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DEVELOPMENT OF SPECTRAL HISTORY METHODS FOR PIN-BY-PIN CORE ANALYSIS METHOD USING THREE-DIMENSIONAL DIRECT RESPONSE MATRIX

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ABSTRACT

Spectral history methods for pin-by-pin core analysis method using the three-dimensional direct response matrix have been developed. The direct response matrix is formalized by four sub-response matrices in order to respond to a core eigenvalue k and thus can be recomposed at each outer iteration in the core analysis. For core analysis, it is necessary to take into account the burn-up effect related to spectral history. One of the methods is to evaluate the nodal burn-up spectrum obtained using the out-going neutron current. The other is to correct the fuel rod neutron production rates obtained the pin-by-pin correction. These spectral history methods were tested in a heterogeneous system. The test results show that the neutron multiplication factor error can be reduced by half during burn-up, the nodal neutron production rates errors can be reduced by 30% or more. The root-mean-square differences between the relative fuel rod neutron production rate distributions can be reduced within 1.1% error. This means that these methods can accurately reflect the effects of intra- and inter-assembly heterogeneities during burn-up and can be used for core analysis. Core analysis with the DRM method was carried out for an ABWR quarter core and it was found that both thermal power and coolant-flow distributions were smoothly converged.

Key Words: BWR, nuclear characteristic, neutron current, response matrix, direct response matrix, Monte Carlo method, core analysis, fuel rod, burn-up correction

1. INTRODUCTION

As nuclear reactor core design has become more advanced, more precise neutronic calculation modeling is in demand. Needed are more precise representations of the intra- and inter-assembly heterogeneity effects on core calculations, such as for complex geometrical configurations in advanced fuel design and for the initial loading with various enrichment fuel assemblies. For their representations, a Mote Carlo calculation [1,2] is one of the most suitable methods because a precise geometrical treatment is possible and no homogenization process is needed in the calculation. A disadvantage of the Monte Carlo calculation, however, is the longer computational time needed to reduce statistical uncertainty, especially for large systems compared with the neutron mean-free-path approach. Consequently, the Monte Carlo calculation is seldom applied for BWR core analysis, in which neutronic calculations and thermal-hydraulic calculations need to be iterated for the entire core configuration until both the thermal power and coolant flow distributions are converged.

To take the advantage of the Monte Carlo calculation for the core analysis within practical computational times, a direct response matrix (DRM) method using the Monte Carlo calculation has been developed [3]. The basic idea of the DRM method is to classify neutron behaviors in a fuel assembly into four fundamental reaction processes: transmission across the fuel assembly, neutron production by neighbor-induced neutrons, neutron production by self-induced neutrons and escape from the fuel assembly. These four reaction processes are represented by four separate types of sub-response matrices. With this scheme the sub-response matrices become independent of neighboring lattice conditions and can be obtained by an infinite lattice calculation using the Monte Carlo method. A nodal response matrix, which is dependent on a core eigenvalue, k, is reconstructed from the four sub-response matrices with the assumed k. The k is treated as a unique adjustment parameter for neutron production rates of all nodes in the core so that overall incoming and outgoing partial neutron currents are balanced. Since the outgoing partial neutron current on each nodal surface is calculated with incoming partial neutron currents into the corresponding node and the nodal response matrix, the k can be evaluated by iterating reconstructions of nodal response matrices and calculations of neutron currents. Since the Monte Carlo calculation is needed only for the lattice calculation, the computational time for the core analysis becomes practical.

In this paper, we described the three-dimensional direct response matrix method using Monte Carlo calculation and spectral history methods for the core burn-up. And then, we compared spectral history methods with the direct calculations using a Monte Carlo method.

2. CORE ANALYSIS METHOD USING DRM

2.1. Three-Dimensional Direct Response Matrix Method

To use the DRM method for core analysis (DRM analysis method), the sub-response matrices need to be extended to three dimensions. In the three-dimensional DRM method, the four sub-response matrices are defined [4,5] as follows.

- Transmission probability ($T_{ii,gi\rightarrow io,go}$): probability that a neutron entering from the *ii*'th face at the *gi*'th group eventually exits to the *io*'th face at the *go*'th group.
- Neighbor-induced production probability ($S_{ii,gi\rightarrow je,m}$): expected number of neutrons born in the axial zone m of the fuel rod *je* by a fission reaction of a neutron entering from the *ii*'th face at the *gi*'th group
- Self-induced production probability ($A_{js,l\rightarrow je,m}$): expected number of neutrons born in the axial zone m of the fuel rod *je* by a fission reaction of a neutron born in the axial zone l of the fuel rod *js*
- Escape probability ($L_{js,l\rightarrow io,go}$): probability that a neutron born in the axial zone l of the fuel rod *js* eventually exits to the *io*'th face at the *go*'th group

The face denoted above means not only a mesh obtained by dividing the nodal surfaces but also an angle bin, along by which neutrons comes in or go out. With these sub-response matrices, a k-dependent response matrix, $R(k)_{ii,gi\rightarrow io,go}$, is represented as follows:

$$R(k)_{ii,gi\to io,go} = T_{ii,gi\to io,go} + \frac{1}{k}\sum_{js1=1}^{js\max}\sum_{l1=1}^{l\max} S_{ii,gi\to js1,l1}\Bigg[L_{js1,l1\to io,go} + \frac{1}{k}\sum_{js2=1}^{js\max}\sum_{l2=1}^{l\max} A_{js1,l1\to js2,l2}\Big[L_{js2,l2\to io,go} + \frac{1}{k}\sum_{js3=1}^{js\max}\sum_{l3=1}^{l\max} A_{js2,l2\to js3,l3}\Big[L_{js3,l3\to io,go} + \frac{1}{k}\sum_{js4=1}^{js\max}\sum_{l4=1}^{l\max} A_{js3,l3\to js4,l4}\Big[L_{js4,l4\to io,go} + \dots \Big]\Big]\Big]\Bigg] \quad (1)$$

where *js*max denotes the number of fuel rods in the corresponding bundle and *l*max denotes the number of axial zones in the corresponding fuel rod. $R(k)_{ii,gi\to io,go}$ is an expected number of neutrons which are induced by a neutron entering from the *ii*'th face at the *gi*'th group and which eventually exit to the *io*'th face at the *go*'th group. Then incoming and outgoing neutron currents are related with the response matrix as follows:

$$J^{out}_{io,go} = \sum_{ii=1}^{i\max}\sum_{gi=1}^{g\max} R(k)_{ii,gi\to io,go}\, J^{in}_{ii,gi} \quad (2)$$

where $J^{out}_{io,go}$ denotes the outgoing partial current through the *io*'th face at the *go*'th group, $J^{in}_{ii,gi}$ denotes the incoming partial current through the *ii*'th face at the *gi*'th group, and *i*max and *g*max denote the number of faces surrounding the corresponding node and energy groups, respectively. The k is evaluated with the condition that overall incoming and outgoing neutron currents are balanced. A pin-wise neutron production rate for an axial zone l of a fuel rod i, $P(i,l)$, can be reconstructed as follows:

$$P(i,l) = \sum_{ii=1}^{i\max}\sum_{gi=1}^{g\max} J^{in}_{ii,gi}\Bigg[S_{ii,gi\to i,l} + \sum_{js1=1}^{js\max}\sum_{l1=1}^{l\max} \frac{S_{ii,gi\to js1,l1}}{k}\Bigg[A_{js1,l1\to i,l} + \sum_{js2=1}^{js\max}\sum_{l2=1}^{l\max} \frac{A_{js1,l1\to js2,l2}}{k}\Big[A_{js2,l2\to i,l} + \sum_{js3=1}^{js\max}\sum_{l3=1}^{l\max} \frac{A_{js2,l2\to js3,l3}}{k}\big[\ \cdots\cdots\big]\cdots\Big]\Big]\Bigg]\Bigg] \quad (3)$$

Although eqs. (1) and (3) include summations which continue for an infinite number of generations, the result will become saturated as the generations grow in number. Thus the summation of the equations can be limited to a practical number of generations.

2.2. Core Analysis Method

Differences between the core analysis method using the DRM method and a conventional BWR core analysis method based on a nodal diffusion calculation (conventional analysis method) are as follows.

- Fitting coefficients of the sub-response matrices for node condition parameters (void fraction, exposure, fuel temperature, etc.) are prepared before core calculations in the DRM analysis method, whereas fitting coefficients of homogenized macro cross sections are prepared in the conventional analysis method.

- A core power distribution is reconstructed by summing pin powers, which are evaluated with neutron production rates calculated by eq. (3) in the DRM analysis method, whereas the distribution is estimated from neutron flux distributions in the conventional analysis system.

- Each element of the sub-response matrices is corrected for changes in the fuel temperature, which is evaluated from the nodal power density, to take the Doppler effect into consideration. To achieve this, the dependence of each element of the sub-response matrices on fuel temperature is represented by the linear correlation to the root of the fuel temperature in the DRM analysis method, whereas the dependence of homogenized macro cross sections is represented by the same correction in the conventional analysis method.

The second feature is an advantage of the DRM analysis method. To evaluate thermal margins, such as the maximum linear heat generating ratio and the minimum critical power ratio, the pin-by-pin power distribution is needed and can be evaluated directly with the pin-by-pin neutron production rates in the DRM analysis method [6].

3. DEVELOPMENT OF SPECRTAL HISTORY METHOD

3.1. Spectral History Method Using Out-going Neutron Current

For the burn-up analysis of highly heterogeneous cores, the historical effect of neutronic interference between neighboring assemblies must be taken into account. However, the method which regards the moderator density history as the nodal spectral history is sufficient for this. Then, the spectral history method was proposed for the conventional analysis method [7]. In this method, the burn-up average of nodal thermal-to-fast flux ratio is used to represent neutron spectral conditions during fuel burn-up (spectral history). For the DRM analysis method, assuming that the out-going current is much influenced by nodal neutron spectral conditions, the following $\bar{s}$ was introduced to represent spectral history:

$$\bar{s} = \frac{1}{E}\int_0^E J^{out}_{thermal} \Big/ J^{out}_{fast}\,(E)dE \tag{4}$$

where $J^{out}_{thermal} \Big/ J^{out}_{fast}$ denotes the node boundary averaged thermal-to-fast out-going current ratio, and E denotes nodal average exposure. To reproduce sub-response matrices, it is necessary to decide the effective historical void fraction taking account nodal spectral history. That is the void fraction at the nodal $\bar{s}$ in infinite lattice burn-up equivalent to the $\bar{s}$ in core analysis.

3.1.1. Verification for spectral history method

For verification of the spectral history method, the heterogeneous system was tested. The geometrical configuration of this system is shown in Fig.1. It consisted of two low enrichment fuel assemblies and two high enrichment fuel assemblies. Each assembly was a typical BWR fuel assembly consisting of an 8 × 8 fuel rod array and one large central water rod. The void fractions of the moderator in the low and high enrichment nodes were 0.4, and no control rod

was inserted. The boundary condition for the system was reflective, so it was an extremely heterogeneous system.

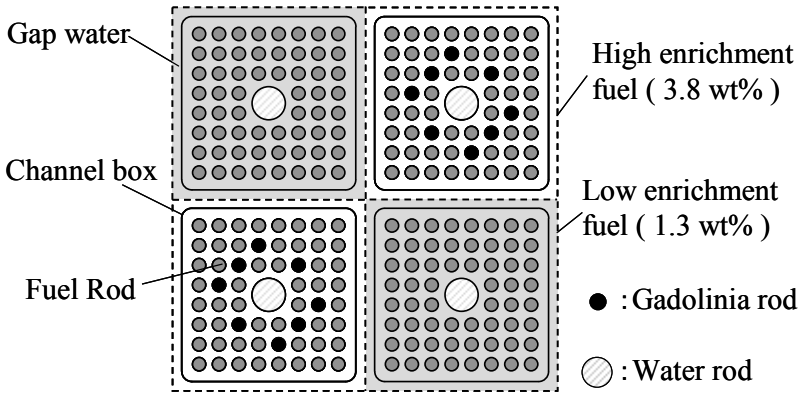

Figure 1. The heterogeneous system.

In the DRM method, the sub-response matrices were calculated by the fuel assembly analysis code VMONT [8] which is based on a Monte Carlo neutron transport method. The VMONT code uses a multi-group model for the neutron spectrum calculation, and the total number of energy groups is 190. There were three energy groups of the produced sub-response matrices: Group 1 is from 5.53 keV to 10.0MeV, Group 2 was from 0.625 eV to 5.53 keV, and Group 3 was from 0.0 to 0.625 eV. The number of tracked neutrons was 2×10^6. The statistical uncertainty of the neutron infinite multiplication factor was about 0.03%Δk and that of the fuel rod neutron production rate was about 0.3%. The number of tracked neutrons was set so that the statistical uncertainty of the neutron infinite multiplication factor was within about 0.03%Δk. Each surface was subdivided by 4 for transverse segments, by 4 for angular segments and by 4 axial zone in a node. In the spectral history correction, the fast neutron group was Group 1, and the thermal neutron group is Group 3.

The reference results were obtained by the direct calculation of the whole system with the VMONT code. The number of tracked neutrons was 2×10^7. The statistical uncertainty of the neutron infinite multiplication factor was about 0.01%Δk and that of the fuel rod neutron production rate was about 0.1%.

At first, for verification of the assumption of the spectral history with the outgoing neutron current, the effective historical void fraction was compared between the nodal outgoing and incoming current and the nodal averaged neutron flux. In this verification, results were obtained by the direct calculation with the VMONT code. The effective historical void fraction with each case is shown in Fig.2. In low enrichment fuel, the void fraction of out-going neutron current was corresponding to that of neutron flux within 3%. In high enrichment fuel, the void fraction of out-going neutron current was also corresponding to that of neutron flux within 3%. As a result, it is appropriate to use out-going neutron current for the spectral history method.

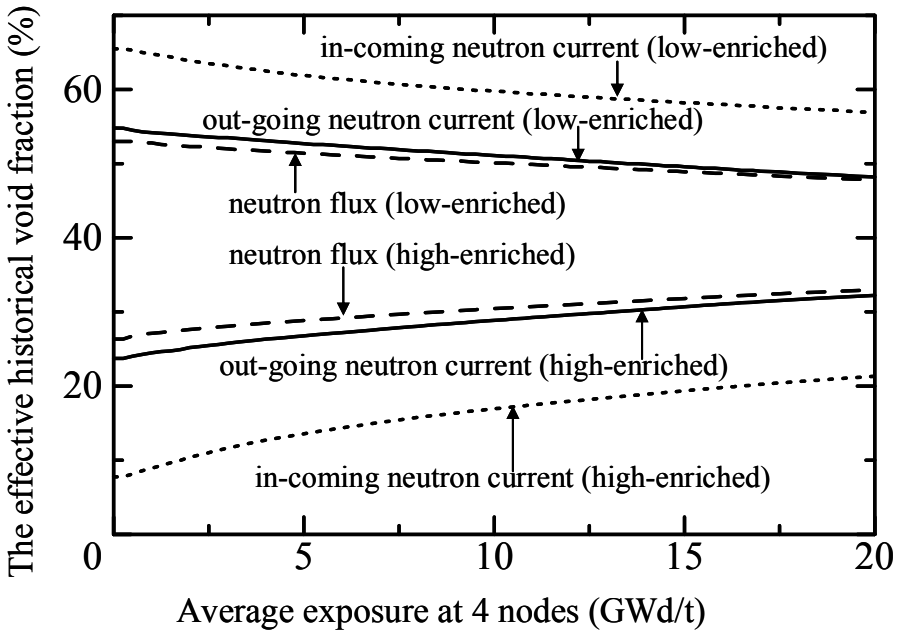

Figure 2. Effective void fraction changes during burn-up.

Using this spectral history method for DRM analysis method, the burn-up results of the neutron infinite multiplication factors for the system are shown in Fig.3. The discrepancy increased with burning gadolinia because the spectral transition which accompanies gadolinia burn-up could not be evaluated using the moderator density history. As the spectral history method took into account the nodal spectral changes, this method could reduce the discrepancy by half during core burn-up.

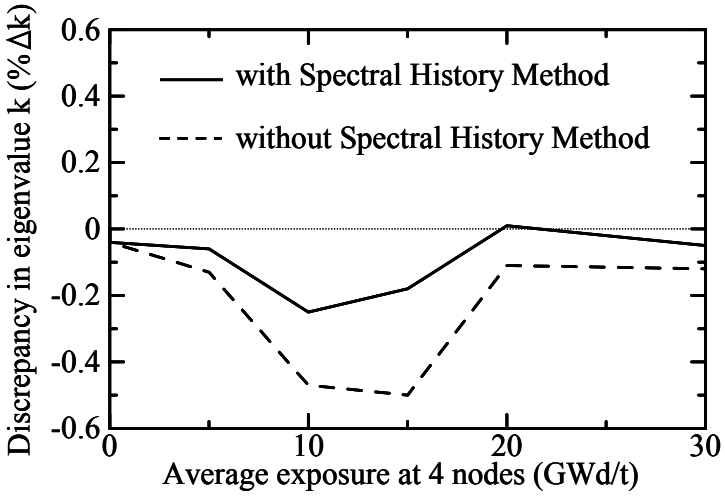

Figure 3. Discrepancy in eigenvalue k.

The results of the nodal neutron production rates for the low and high enrichment fuel nodes are shown in Fig.4. The spectral history method could also reduce the discrepancy in nodal neutron production rates, and the reductions in low and high enrichment fuel nodes were 34% and 40%, respectively, during burn-up.

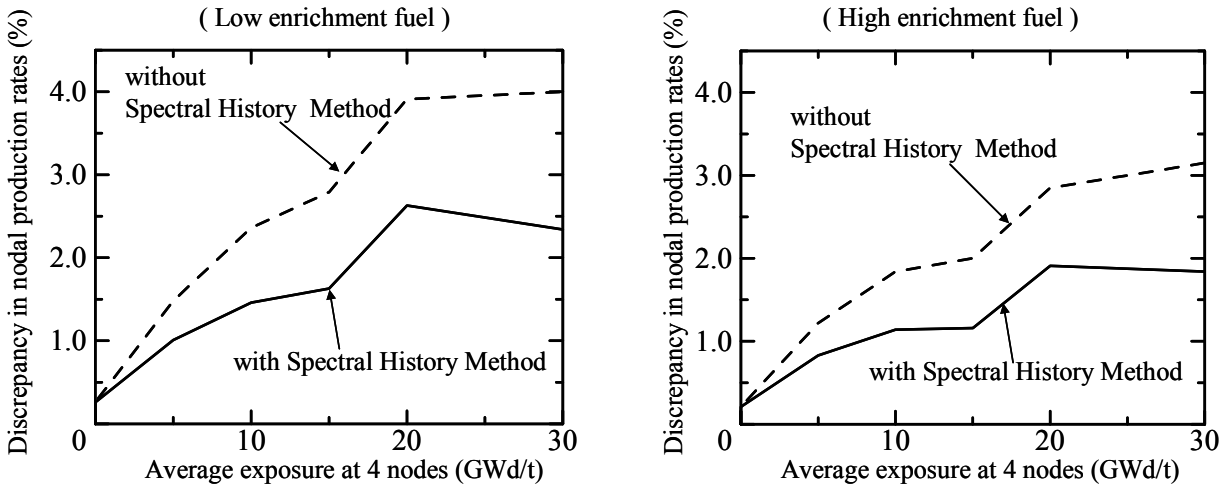

Figure 4. Discrepancy in nodal neutron production rates.

3.2. Correction of Pin Power Discrepancy Due to Burn-up

The most advanced feature of the DRM method is to be able to evaluate the pin power of every fuel rod during burn-up. That is, this method can evaluate the exposure of every fuel rod. Since pin power is evaluated with the sub-response matrices obtained by infinite lattice calculation, of which pin exposures are different from actual ones, discrepancies of pin powers from actual ones arise with burn-up. Then, pin powers were corrected by assuming that pin power is approximately proportional to fissile material quantity as follows:

$$P_i \cong \frac{N_{\inf} + \Delta N_i}{N_{\inf}} P_{i,DRM} \tag{5}$$

$$\Delta N_i = \frac{dN_{\inf}}{dE_{\inf}} \Delta E_i \tag{6}$$

where P_i denotes the corrected neutron production rate of the i-th pin, $N_{\inf}$ denotes number density of fissile materials in the i-th pin evaluated by the infinite lattice calculation, $P_{i,DRM}$ denotes neutron production rate of the i-th pin evaluated by the DRM method and ΔN_i denotes correction for number density of fissile materials in the i-th pin. The term of $dN_{\inf}/dE_{\inf}/N_{\inf}$ in substituting eq. (6) into eq. (5) could be evaluated for every burn-up state by the infinite lattice calculation. The correction of pin exposure, ΔE_i, could be evaluated as follows:

$$\Delta E_i = E - E_{\inf} \tag{7}$$

where E denotes corrected exposure of the i-th pin, $E_{\inf}$ denotes exposure evaluated with the infinite lattice calculation of the i-th pin. The corrected exposure, E, was evaluated with pin powers by eq. (5) of the former exposure steps. The term $dN_{\inf}/dE_{\inf}/N_{\inf}$ which must be calculated denotes the burn-up change in the density of fissile materials for each density of fissile materials. In this method, the burn-up change in the density of fissile materials for each fuel rod was assumed to be equivalent to the change for nodal average. As a result, pin powers could be corrected using just the stored nodal averaged $dN_{\inf}/dE_{\inf}/N_{\inf}$, without much added CPU-time and memory source and without re-calculating sub-response matrices corresponding to the node conditions.

3.2.1. Verification for the correction of pin power discrepancy

For verification of the pin power correction method, the heterogeneous system shown in Fig.1 was tested. The verification condition in the DRM method was same in section 3.1.1. In this test, the nodal exposure was evaluated in a direct Monte Carlo calculation.

The results of the root-mean-square (RMS) differences between the relative fuel rod neutron production rate distributions are shown in Fig.5. The correction method could lower the RMS differences, and the differences in low and high enrichment fuel nodes were 0.9% and 1.1%, respectively, during burn-up. Further this method could decrease the transition of the RMS differences below 0.5% from the beginning of burn-up compared with the method without correction in which the transition was more than 1.0%.

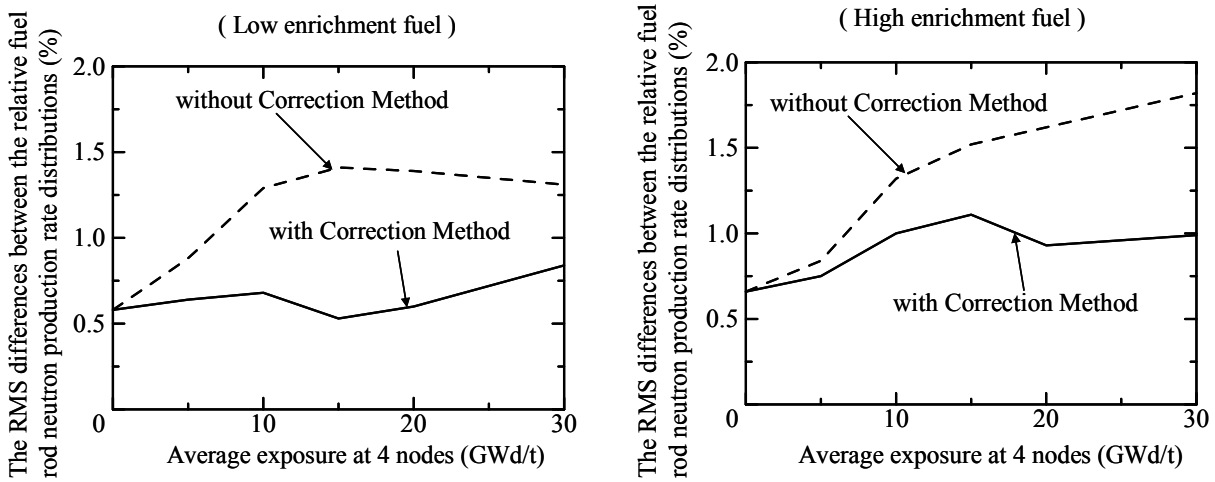

Figure 5. RMS differences in neutron production rate distributions.

1.035	0.967	0.931	1.001	1.006	1.024	0.984	1.074
1.40	0.13	-0.01	-0.17	0.30	0.35	-0.80	-0.69
2.89	1.45	1.11	1.18	1.45	1.10	-0.59	-1.09
0.967	0.947	0.990	0.985	0.987	1.000	0.979	0.985
0.48	1.05	-0.68	0.00	0.15	0.45	-0.65	-0.88
1.69	2.44	0.60	0.87	0.89	0.84	-0.89	-1.72
0.931	0.990	0.989	1.014	1.015	0.987	1.004	1.033
0.17	-0.83	-0.16	0.16	0.74	0.14	0.43	-0.28
1.34	0.28	0.94	1.20	1.59	0.58	-0.02	-1.88
1.001	0.985	1.014			1.020	0.993	1.022
0.14	0.19	-0.27			0.53	0.83	-0.16
1.43	1.11	0.86	water rod		0.97	0.17	-1.76
1.006	0.987	1.015			1.017	1.002	1.018
0.34	0.02	0.55			1.64	-0.16	0.13
1.35	0.84	1.36			1.92	-0.83	-1.56
1.024	1.000	0.987	1.020	1.017	0.996	1.010	0.947
0.10	0.42	0.50	0.92	0.67	0.13	0.05	-0.42
0.80	0.74	0.75	1.03	0.92	-0.16	-1.19	-1.61
0.984	0.979	1.004	0.993	1.002	1.010	0.984	0.992
-0.08	-0.62	0.50	0.21	-0.44	0.00	-0.85	-0.73
0.09	-0.86	-0.29	-0.43	-1.19	-1.19	-2.10	-2.08
1.074	0.985	1.033	1.022	1.018	0.947	0.992	1.077
-0.09	-0.14	-0.03	-1.09	-0.58	-0.47	-1.55	-1.06
-0.53	-0.99	-1.60	-2.66	-2.25	-1.78	-2.77	-2.51

(a) Low enrichment fuel

0.937	1.000	1.038	1.027	1.026	1.036	0.997	0.930
1.13	0.93	1.55	1.46	0.83	1.17	1.63	2.58
1.89	2.07	2.86	2.74	2.09	2.61	2.98	4.10
1.012	0.998	1.032	0.892	0.946	1.024	1.000	0.997
-0.36	0.02	-0.31	-0.29	-1.12	-1.30	-1.00	1.81
-0.30	0.05	-0.11	-0.36	-1.00	-0.95	-0.14	3.21
1.056	1.039	0.886	1.048	1.037	0.878	1.024	1.036
-0.51	-0.61	-0.90	-0.89	-1.13	-1.03	-1.16	1.29
-0.98	-0.95	-1.12	-1.58	-1.64	-1.09	-0.73	2.62
1.039	0.891	1.051			1.037	0.946	1.026
-0.26	-1.17	-0.34			-0.21	-0.08	1.12
-1.03	-1.39	-1.16	water rod		-0.71	0.07	2.47
1.044	0.954	1.043			1.048	0.892	1.027
-0.83	-0.67	-0.93			0.28	0.12	1.45
-1.75	-1.36	-1.81			-0.43	0.09	2.89
1.056	1.030	0.872	1.043	1.051	0.886	1.032	1.038
0.33	0.01	-0.01	-0.12	-0.73	-0.38	0.41	1.09
-0.80	-0.72	-0.21	-0.96	-1.47	-0.56	0.56	2.49
1.030	1.002	1.030	0.954	0.891	1.039	0.998	1.000
0.16	0.50	-0.89	-0.06	0.19	-0.98	-0.30	0.90
-0.86	-0.31	-1.66	-0.83	-0.03	-1.36	-0.27	2.12
0.977	1.030	1.056	1.044	1.039	1.056	1.012	0.937
-2.01	0.24	0.54	-0.59	-0.13	-0.79	-0.59	1.00
-2.91	-0.81	-0.62	-1.56	-0.90	-1.33	-0.47	1.91

(b) High enrichment fuel

Upper numbers: reference, Middle numbers: relative difference with the correction method (%),
Lower numbers: relative difference without the correction method (%)
Relative difference: (each method – reference) / reference
The gray areas mark the maximum difference

Figure 6. Pin-by-pin neutron production rates at 20GWd/t.

Pin-by-pin neutron production rate distributions and the relative differences with or without the correction methods at the average exposure of 20GWd/t is shown in Fig.6. The maximum differences in the fuel with low and high enrichments with the correction method were 1.6% and 2.6% respectively. In particular, the correction method could decrease the relative differences in the boundary neighborhood fuel rods where the differences tended to increase in the heterogeneous system.

4. APPLICATION OF DRM FOR CORE ANALYSIS

A quarter core of the ABWR was analyzed with the coupled analysis of the neutronic and thermal hydraulic calculations by using the DRM analysis method. The quarter core of the ABWR consists of 218 fuel assemblies. A 9×9 lattice fuel containing 66 full-length fuel rods, eight partial-length rods, and two large water rods was assumed. The calculation conditions for thermal-hydraulics such as the core thermal power, total coolant-flow, core pressure, and the thermal-hydraulic properties of the 9×9 lattice fuels were set to those in a practical plant. The DRM calculation was carried out with the 2×2 sub-surface, 4 angle divisions and 2 axial zones in a node.

Changes in the k-effective difference the root mean square differences of the nodal power and coolant-flow distributions from values after 20 iterations versus the number of iterations of neutronic and thermal-hydraulic calculations are shown in Fig.7. The k-effective, nodal power and coolant-flow distributions were smoothly converged. The k-effective difference became less than 0.01 % k after 4 iterations and the root mean square differences of the nodal power and coolant-flow distributions became less than 0.1 % after 12 and 7 iterations, respectively.

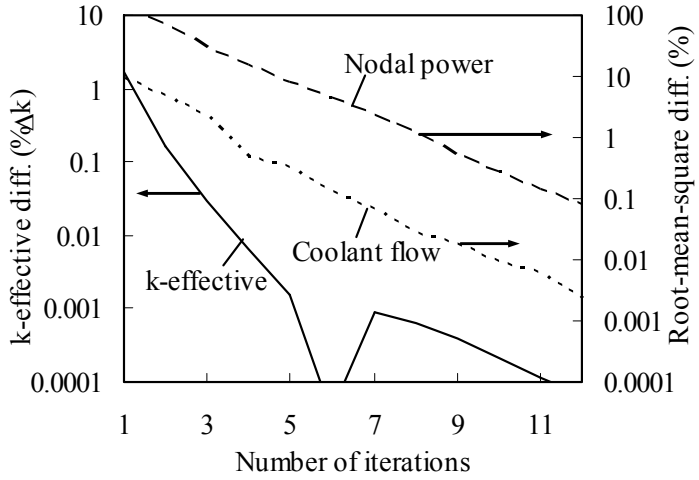

Figure 7. Difference of k-effective, nodal power and coolant flow distribution from ones after 20 iterations.

The three-dimensional thermal power and void fraction distribution evaluated by the analysis are shown in Fig.8. Heterogeneity in the nodal and pin-by-pin power distributions around the control rods was well represented. The test calculation was carried out with Hitachi's SR11000 super computer system. The computation time was within 0.5 hours for each iteration. The required memory capacity was about 6 GBytes.

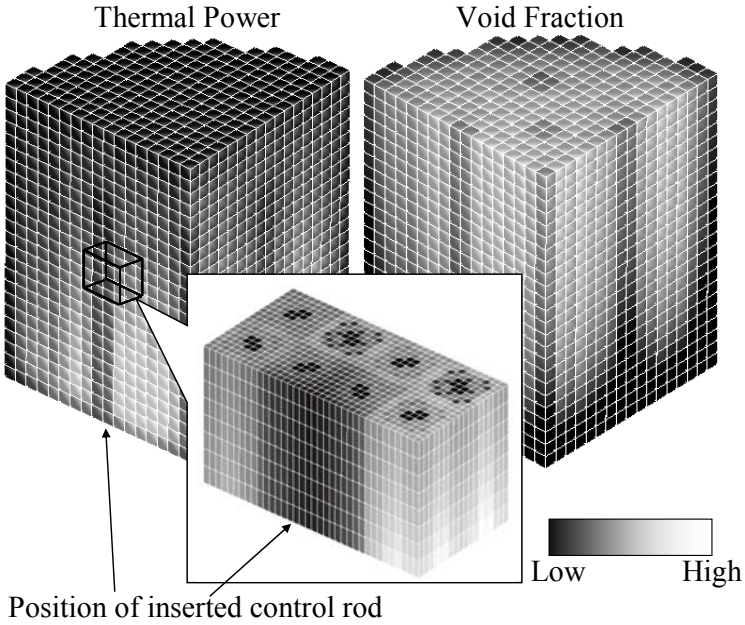

Figure 8. Thermal power and void fraction distribution of ABWR quarter core obtained by the DRM analysis.

5. CONCLUSIONS

Spectral history methods for pin-by-pin core analysis method using the three-dimensional direct response matrix have been developed. As the sub-response matrices were obtained by pin-by-pin evaluation in the Monte Carlo method, the pin-by-pin correction could be performed. To reflect the burn-up effect, spectral history methods can treat the nodal burn-up spectrum using the spectral history method and correct the fuel rod neutron production rates using burn-up change of fissile materials and pin-by-pin exposure. These developed methods were tested in a heterogeneous system. The test showed that the neutron multiplication factor error could be reduced by half error during burn-up, the nodal neutron production rates errors could be reduced by 30% or more. The root-mean-square differences between the relative fuel rod neutron production rate distributions could be reduced within 1.1% error. These good agreements meant that those spectral history methods for the three-dimensional direct response matrix method accurately reflected the effects of intra- and inter-assembly heterogeneities in heterogeneous systems during burn-up and thus can be used for core analysis. Core analysis with the DRM

method was carried out for an ABWR quarter core and it was found that both thermal power and coolant-flow distributions were smoothly converged.

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