International Conference on Mathematics, Computational Methods & Reactor Physics (M&C 2009)
Saratoga Springs, New York, May 3-7, 2009, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2009)

Monte Carlo Anchoring Method for Loosely-Coupled k-Eigenvalue Problems

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ABSTRACT

This paper describes a new Monte Carlo anchoring method which provides more accurate fission source distribution using a small number of histories per generation. In this Monte Carlo anchoring method, the fission source term is decomposed into a conventional fission distribution and an "anchoring" distribution. The anchoring distribution is updated by a deterministic method and it plays a role of stabilizing the fission source distribution. By introducing anchoring factor α, the biasing effect owing to a potentially inaccurate deterministic solution is minimized. Numerical tests of the new method are performed on loosely-coupled two-group rod problems, providing encouraging results.

Key Words: Monte Carlo, eigenvalue, loosely-coupled problem, fission source distribution, anchoring

1. INTRODUCTION

Although advantages in describing continuous energy and complex geometry, the Monte Carlo method is still difficult to apply to realistic eigenvalue problems due to its huge computational burden. One of the difficulties is originated by its slow convergence of fission source distribution, especially in loosely-coupled fissile systems.

The speedup of the Monte Carlo method by providing accurate fission source distribution, generated by deterministic acceleration method, was proposed by the authors [1]. However, the biasing effect on the eigenvalue was reported in cases of using a small number of histories per generation. Similar disadvantage was also observed in loosely-coupled fissile array problems [2, 3].

In this paper, we propose a new Monte Carlo method that will provide more accurate fission source distribution even using a small number of histories per generation. A portion of the fission source distribution is provided by a deterministic method, thus stabilizing (or anchoring) the resulting fission source distribution. The updated fission source distribution also enables better estimation in eigenvalue with significantly reduced bias.

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2. MONTE CARLO ANCHORING METHOD

We start with the following steady-state neutron transport equation in operator form:

$$H\psi = S\psi + \frac{1}{k_{eff}} F\psi, \tag{1}$$

where

$$H\psi = \vec{\Omega}\cdot\nabla\psi\left(\vec{r},\vec{\Omega},E\right) + \sigma_t\left(\vec{r},E\right)\psi\left(\vec{r},\vec{\Omega},E\right), \tag{2}$$

$$S\psi = \int d\vec{\Omega}'\int dE'\sigma_s\left(\vec{r},E'\to E,\vec{\Omega}'\cdot\vec{\Omega}\right)\psi\left(\vec{r},\vec{\Omega}',E'\right), \tag{3}$$

$$F\psi = \frac{\chi(E)}{4\pi}\int d\vec{\Omega}'\int dE'\nu\sigma_f\left(\vec{r},E'\right)\psi\left(\vec{r},\vec{\Omega}',E'\right). \tag{4}$$

In a conventional method for the Monte Carlo eigenvalue problem, Eq. (1) is solved with power iteration index l as:

$$H\psi^{l+1} = S\psi^{l+1} + \frac{1}{k_{eff}^{l}} F\psi^{l}. \tag{5}$$

In the Monte Carlo anchoring method proposed in this paper, the fission source term in Eq. (5) is decomposed into a conventional fission source distribution and an anchoring distribution $Q^L\left(\vec{r}\right)$:

$$\frac{1}{k_{eff}^{l}} F\psi^{l} \simeq \left(1-\alpha\right)\frac{1}{k_{eff}^{l}} F\psi^{l} + \alpha \mathrm{Q}^L, \text{ for } l\in L, \tag{6}$$

where

$$\mathrm{Q}^L = \frac{\chi(E)}{4\pi} Q_{\det}^L\left(\vec{r}\right), \tag{7}$$

and L is the iteration index for the anchoring distribution, and α an appropriate anchoring factor. The anchoring distribution $Q_{\det}^L\left(\vec{r}\right)$ is updated by a deterministic method at each L-th generation. In this paper, the partial current-based Coarse Mesh Finite Difference (p-CMFD) method [1, 4] is used as the deterministic method, where coefficients are formulated based on Monte Carlo track-length tally results for $l\in L$.

For L-th generation, the update of the anchoring distribution starts with the following definitions:

$$\phi_{m,g}^{L-1/2} \equiv \frac{1}{N_l}\sum_{l\in L-1}\int_{\vec{r}\in V_m} dV \int_{E\in E_g} dE \int d\vec{\Omega}\,\psi^l\left(\vec{r},\vec{\Omega}',E\right), \tag{8}$$

$$J_{M+1/2,g}^{+,L-1/2}\left(\vec{r}\right) \equiv \frac{1}{N_l}\sum_{l\in L-1}\int_{\vec{r}=M+1/2} d\Gamma \int_{E\in E_g} dE \int_{\vec{\Omega}\cdot\vec{n}>0} d\vec{\Omega}\left|\vec{\Omega}\cdot\vec{n}\right|\psi^l\left(\vec{r},\vec{\Omega}',E\right), \tag{9}$$

$$J_{M+1/2,g}^{-,L-1/2}\left(\vec{r}\right) \equiv \frac{1}{N_l}\sum_{l\in L-1}\int_{\vec{r}=m+1/2} d\Gamma \int_{E\in E_g} dE \int_{\vec{\Omega}\cdot\vec{n}<0} d\vec{\Omega}\left|\vec{\Omega}\cdot\vec{n}\right|\psi^l\left(\vec{r},\vec{\Omega}',E\right), \tag{10}$$

$$\sigma_{x,M,g} = \frac{\sum_{m\in M}\frac{1}{N_l}\sum_{l\in L-1}\int_{\vec{r}\in V_m} dV \int_{E\in E_g} dE \int d\vec{\Omega}\,\sigma_x\left(\vec{r},\vec{\Omega},E\right)\psi^l\left(\vec{r},\vec{\Omega},E\right)}{\phi_{M,g}^{L-1/2}}, \tag{11}$$

$$\phi_{M,g}^{L-1/2} = \frac{\sum_{m\in M} \phi_{m,g}^{L-1/2} V_m}{\sum_{m\in M} V_m}, \tag{12}$$

where N_l is number of ls within L-1, m is fine-mesh cell index in coarse-mesh cell M, $\vec{n}$ is the normal vector at coarse-mesh cell surface M+1/2.

The p-CMFD equations are then formulated for each coarse-mesh cell and group g, using the above definitions:

$$\sum_{M'} \int_{MM'} d\Gamma \hat{n}\cdot \vec{J}_g^{\,L} + \sum_{m\in M} \sigma_{a,g}\phi_{m,g}{}^{L} V_m = \frac{1}{k_{eff,\det}} \sum_{g'} \sum_{m\in M} \nu\sigma_{f,g'}\phi_{m,g'}{}^{L} V_m + \sum_{g'\neq g} \sum_{m\in M} \sigma_{s,g'\to g}\phi_{m,g'}{}^{L} V_m, \tag{13}$$

where

$$J_{M+1/2,g}^{L} = J_{M+1/2,g}^{+,\,L} - J_{M+1/2,g}^{-,\,L}, \tag{14}$$

$$J_{M+1/2,g}^{+,\,L} = \frac{-\tilde{D}_{M+1/2,g}(\phi_{M+1,g}^{L} - \phi_{M,g}^{L}) + 2\hat{D}_{M+1/2,g}^{+}\phi_{M,g}^{L}}{2}, \tag{15}$$

$$J_{M+1/2,g}^{-,\,L} = \frac{\tilde{D}_{M+1/2,g}(\phi_{M+1,g}^{L} - \phi_{M,g}^{L}) + 2\hat{D}_{M+1/2,g}^{-}\phi_{M+1,g}^{L}}{2}, \tag{16}$$

$$\tilde{D}_{M+1/2,g} = 2\frac{(D_{M,g}/\Delta_M)(D_{M+1,g}/\Delta_{M+1})}{D_{M,g}/\Delta_M + D_{M+1,g}/\Delta_{M+1}}, \tag{17}$$

$$\hat{D}_{M+1/2,g}^{\pm} = \frac{2J_{M+1/2,g}^{\pm,L-1/2} + \tilde{D}_{M+1/2,g}(\phi_{M+1,g}^{L-1/2} - \phi_{M,g}^{L-1/2})}{2\phi_{M+1/2\mp1/2,g}^{L-1/2}}, \tag{18}$$

and $k_{eff,\det}$ is the eigenvalue of the p-CMFD equations.

The system of equations (13) for the whole problem provides $k_{eff,\det}$ and coarse-mesh cell average scalar fluxes. The anchoring distribution is then updated as,

$$Q_{\det}^{L}(\vec{r}) \simeq Q_m^L = \sum_{g'} \nu\sigma_{f,m,g'}\phi_{m,g'}^{L-1/2}\frac{\phi_{M,g'}^{L}}{\phi_{M,g'}^{L-1/2}}, \text{ for } m\in M. \tag{19}$$

The Monte Carlo L-th generation solves Eqs. (5) and (6), by sampling $(1-\alpha)N$ histories from the (L-1)-th generation fission source and αN histories from the anchoring distribution, where N is the number of histories per generation.

3. NUMERICAL RESULTS

We applied the Monte Carlo anchoring method to loosely-coupled rod problems. Boundary conditions are all vacuum and the two-group cross sections in Ref. [5] are used.

3.1. Test Problem 1: Symmetric Problem

For test problem 1, three moderator regions and two fuel regions are configured symmetrically and each region is of 40 cm thickness as shown in Fig. 1.

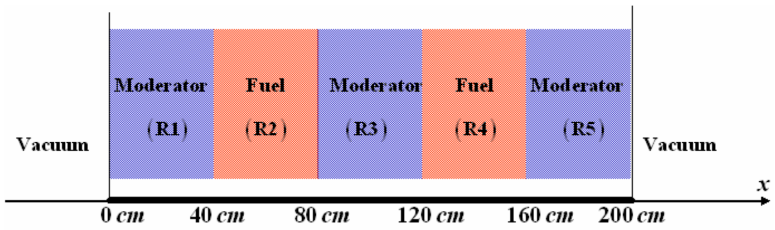


Figure 1. Problem configuration of test problem 1

20,000 histories per generation are used with 6,000 inactive generations and 13,000 active generations. For p-CMFD acceleration calculation, averaged track-length estimator results in 200 cell bins over 500 generations are used with 10^{-10} iteration stop criterion. The flat distribution is used for both of initial source distribution and initial anchoring distribution. The ratio (S_2/S_4) of fission sources in the two fuel regions and the eigenvalues are shown in Fig. 2, while the eigenvalue k_{eff} is shown in Fig. 3. The normalized flux distributions are also shown in Figs. 4 and 5.

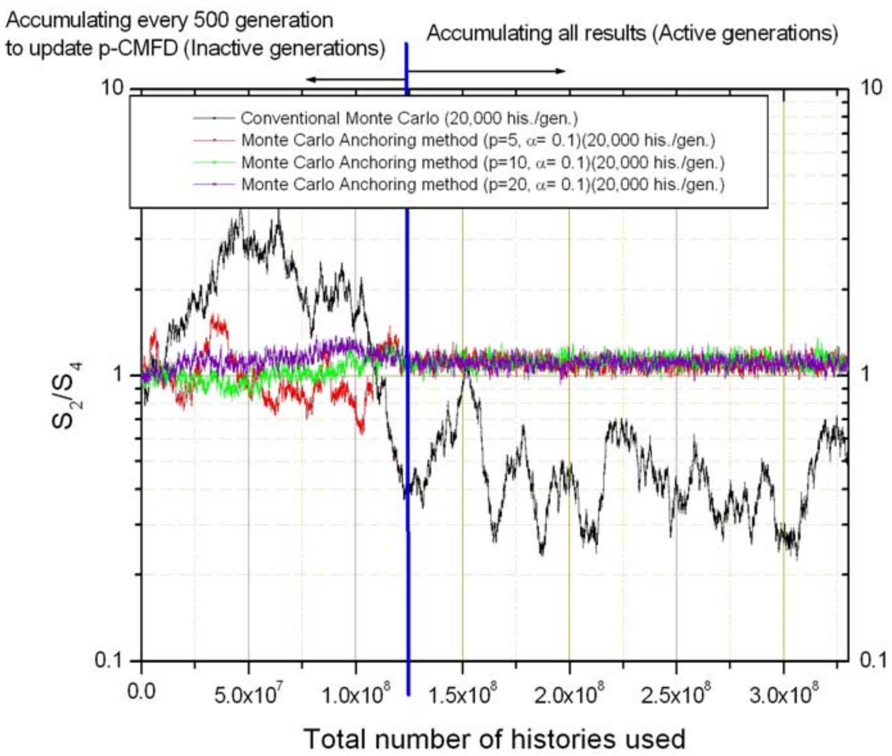


Figure 2. Fission source ratio of test problem 1

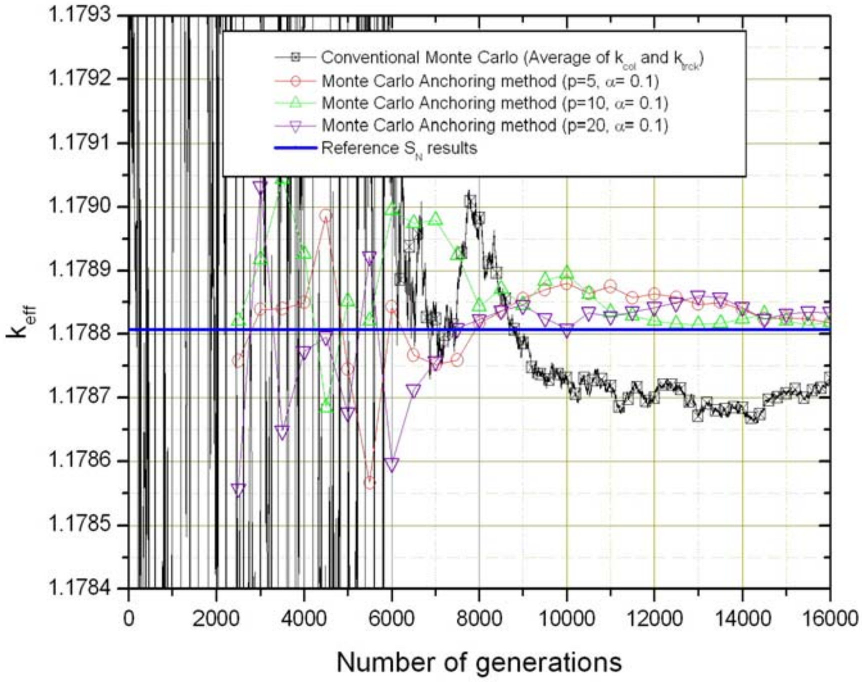


Figure 3. k_{eff} of test problem 1

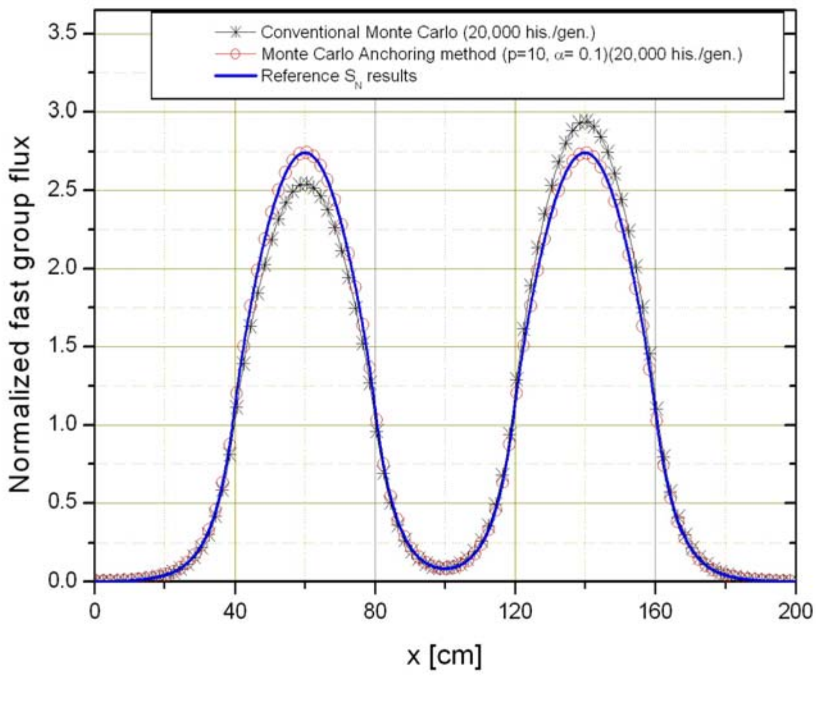


Figure 4. Normalized fast group flux distributions of test problem 1

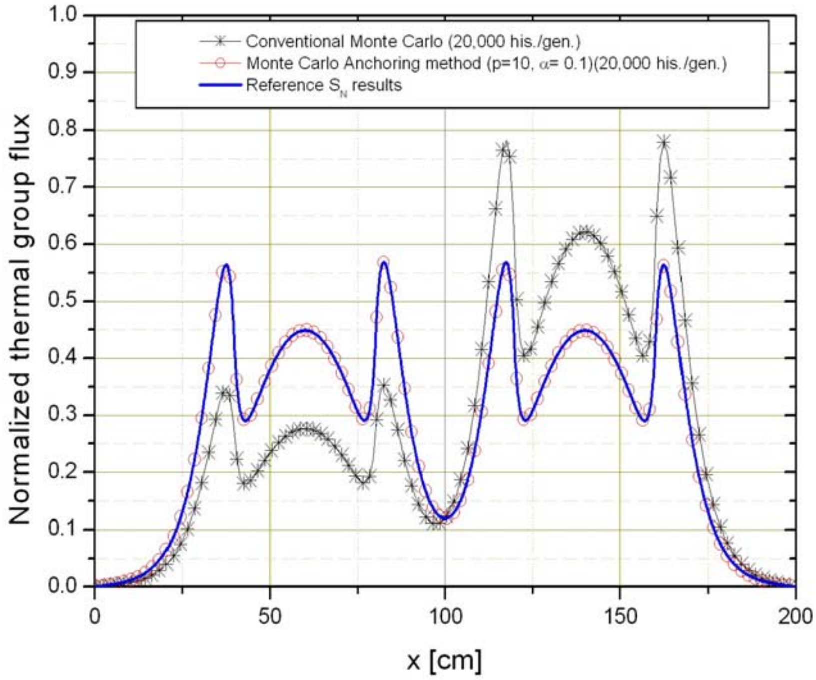


Figure 5. Normalized thermal flux distributions of test problem 1

For the comparison of eigenvalues, we performed one more conventional Monte Carlo calculation with 4,000,000 histories per generation, 30 inactive generations and 70 active generations. As shown in Tables I and II, the two conventional Monte Carlo calculations show slightly lower values while the Monte Carlo anchoring method shows better agreement with the reference S_N calculation, with the same number of total histories (number of generations times histories per generation) and similar total computation time.

Table I. Eigenvalues in test problem 1

Methods	k_{eff}	Relative difference [%]	Methods	k_{eff}	Relative difference [%]
Conventional Monte Carlo (20,000 his./gen.)	1.1786036[a] 1.1787657[b] 1.1786846[c]	1.72717 10^{-2} 3.52051 10^{-3} 1.04003 10^{-2}	**Monte Carlo Anchoring method (p=5, α=0.1)**[e]	1.178818	9.16180 10^{-4}
Conventional Monte Carlo (4,000,000 his./gen.)[d]	1.1786924[a] 1.1787143[b] 1.1787034[c]	9.73866 10^{-3} 7.88085 10^{-3} 8.80551 10^{-3}	**Monte Carlo Anchoring method (p=10, α =0.1)**[e]	1.178805	1.86629 10^{-4}
Reference S_N results	1.1788072 (Δx = 0.1 cm, iteration stop criterion = 10^{-15})				

[a] :collision estimator, [b]: track-length estimator, [c] : averaged k_{eff}, [d] : 30 inactive, 70 active generations are used.
[e]: "p" represents the number of fine-mesh cells within one coarse-mesh cell and "α" the anchoring factor.

Table II. Computation times of each method in test problem 1

Methods	*Computation time [sec]*[a]	Methods	*Computation time [sec]*
Conventional Monte Carlo (20,000 his./gen.)	2436	**Monte Carlo Anchoring method (p=5, α=0.1)**	2759
Monte Carlo Anchoring method (p=1, α=0.1)	2277	**Monte Carlo Anchoring method (p=10, α =0.1)**	1957

[a] : Three Core 2 Duo Dual-Core Processor E8400 CPUs are used

3.2. Test Problem 2: Asymmetric Problem

a) With 20,000 histories per generation

Test problem 2 has same geometrical configuration with test problem 1, while the group 2 fission cross section is increased by 0.4 % in region 4, rendering the problem asymmetric. 20,000 histories per generation are used with 6,000 inactive generations and 14,000 active generations. For the p-CMFD acceleration calculation, averaged track-length estimator results in 200 cell bins over 200 generations are used with 10^{-10} iteration stop criterion. The flat initial distributions are used as in test problem 1. The ratio of fission sources in the two fuel regions is shown in Fig. 6.

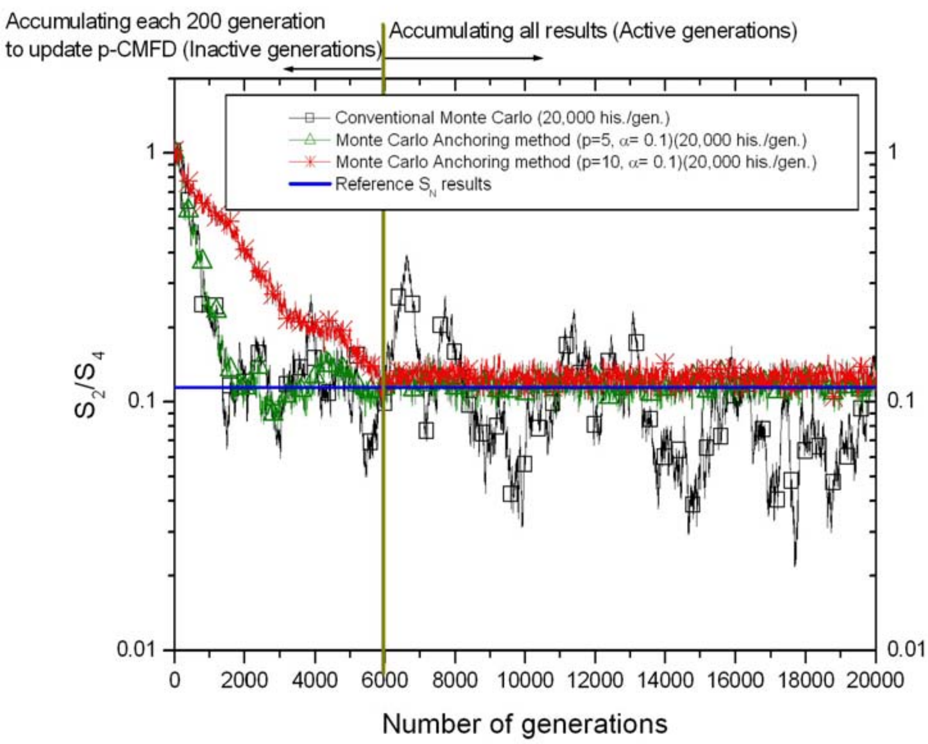
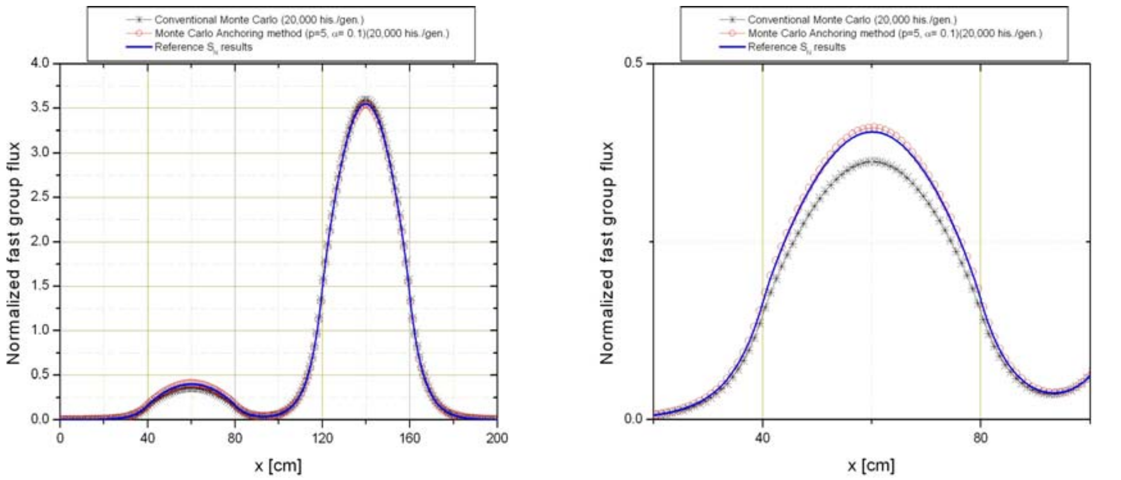


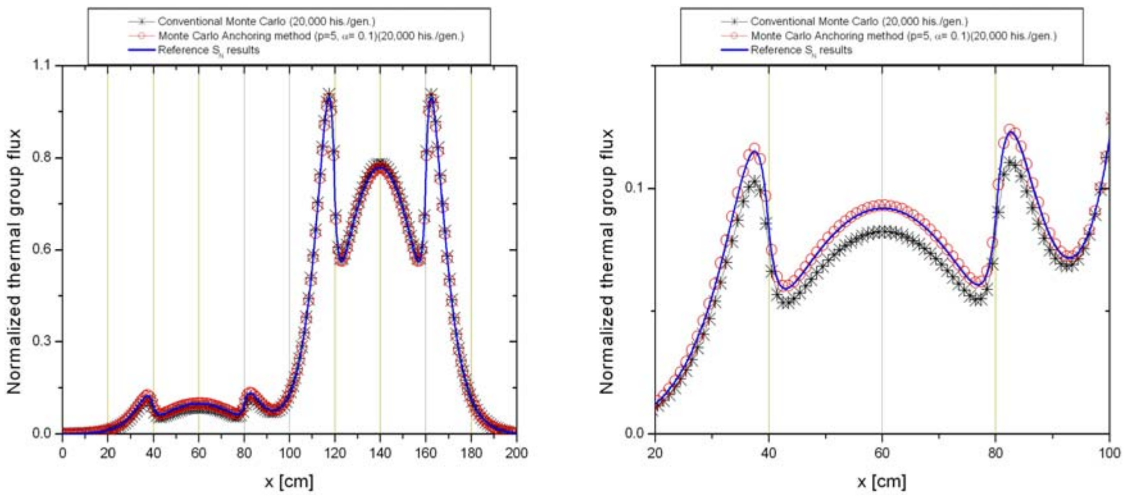
Figure 6. Fission source ratio of test problem 2

As shown in Figure 6 and Table III, the increase of coarseness, i.e., larger p, causes smaller fluctuation due to relatively small stochastic errors in p-CMFD coefficients, while the dominance ratio becomes worse due to the characteristic of the p-CMFD method. The total computation time is also shown in Table III. In case of small coarseness, the computing time is increased owing to the increase of calculation burden in p-CMFD calculation. However the increased burden of the p-CMFD calculation may not be significant since it can be performed less frequently. Thus, choice of an appropriate coarseness results in faster convergence of fission source distribution with smaller stochastic error than the conventional Monte Carlo method.



(a) Fast flux distribution in whole problem domain (b) Fast flux distribution between 20 cm and 100 cm

Figure 7. Normalized fast flux distributions of test problem 2



(a) Thermal flux distribution in whole problem domain (b) Thermal flux distribution between 20 cm and 100 cm

Figure 8. Normalized thermal flux distributions of test problem 2

In the conventional Monte Carlo method, the normalized flux distributions show discrepancies with reference S_N results due to the stochastic uncertainty as shown in Figs. 7 and 8. In the Monte Carlo anchoring method, the normalized flux distributions are in good agreement with reference S_N results. The fission source ratio in the Monte Carlo anchoring method shows 1.33462 % discrepancy with reference solution in the p=5, α =0.1 case while that in the conventional Monte Carlo method shows 30.36337 % discrepancy as shown in Table III.

Table III. Eigenvalues and fission source ratio in test problem 2

Methods	k_{eff}	**Relative difference in k_{eff} [%]**	S_2/S_4	**Relative difference in S_2/S_4 [%]**	***Computation time [sec]***[f]
Conventional Monte Carlo	1.1801018[a] 1.1800524[b] 1.1800771[c]	$6.9076\ 10^{-4}$ $3.4903\ 10^{-3}$ $1.3998\ 10^{-3}$	0.164020[g]	30.36337	2376
Monte Carlo Anchoring method (p=1, α =0.1)[d]	1.180061	$2.7626\ 10^{-3}$	0.121487	5.9835	4861
Monte Carlo Anchoring method (p=5, α =0.1)[d]	1.180113	$1.6439\ 10^{-3}$	0.115763	1.33462	3423
Monte Carlo Anchoring method (p=10, α =0.1)[d]	1.180072	$1.8304\ 10^{-3}$	0.125393	8.91198	2552
Reference S_N results[e]	1.1800936	-	0.114218	-	

[a] :collision estimator, [b]: track-length estimator, [c] : averaged k_{eff},

[d]: "p" represents the number of fine-mesh cells within one coarse-mesh cell and "α" the anchoring factor.

[e]: Δx = 0.1 cm, iteration criterion = 10^{-15}

[f]: Three Core 2 Duo Dual-Core Processor E8400 CPUs are used.

[g]: S_2/S_4 is obtained from track-length tally results.

The fission source ratios and eigenvalues versus various anchoring factors are shown in Figures 9 and 10, respectively. The conventional Monte Carlo method shows good agreement in k_{eff} with the reference solution, while it gives large fluctuating and biased fission source ratio (as shown in Table III and Fig. 9). The Monte Carlo anchoring method shows good agreement in fission source ratio for the anchoring factors used, while it shows smaller bias in k_{eff} with sufficiently small α (less than 0.2 in test problem 2 as shown in Fig. 10).

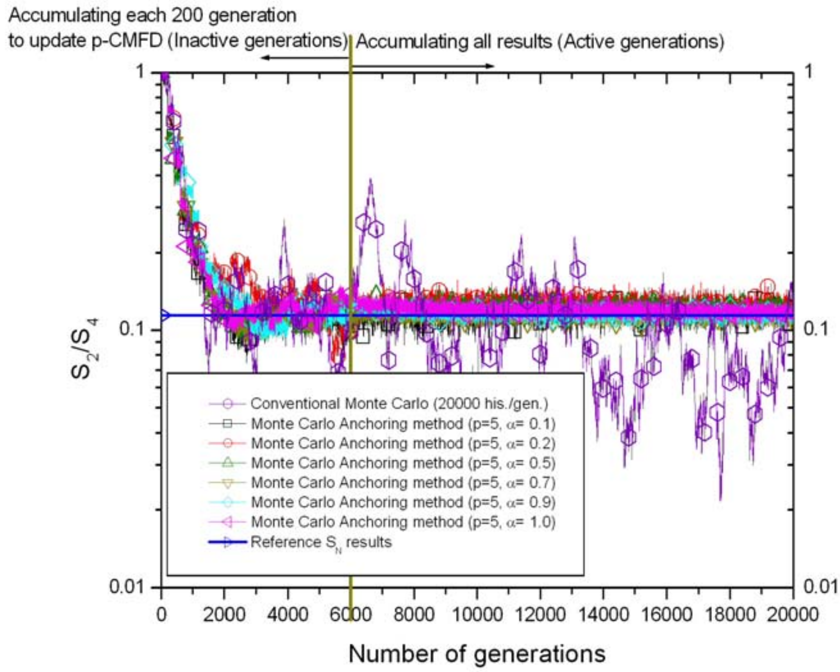


Figure 9. Fission source ratios versus various anchoring factors in test problem 2

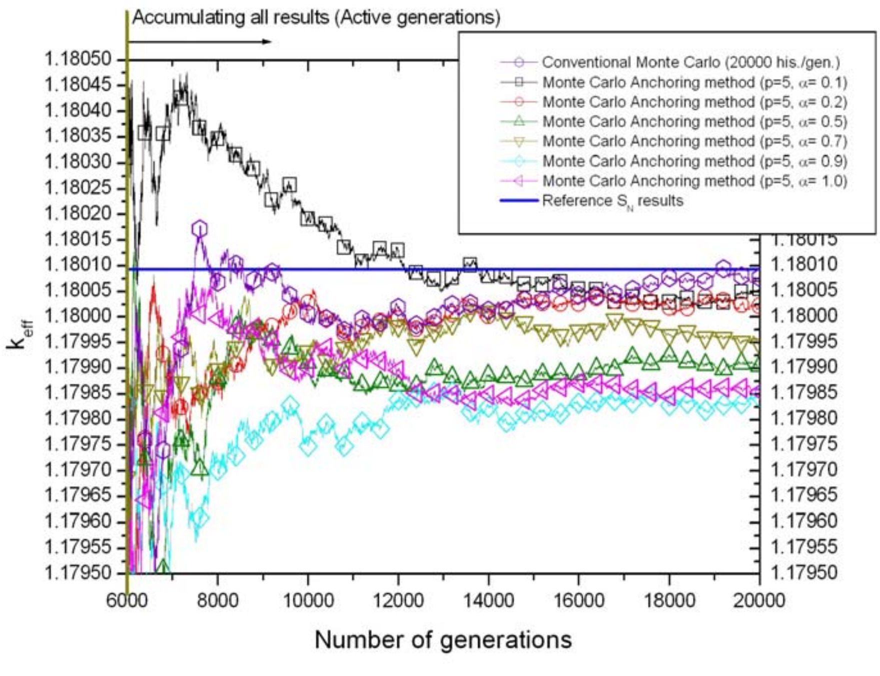


Figure 10. k_{eff} versus various anchoring factors in test problem 2

b) With 2,000 and 200,000 histories per generation

In Monte Carlo methods, a small number of histories per generation causes large stochastic fluctuation and bias in fission source, while a large number of histories per generation causes huge computational burden. Thus, two more cases for different computational conditions are studied in test problem 2.

2,000 histories per generation are used with 60,000 inactive generations and 140,000 active generations for case 1, while 200,000 histories per generation are used with 600 inactive generations and 1400 active generations for case 2. The total histories used in Monte Carlo calculation are identical in the two cases (as in the previous cases of section 3.2.a)). The ratio of fission sources in the two fuel regions and the eigenvalues are shown in Figs. 11, 12 and Table IV.

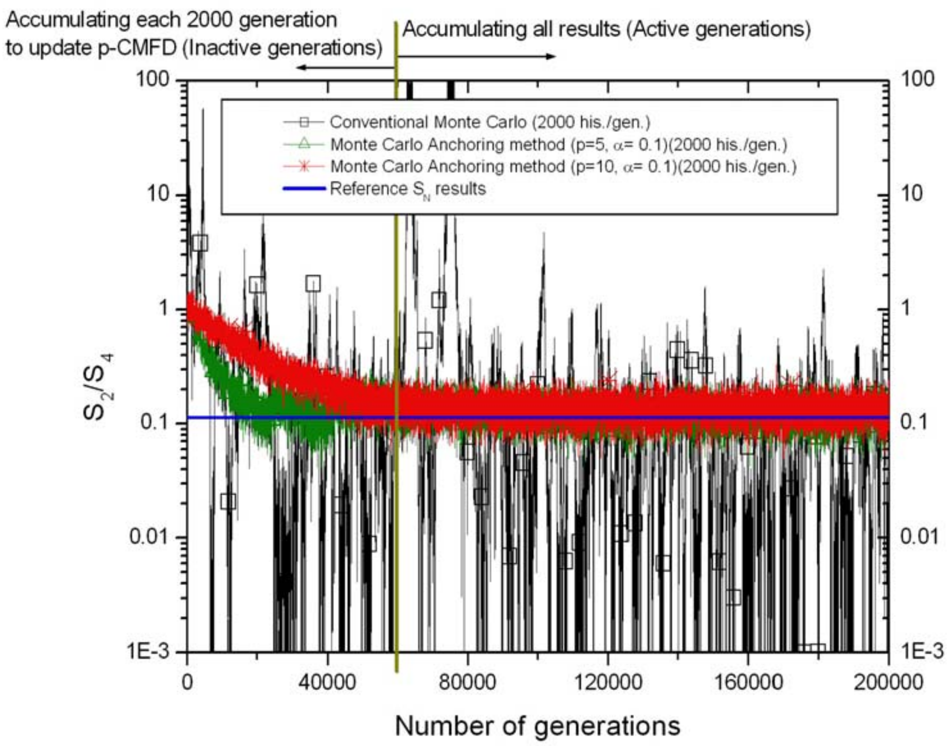


Figure 11. Fission source ratios of 2,000 his./gen. case in test problem 2

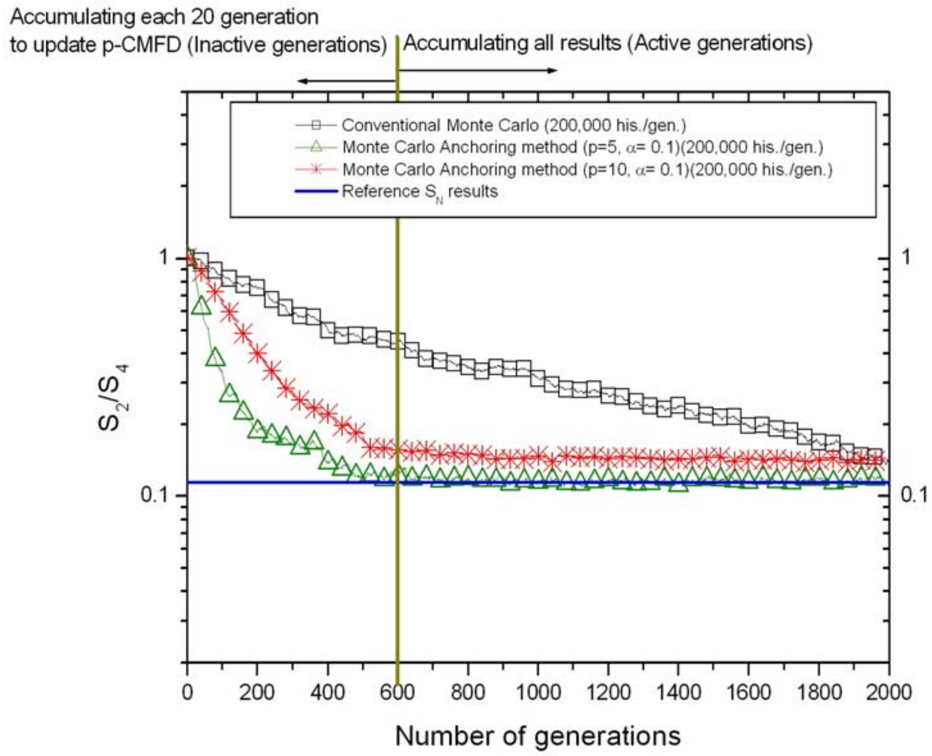


Figure 12. Fission source ratios of 200,000 his./gen. case in test problem 2

Since the deterministic distribution in the Monte Carlo anchoring method plays an anchoring role, the biasing effect due to the stochastic uncertainty is smaller than that in the conventional Monte Carlo method and it gives better results in the case of 2,000 histories per generation.

In the case of 200,000 histories per generation, the conventional Monte Carlo method shows smaller stochastic fluctuation, while slower convergence in outer iteration. However, the Monte Carlo anchoring method shows faster convergence in fission source due to the deterministic acceleration effect.

Table IV. Eigenvalues and fission source ratios of two different his./gen. cases in test problem 2

Case of 2,000 his./gen. with 60,000 inactive generations and 140,000 active generations					
Methods	k_{eff}	**Relative difference in k_{eff} [%]**	S_2/S_4	**Relative difference in S_2/S_4 [%]**	***Computation time [sec]*** [f]
Conventional Monte Carlo	1.1799156 [a] 1.1800497 [b] 1.1799827 [c]	$2.6345\ 10^{-2}$ $3.1718\ 10^{-3}$ $2.9032\ 10^{-3}$	0.185494 [g]	62.4035	5270
Monte Carlo Anchoring method (p=1, α =0.1) [d]	1.180097	$2.8811\ 10^{-4}$	0.113291	0.8116	5917
Monte Carlo Anchoring method (p=5, α =0.1) [d]	1.180068	$2.1693\ 10^{-3}$	0.125910	10.2366	5296
Monte Carlo Anchoring method (p=10, α =0.1) [d]	1.180088	$4.7454\ 10^{-4}$	0.136080	19.1406	3870
Case of 200,000 his./gen. with 600 inactive generations and 1,400 active generations					
Methods	k_{eff}	**Relative difference in k_{eff} [%]**	S_2/S_4	**Relative difference in S_2/S_4 [%]**	***Computation time [sec]*** [f]
Conventional Monte Carlo	1.1797827 [a] 1.1797193 [b] 1.1797510 [c]	$1.5084\ 10^{-2}$ $3.7200\ 10^{-2}$ $9.3976\ 10^{-2}$	0.259963 [g]	127.6025	2909
Monte Carlo Anchoring method (p=1, α =0.1) [d]	1.180086	$6.4402\ 10^{-4}$	0.102884	9.9231	3708
Monte Carlo Anchoring method (p=5, α =0.1) [d]	1.180069	$2.0846\ 10^{-3}$	0.116690	2.1643	2510
Monte Carlo Anchoring method (p=10, α =0.1) [d]	1.180054	$3.3557\ 10^{-3}$	0.142902	25.1134	2552
Reference S_N results [e]	1.1800936	-	0.114218	-	

[a] :collision estimator, [b]: track-length estimator, [c] : averaged k_{eff},
[d]: "p" represents the number of fine-mesh cells within one coarse-mesh cell and "α" the anchoring factor.
[e]: Δx = 0.1 cm, iteration criterion = 10^{-15}
[f]: Three Core 2 Duo Dual-Core Processor E8400 CPUs are used
[g]: S_2/S_4 is obtained from track-length tally results.

4. CONCLUSIONS

In this paper, the Monte Carlo anchoring method was presented, in which the fission source in the Monte Carlo power iteration procedure is decomposed into conventional fission source distribution and anchoring distribution. The anchoring distribution is updated by deterministic p-CMFD acceleration method and used in conventional Monte Carlo power iteration procedure with the anchoring factor α.

For the problems tested, the Monte Carlo anchoring method showed much better fission source distribution (and final flux distribution) than the conventional Monte Carlo method. The eigenvalue k also converges faster and it is estimated with significantly reduced bias.

Although the present paper presented numerical tests in the multi-group rod transport problem, extension to the continuous-energy and general three-dimension Monte Carlo problem should be straightforward and it is currently in progress.

ACKNOWLEDGMENT

This work was supported in part by the Ministry of Education, Science and Technology of the Republic of Korea through the Nuclear Technology Infrastructure and Basic Research Program.

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