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ON REACTIVITY EFFECTS CALCULATIONS

Abramov B.D.
Institute for Physics and Power Engineering
Russia, Kaluga region, Obninsk, Bondarenko Sq.1
abramov@ippe.ru

ABSTRACT

In the paper, the following methods of nuclear reactors (NR) calculations are considered:
1. methods of evaluation of neutron field perturbation caused by local perturbations of macroscopic cross sections of neutrons interaction with nuclei;
2. methods of evaluation of reactivity effects (RE) caused by the above cross sections perturbations.

Methods of analysis of neutron fields under consideration are based on synthesis of perturbation theory and albedo method. In these methods solution of perturbed problem is reduced to solution of non-perturbed problem in the entire volume of nuclear reactor and to solution of perturbed problem in selected NR subdomain containing perturbation, with albedo type boundary condition on its boundary, which is not known in advance. This boundary condition is then revised and later on, unknown perturbed neutron field in the subdomain under consideration is determined.

Considered methods of RE evaluation are in turn a summary of the above methods of solution of uniform conditionally critical problems of nuclear reactors theory. These methods are capable of evaluating RE on the basis of solutions of direct and adjoint unperturbed conditionally critical problems for the entire NR volume and solutions of sequence of non-uniform boundary problems in selected subdomain of perturbation localization.

Methods proposed in the paper are new and highly effective. These are capable of evaluating local perturbations without solving perturbed equations in the entire NR volume.

Key Words: nuclear reactors, local effects calculations

1. INTRODUCTION

Following [1-6], in the paper considered are methods of analysis of perturbations of neutron fields and accompanying reactivity effects caused by such locally strong, although globally weak, perturbations that result in significant changes of neutron flux in the area of perturbations localization but have slight effect on the general pattern of neutron flux distribution in the entire reactor. Such effects are usually hard to analyze using either small-perturbation theory (which often causes even errors in the effect sign), or exact perturbation theory requiring sometimes unreasonably high computer intensity for obtaining some reliable result.

Offered paper is devoted to the solution of these problems. In Section 1 of the paper, presented is mathematical statement of the problems under consideration, in Section 2, proposed methods of analysis of local perturbations of neutron fields are formulated and Section 3 is devoted to proposed methods of reactivity effects evaluation.

1.1. Neutron Transport Equation

Let us consider the problem of solving the neutron transport equation [1-6]

$$\Omega\nabla\psi + C\psi = Q \tag{1.1a}$$

in a bounded domain $G \in R^3$ with a boundary condition:

$$\psi(x,E,\Omega) = f(x,E,\Omega), \quad \Omega\breve{n}(x) < 0, \quad x \in \Gamma, \tag{1.1b}$$

on its piecewise smooth surface Γ of class $C^{(1)}$, where $\psi(x,E,\Omega)$ is the density of neutrons at $x \in R^3$ that possess the energy $E \in [\underline{E}, \overline{E}]$ and move in the direction $\Omega \in S^1$; S^1 is the unit sphere in R^3; $0 \le \underline{E} \le E \le \overline{E} = E_o < \infty$ is the interval of admissible neutron energies E; f, Q are known functions; $\breve{n}(x)$ is a unit outward normal to Γ; $\Omega\breve{n}(x)$ is the scalar product of vectors in R^3;

$$C = \Sigma - K, \quad K = K_s + K_f, \tag{1.2a}$$

$$K_b\psi = \int dE' \int d\Omega' \omega_b(x,E,E',\Omega,\Omega')\psi(x,E',\Omega'), \quad b = s,f, \tag{1.2b}$$

$$\omega_{b'}(x,E,E',\Omega,\Omega') = \sum_l (E'/E)^{1/2} \nu_{b'l}(E')\Sigma_{b'l}(x,E')W_{b'l}(E',E,\Omega',\Omega); \tag{1.2c}$$

$\nu_{b'l}(E)$ is the number of secondary neutrons formed in a single type b' reaction with a nucleus of a nuclide of type l; $W_{b'l}(E',E,\Omega',\Omega)$ is the probability density of their distribution over the energies E and the scattering directions Ω; $\Sigma_{b'l} = \widetilde{N}_l(x)\sigma_{b'l}(E')$ is the macroscopic cross-section of this reaction; $\widetilde{N}_l(x)$ is the density of nuclei of the l-th nuclide; moreover, one has

$$\int dE \int W_{b'l}(E',E,\Omega',\Omega)d\Omega = 1; \qquad \omega_s = \sum_{b' \ne c,f} \omega_{b'}; \qquad \Sigma(x,E) = \sum_{b'} \sum_l \Sigma_{b'l}(x,E).$$

The summation in the last formula is taken over all nuclides l and all processes b' of interaction between neutrons and nuclei: elastic scattering $(b'=e)$, inelastic scattering $(b'=i)$, fission $(b'=f)$, radiative capture $(b'=c)$, and so on [2,3]. With regard to the quantities $\nu_{b'l}, \sigma_{b'l}, W_{b'l}, \widetilde{N}_l$ it is assumed that they satisfy conditions 3.1 of [2], which reflect a certain complex of general laws governing the neutron-matter interaction in nuclear reactor theory.

1.2. Domain and Subdomains

Suppose that G is conditionally divided into subdomains G_n, $n = \overline{1,N}$. Following [1-3], the subdomain G_n (or omitting the index n, the domain G itself) is any open connected measurable subset $G_n \subseteq G$ with a piecewise-smooth boundary Γ_n of class $C^{(1)}$ such that almost all (with respect to the standard measure in four-space $S^1 \times R^2$) straight lines

$x_o + \Omega t,\ x_o \in \pi_{\Omega n}, t \in (-\infty, \infty)$ having a common point with G_n intersect G_n in a finite number N_n of intervals (t_{nk}^-, t_{nk}^+), where $\pi_{\Omega n}$ and x_o are the orthogonal projections of G_n and a vector $x \in G$ onto the plane perpendicular to a vector $\Omega \in S^1$, and where the dependence of N_n and $t_{nk}^{\pm}$ ($k = \overline{1, N_n},\ n = \overline{1, N}$) on Ω and x_o is not indicated. For convex subdomains, the index $k = 1$ of the only interval (t_{nk}^-, t_{nk}^+) is omitted. It is assumed that any set G_n contains a convex subset of a nonzero measure $\mu > 0$ in R^3.

1.3. Functional Spaces and Norms

Let $Y_n = G_n \times \left[\underline{E}, \overline{E}\right] \times S^1$, $\breve{n}_n(x)$ be the outward normal to Γ_n, and $\Gamma_n^{\pm} = \left\{(x, E, \Omega) \in \Gamma_n \times \left[\underline{E}, \overline{E}\right] \times S^1 : x = x_o + \Omega t_{nk}^{\pm},\ x_o \in \pi_{\Omega n},\ k = 1,2,..., \widetilde{N}_n \right\}$. Following [1-3], we introduce the Banach spaces $L_p(Y_n), L_p(\Gamma_n^{\pm})$ of real functions $\psi(x, E, \Omega)$, summable with the p-th power, $1 \le p \le \infty$, on the sets Y_n and $\Gamma_n^{\pm}$ with norms

$$\|\psi\|_{L_p(Y_n)} = \left\{ \int_{\underline{E}}^{\overline{E}} dE \int_{S^1} d\Omega \int_{\pi_{\Omega n}} dx_o \left(\sum_{k=1}^{N_n} \int_{t_{nk}^-}^{t_{nk}^+} dt \left| \psi(x_o + \Omega t, E, \Omega) \right|^p \right) \right\}^{1/p},$$

$$\|\psi\|_{L_p(\Gamma_n^{\pm})} = \left\{ \int_{\underline{E}}^{\overline{E}} dE \int_{S^1} d\Omega \int_{\pi_{\Omega n}} dx_o \left(\sum_{k=1}^{N_n} \left| \psi(x_o + \Omega t_{nk}^{\pm}, E, \Omega) \right|^p \right) \right\}^{1/p}, \quad 1 \le p < \infty,$$

$$\|\psi\|_{L_\infty(Y_n)} = \operatorname*{vrai\,max}_{(x,E,\Omega) \in Y_n} |\psi(x, E, \Omega)|, \quad \|\psi\|_{L_\infty(\Gamma_n^{\pm})} = \operatorname*{vrai\,max}_{(x,E,\Omega) \in \Gamma_n^{\pm}} |\psi(x, E, \Omega)|,$$

The generalized solutions of equation (1.1) inside of the subdomains G_n and on their borders Γ_n are searched in spaces $L_p(Y_n)$ and $L_p(\Gamma_n^{\pm})$ of the functions $\psi(x, E, \Omega)$ defined almost everywhere in $Y_n, \Gamma_n^{\pm}$.

1.4. Direct and Inverse Neutron Transport Operators

Following [1-3], we define the operators L_n, L_n^{-1} and H_n for almost all $(E, \Omega, x_o) \in \left[\underline{E}, \overline{E}\right] \times S^1 \times \pi_{\Omega n}$ on the intervals $[t_{nk}^-, t_{nk}^+]$ by the formulas

$$L_n \psi = \left[\frac{d}{dt} + \Sigma(x_o + \Omega t, E) \right] \psi(x_o + \Omega t, E, \Omega), \quad \psi \in D_{on}^p, \tag{1.3}$$

$$L_n^{-1}\Phi = \int_{t_{nk}^-}^{t} dt'\Phi(x_o + \Omega t', E, \Omega)\exp[-\int_{t'}^{t} dt''\Sigma(x_o + \Omega t'', E)], \quad \Phi \in L_p(Y_n), \qquad (1.4)$$

$$H_n(f) = f_n(x_o + \Omega t_{nk}^-, E, \Omega)\exp[-\int_{t_{nk}^-}^{t} dt'\Sigma(x_o + \Omega t', E)], \quad f_n \in L_p(\Gamma_n^-), \qquad (1.5)$$

where $D_{on}^p \subset L_p(Y_n)$ is a set of functions ψ of the form (1.4) (for every $\Phi \in L_p(Y_n)$ and any $0 < \sqrt{E}\Sigma(x,E) \in L_\infty(Y)$) and L_n^{-1} is the inverse of the linear unbounded closed operator L_n [1-3] generated by the linear differential expression $(\Omega\nabla + \Sigma)\psi$ on functions $\psi \in D_{on}^p$ according to formula (1.3); L_n^{-1} is an operator from $L_p(Y_n)$ into $D_{on}^p \subset L_p(Y_n)$ and $L_p(\Gamma_n^\pm)$ that is linear and continuous for all $1 \le p \le \infty$.

1.5. Some Extensions

For almost all $(E,\Omega) \in \left[\underline{E}, \overline{E}\right] \times S^1$ the functions $\psi \in D_{on}^p$ have the first generalized derivative in the direction of Ω, which coincides with $\Omega\nabla\psi$ and belongs $L_p(Y_n)$ [1]. Moreover, for almost all $(E,\Omega,x_o) \in \left[\underline{E}, \overline{E}\right] \times S^1 \times \pi_{\Omega n}$ the functions $\psi \in D_{on}^p$ posses, as $t \to t_{nk}^\pm$ ($t \in (t_{nk}^-, t_{nk}^+)$), the limit values $\psi_{nk}^\pm = \psi(x_o + \Omega t_{nk}^\pm, E, \Omega)$ (traces) on $\Gamma_n^\pm$, which form a multifunction $\psi_n^\pm = (\psi_{n1}^\pm, \psi_{n2}^\pm, ..., \psi_{nN_n}^\pm) \in L_p(\Gamma_n^\pm)$ of boundary values. It is obvious that $\psi_n^- = 0$. A wider class of function $D_{fn}^p \supseteq D_{on}^p$ with a given (not necessarily zero) trace $\psi_n^- = f_n \in L_p(\Gamma_n^-)$ yields the representation $\psi = L_n^{-1}\Phi + H_n(f)$, where $\Phi \in L_p(Y_n)$, $f_n = (H_n(f))_n^- \in L_p(\Gamma_n^-)$, H_n .is linear and continuous (for $1 \le p \le \infty$) operator from $L_p(\Gamma_n^-)$ into $D_{fn}^p \subset L_p(Y_n)$ and $L_p(\Gamma_n^\pm)$ defined by (1.5). Moreover, $(\Omega\nabla + \Sigma)H_n(f) = 0$.

The set D_{fn}^p is not linear if $f_n \ne 0$. We obtain a linear set $D_n^p \supseteq D_{fn}^p$ of functions with an arbitrary trace $\psi_n^- \in L_p(\Gamma_n^-)$ by considering D_{fn}^p for arbitrary (not necessary fixed) $f_n \in L_p(\Gamma_n^-)$. The general form of functions from D_n^p is given by the formula (see [3,6]) $\psi = L_n^{-1}\widetilde{L}_n\psi + H_n(\psi)$, $\psi \in D_n^p$, where $\widetilde{L}_n$ is the extension of L_n to D_n^p.

1.6. Boundary Value Problems in Subdomains

Since the operator K_n defined by the linear integral expression (1.2) on $\psi \in L_p(Y_n)$ is linear and continuous in $L_p(Y_n)$ with $p \in [1,\infty]$, it is possible to state a rather general boundary value problem for Eq. (1.1) in the subdomain G_n in the form [3,6]:

$$\widetilde{L}_n\psi = K_n\psi + Q_n, \quad Q_n \in L_p(Y_n), \ \psi \in D_n^p, \tag{1.6a}$$

or, equivalently, in the form

$$\psi = L_n^{-1}K_n\psi + L_n^{-1}Q_n + H_n(\psi), \quad Q_n \in L_p(Y_n), \ \psi_n^- \in L_p(\Gamma_n^-). \tag{1.6b}$$

1.7. The Basis Theorem

Returning to the problem (1.1), we denote by $Y, \Gamma_\pm, L_p(Y), L_p(\Gamma_\pm), D_o^p, D_f^p$, $D^p, L, L^{-1}, \widetilde{L}, H, K, \|.\|_p = \|.\|_{L_p(Y)}, \|.\|_p^\pm = \|.\|_{L_p(\Gamma_\pm)}$ the corresponding sets, classes of functions, operators and norms for $G_n = G$. Assuming that $Q \in L_p(Y), f \in L_p(\Gamma_-)$ for some $p \in [1,\infty]$, this problem can be formulated, by analogy with (1.6), as an equation

$$\widetilde{L}\psi = K\psi + Q, \quad \psi \in D_f^p, \tag{1.7a}$$

or, equivalently, as

$$\psi = L^{-1}K\psi + L^{-1}Q + H(f), \quad \psi \in L_p(Y). \tag{1.7b}$$

The following statements are true [1-3,6].

Theorem 1.1 The operator $L^{-1}K$ is linear and continuous in $L_p(Y)$, compact for $1 < p < \infty$ (and its square is compact for $1 \le p \le \infty$), Riesz-Schauder's theory is valid for the equation $\lambda\psi = L^{-1}K\psi$; there exists a unique (normalized) eigenelement ψ_o, $\theta < \psi_o \in L_p(Y)$, $\lambda_o\psi_o = L^{-1}K\psi_o$, its corresponding eigenvalue $\lambda_o > 0$ is simple, and it exceeds in modulus the other eigenvalues. Finally, $\lambda_o = r(L^{-1}K)$, where $r(L^{-1}K)$ is the spectral radius of the operator $L^{-1}K$. Under the condition

$$r(L^{-1}K) < 1 \tag{1.8}$$

there exists a unique solution to Eq. (1.7), and that solution can be found by the method

$$\psi^{(m+1)} = L^{-1}K\psi^{(m)} + L^{-1}Q + H(f), \quad \psi^{(m)} \in L_p(Y), \quad m = 0,1,,\ldots., \tag{1.9a}$$

which converges in $L_p(Y)$ $\forall \psi^{(0)} \in L_p(Y)$ at the rate of a geometric progression with a multiplier $r(L^{-1}K)$; the solution can also be found by the equivalent method

$$(\Omega\nabla + \Sigma)\psi^{(m+1)} = K\psi^{(m)} + Q, \quad \psi^{(m+1)} \in D_f^p, \quad m = 0,1,.... \tag{1.9b}$$

1.8. The Albedo Formulation of Boundary Value problems

Let P_n denote the projection operators onto the subdomains, with the properties

$$P_m P_n = P_n \delta_{mn}; \quad \|P_n\|_p = 1; \quad P_n\psi = \begin{Bmatrix} \psi(x,E,\Omega), x \in G_n, \\ 0, \quad x \notin G_n \end{Bmatrix}.$$

Let $\psi_n = P_n\psi,\ \psi_l = P_l\psi$ for a given $\psi \in D_f^p$. Since functions in D_f^p are absolutely continuous along almost all neutron propagation paths (i.e. absolutely continuous with respect to $t \in [t_k^-, t_k^+]$ for almost all $E,\Omega,x_o \in \left[\underline{E},\overline{E}\right] \times S^1 \times \pi_\Omega$), it follows that for almost all paths

$$\psi_n^- = \psi_l^+ \ on \ \ \Gamma_n^- \cap \Gamma_l^+, \quad \psi_n^- = f \ \ on \ \ \Gamma_n^- \cap \Gamma_-, \quad n \neq l, \ n,l = \overline{1,N}, \tag{1.10}$$

provided, of course, that the sets $\Gamma_n^-, \Gamma_l^+, \Gamma_-$ in question have a non-empty intersection.
We shall call (1.10) the sewing conditions [3,6].

The solution of problem (1.1) in D_f^p may be reduced to the solution of the equations (1.6a) with conditions (1.10). Because

$$\psi_n = (1 - L_n^{-1}K_n)^{-1} H_n(\psi_n) + (1 - L_n^{-1}K_n)^{-1} L_n^{-1} Q_n, \tag{1.11}$$

so we need only determine the values $\psi_n^{\pm}$ of the functions ψ_n at the subdomain interfaces. Introducing the albedo operators

$$R_n = P_n^+ (1 - L_n^{-1}K_n)^{-1} H_n(\cdot), \qquad S_n = P_n^+ (1 - L_n^{-1}K_n)^{-1} L_n^{-1}, \tag{1.12}$$

we arrive at the so-called albedo formulation of problem (1.1), as a set of equations

$$\psi_n^+ = R_n\psi_n^- + S_n Q_n, \quad \psi_n^{\pm} \in L_p(\Gamma_n^{\pm}), \ n = \overline{1,N} \tag{1.13}$$

with conditions (1.10) [3,6]. In this formulation, the solution of problem (1.1) reduces to constructing the albedo operators R_n, S_n of the subdomains, the determination of the neutron densities $\psi_n^{\pm}$ at the subdomain interfaces, and subsequently the determination (if necessary) of the values of ψ inside the subdomains by formula (1.11).

2. THE ALBEDO PERTURBATIONS THEORY METHODS FOR INHOMOGENEOUS PROBLEMS

2.1. Perturbed Inhomogeneous Problem Formulation

Let us consider some special methods of the perturbation theory based on different techniques of data transfer from an unperturbed problem (1.1) to a perturbed one

$$\Omega\nabla\widetilde{\psi} + \widetilde{C}\widetilde{\psi} = \widetilde{Q}, \qquad \widetilde{\psi} = \widetilde{f} \ \ on \ \ \Gamma_-, \tag{2.1}$$

where $\widetilde{\psi} = \psi + \Delta\psi, \widetilde{C} = C + \Delta C, \widetilde{Q} = Q + \Delta Q, \widetilde{f} = f + \Delta f$; $\Delta C, \Delta Q, \Delta f$ are the perturbations of C , Q , f ; and $\Delta\psi$ is the perturbation of ψ which satisfies the following equation

$$(\Omega\nabla + \widetilde{C})\Delta\psi \ = \Delta Q - \Delta C\psi, \quad \Delta\psi = \Delta f \ on \ \ \Gamma_- \tag{2.2}$$

It is supposed that the statement of the Theorem 1.1 extends to the perturbed problem (2.1) with the necessary changes in notation.

2.2. The DTV Method

Let the following conditions be in effect: $\Delta f = 0$, $G = G_1 \cup G_2 \cup \Gamma_1 \cap \Gamma_2$, $\Gamma_1 \cap \Gamma = 0$ and let $\Delta C, \Delta Q = \Delta Q_1$ be located in the subdomain G_1 . Then, if the solution ψ of the unperturbed problem (1.1) is known, the solution $\Delta\psi_1$ of the perturbed problem (2.2) in G_1 can be found by the following scheme (data transfer through a volume, the DTV method [5,6]):

$$(\Omega\nabla + \widetilde{C})\Delta\psi_1 \ = \Delta Q_1 - \Delta C\psi_1, \quad \Delta\psi_1^- = \ R_2\,\Delta\psi_1^+, \tag{2.3}$$

with R_2 being the albedo operator for the subdomain G_2 , and $\Delta\psi_1^{\pm}$ the values (traces) of the $\Delta\psi$ function on $\Gamma_1^{\pm}$. In this method the initial sources Q and f are excluded from consideration and all necessary information, which is contained in them, is included into the volume source $\Delta C\psi$ distributed over the subdomain G_1 . From previous results it follows

Theorem 2.1. Let $\widetilde{L}, \widetilde{K}$ be the operators of the perturbed problem (2.1) and $r(\widetilde{L}^{-1}\widetilde{K}) < 1$, $r(L^{-1}K) < 1$. Then a solution of problem (2.3) in $L_p(Y_1)$ exists, it is unique and can be found by successive approximations

$$(\Omega\nabla + \widetilde{C})\Delta\psi_1^{(m+1)} = \Delta Q_1 - \Delta C\psi_1, \quad \Delta\psi_1^{(m+1)-} = R_2\Delta\psi_1^{(m)+}, \quad m = 0,1,...,$$

which converge in $L_p(Y_1)$ and $L_p(\Gamma_1^{\pm})$ for any $\Delta\psi_1^{(0)+} \in L_p(\Gamma_1^+)$.

2.3. The DTB Method

Let us consider method of data transfer through boundaries (the DTB method [5,6]):

$$(\Omega\nabla + \widetilde{C})\widetilde{\psi}_1 = \widetilde{Q}_1, \qquad \widetilde{\psi}_1^- = \psi_1^- + R_2(\widetilde{\psi}_1^+ - \psi_1^+), \tag{2.4}$$

with $\psi_1^{\pm}$ being the value of unperturbed function ψ on $\Gamma_1^{\pm}$. For DTB method, the following theorem is valid:

Theorem 2.2. Let the conditions of Theorem 2.1 be satisfied. Then a solution of problem (2.4) in $L_p(Y_1)$ exists, it is unique and can be found by successive approximations

$$(\Omega\nabla + \widetilde{C})\widetilde{\psi}_1^{(m+1)} = \widetilde{Q}_1, \quad \widetilde{\psi}_1^{(m+1)-} = \psi_1^- + R_2(\widetilde{\psi}_1^{(m)+} - \psi_1^+), \quad m = 0,1,...,$$

which converge in $L_p(Y_1)$ and $L_p(\Gamma_1^{\pm})$ for any $\widetilde{\psi}_1^{(0)+} \in L_p(\Gamma_1^+)$.

2.4. Some Remarks

It should be noted that DTV and DTB methods are capable of solving disturbed problem inside perturbation localization domain G_1 without resorting to solution of perturbed problem in the entire G domain, this in some cases being quite essential. The quality of numerical approaches to the implementation of these methods depends on the choice of sub-domain G_1 and approximation $\widetilde{R}_2$ of operator R_2. As G_1 dimensions increase (with the increase of the distance of its boundary Γ_1 from perturbation), requirements on $\widetilde{R}_2$ usually become so low that in some cases even the simplest approximation $\widetilde{R}_2 = 0$ is acceptable [4,5]. In the latter case, proposed methods are converted into some versions of those presented in [4].

3. THE ALBEDO PERTURBATIONS THEORY METHODS FOR REACTIVITY EFFECTS CALCULATIONS

3.1. Homogeneous Conditional Critical Equations

Let us consider homogeneous conditional critical problems. Assuming that

$$K = S + F, \quad S = K_s, \quad F = K_f, \tag{3.1}$$

conditionally critical problem of second kind can be formulated [2] on finding solution $\theta < \psi \in D_o^p$ and related eigenvalue $k_{eff} > 0$ in the following equation:

$$M\psi = F\psi / k_{eff}, \quad M = L - S, \tag{3.2}$$

which is equivalent (on condition $r(L^{-1}S) < 1$) to any one of the following equations:

$$k_{eff}\psi = (1 - L^{-1}S)^{-1} L^{-1} F\psi, \quad k_{eff}\varphi = (1 - SL^{-1})^{-1} FL^{-1}\varphi, \quad \varphi = L\psi. \tag{3.3}$$

It should be noted that reactions $(n,2n), (n,3n),...$ are included in the definition of operator S and, hence, correctness of condition $r(L^{-1}S) < 1$ is no more apparent. However it can always be considered met for existing reactor designs.

Also it should be noted that the problem (3.2) is meaningful, if only $F \neq 0$, i.e. if the measure $\mu(G_f)$ of multitude $G_f = \{x \in G : \sum_l \Sigma_{fl}(x,E) > 0\}$ is different from zero.

Taking into account the above considerations the following propositions are correct.

Theorem 3.1. *Let us assume that* $r(L^{-1}S) < 1$ *and* $\mu(G_f) > 0$. *Then* $A = (1 - SL^{-1})^{-1} FL^{-1}$ - *is linear, compact in* $L_p(Y)$ *with* $1 < p < \infty$ *(and its squared number – with all* $1 \le p \le \infty$*)* u_o - *positive operator, and appropriate statements of Theorem 1.1 concerning qualitative characteristics of solutions of equation* $\lambda\psi = L^{-1}K\psi$, *with* $k_{eff} < \lambda_o$ *extend to equations (3.2) and (3.3). Unknown* k_{eff}, ψ *can be determined by method of fission source iterations (FSI):*

$$M\psi^{(l+1)} = F\psi^{(l)}, \quad l = 0,1,...; \quad k_{eff} = \lim_{l\to\infty} \frac{(g, F\psi^{(l)})}{(g, F\psi^{(l-1)})}, \tag{3.4}$$

converged into $L_p(Y)$ *with any* $\theta < \psi^{(0)} \in L_p(Y)$ *and such* $\theta < g \in L_\infty(Y)$ *that* $(g, F\psi^{(0)}) > 0$. *Here* $\psi^{(l)}(\psi^*,\psi)/k_{eff}^l \to (\psi^*,\psi^{(0)})\psi$ *with* $l \to \infty$, *where* ψ^* - *positive solution of the following equation:*

$$M^*\psi^* = F^*\psi^* / k_{eff}, \quad M^* = L^* - S^*, \tag{3.5}$$

conjugate to equation (3.2), (,) – symbol of integration over $x, E, \Omega \in Y$.

3.2. Some Reformulations of the Fission Source Iterations (FSI) Method

Let us turn to methods of evaluation of reactivity effects $\Delta\rho = \widetilde{\rho} - \rho$, where $\rho = 1 - 1/k_{eff}$ - reactivity of unperturbed reactor state described by equation (3.2), and $\widetilde{\rho} = 1 - 1/\widetilde{k}_{eff}$ - reactivity of perturbed reactor state described by the following equation:

$$(\Omega\nabla + \widetilde{C})\widetilde{\psi} = \widetilde{F}\widetilde{\psi} / \widetilde{k}_{eff}, \quad \widetilde{\psi} \in D_o^p \tag{3.6}$$

with perturbed operators $\widetilde{C}, \widetilde{F}$, similar to operators C, F.

It is often recommended to make $\Delta\rho$ estimates by the following relationship:

$$\Delta\rho = (\psi^*, (\Delta F / k_{eff} - \Delta C)\widetilde{\psi}) / (\psi^*, \widetilde{F}\widetilde{\psi}) \tag{3.7}$$

of (even) perturbation theory on the basis of known $\Delta F, \Delta C$ and evaluated $k_{eff}, \psi^*, \widetilde{\psi}$, but not by the relationship $\Delta\rho = (k_{eff})^{-1} - (\widetilde{k}_{eff})^{-1}$ on the basis of preliminarily determined values of $k_{eff}, \widetilde{k}_{eff}$. It is assumed that this method is better, since it does not lead to the problem of accurate calculation of the difference of two values close to one: $1/k_{eff}, 1/\widetilde{k}_{eff}$, which is difficult to realize in practice (because of approximation errors).

On the other hand, the problem of optimum organization of $\Delta\rho$ calculation on the basis of equations (3.6) and (3.7) obviously remains undetermined. Below proposed is iteration method for its solution:

$$\Delta\rho^{(l+1)} = \left(\frac{1}{k_{eff}} - \frac{1}{k_{eff}^{(l+1)}}\right) = \frac{(\psi^*, (\Delta F / k_{eff} - \Delta C)\psi^{(l+1)})}{(\psi^*, \widetilde{F}\psi^{(l+1)})}, \tag{3.8a}$$

$$(\Omega\nabla + \widetilde{C})\psi^{(l+1)} = \widetilde{F}\psi^{(l)} / k_{eff}^{(l)}, \quad \psi^{(l+1)} \in D_o^p, \tag{3.8b}$$

where $l = 0,1,...$ and, for instance, $k_{eff}^{(0)} = k_{eff}$, $\psi^{(0)} = \psi$.

Proposed method is optimal, because calculation of $\psi^{(l+1)}$ and $k_{eff}^{(l+1)}$ (and, hence, $\Delta\rho^{(l+1)}$) is made in coordination, namely: $k_{eff}^{(l+1)}$ is determined from Eq. (3.8a) on the basis of specified $\psi^{(l+1)}$ value, and then $\psi^{(l+2)}$ is calculated from Eq. (3.8b), and so on.

It follows from Theorem 3.1 and recursion relationship:

$$k_{eff}^{(l+1)} = k_{eff}^{(l)} \frac{(\psi^*, \widetilde{F}\psi^{(l+1)})}{(\psi^*, \widetilde{F}\psi^{(l)})} = \frac{(g, \widetilde{F}D^{l+1}\psi^{(0)})}{(g, \widetilde{F}D^{l}\psi^{(0)})}, \quad l = 0,1,..., \tag{3.9}$$

arising from Eq. (3.8) with $g = \psi^*$, $D = (\widetilde{L} - \widetilde{S})^{-1}\widetilde{F}$:

Theorem 3.2. *Iterations (3.8) converge into* $L_p(Y)$*: with* $l \to \infty$

$$k_{eff}^{(l)} \to \widetilde{k}_{eff}, \quad \psi^{(l)}(\widetilde{\psi}^*, \widetilde{\psi}\ ) \to (\widetilde{\psi}^*, \psi)\widetilde{\psi}\ , \tag{3.10}$$

where $\widetilde{\psi}^*$ *- positive solution of corresponding perturbed problem.*

It should be noted that effectiveness of method (3.8) is determined in many instances by the extent of similarity of unperturbed and perturbed problems, i.e., in particular, by the correctness of choice of $\psi^{(0)} = \psi$ (since with $\psi = \widetilde{\psi}$ method (3.8) converges in one iteration). In the applied problems of nuclear reactors theory it is usually sufficient to make two or three iterations of this method.

3.3. The Albedo Formulation of the FSI Method for Local Reactivity Effects Calculations

Let us consider albedo definitions of perturbation theory methods for evaluation of reactivity effects caused by local perturbations in G_1 subdomain of the reactor. Assuming that $\widetilde{F} \neq 0$ in G_1 and changing from Eq. (3.6) in the whole domain G to the following equivalent relationship:

$$(\Omega\nabla + \widetilde{C}\ )\widetilde{\psi}_1\ = \widetilde{F}\ \widetilde{\psi}_1\ / \widetilde{k}_{eff}, \qquad \widetilde{\psi}_1^- = \widetilde{R}_2\ \widetilde{\psi}_1^+ \tag{3.11}$$

in subdomain: G_1, where $\widetilde{R}_2\ = P_2^+[1 - L_2^{-1}(S_2 + F_2\ / \widetilde{k}_{eff}\ )]^{-1} H_2(\cdot)$ - albedo operator of domain $G_2 = G \setminus G_1 \cap \Gamma_1$ depending on $\widetilde{k}_{eff}$, evaluation of $\Delta\rho$ is decreased to solution of a sequence of the boundary problems:

$$(\Omega\nabla + \widetilde{C})\psi_1^{(l+1)} = \widetilde{F}\psi_1^{(l)} / k_{eff}^{(l)}, \qquad \psi_1^{(l+1)-} = R_2^{(l)}\,\psi_1^{(l)+} \tag{3.12a}$$

in subdomain G_1 with $\Delta\rho^{(l+1)}$ evaluated by (3.8a) type relationships:

$$\Delta\rho^{(l+1)} = \frac{(\psi^*,(\Delta F / k_{eff} - \Delta C)\psi^{(l+1)})_1 - (1,\Omega\nabla(\psi^*\psi^{(l+1)}))_1}{(\psi^*,\widetilde{F}\psi^{(l+1)})_1} \tag{3.12b}$$

equivalent to recursion equations of (3.9) type:

$$k_{eff}^{(l+1)} = k_{eff}^{(l)}(\psi^*,\widetilde{F}\psi_1^{(l+1)})_1 /(\psi^*,\widetilde{F}\psi_1^{(l)})_1, \tag{3.12c}$$

where $R_2^{(l)}$ - operator $\widetilde{R}_2$ making change $\widetilde{k}_{eff} \to k_{eff}^{(l)}$, $(,)_n$ - symbol of integration over Y_n, $k_{eff}^{(0)} = k_{eff}$, $\psi^{(0)} = \psi$, $l = 0,1,...$, $(1,\Omega\nabla\varphi) = (\int_{\Gamma_1^+} - \int_{\Gamma_1^-})|\Omega n_1|\varphi\, d\gamma dE d\Omega$.

The following theorem is correct:

Theorem 3.3. *Let there be given* $\mu(\widetilde{G}_{1f}) > 0$, *where* $\widetilde{G}_{1f} = \{x \in G_1 : \sum_l \widetilde{\Sigma}_{fl}(x,E) > 0\}$. *Then iterations (3.12) converge into* $L_p(Y_1)$, *and the following relationships exist:*

$$k_{eff}^{(l)} \to \widetilde{k}_{eff} \text{ and } \psi_1^{(l)}(\widetilde{\psi}^*,\widetilde{\psi})_1 \to (\widetilde{\psi}^*,\psi)_1\widetilde{\psi}_1. \tag{3.13}$$

3.4. Some Approximations

Method (3.12) makes it possible to find solution of perturbed problem in subdomain G_1 of perturbation localization without solving perturbed problem in the entire domain G, this being quite important in some cases.

Quality of schemes of numerical realization of this method depends on the choice of approximation $\widetilde{R}_2$ of albedo operator R_2 and dimensions of subdomain G_1. As the latter increases, the requirements for $\widetilde{R}_2$ usually decrease.

It should be noted that in addition to (3.12) scheme, other options of albedo perturbation theory schemes are possible that can be used for evaluation of reactivity effects caused by globally weak but locally strong perturbations. In these schemes, it is expedient to replace operator $R_2^{(l)}$ by $\widetilde{R}_2 \approx R_2^{(0)}$ determined from relationship $\psi_1^- = \widetilde{R}_2\,\psi_1^+$ and replace equation (3.12b) by the following relationship:

$$\Delta\rho^{(l)} \approx [(\psi^*,(\Delta F / k_{eff} - \Delta C)\psi^{(l)})_1]/[(\psi^*,\widetilde{F}\psi^{(l)})_1 + (\psi^*,F\psi)_2] \tag{3.14}$$

arising from (3.8a) in the approximation $(\psi^*, F\Delta\psi)_2 \approx 0, (\psi^*, \widetilde{F}\widetilde{\psi})_1 / (\psi^*, F\psi)_2 << 1$ of "global weakness" of perturbation, this significantly simplifying realization of these schemes owing to additional removal of limitation $\widetilde{F} \neq 0$ in G_1.

3.5. Comparison with Low Perturbation Theory. Illustrative Example

It should be noted also that in the employments, there may become important even the first approximation $\Delta\rho^{(1)}$ of such schemes requiring no albedo operators calculation at all (because $\psi_1^{(1)-} = R_2^{(0)}\psi_1^{(0)+} = \psi_1^-$) and making it possible in some cases to significantly correct forecasting of $\Delta\breve{\rho}$ reactivity effect using low perturbation theory:

$$\Delta\breve{\rho} = (\psi^*, (\Delta F / k_{eff} - \Delta C)\psi) / (\psi^*, F\psi), \tag{3.15}$$

this arising from (3.14) and (3.15), as well as from the fact that the functions $\psi_1^{(1)}, \widetilde{F}\psi_1^{(1)}$ usually give better approximation of perturbed functions $\widetilde{\psi}, \widetilde{F}\widetilde{\psi}$ than $\psi, F\psi$ functions.

These propositions can be illustrated by the example of evaluation of reactivity effect caused by replacement of absorber layer $(-h, h)$ in central area of symmetrical multiplication slab $(-H, H)$ by vacuum. Considering only one-velocity problem in plane geometry, solutions of such problems in the layer $z \in (-h, h)$ can be presented as follows:

$$\psi(z,\mu) = \theta(\mu)\psi(-h,\mu)e^{-\frac{z+h}{\mu}\Sigma} + \theta(-\mu)\psi(h,\mu)e^{\frac{h-z}{\mu}\Sigma},$$
$$\psi^*(z,\mu) = \theta(\mu)\psi^*(h,\mu)e^{-\frac{h-z}{\mu}\Sigma} + \theta(-\mu)\psi^*(-h,\mu)e^{\frac{z+h}{\mu}\Sigma},$$

$$\widetilde{\psi}(z,\mu) = \theta(\mu)\widetilde{\psi}(-h,\mu) + \theta(-\mu)\widetilde{\psi}(h,\mu),$$
$$\psi^{(1)}(z,\mu) = \theta(\mu)\psi(-h,\mu) + \theta(-\mu)\psi(h,\mu), \tag{2.16}$$

where θ - Heaviside unit step. As follows from (2.7),(2.14)-(2.16):

$$\Delta\rho = 2\int_0^1 d\mu\mu\widetilde{\xi}(\mu)(1-e^{-2\Sigma h/\mu}), \quad \Delta\breve{\rho} = 4\Sigma h\int_0^1 d\mu\xi(\mu)e^{-2\Sigma h/\mu},$$

$$\Delta\rho^{(1)} = 2\int_0^1 d\mu\mu\xi(\mu)(1-e^{-2\Sigma h/\mu}), \quad \frac{\Delta\rho}{\Delta\breve{\rho}} \underset{\Sigma\to\infty}{\to} \infty, \quad \frac{\Delta\rho^{(1)}}{\Delta\breve{\rho}} \underset{\Sigma\to\infty}{\to} \infty,$$

$$\Delta\rho / \Delta\rho^{(1)} \underset{\Sigma\to\infty}{\to} \int_0^1 d\mu\mu\widetilde{\xi}(\mu) / \int_0^1 d\mu\mu\xi(\mu) < \infty,$$

where $\widetilde{\xi}(\mu) = \psi^*(h,\mu)\widetilde{\psi}(h,-\mu)/(\psi^*, F\widetilde{\psi})_2$, $\xi(\mu) = \psi^*(h,\mu)\psi(h,-\mu)/(\psi^*, F\psi)_2$ and as G_1 the layer $(-h,h)$ is chosen directly: $G_1 = \{z \in (-h_1, h_1), h_1 = h\}$.

So, in case of absorber layer replacement with vacuum, according to low perturbation theory, reactivity effect error increases with no limit for $\Sigma \to \infty$, while in the first approximation of proposed method this remains finite value and, moreover, it tends to zero with the increase of size h_1 of domain $G_1 = \{z \in (-h_1, h_1), h < h_1 < H\}$: $\Delta\rho / \Delta\rho^{(1)} \to 1$ for $h_1 \to H$.

This example shows comparatively high accuracy reached already in the first approximation in the proposed method even as applied to some up-scale problems, for which the error of traditional low perturbation theory can be unrestrictedly high.

4. CONCLUSIONS

In the paper, there are statement and solution of the problem of development and justification of methods of analysis of the local neutron fields perturbations and accompanying reactivity effects without resorting to solution of perturbed problem in the entire reactor. Mathematical statements of such problems are considered, theorems of existence and uniqueness of solutions of the equations under study are formulated and methods of approximate solutions of these equations are theoretically justified.

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