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ENHANCEMENT OF XENON DYNAMICAL FORECASTS FOR PWR GUIDING SYSTEMS USING VARIATIONAL DATA ASSIMILATION

Angélique Ponçot, Patrick Erhard and Jean-Philippe Argaud
EDF R&D
1 avenue du Général de Gaulle, F-92141 Clamart Cedex France
angelique.poncot@edf.fr

Serge Gratton, Bertrand Bouriquet, Sophie Ricci, Sébastien Massart
CERFACS
42 avenue Gaspard Coriolis, F-31057 Toulouse Cedex 01 France

Olivier Thual
Université de Toulouse , INPT, UPS, IMFT (Institut de Mécanique des Fluides de Toulouse)
Allée Camille Soula, F-31400 Toulouse, France

ABSTRACT

In pressurised water reactors, the concentration of the nuclear fission product xenon-135 can exhibit axial oscillations which have to be controlled for safety reasons, related to spatial oscillations of in-core power. The evolution of the xenon concentration depends non-linearly on the concentration of the fission product iodine-135. This non-linearity represents a challenge for the prediction of oscillations. We investigate two variational data assimilation methods to improve the accuracy of the prediction of the xenon concentration. Data assimilation techniques are widely used in meteorology and oceanography to obtain more precise estimates of initial states. These methods combine all available pieces of information about the system such as the model prediction, a prior estimate of the true state, and physical measurements. The different sources of information are weighted according to their accuracy by the means of error covariance matrices. In our work, we propose one 3DVAR and one 4DVAR scheme for xenon dynamics. These schemes were tested in a twin experiment setting using the CIREP1D simulation code. CIREP1D implements a monodimensional coupled model of xenon dynamics, thermal, and thermal-hydraulic processes. Numerical experiments show that the proposed data assimilation schemes improve the accuracy of the xenon forecast.

Key Words: xenon dynamics, variational assimilation, monodimensional nuclear core modelling

1. INTRODUCTION

Xenon-135 is known to be at the origin of axial power oscillations of about one day period in pressurised water reactors (PWRs). These oscillations do not change the overall power produced by the nuclear plant but they are undesirable from a safety point of view. As soon as oscillations are detected, they are damped using appropriate control rod movements inside the core. Detection as well as prediction of xenon induced oscillations are an important part in the operation of a nuclear power plant.

No direct measurements of the concentration of xenon in the reactor core are available. The simulation of the nonlinear xenon dynamics still represents a challenge. Several models have been

proposed for the real time estimation of xenon concentration. They include flux and iodine-135 dynamics modelling. Some of them require an estimation of parameters such as presented in [6]. Few of these models take into account the initial conditions of the coupled flux-iodine-xenon and most of them assume equilibrium concentrations. Note, however, that the initial distributions of iodine and xenon have a significant impact on the power transient. Song and Cho [8] determined an analytic initialisation of iodine and xenon of an out-of-equilibrium state which consists of adding a corrective term with a sinus shape to the 1D equilibrium concentrations. The amplitude of the sinus is fitted with axial offset power measurements. These measurements are considered to be perfect, in the same way as the xenon dynamics model is considered exact. This approach is based on analytical developments which limit the shape of the added correction and does not take into account the errors in the measurements. In order to overcome these difficulties, we investigate the feasibility of using variational data assimilation methods for estimating xenon and iodine concentrations.

Data assimilation techniques are widely used in earth sciences such as meteorology [1] and oceanography. The aim is to provide a satisfactory estimation of the unknown true state of a dynamical system by combining all pieces of information about the system. This information, obtained from measurements (called observations) and simulation, is weighted according to its reliability expressed in terms of error covariance matrices. In practice, the model gives a simulated state called the background state. The purpose of data assimilation is to determine a state, called the analysis state, which is closer to the true state than the one described solely by the observations or the model. Thus, the analysis state can be used to compute a forecast.

In this study, we present two variational data assimilation schemes developed in order to improve the estimation of xenon and iodine concentrations in a nuclear core. In Section 2, we introduce the model developed for this purpose, a monodimensional xenon dynamics model which includes neutron and thermal-hydraulic processes. Then in Section 3, we give a brief overview of data assimilation methods. In Section 4 we present the settings of the assimilation scheme for the xenon dynamics. Finally, in Section 5, we propose an industrial application for xenon dynamics fitting.

2. AXIAL XENON DYNAMICS MODEL

Since 3D operational industrial codes are time consuming, we decided to develop a monodimensional axial xenon/iodine dynamics model coupled with a monodimensional thermal/thermal-hydraulic model, named CIREP1D. The agreement between 1D and 3D models has been studied in [7]. CIREP1D simulates axial xenon dynamics according to the overall power and the control rod insertion records in a given time window. On the one hand, CIREP1D is a low time computing code: it takes a few seconds to simulate a xenon oscillation of an one-week time range. On the other hand, contrary to simpler models, it gives access to quantities measured in core: axial power, axial xenon, axial iodine, axial flux and boron concentration.

Iodine and xenon balance equations are:

$$\begin{cases} \dfrac{\partial C_I}{\partial t}(z,t) = \gamma_I \Sigma_f(z,t)\Phi(z,t) - \lambda_I C_I(z,t), \\ \dfrac{\partial C_{Xe}}{\partial t}(z,t) = \gamma_{X_e} \Sigma_f(z,t)\Phi(z,t) + \lambda_I C_I(z,t) - \left(\lambda_{X_e} + \sigma_{Xe}(z,t)\ \Phi(z,t)\right) C_{Xe}(z,t), \end{cases} \tag{1}$$

where the z-coordinate is measured from the bottom of the 1D reactor. Σ_f is the fission cross section of the fuel and σ_{Xe} the absorption cross section of xenon-135. γ_I and γ_{Xe} are the fractional fission yield of iodine and xenon. Finally, λ_I and λ_{Xe} are decay constants of iodine and xenon.

The neutronic flux $\Phi = (\Phi_1, \Phi_2)$ is identified by solving two-group diffusion equations. We assume that the time step of flux simulation (a few seconds) is shorter than the xenon oscillations. As a consequence, at each time step for the resolution of xenon equation, the flux can be computed using the stationary diffusion equations:

$$\begin{cases} -\partial_z D_1(z)\partial_z \Phi_1(z) + [\Sigma_{a1}(z) + \Sigma_r(z)]\,\Phi_1(z) = \dfrac{1}{k}\left(\nu_1 \Sigma_{f1}\Phi_1(z) + \nu_2 \Sigma_{f2}(z)\Phi_2(z)\right) \\ -\partial_z D_2(z)\partial_z \Phi_2(z) + [\Sigma_{a2}(z) + D_2(z)]\,\Phi_2(z) - \Sigma_r(z)\Phi_1(z) = 0, \end{cases} \tag{2}$$

where Φ_1 et Φ_2 are groupwise neutron axial flux distribution, Σ_r is the scattering cross section and Σ_{ag}, D_g and $\nu_g \Sigma_{fg}$ are the groupwise absorption cross section, the groupwise diffusion coefficient, and the groupwise neutron emitted in fission cross section. The balance is obtained by looking for boron concentration such that the eigenvalue k is equal to one (critical boron concentration computation). The boron influence does not appear explicitly in the previous equations but is linked to cross section values through the feedback model.

Thermal effects are not lumped in a prompt power feedback parameter as done in [6]. The developed feedback model is a linear interpolation model relying on assumption that the cross sections depend on six quantities: fuel irradiation, xenon concentration C_{Xe}, boron concentration C_B, moderator density ρ_{mod}, moderator temperature T_{mod} and fuel temperature T_f. Therefore, CIREP1D includes a thermal/thermal-hydraulic model, as described below.

Since the speed of the water flowing upwards through the reactor is high, we can assume that the thermal-hydraulic problem is an axial monodimensional problem for the slow transients which are common in the normal operational mode. The moderator temperature T_{mod} is then described by the following equation:

$$Q\,\partial_z T_{mod}(z,t) = \frac{1}{\rho_{mod} c_{mod}} \left[P_f^{lin}(z,t) + P_{mod}^{lin}(z,t)\right], \tag{3}$$

where Q, c_{mod} and ρ_{mod} respectively represent the volume flow rate, the moderator specific heat capacity and the moderator density. Lineic power P_f and P_{mod} released in both fuel and moderator are computed from the known two-group flux.

Contrary to the thermal-hydraulic model, we employ a radial model for the thermal fuel model. Thus, a radial description of the fuel is then required. We neglect the axial conduction in fuel pin and assume rotational symmetry of the problem. Under these assumptions, the thermal problem can be described by a monodimensional model in the radial variable r:

$$-\frac{1}{r}\lambda_f\,\partial_r T_f(r,z) - \lambda_f\,\partial_r^2 T_f(r,z) = P_f(z)/\mathcal{A}. \tag{4}$$

The variable λ_f represents the fuel thermal conductivity and $\mathcal{A}$ corresponds to the pin section. This equation is coupled with the neutron equation through the lineic power P_f and to the thermal-hydraulic problem through the boundary condition expressed on edge Γ:

$$\forall r \in \Gamma, \quad \Phi_{th}(r,z) = h_{tot}(z)\left[T_f(r,z) - T_{mod}(z)\right], \tag{5}$$

where the variable Φ_{th} is the thermal flux and h_{tot} is the thermal exchange parameter. The thermal and thermal-hydraulic parameters ρ_{mod}, c_{mod}, λ_f and h_{tot} depend on moderator and fuel temperatures. Therefore, the coupled thermal/thermal-hydraulic problem is nonlinear.

Globally speaking, CIREP1D solves a nonlinear system of ordinary differential equations:

$$\frac{\partial(C_{Xe}, C_I)}{\partial t}(z,t) = \mathcal{G}(C_{Xe}, C_I)(z,t), \tag{6}$$

by using an implicit Euler scheme. Each time step requires a critical boron concentration computation corresponding to the assumption that neutron and thermal-hydraulic effects may be treated as prompt with stationary coupled equations. The xenon dynamics can be initialised by either a given xenon and iodine concentrations or by equilibrium concentrations.

As an example of CIREP1D simulation, we present results from a computation with the following characteristics: time range of 200 hours, load following during 30 minutes and, then until the end of simulation, no further rod movement and no power change. The simulated core is in the middle of a burnup cycle and then is moderately irradiated. Fig.1 shows a xenon ocscillation which disappears without any external intervention after 100 hours.

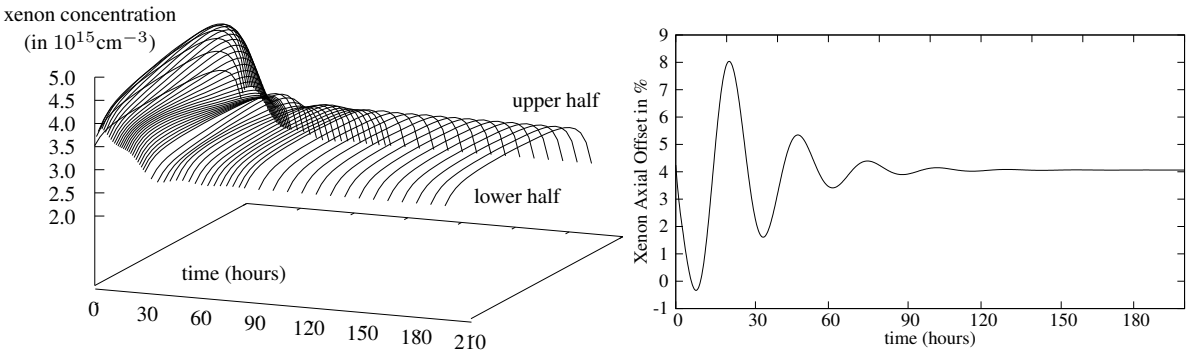

Figure 1. Xenon dynamics simulation on a time range of 200 hours.

3. A BRIEF OVERVIEW OF DATA ASSIMILATION TECHNIQUES

The computation of the state of a physical system, such as the xenon state, requires a model and data (model parameters and/or initial conditions). Error sources in this computation are various: physical approximation such as monodimensional modelling, discretisation error and uncertainty in parameters, such as fuel irradiation. To obtain a better estimation of the state of the system, we can improve modelling precision but it may be not convenient in computational time for real time applications such as monitoring operator system. We can also improve precision of model parameters as described by Onega and Kisner for the xenon transient control in [6]. This approach is a first step in improving the state estimation but it does not take into account errors in the measurements used to adjust the model parameters: thus bad measurements can affect adversely the quality of the

computed state. In addition, it does not allow to correct initial conditions of a dynamical system such as the one describing the evolution of xenon concentration. Data assimilation techniques have been developed to overcome these difficulties.

We will now introduce some concepts and definitions. A discrete model for the evolution of physical system, such as xenon dynamics, from time t_i to time t_{i+1} is described by:

$$\mathbf{X}(t_{i+1}) = \mathcal{M}_{i+1,i}(\mathbf{X}(t_i)),$$

where $\mathbf{X}$ and $\mathcal{M}$ are the model's state vector and its corresponding dynamics operator, respectively. The dynamics $\mathcal{M}$ of the model evolution is commonly nonlinear. The state vector $\mathbf{X}$ is obtained usually by discretisation of physical fields on a grid. Its dimension is denoted by n. The aim is to evaluate the best estimate of the unknown *true state*, denoted $\mathbf{X}^t$ which is defined by the best possible representation of reality as a state vector. The best estimate that we are looking for in the data assimilation process is called *analysis* and is denoted by $\mathbf{X}^a$.

The information about the system that can be used to produce the analysis is listed below:

measurements in the core gathered into an *observation vector* $\mathbf{Y}^o$. Its dimension is p.

the observation operator. The key to data analysis is to take advantages of the discrepancies between observations and state vector. Often, observation vector and state vector are not defined in the same space. This difficulty is overcome through the use of a function from model state space to observation space called *observation operator* and denoted $\mathcal{H}$. Operator $\mathcal{H}$ can be nonlinear.

an a priori estimate of the true state before the analysis is carried out. This estimate is called *background state* and is denoted $\mathbf{X}^b$. In most cases, the analysis problem is under-determined because observations are sparse and only indirectly related to the model variables. The use of this background information helps to make it a well-posed problem. Usually, this background state is generated from the output of a previous analysis.

uncertainties in the previous data. Background and observation errors are defined by:

$$\epsilon^b = \mathbf{X}^b - \mathbf{X}^t \quad \text{and} \quad \epsilon^o = \mathbf{Y}^o - \mathcal{H}(\mathbf{X}^t).$$

The covariance matrices of these errors respectively are denoted by $\mathbf{B}$ and $\mathbf{R}$. Error modelling is a difficult task, mostly because true state $\mathbf{X}^t$ is unknown and the knowledge of the error covariances is approximative. But it is a very important step which acts on the quality of the analysis.

Basically, two families of data assimilation methods have been developed: the stochastic methods and the variational methods. The most famous stochastic method is probably the Kalman filter. This method is still considered as a reference but its application for real data assimilation problems is limited to problems of small size due to its huge computational cost. Several variants of this method have been developed either to reduce its computational cost or to remove the assumption on linearity of the used operators. Variational methods are based on the minimisation of a cost function. These methods, 3DVAR and 4DVAR, well-suited to nonlinear cases and problems of large size, are mainly used in operational meteorology and oceanography since the 1990s. Each

variational method is equivalent to a filter method under linear assumptions. The system presented in this paper is small enough to be handled by a filter method. Nevertheless, variational methods have been chosen for the present case in order to be able to generalise results to a 3D industrial code. We now give here some elements on variational methods. The 4DVAR cost function measures the weighted sum of the square of distances $\mathcal{J}^b$ to background state $\mathbf{X}^b$ and $\mathcal{J}^o$ to the observations $\mathbf{Y}^o$ over a time interval $[t_0, t_n]$:

$$\begin{aligned} \mathcal{J}_{4DVAR}(\mathbf{X}(t_0)) = &\ \mathcal{J}^b(\mathbf{X}(t_0)) + \mathcal{J}^o_{4DVAR}(\mathbf{X}(t_0)) \\ \text{with } \mathcal{J}^b(\mathbf{X}(t_0)) = &\ \tfrac{1}{2}\left[\mathbf{X}(t_0) - \mathbf{X}^b(t_0)\right]^T \mathbf{B}^{-1}\left[\mathbf{X}(t_0) - \mathbf{X}^b(t_0)\right] \\ \text{and } \mathcal{J}^o_{4DVAR}(\mathbf{X}(t_0)) = &\ \tfrac{1}{2}\sum_{i=0}^{n}\left[Y_i^o - \mathcal{H}(\mathcal{M}_{i,0}(\mathbf{X}(t_0)))\right]^T \mathbf{R}_i^{-1}\left[Y_i^o - \mathcal{H}(\mathcal{M}_{i,0}(\mathbf{X}(t_0)))\right], \end{aligned} \tag{7}$$

where weight matrices $\mathbf{B}^{-1}$ and $\mathbf{R}_i^{-1}$ are the inverse of the background and observation error covariance matrices at time t_i. Minimisation of (7) is done with respect to initial state $\mathbf{X}(t_0)$. In practice, the starting point of the minimisation is taken equal to the background $\mathbf{X}^b$. Evaluations of gradient of $\mathcal{J}_{4DVAR}$:

$$\nabla\mathcal{J}_{4DVAR}(\mathbf{X}(t_0)) = \mathbf{B}^{-1}\left[\mathbf{X}(t_0) - \mathbf{X}^b(t_0)\right] - \sum_{i=0}^{n}\mathbf{M}_{i,0}^T\mathbf{H}^T\mathbf{R}_i^{-1}\left[Y_i^o - \mathcal{H}(\mathcal{M}_{i,0}(\mathbf{X}(t_0)))\right], \tag{8}$$

are required by most minimisation methods which implies that the adjoint operator $\mathbf{M}_{i,0}^T$ and $\mathbf{H}^T$ are available. 3DVAR method is a cheaper alternative to 4DVAR because it does not require the evaluation of the model evolution and its adjoint. 3DVAR cost function is very close to 4DVAR one but the time sum disappears:

$$\begin{aligned} \mathcal{J}_{3DVAR}(\mathbf{X}(t_0)) = &\ \mathcal{J}^b(\mathbf{X}(t_0)) + \mathcal{J}^o_{3DVAR}(\mathbf{X}(t_0)) \\ \text{with } \mathcal{J}^o_{3DVAR} = &\ \frac{1}{2}\left[Y^o - \mathcal{H}(\mathbf{X}(t_0))\right]^T \mathbf{R}^{-1}\left[Y^o - \mathcal{H}(\mathbf{X}(t_0))\right]. \end{aligned} \tag{9}$$

4. VARIATIONAL ASSIMILATION SCHEME FOR XENON DYNAMICS

We develop two variational schemes, 3DVAR and 4DVAR, in order to improve xenon and iodine concentration estimation in core. The xenon dynamics model was previously described. First, we present the context of assimilation experiments and then, we describe different components of the assimilation system.

Twin experiments

In the framework of twin experiments, the true state $\mathbf{X}^t$ is known. It is a simulated state usually obtained by the model used for assimilation. Twin experiments offer a convenient framework to validate assimilation schemes independently of the model. It also offers opportunity to compare the analysis to the true state. True state can be used to build background state, for example by adding a noise to $\mathbf{X}^t$. It can also be used to build synthetic observations applying observation operator $\mathcal{H}$ to $\mathbf{X}^t$.

Observations used further in the analysis process are not coming from real core measurements. They come from numerical simulations with CIREP1D, following this scheme:

1. We compute xenon dynamics initialised by equilibrium concentrations, in a time range of one hour for example. The concentrations obtained after this hour are defined as the real state $\mathbf{X}^t$ at the initial time t_0 of the future analysis process.

2. We do a reference simulation with CIREP1D to make the real state $\mathbf{X}^t$ evolve from t_0 to t_n:

$$\mathbf{X}^t(t_i) = \mathcal{M}_{i,0}(\mathbf{X}(t_0)).$$

3. Observations over time range are obtained by introducing a measurement noise ϵ on real data:

$$\mathbf{Y}_i^o = \mathcal{H}(\mathbf{X}^t(t_i)) + \epsilon.$$

Model

The evolution model corresponds to the xenon dynamics model implemented in CIREP1D. This model is based on the resolution of the xenon and iodine monodimensional time equation. Each iteration time step requires a critical boron concentration computation which includes successive stationary neutron/thermal/thermal-hydraulic computations. For such a computation, CIREP1D inputs are:

- initial and final times t_0 and t_n of transient,
- initial xenon and iodine concentrations at time t_0,
- transient data: overall power and control rod position variations over the time interval $[t_0; t_n]$.

State vector

The state vector corresponds to xenon and iodine axial concentrations discretised on the 30 nodes of the 1D spatial mesh used in CIREP1D. The dimension of $\mathbf{X}$ is then 60. The analysis problem is to find a correction $\delta\mathbf{X}$ such that $\mathbf{X}^a = \mathbf{X}^b + \delta\mathbf{X}$ is as close as possible to $\mathbf{X}^t$. This correction is searched in the same space as the state vector one. Thus, the minimisation problem has dimension 60.

Observations

The observation vectors $\mathbf{Y}_i^{obs}$ at different observation time t_i are composed of 3 different measurements: 6 integrated powers over several nodes, 1 power axial offset data and 1 boron concentration data. This observation vector has dimension 8. The simulated observation data are computed through the resolution of the neutron/thermal/thermal-hydraulic coupled stationary equations. Thus, the nonlinear observation operator $\mathcal{H}$ roughly corresponds to a critical boron calculation. Therefore, it depends on xenon but not on iodine. Since the 3DVAR scheme does not involve evolution model, it cannot control iodine concentration. Another important characteristic of this scheme, compared to meteorological schemes, is the quasi-equivalence in computational cost of evaluation of the model $\mathcal{M}$ and observation operator $\mathcal{H}$.

Error covariance matrices

To build the observation error covariance matrix $\mathbf{R}$, we assume that measurement errors are Gaussian, that they are not correlated in space, and that they does not depend on time. In this case $R_i = R$ and R is diagonal. To build the background error covariance matrix $\mathbf{B}$, we also assume that background error is Gaussian. We assume too that there is no correlation between xenon and iodine background errors (univariate modelling). This is a constraint that strongly impact analysis and more specially result in a univariate correction in xenon. Spatial correlation between species errors are modeled using Balgovind correlation based on exponential function [3]. The resulting $\mathbf{B}$ matrix is then block-diagonal, each block corresponding to species xenon or iodine.

Minimisation

Finally, to solve the non-linear minimisation problem, we use the quasi-Newton method LBFGS [4]. This method requires the computation of the gradient of $\mathcal{J}$ which is done using the adjoint of the xenon dynamics model. In our case, the adjoint is obtained by automatic differentiation using TAPENADE software [2]. Both $\mathcal{J}$ and $\nabla\mathcal{J}$ are computed in the framework of the PALM assimilation coupler [5].

5. INDUSTRIAL APPLICATION

We describe here an industrial application where xenon dynamics control could be useful. Our goal is to obtain a better estimation of the xenon concentration after a simulation based on wrong data. The aim here is to reduce the required delay before producing a forecast simulation. Nowadays, this required time delay is currently around one day and a half after input data perturbation.

5.1. Context

Nuclear reactors are monitored by devices measuring physical fields such as temperature or activities and assuring that the reactor remains in a regular transient. Improvement in modelling field favours use of online simulation tools, such as xenon numerical models, in addition to measurements, in order to improve nuclear plant monitoring. Xenon oscillation prediction could allow a more appropriate control of the rod movements and then destroy these oscillations even before they are detected. Real time xenon dynamics simulation tools require the use of continuous input data. However, for safety reasons, direct access to control system data is not possible. The only accessible data are those delivered by monitoring systems that can unfortunately be temporarily unavailable. This unavailability can last from a few hours to several days and results in a non reliable prediction from the simulation tool. Typically, the simulation tool remains non reliable for 36 hours after the unavailability of data because of xenon dynamics memory. To reduce this delay, we propose to use data assimilation to estimate xenon and iodine concentrations at the time defined by the return of data.

For the presented computations, the unavailability is replaced by a wrong data concerning overall power control rod position as shown in Fig. 2. These computations are done within the framework of twin experiments. Errors introduced in measurements are set to 10%, 5% and 1% respectively for axial power, power axial offset (AO) and boron concentration measurements. The background error is set to 5%.

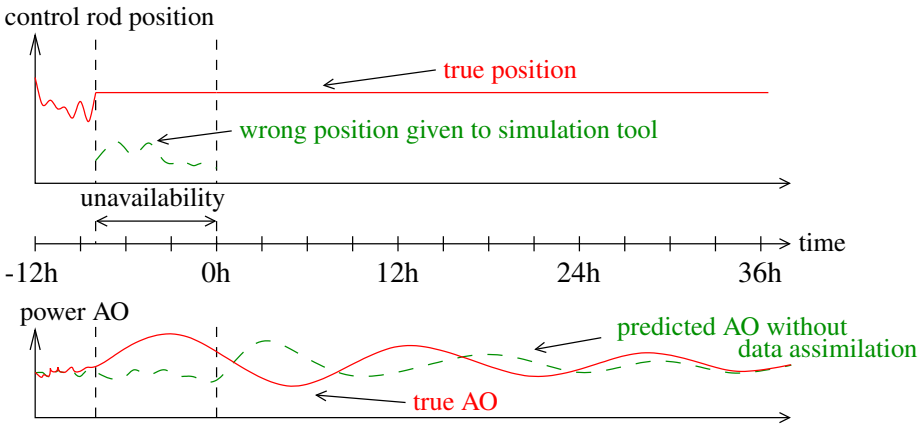

Figure 2. Illustration of the effects of wrong data on a simulation tool.

5.2. 3DVAR results

In Fig. 3, we show analysis results in terms of increment added to the background vector: true increment is defined by $\delta\mathbf{X}^t = \mathbf{X}^b - \mathbf{X}^t$ and analysis increment by $\delta\mathbf{X}^a = \mathbf{X}^b - \mathbf{X}^a$. We observe that the true increment on xenon is well estimated by the analysis increment. But we notice that the iodine analysis increment is null. Since 3DVAR scheme does not involve evolution model $\mathcal{M}$, the observation operator $\mathcal{H}$ does not depend on iodine and no correlation between xenon and iodine errors are assumed, therefore iodine concentration cannot be corrected by such an assimilation scheme.

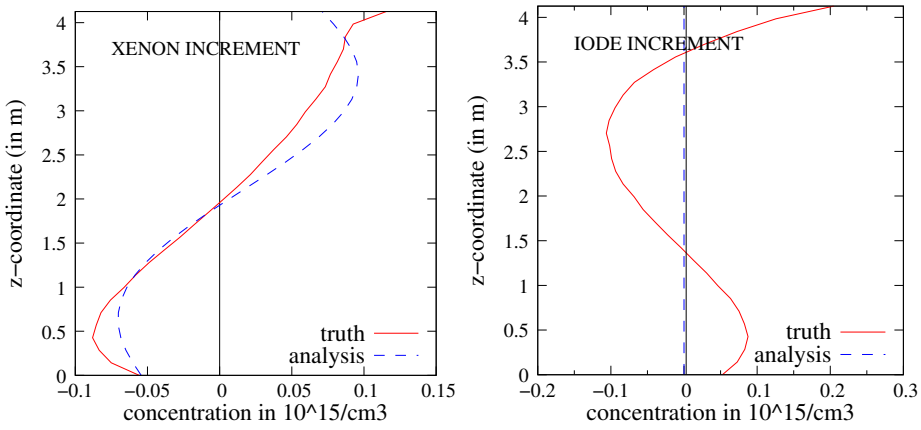

Figure 3. Xenon and iodine analysis increments computed by a 3DVAR scheme.

In Fig. 4, we compare true axial power shape to, respectively, background and analysis axial power shape at analysis time t_0 and after 6 hours. The power shape is well predicted thanks to the 3DVAR scheme at analysis time t_0, but the delay of the prediction reliability is short: whereas at time t_0

true and analysis power shapes are very close (Fig. 4(a)), after few hours, analysis power shape is no more better than background power shape (Fig. 4(b)). This happens because xenon is essentially produced by radioactive decay of iodine. Therefore, a bad estimation of the initial iodine concentration will affect the xenon concentration estimation later. Then, iodine concentration has to be adjusted, which is feasible only through a 4DVAR scheme.

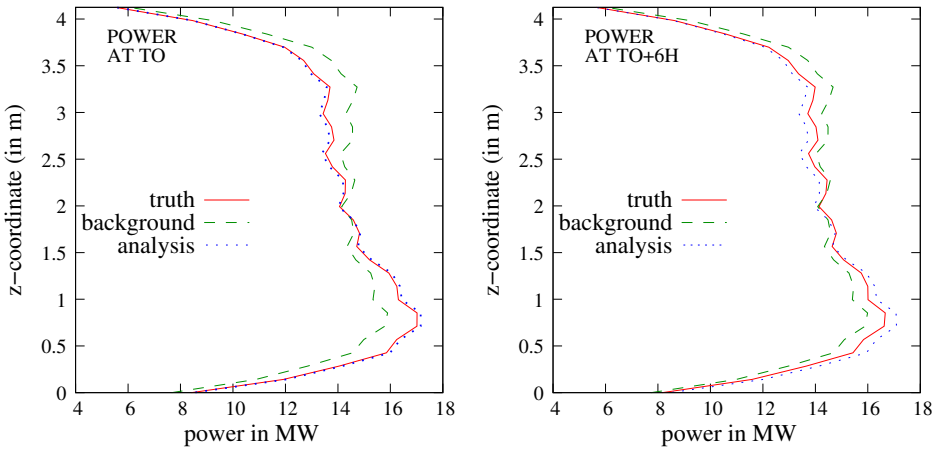

Figure 4. Axial power distributions at initial time t_0 and after 6 hours at $t_0 + 6\,h$. The power analysis comes from a 3DVAR scheme.

5.3. 4DVAR results

To implement a 4DVAR scheme, one has to choose the time interval $[t_0; t_n]$ considered for assimilation and the observation frequency. In Fig. 5, we show dependency of xenon and iodine analysis to the time interval. Xenon analysis is not quite sensitive to this parameter, because observations, especially axial power measurements, are strongly related to xenon concentration. Therefore xenon concentration needs very few data to be well adjusted. This is not the case for iodine which is only related to the observations through the dynamical model. Thus iodine analysis is really improved with a 6 hour time interval compared to a 15 minute time interval.

We obtain similar conclusions with the dependency of xenon and iodine analyses to observation frequency. In practice, it seems that an observation frequency of 2 hours is enough to correct the dynamical system. In Fig. 6, we compare relative errors on the predicted axial powers with a 3DVAR scheme and a 4DVAR scheme based on 6 hour time interval and an observation frequency of 2 hours. It appears that the 4DVAR scheme enables to reduce relative errors on predicted power from 10% to less than 2%.

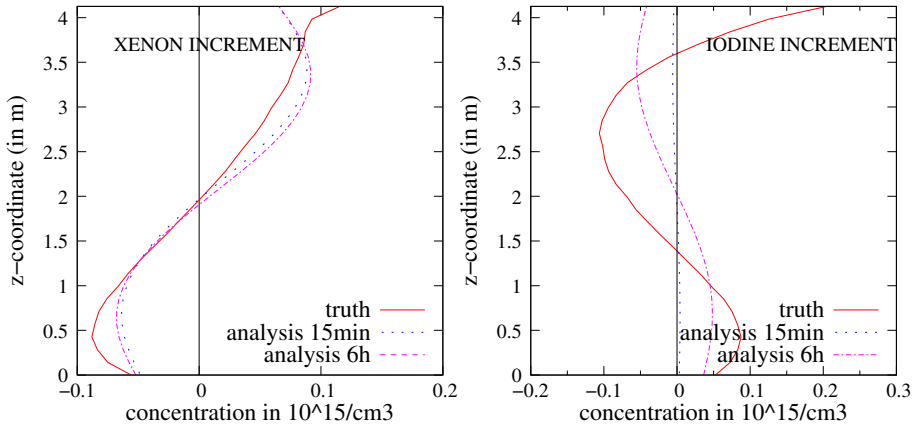

Figure 5. Dependency of the xenon and iodine analysis increments to the time interval in a 4DVAR scheme.

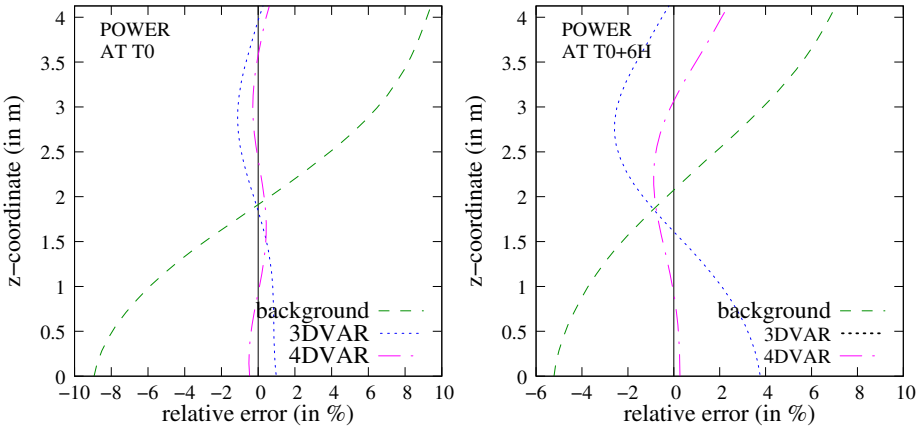

Figure 6. Comparison of 3DVAR and 4DVAR scheme performance on predicted axial power distributions.

In conclusion, the 3DVAR scheme is enough to obtain a good representation of core state at analysis time t_0: monodimensional power and xenon map are well adjusted. This scheme, cheaper in computation time than the 4DVAR scheme, is then interesting to produce a picture of reactor at a given time. But this 3DVAR scheme does not take into account the model dynamics which implies that iodine concentration estimation cannot be improved by this way. To improve the prediction, we must use the 4DVAR scheme. With this scheme, we show than a delay less than half a day (compared to the previous 36 hours) is required to fit the xenon dynamics and to obtain a better prediction of xenon effects.

6. CONCLUSIONS

In this paper, we have shown how variational data assimilation methods can be used to improve the accuracy of the prediction of xenon concentration in PWR. A monodimensional xenon dynamics code CIREP1D was developed for this purpose. In twin experiments, 3DVAR and 4DVAR schemes performed so well that their use in continuously adjusted online simulation tools can be considered. We showed that a 3DVAR scheme was sufficient to obtain a good representation of the core state at the analysis time. Thus, it could be useful for a re-analysis step for example. However, a 4DVAR scheme is to be prefered over the 3DVAR scheme for the reliability of the forecasts of xenon concentrations. Experiments showed that a 6 hour time interval with a time step of 2 hours was sufficient to obtain a reliable prediction after the unavailability of data.

These encouraging results can still be improved, especially through a better modelling of error covariance matrices. The background error covariance matrix $\mathbf{B}$ could be estimated by ensemble methods. These methods lead to an improvement in the covariance matrix estimation through iterations on the analysis. The use of model error should also improve the quality of the analysis: approximations are done in the physical modelling, more precisely in the cross section values and in the fuel irradiation map. However, the description of the model error is a difficult task.

REFERENCES

[1] R. Daley, *Atmospheric Data Analysis*, Cambridge University Press (1991).

[2] INRIA, "TAPENADE 2.1 user's guide", http://tapenade.inria.fr:8080/tapenade/index.jsp.

[3] G. Gaspari and S.E. Cohn "Construction of correlation functions in two and three dimensions", *Q. J. R. Meteorol. Soc. (UK)*, **125**, pp. 723-57 (1999).

[4] D. Liu and J. Nocedal, "On the limited memory BFGS method for large scale optimization," *Mathematical Programming, Series B*, **45**, pp. 503-528 (1989).

[5] T. Lagarde, A. Piacentini and O. Thual, "A new representation of data-assimilation methods: the PALM flow-charting approach," *Quarterly Journal of the Royal Meteorological Society*, **127**, pp. 189-207 (2001), www.cerfacs.fr/~palm/.

[6] R.J. Onega and R.A. Kisner, "Parameter identification in the xenon oscillation problem using the method of maximum likelihood," *Trans. Am. Nucl. Soc. (USA)*, **30**, pp. 745-747 (1978).

[7] A. Ponçot, "Assimilation de données pour la dynamique du xénon dans les cœurs de centrale nucléaire", Ph.D. thesis, Université de Toulouse, INPT, France (2008).

[8] J.S. Song and N.Z. Cho and H.L. Byung,"Analytic initialization of nonequilibrium iodine and xenon distributions for core transient simulation," *Nuclear Technology*, **116**, pp. 137-145 (1996).