to this problem. Results are summarised in Table IV and depict three calculational cases.

- Nodal - "mapped" leakage approximation (termed Nodal-mapped): This case represents the application of the "mapped" quadratic leakage approximation, as described in Section 3.1, throughout the system.

- Nodal - mixed leakage approximation (termed Nodal-mixed): This case represents a mixed leakage approximation in the axial direction, by starting the axial quadratic leakage approximation at the radial edge of the inner control rod, and utilizing the axial flat leakage approximation within the rod. This is done in these regions since the nodal aspect ratios are skewed.

- A somewhat refined nodal mesh (termed Nodal-mixed-refined), with two additional radial meshes within the inner 20 cm fuel zone, and one additional radial mesh in the second, partially inserted, control rod: This case serves to limit the inner mapped radial mesh sizes. The mapped node sizes in radial direction, in the conformal mapping approach, are given as $\ln\left(\frac{r_n^+}{r_n^-}\right)$, which, within the original meshing scheme, may be deemed to be large in these inner nodes.

Table IV. Summary results from nodal extensions applied to the IAEA

Method (R×Z×θ)	k_{eff}	k_{eff} Error	RZ(Ave,Max)	Rθ(Ave,Max)
Extrapolated	1.02420	—	—	—
Nodal - original	1.02424	4 pcm	0.5% (6.9%)	0.5% (2.8%)
Nodal - mapped	1.02425	5 pcm	0.5% (4.5%)	0.5% (2.8%)
Nodal - mixed	1.02424	4 pcm	0.5% (3.1%)	0.5% (2.8%)
Nodal - mixed - refined	1.02428	8 pcm	0.4% (2.1%)	0.3% (0.6%)

From the results in this table we may deduce that all the proposed extensions improve the errors in power distribution. The greatest improvement may be attributed to the “mapped” leakage approximation and this is seen as an important ingredient of this nodal method. The other, less formal approaches, such as the “mixed” leakage approximation and “nodal refinement”, further improves results untill all errors are within typical nodal limits, as compared to results for the Cartesian equivalent of this IAEA benchmark problem.

7 Conclusions

We have constructed two neutronically difficult problems in order to evaluate the conformal mapping approach to the Analytic Nodal Method (ANM), and found that the inhomogeneities within the central radial regions of both of these problems pose challenges to this method. In order to address these shortcomings, we have developed a range of nodal extensions.

We conclude that the “mapped” leakage approximation is most accurate, especially in situations where strong material inhomogeneities exist near the radial center. Furthermore, we found that the finite-difference / nodal hybrid offers a solution for determining the flux distribution arbitrarily close to the coordinate origin, where the Logarithmic conformal mapping function exhibits a singularity and cannot be applied. This methodology will be extended to 3-dimensions in future.

Finally we note, from the “Nodal - mixed - refined” case of the IAEA 3D problem, that for the application of the conformal mapping ANM radial calculational nodal mesh sizes should be restricted. We suggest that the ratio of consecutive node boundaries, $\left(\frac{r_n^+}{r_n^-}\right)$, preferably should not exceed a factor of 2. This restriction is only of concern near the coordinate origin, since this ratio tends toward 1 radially further into the system.

ACKNOWLEDGEMENTS

The authors would like to acknowledge various members of the Radiation and Reactor Theory group at Necsa for their support in this work, and thank especially Dr. Wessel Joubert for his contribution.

REFERENCES

[1] K. S. Smith, ”An Analytic Nodal Method for Solving Two-Group, Multidimensional Transient and Static Neutron Diffusion Equations,” Nucl. Eng. Thesis, Department of Nucl. Eng., MIT (1979).

[2] R. H. Prinsloo, et al., ”Development of the Analytic Nodal Method in Cylindrical Geometry,” *Proceedings of Topical Meeting on Mathematics and Computation 2005,* M&C 2005 on CD (2005).

[3] R. H. Prinsloo, et al., ”Development of the Analytic Nodal Method for Cylindrical Geometry for PBMR Application,” *Proceedings of High Temperature Reactor 2006 Meeting,* HTR 2006 on CD (2006).

[4] R. R. Lee, et al., “Argonne Code Center: Benchmark Problem Book”, ANL-7416, Suppl. 2, ANL (1977).

[5] A. M. Ougouag, et al., ”Analytic Solution of the Neutron Diffusion Equation in 3D Cylindrical Geometry for Application in a Nodal Method”, *Proceedings of Topical Meeting on Mathematics and Computation 2003,* M&C 2003 on CD (2003).

[6] R. H. Prinsloo, ”Solution of the Multigroup Analytic Nodal Diffusion Equations in 3-Dimensional Cylindrical Geometry,” Reactor Science Thesis, Department of Physics., North-West University, Potchefstroom, South Africa (2006).