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DEVELOPMENT OF NEW PIN POWER RECOVERY METHODOLOGY

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ABSTRACT

To overcome the weakness of the traditional methodology in predicting pin power for the core operated with control rod insertion, a new pseudo pin by pin calculation (P3C) methodology has been developed in Westinghouse and implemented in the ANC9 code. Unlike the traditional method, whose accuracy depends on what pre-defined control rod sequences are modeled for the data generation and how the actual operation condition is close to the pre-defined the scenario, the new methodology can follow the history of each fuel rod and directly take the history and moment effect into the pin-cell cross-sections and the local fluxes. Therefore, the new methodology can handle any complicated control rod insertion situation. A wide range of qualification for both unit assemblies and mini-core case was performed using ANC9. The results show that, for all modeled assembly types and all control rod insertion scenarios, the P3C methodology could follow the fuel rod behavior and excellently reproduced the reference results from the lattice code simulation.

Key Words: pin power recovery, pin-cell macroscopic cross-section, flux form factor

1. INTRODUCTION

A concerted effort to merge and modernize the methods employed in the PWR and BWR steady-state reactor physics codes of Westinghouse was launched a few years ago. In the first phase of the NEXUS project, a new once-through nodal cross-section generation methodology was developed [1]. After completion of the cross-section representation model, a new pin power recovery methodology has been under development for the Westinghouse suite of core analysis codes. One of its main features is to enable capturing the impact of control rod history on pin power distributions in an explicit on-the-fly procedure, thus overcoming an important limitation of the existing BWR implicit methodology that requires complicated data generation procedures. Since the next generation of PWRs such as the AP1000 will be operated with control rods inserted into the core, the effect of control rod history has become relevant also in the PWR field.

This paper presents the new pin power recovery methodology. Its trademark feature is a pseudo pin by pin calculation ("P3C"). The first application of this new method concerns pin power recovery in PWR applications. It is shown in this paper that P3C enables accurate prediction of the pin powers in a PWR by properly accounting for control rod history and spectrum interaction

effects. Beside its accuracy, another key feature of this P3C methodology is that it can be implemented in a computationally efficient fashion, thus making it a viable tool not only for routine core design calculations but also for Westinghouse on-line core monitoring systems.

2. SYNOPSIS OF THE EXISTING PIN POWER METHODOLOGY

In most core design codes based on nodal methods (e.g. ANC [2] and POLCA7 [3]) a modulation method is used to recover heterogeneous pin power distributions from the nodal results. The heterogeneous pin power form factors that are used in this process are either energy group dependent (as in ANC) or group independent (as in POLCA7). The power in a pin-cell k can therefore generally be expressed as:

$$P^k = \sum_g (\kappa\Sigma_{f,g}^{\hom,k} \cdot \phi_g^{\hom,k}) p_g^k \tag{1}$$

Here $\kappa\Sigma_{f,g}^{\hom}$ is the homogeneous intra-node kappa-fission cross-section in energy group g which is represented by a two-dimensional non-separable low order polynomial approximation. The homogeneous intra-node flux $\phi_g^{\hom}$ is often obtained by means of some interpolation scheme or as an approximate solution of the two-dimensional diffusion equation (the latter approach is used in ANC and POLCA7). The method assumes that the heterogeneity can be fully captured by the heterogeneous power form factors p_g^k. These form factors are generated from unit assembly lattice calculations, often as a function of node average burnup and of selected instantaneous state parameters.

In some applications (notably BWR) the form factors are also generated as a function of additional history parameters (e.g., coolant density history and control rod history in the case of POLCA7). Typically, these history cases involve a straight-forward lattice depletion calculation with the given history parameter kept constant for each history case (e.g. fixed void conditions). While control rod history is also often modeled in this way (i.e. control rod continually inserted for a control rod history case), it is not representative of actual control rod history scenarios where repeated insertion and withdrawal of control rods from any given computational node in the nodal core model can be expected. Therefore, in the Westinghouse BWR methodology fixed (pre-determined) sequences of rod insertion and withdrawal are simulated in the control rod history lattice calculations. The problem with this approach is that the actual sequence of control insertions and withdrawals cannot be determined a priori for a given cycle, but is, at best, known only to some partial extent. Furthermore, the cycle-wise determination of nodal cross-sections and form factors conflicts with the once-through nodal cross-section generation philosophy. Besides, application of these control rod history data sets requires very complicated procedures in the simulator to track and combine various periods of rod insertion and withdrawal in each node that has been subjected to such sequences. Experience has shown that this method can induce unphysical spikes (or dips) in the axial profile of pin powers within partially rodded nodes.

It has therefore been concluded that the current BWR control rod history model has too many short-comings for considering its extension to the AP1000. Therefore a complete review of the pin power recovery methodology for both BWR and PWR has been undertaken.

3. PSEUDO PIN BY PIN CALCULATION (P3C) METHODOLOGY

As it follows from the above discussion, the existing pin power recovery method does not directly account for the real depletion history of each individual fuel rod, thereby limiting its predicting capabilities in strongly heterogeneous environments, such as those characterized by repeated insertion and withdrawal of control rods. This limitation is overcome by the new pseudo pin by pin calculation (P3C).

The "ideal" method to obtain pin-wise results would consist of 3D pin by pin transport calculations along with pin-wise isotopic tracking capabilities. For any pin-cell *k*, the pin power could then be computed directly in terms of pin-cell average cross-sections $\kappa\Sigma_{f,g}^{het,k}$ and fluxes $\phi_g^{het,k}$ for the heterogeneous reactor (no assembly homogenization applied) over all energy groups (*g*):

$$P^k = \sum_g \kappa\Sigma_{f,g}^{het,k} \cdot \phi_g^{het,k} \tag{2}$$

Unfortunately, the computing power currently available still precludes implementation of such a scheme in standard core design applications. On the other hand, the fundamental idea of using pin-cell cross-sections and fluxes for heterogeneous assemblies in combination with the tracking of individual fuel rod depletion histories, is exploited in the P3C methodology. Only the manner in which the pin-cell average parameters are obtained is different from the direct (transport) approach. The details of this are presented in the following. For convenience, the notation "*refh*" used in the following discussion will represent data from the reference depletion history.

3.1. Pin-Cell Cross-Section Calculation

By extending the Westinghouse cross-section representation model for nodal cross-sections to pin-cells (with spatial smearing over pin-cell regions), for the reaction type (*x*), one can compute the pin-cell macroscopic cross-section as:

$$\Sigma_{x,g}^k = \Sigma_{x,g}^{k,refh}\left(E^k, \{p\}, SH^{k,ref}(E^k)\right) + \sum_j \left(N_j^k - N_j^{k,ref}\left(E^k\right)\right)\sigma_{x,g}^{k,j} \tag{3}$$

where E^k is the local burnup of the pin-cell (*k*), $\{p\}$ is the set of instantaneous node-average state parameters such as moderator and fuel temperatures, spectrum index [1] and component (control rod, discrete BA) insertion status. $SH^{k,ref}(E^k)$ represents the local pin-cell spectrum history for the reference depletion history case.

The first term on the right-hand side of the equation represents the macroscopic cross-section consistent with the reference depletion history. The second term on the right-hand side accounts for the depletion history correction (the so-called "micro-depletion correction") based on the number density deviation ($N_j^k - N_j^{k,ref}\left(E^k\right)$) of the tracked isotope (*j*). While tracking nuclide depletion on a node-wise basis in the simulator is quite feasible, it is not practical to do so on a pin-wise basis in standard core design calculations. Therefore, a simpler approach is needed. One such approach is to assume that the pin-cell nuclide number densities can be functionalized in terms of the local pin-cell spectrum history parameter, $SH^k(E^k)$, so that Eq.(3) can be re-written as:

$$\begin{aligned}\Sigma_{x,g}^k &= \Sigma_{x,g}^{k,refh}\left(E^k,\{p\},SH^{k,ref}(E^k)\right)\cdot\left(1+\frac{\sum_j\left(N_j^k\left(E^k,SH^k(E^k)\right)-N_j^{k,ref}\left(E^k,SH^{k,ref}(E^k)\right)\right)\sigma_{x,g}^{k,j}}{\Sigma_{x,g}^{k,refh}\left(E^k,\{p\},SH^{k,ref}(E^k)\right)}\right) \\ &= \Sigma_{x,g}^{k,refh}\left(E^k,\{p\},SH^{k,ref}(E^k)\right)\cdot\left(1+H_{x,g}^k\left(E^k,SH^k(E^k)\right)\right)\end{aligned} \tag{4}$$

Investigations have shown that, within the spectrum history range of practical interest, the history correction factor exhibits an approximately linear dependence on the relative deviation of the spectrum history, i.e.,

$$H_{x,g}^k\left(E^k,SH^k(E^k)\right)\approx \mathrm{K}_{x,g}^k\left(E^k\right)\cdot\left[\frac{SH^k(E^k)-SH^{k,ref}(E^k)}{SH^{k,ref}(E^k)}\right] \tag{5}$$

The $\mathrm{K}_{x,g}^k\left(E^k\right)$ is a pin-cell spectrum history correction coefficient, which can be pre-generated through an off-reference lattice depletion calculation with adequate change of the assembly heterogeneity and the neutron spectrum, and then tabulated as a function of the pin burnup.

3.2. Pin-Cell Flux Calculation

With the pin-cell average macroscopic cross-sections available (as reconstructed according to the method described in the above), the possibility exists of determining pin-cell fluxes by means of a pin by pin diffusion calculation. Considering the current computer capability, the direct pin by pin calculation is considered inefficient and not practicable for core design and core monitoring applications. Computational efficiency is therefore attained by retaining the idea of heterogeneous information recovery by modulation, so that the pin-cell flux is computed as

$$\phi_g^{k,het} = \phi_g^{k,\mathrm{hom}}\cdot f_g^k \tag{6}$$

where $\phi_g^{k,\mathrm{hom}}$ is the so-called homogeneous pin-cell flux distribution and f_g^k is the energy-group-wise flux form factor of the pin-cell. The homogeneous pin-cell flux distribution corresponds to the smooth flux distribution within spatially homogenized nodes. The flux form factors are thus burdened with carrying all of the heterogeneous flux distribution information.

3.2.1 Homogeneous pin-cell flux calculation

In ANC9 the homogeneous intra-node flux distribution is obtained by means of an approximate solution of the two-dimensional diffusion equation, namely by using a finite-order Fourier expansion [2] subject to node side flux and node corner flux constraints.

3.2.2 Pin-cell flux form factors

From Eq.(6), it can be identified that the flux form factor is the pin-cell flux ratio of heterogeneous to homogeneous results. Similar to the pin-cell cross-sections, for a given fuel assembly, the flux form factors vary with the local conditions, such as boron concentration, moderator and fuel temperatures, component insertion status. Eventually, the flux form factor with a reference depletion history (e.g. SH^{ref}) can be expressed in the same way as the pin-cell cross-sections:

$$f_g^{k,refh}\left(E,\{p_i\},SH^{k,ref}\right)= f_g^{k,ref}\left(E\right)\cdot\zeta_g^k\left(E,\{p_i\}\right) \tag{7}$$

Both the reference flux form factors ($f_g^{k,ref}$) and the spectrum correction factors ($\zeta_g^k\left(E,\{p_i\}\right)$) can be pre-generated as a partner to the pin-cell cross-sections described in the section 3.1.

Clearly, at a given burnup step, Eq.(7) will give the flux form factors corresponding to the local condition with the reference depletion history. The pin-cell macroscopic cross-section corresponding to the same condition would be $\Sigma_x^{k,refh}$, given in Eq.(4). However, to be consistently carried on, the heterogeneous flux calculation requires the actual flux form factors to be available, since these capture both the local instantaneous condition and the actual depletion history.

Since the change of the flux form factors was caused by the change of the pin-cell cross-sections, the actual flux form factors can be obtained from the factors with the reference depletion history through adjustment based on the cross-section change, i.e.,

$$f_g^k = f_g^{k,refh}\cdot F_g^k\left(\Sigma_x^{k,refh},\Sigma_x^k\right) \tag{8}$$

Here $F_g^k\left(\Sigma_x^{k,refh},\Sigma_x^k\right)$ is the adjustment factor to the flux form factor of the energy group g.

By combining Eq.(4), (6), and (8), the power of pin-cell (*k*) is computed as:

$$P^k = \sum_g \kappa\Sigma_{f,g}^k\cdot f_g^k\cdot\phi_g^{k,\mathrm{hom}} \tag{9}$$

4. VALIDATION OF NEW PIN POWER METHODOLOGY

The pseudo pin by pin calculation (P3C) methodology has been implemented in ANC9 to derive the pin-cell power distribution and validated with unit assembly and 3x3 mini-core simulations with repeated control rod insertion. Several different control rod insertion strategies were modeled. The reference results for all cases are obtained from PARAGON [4], which is able to simulate explicitly the actual fuel depletion. The PARAGON pin by pin power results are then compared against ANC9 with the new as well as the traditional (described in Section 2) pin power methodology. Since the effect of the past depletion history and instantaneous fuel conditions is now captured in the pin-cell macroscopic cross-sections and flux form factors, the P3C methodology will be shown to perform remarkably well, and substantially better than the traditional methodology, over all control rod insertion scenarios analyzed.

4.1 Unit Assembly Calculations

Calculations in unit assembly (UA) geometry were performed as a general test of the new methodology and in particular of the accuracy of the pin-wise spectral history correction to the pin form factors and cross-sections. The fuel assembly design used in the set of simulations presented here is Standard 17x17 Westinghouse Fuel with 4.95 w/o ^{235}U with and without burnable absorbers. Several control rod history sequences were simulated, as shown in Fig. 1. These scenarios yield severe heterogeneity changes for the assembly and challenging conditions for the new methodology. Calculations performed at reduced fuel enrichments showed no substantial sensitivity in the obtained accuracy. Various control rod absorber materials were modeled during the qualification, also showing similar, overall accuracy in the pin power prediction for every control rod type. Furthermore, results show consistent behavior and compare well for all the simulated control sequences.

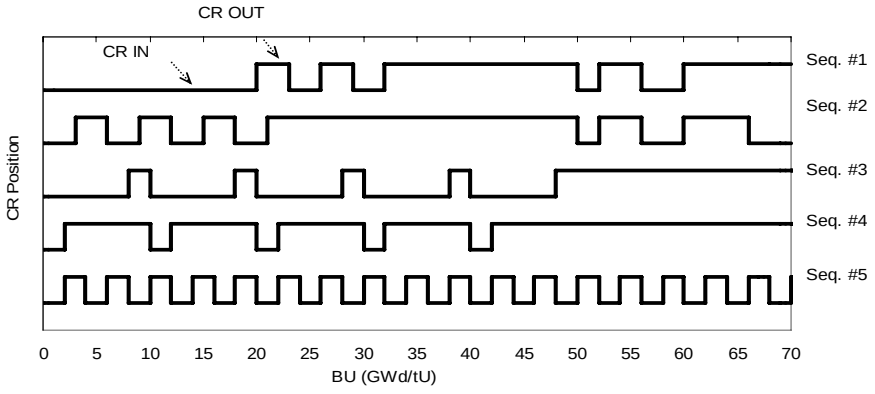

Figure 1 Control rod sequences modeled for unit assembly calculations

As an example, the results obtained for sequence 2 with 24 Ag-In-Cd control rodlets and 4.95 w/o ^{235}U fuel pins without any burnable absorbers (non-BA) are displayed in Fig. 2. The results presented refer to the corner pin (top-left plot), pin in position (5,4) of the assembly (top-right plot), which is close to the guide thimble, and the limiting pin (bottom-left plot). The corner pin is selected since the power typically migrates to the corner of the assembly following control rod insertion. Pin (5,4) is of interest since this is the location where the power redistributes and peaks

when the control rods are extracted. In addition, the average pin power difference for the assembly, which is indicative of the overall accuracy of the methodology, is presented in the bottom-right plot. It should be noted that the pin power difference throughout this paper refers to the difference between ANC9 and PARAGON results multiplied by 100.

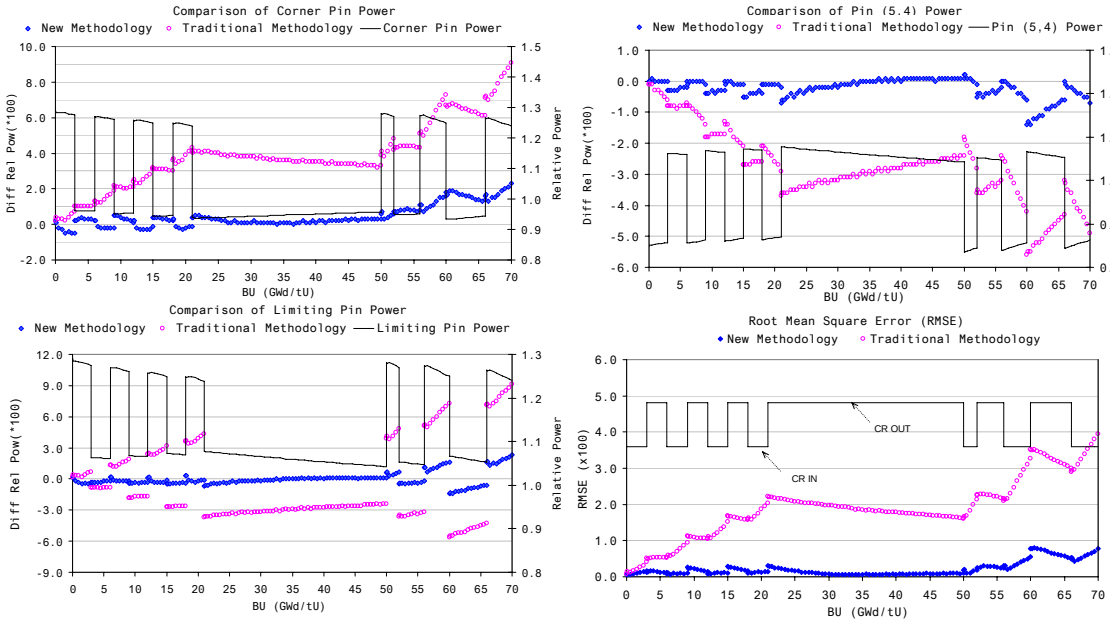

Figure 2 Pin power comparisons for non-BA fuel with 4.95 w/o ^{235}U (ANC9 vs. PARAGON, Sequence #2)

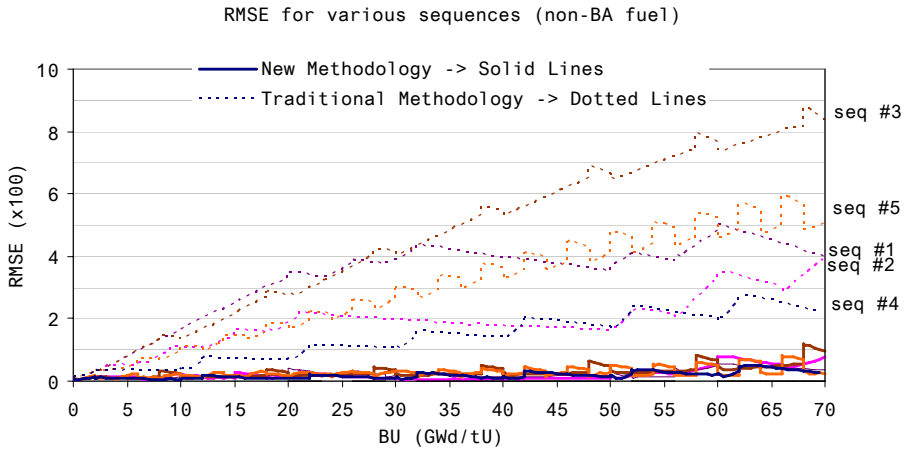

Figure 3 Root Mean Square Error (RMSE) for non-BA fuel with 4.95 w/o ^{235}U (ANC9 vs. PARAGON, CR sequences in Fig. 1)

Fig. 3 shows a summary comparison of the RMSE between new and traditional methodology for non-BA fuel cases and Fig. 1 simulations. While RMSE up to 9% are observed with the traditional methodology, RMSE is within 1% with the new methodology, and typically below 0.5%. Furthermore, there is a consistent behavior and accuracy throughout the control rod

histories simulated. This makes the agreement in the control rod simulations performed with the new methodology as accurate as that observed for a standard unrodded depletion calculation.

Assemblies containing BAs were also analyzed. The calculations cover typical BA types, such as the Westinghouse ZrB_2 IFBA, the Wet Annular Burnable Absorber (WABA), and gadolinia (Gd_2O_3) rods. Several loading patterns, including enrichment zoning have been considered. The BA concentrations chosen exceed those used in standard PWR core designs. The high BA concentration is intended to further challenge the new pin power methodology and the associated spectral history corrections. The strong local power suppression and typical collocation of BA bearing pins close to the water holes/control rods (or in place of control rods before WABA rods are removed) enhances the heterogeneity of the assembly and the severity of the depletion environment. As an example of BA simulations, the results for an assembly with 156 IFBA rods are presented in respectively Fig. 4 and Fig. 5. A summary of the RMSE for all the IFBA and WABA cases analyzed is given in Fig. 6. Results for a gadolinia fuel assembly (8 w/o Gd_2O_3 in 16 gadolinia rods) are given in some detail in Fig. 7.

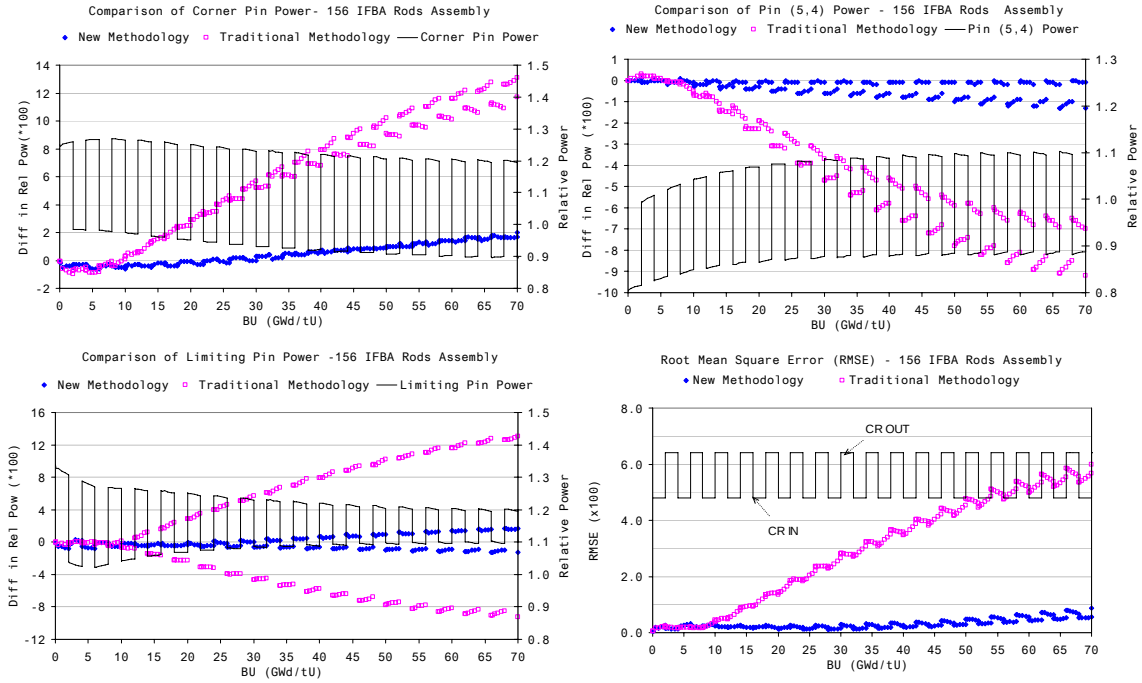

Figure 4 Comparison of pin power for 156 IFBA fuel assemblies (ANC9 vs PARAGON)

The pin power results obtained with the new methodology for IFBA and WABA assemblies are similar to those described earlier for non-BA assemblies (while the disagreement with the traditional methodology becomes even larger due to the increased heterogeneity effect of the BAs), as shown in Fig. 4 for 156 IFBA rods and in summary results in Fig. 5. This confirms a consistent improvement in the pin power prediction with the new methodology compared to the traditional methodology for these standard Westinghouse BA designs.

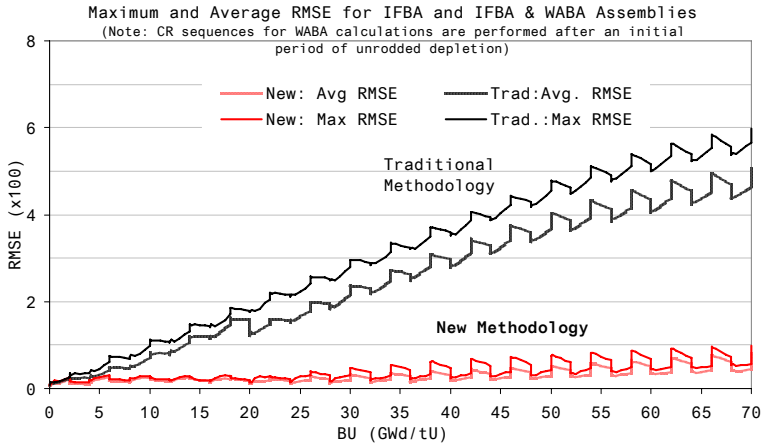

Figure 5 Root mean square error (average and maximum) for all IFBA and WABA cases (ANC9 vs PARAGON)

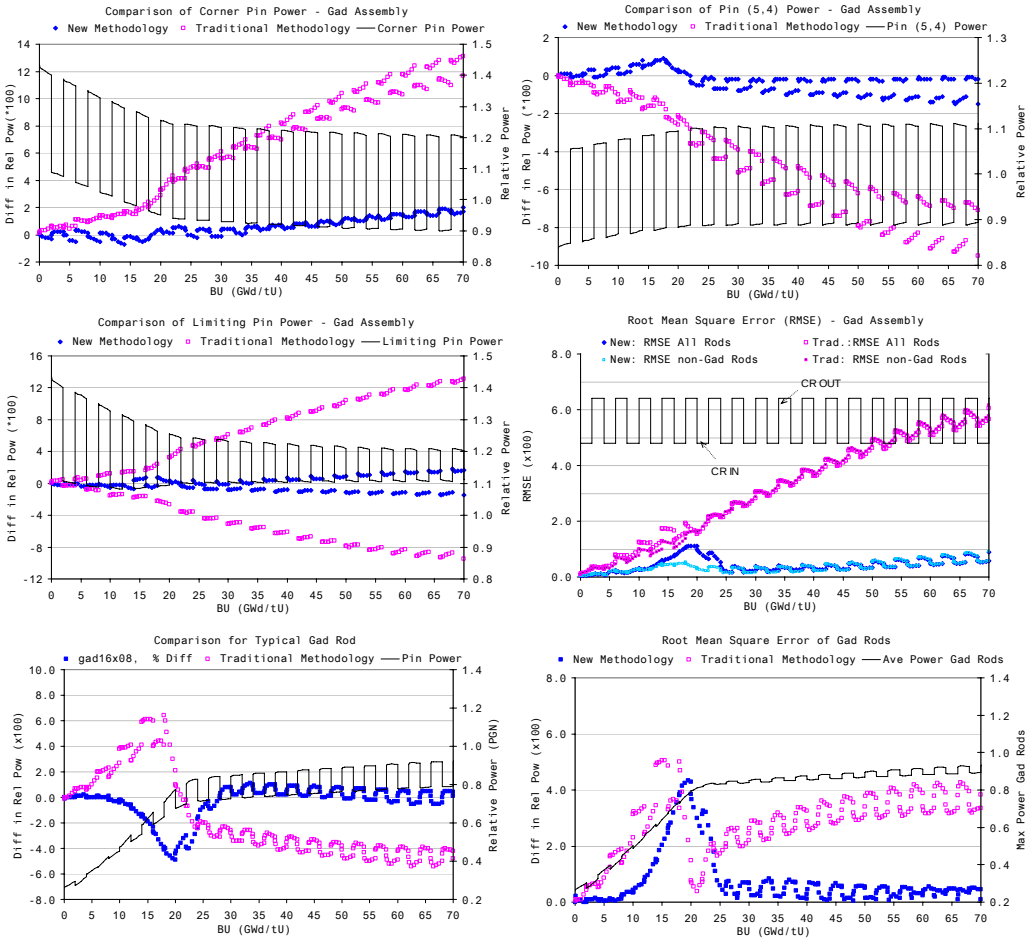

Figure 6 Comparison of pin power for gadolinia fuel assembly (ANC9 vs PARAGON)

Results for the gadolinia fuel assembly in Fig. 6 also show a substantial improvement with respect to the traditional methodology. However slightly larger RMSE are observed in the first

part of the depletion, driven by the misprediction of the gadolinia pin power during the gadolinium burnout depletion range (compare RMSE of the gadolinia assembly with and without taking Gad rods into account, second plot on the right in Fig. 6). It is important to note that the new methodology yields substantial improvements with respect to the traditional methodology also for the gadolinia assembly and that the gadolinia rod power is still considerably lower than the limiting rod power when the larger differences occur and thereby do not represent a safety or an operational concern (plots in the bottom of Fig. 6, showing the behavior of a typical Gad rod, on the left, and the RMSE for all Gad rods, on the right). While the new methodology predicts the gadolinia rods power with same accuracy as that of the non-BA rods after gadolinium burns out (i.e. max difference <1%, RMSE <0.5%), the error remains substantial throughout the rods depletion with the traditional methodology. Therefore, even though further investigation to improve the gadolinium fuel pin power prediction will be performed, the new methodology is already a considerable improvement with respect to the conventional methodology also for control rod sequences in gadolinia assemblies.

4.2 Mini-Core Calculations

The fuel assembly design used in the 3x3 (9 assemblies) mini-core model is the same as for the UA study, i.e. Standard 17x17 Westinghouse PWR Fuel. The same 4.95 w/o ^{235}U and no BA fuel is employed. A bank of 24 full-size Ag-In-Cd rodlets can be inserted in the center assembly, while the other assemblies are assumed to be unrodded. The depletion history of the model is as follows: rodded depletion from 0 to 11 GWd/tU and from 22 to 32 GWd/tU, while the control rods are extracted between 11 and 22 GWd/tU.

The pin power comparison between ANC9 and PARAGON for the center assembly is shown in Fig. 7 at 11 GWd/tU before (left plot) and after (right plot) the change in the control rod position. Fig. 8 shows the comparison at 22 GWd/tU, i.e. 11 GWd/tU after the CR have been extracted (left) and at 32 GWd/tU (right), with ~20 GWd/tU of CR history cumulated in the center assembly. Results obtained with the new and the traditional methodologies are shown. The pin power predicted by the new methodology is clearly significantly more accurate than that of the traditional method and again indicates a considerable reduction in pin power prediction uncertainty, as supported by summary RMSE in Table 1. The traditional methodology shows a noticeable weakness in capturing control rod history effects. While the new methodology performs quite well in this respect, it is evident from Fig. 7 that further improvement would not be unwelcome and current efforts endeavoring to achieve this are underway at Westinghouse.

Table 1 Summary RMSE for 3x3 mini-core (ANC9 vs. PARAGON)

BU (GWd/tU)	11	11	22	22	32
CR position (IN:inserted/OUT:extracted)	IN	OUT	OUT	IN	IN
RMSE New/Traditional methodology					
Center assembly pins only	0.8/1.6	1.4/2.0	0.4/1.4	1.5/1.9	0.7/3.2
All 9 assembly pins	0.4/2.5	0.7/0.9	0.3/0.6	1.1/1.2	0.4/1.2

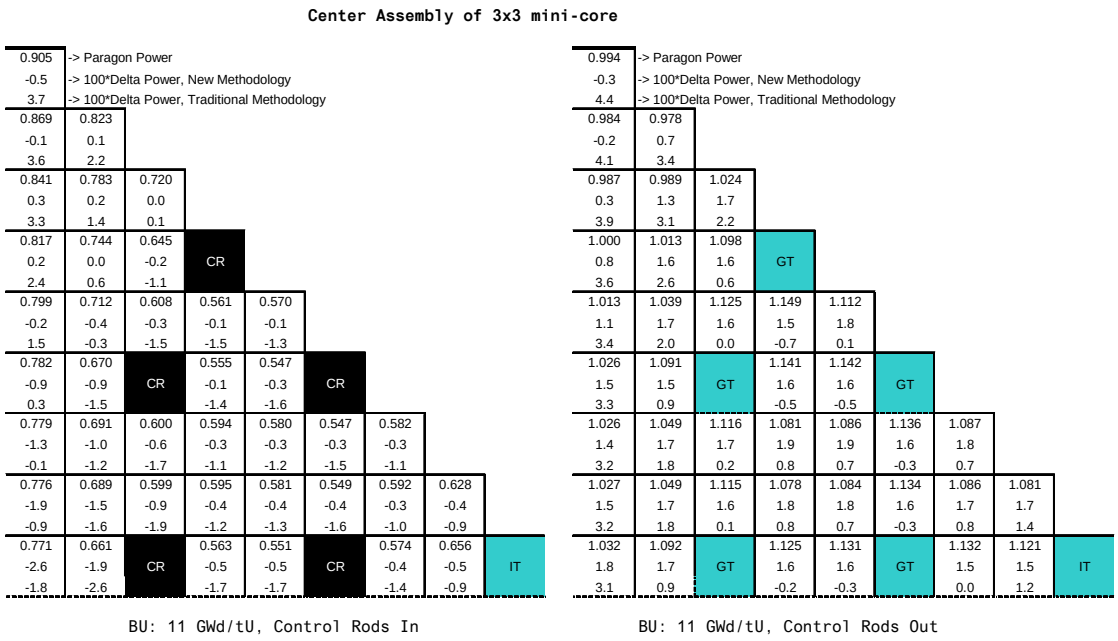

Figure 7 Pin power difference for the mini-core center assembly at 11 GWd/tU with CR inserted (left) and extracted (right) (ANC9 vs PARAGON)

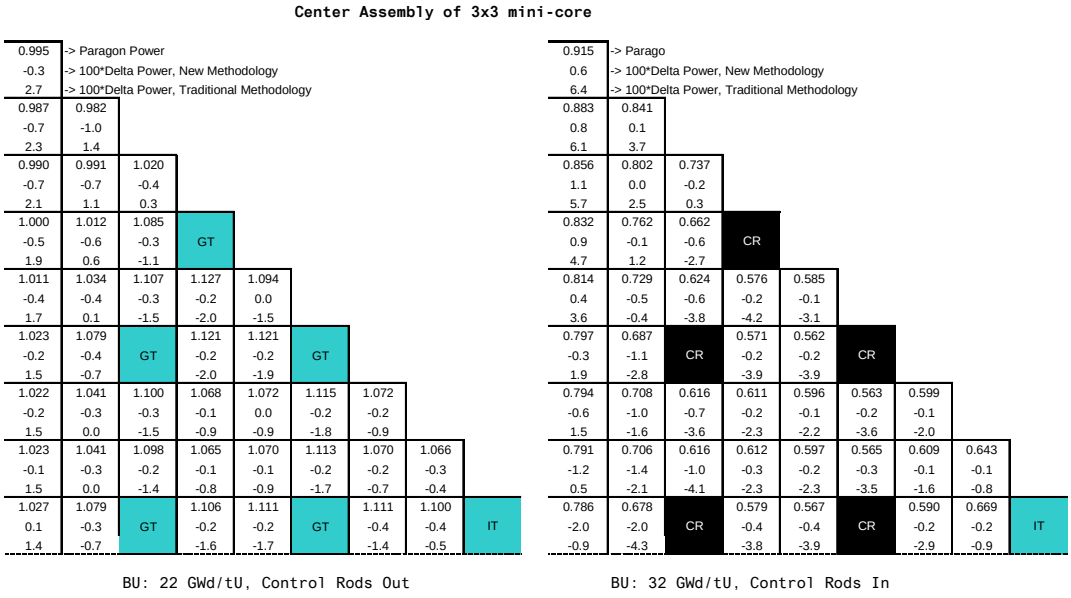

Figure 8 Pin power difference for the mini-core center assembly at 22 (left) and 32 (right) GWd/tU (ANC9 vs PARAGON)

On the whole, the capability of the new methodology to adequately predict the pin power during repeated control rod insertion and withdrawal sequences is confirmed also when the model is extended to a larger geometry (than a unit assembly) where the interaction between multiple assemblies must be properly taken into account.

5. CONCLUSIONS

A pseudo pin by pin calculation (P3C) methodology has been developed and implemented in the Westinghouse nodal code ANC9. Both unit assembly and mini-core calculations have demonstrated that the P3C methodology can adequately predict fuel rod powers for any control rod insertion scenario, thereby verifying that the P3C methodology well captures control rod history effects. While the potential for further improvement was noticed, it is concluded that the P3C methodology satisfies the main technical requirement of being able to model control history in a computationally efficient and physically realistic manner.

Westinghouse believes that the P3C methodology has brought the current nodal code to a new level, and provides a refreshed view of issues related to assembly spectrum interaction. Further development in Westinghouse will focus on the extension of the P3C methodology to better capture the environment effect on nodal cross-section calculations. Future work will also consider application of the methodology to BWR and MOX/UO_2 loaded cores.

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