International Conference on Mathematics, Computational Methods & Reactor Physics (M&C 2009)
Saratoga Springs, New York, May 3-7, 2009, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2009)

SUBCRITICALITY DETERMINATION BY NEURAL-BASED INVERSION OF SPACE-ENERGY NEUTRON KINETIC EQUATIONS

P. Picca
Dipartimento di Energetica, Politecnico di Torino
Corso Duca degli Abruzzi, 24, Torino, Italy
paolo.picca@polito.it

R. Furfaro, B.D. Ganapol
Department of Aerospace and Engineering, University of Arizona
Tucson AZ 85721
robertof@email.arizona.edu, ganapol@cowboy.ame.arizona.edu

S. Dulla, P. Ravetto
Dipartimento di Energetica, Politecnico di Torino
Corso Duca degli Abruzzi, 24, Torino, Italy
sandra.dulla@polito.it, piero.ravetto@polito.it

ABSTRACT

The determination of the reactivity level in ADS represents nowadays an important open issue for the development of subcritical systems. Several approaches were developed in the past based on simplified models, such as point kinetics, for which an analytical inversion is feasible. On the other hand, numerical simulations shows that the evolution of the power and detector signals are not point-like. In the present work, we propose a neural-based inversion that can invert more realistic physical models. As a result, the spatial and spectral effects connected to the position and the energy selectivity of detectors are directly accounted for. An outline of the theory is presented in the following and some preliminary results show the performance of the technique proposed in the interpretation of pulsed experiments.

Key Words: Accelerator-Driven Systems; reactivity measurement; inverse techniques; neural networks

1. INTRODUCTION

The experimental estimate of integral parameters in accelerator-driven systems (ADS) constitutes an important issue for the development of the subcritical technology. In particular, the assessment of a methodology for inferring the reactivity level was and still is subject of many theoretical and experimental studies [1, 2]. Typically, the dynamic properties of the subcritical structures are explored by performing flux measurements in pulsed experiments and adopting inversion techniques for their interpretation. The inverse techniques are mainly based on simplified mathematical models, such as point kinetics (PK), since the inversion can be performed by analytical means. These methods are based on the determination of the subcriticality by using the thermal power as an input and assume the behaviour of the power in the source transient to be point-like. Experimentally, the fission power is not measurable in nowadays subcritical facilities as only neutron fluxes can be recorded by detectors. Even if point kinetic inversion techniques are largely applied to invert detector signals [3], they suffer from an inherent shortcoming by not accounting for the spatial and energy features of the transient. Several *ad hoc* solutions have been proposed to overcome the problem. For instance, in a recent paper [4], the spatial dependency in PK model is introduced through a set of coefficients, which are evaluated from reference steady-state computations.

The present work proposes an alternative methodology to determine the subcriticality level, which approaches the problem from a different perspective. Instead of using a lumped parameter model and accounting for spatial features with corrective factors, the dynamic of the multiplying system is analyzed starting from first principles, i.e. the time-dependent neutron and precursor balance equations. The evolution of the local signal can therefore be simulated accurately and the energy variable can also be included in the analysis to take into account energy selective detectors. The connection between experimental data and numerical results is then established through a neural-based inversion. An artificial neural network (ANN) is a biologically inspired structure that is capable of learning patterns and functional relationships present in a set of data. In our work, the ANN is trained to recognize the subcriticality level from simulated detector responses and can be directly applied to experimental measures to infer the reactivity. Recent investigations performed on the neural inversion of PK [5] show that ANN has the capability to deal with noise on the data set. In the following, a description of the methodology is given, by specifying the different steps in the construction of the training set and in the optimization of the neural network design. After training the ANN, its generalization performances are tested in several cases not included in the training set.

2. OVERVIEW OF THE METHODOLOGY

A scheme of the overall procedure is reported in Fig. 1. Firstly, a mathematical model for the simulation of a source pulse in a subcritical system is chosen. The geometric parameters, the detector positions and the pulse duration are set. The detector responses are in principle function of the whole set of cross sections, but it is not sure that there is a unique combination of the material parameters which gives the given detector evolutions. At a first stage, it is therefore more convenient trying to assume that the local measures of neutron flux depend on a single parameter, the multiplication constant. Future work will focus on the problem of the uncertainties on cross sections.

Following this one-parameter inversion philosophy, the cross sections are given as an input and a preliminary evaluation of the multiplication constant is performed by solving the corresponding eigenvalue problem. Then, for each multiplication constant in the training set, the fission cross sections are uniformly changed to impose the desired value of k_{eff}. The training set is generated by solving the time-dependent mathematical model, as sketched in Fig. 1 (a). A number of pulsed transients are simulated for systems with different levels of subcriticality. A time sampling is chosen and the detector responses are collected at each selected time together with the corresponding subcriticality level. These data constitute the training set for the ANN (Fig. 1 (b)).

A suitable neural network needs now to be designed. Since there is not a linear relation between the multiplication constant and the meter readings, a multilayer structure needs to be adopted in the ANN design [6]. The architecture of the network is to be specified by defining the neurons per layer and the transfer function to be implemented per neuron. It is also very important that the total number of neurons, and therefore the number of unknown parameters, is properly chosen on the basis of the training set dimension, otherwise under- or over-constrained problems can occur. In general, an iterative process is necessary for an optimized design of the neural network, since normally one starts from a trial training set, trains the network and checks if the structure is suitable by testing its performance, and, if it is needed, changes the training set and the ANN structure.

The ANN is then trained in a supervised way, since the target value of the network (k_{eff}) is known for each set of detector responses. The supervised training is an optimization process where weights and biases of the neurons are successively changed in order to minimize a cost function, which is a functional of the difference between the output of the network and the corresponding target value. In the case of a multilayered network, a backpropagation algorithm is used [6]. By applying the steepest gradient method, classical backpropagation provides the direction in which the unknowns parameters in the ANN are to be changed in order to reduce the value of the cost function. As the cost function depends on several hundreds of variables, the steepest gradient optimization may easily result in a very time consuming operation. In literature, several algorithms are available to overcome this problem. For the present application, about one hundred neurons are used and accelerations for the training are strongly suggested for limiting the computational time. In the following a one-step secant (OSS) method is chosen, which provides a compromise between the steepest gradient method (short time for a single computation and memory storage requirements) and the Newton methods (convergence in fewer epochs).

A crucial point in assessing ANN methodology is the possibility to generalize its performance, i.e. the capability of the network to predict the multiplication constant on simulations not included in the training set. For example, it can happen that an under-constrained ANN meets the performance goal but can hardly deal with new data. The capability of generalization represents an indication of a good network design and, at the same time, assesses the power of the tool that can eventually deal with experimental data.

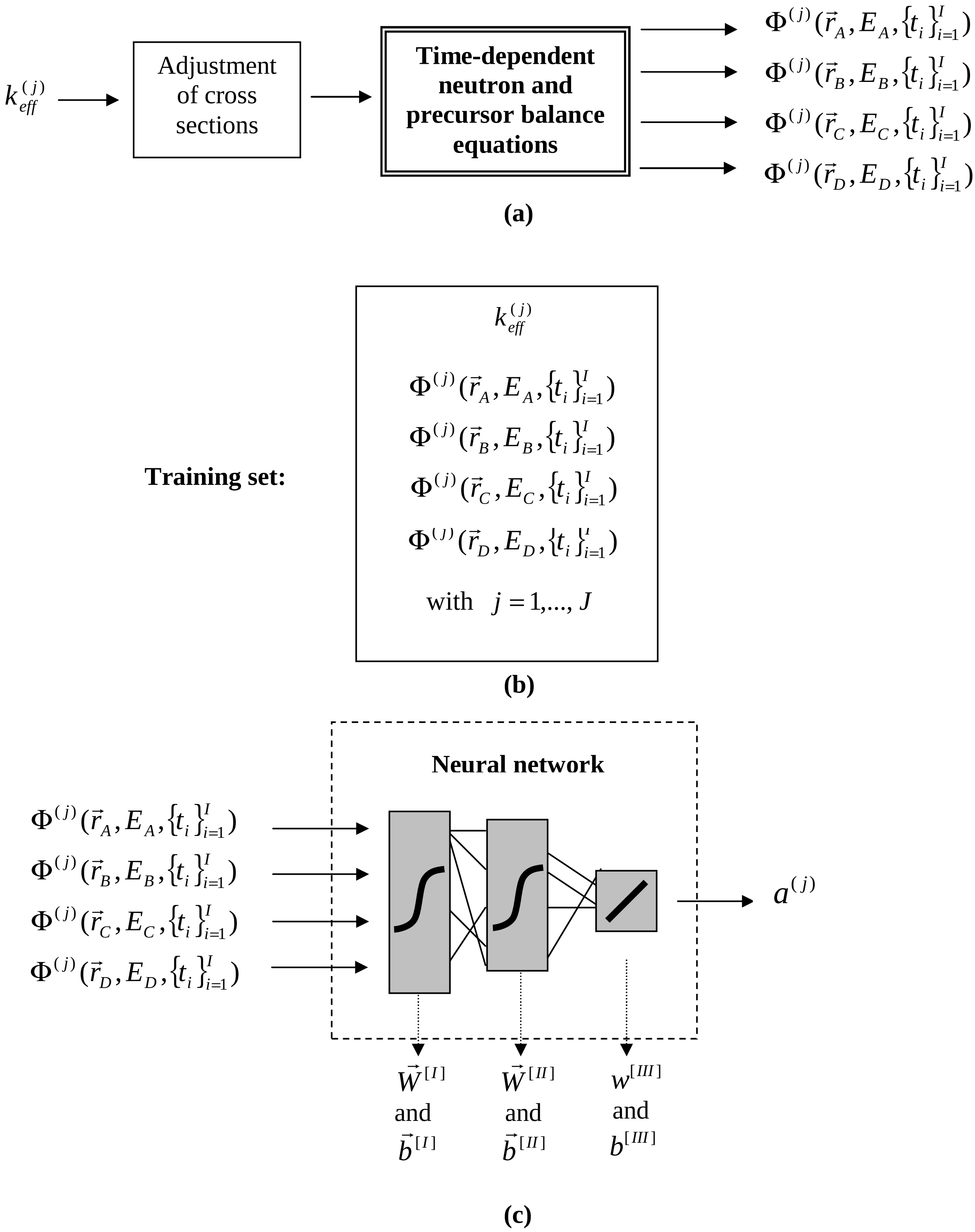

Figure 1: Scheme of the neural-based inversion of space-energy neutron kinetic equations.

3. RESULTS

The system considered in this work for the training process is a 1D heterogeneous multiplying structure, whose layout and cross sections are taken from [7] and are representative of a media similar to Yalina subcritical facility [8]. Figure 2 sketches a half of the reactor and gives some more details about the material composition of the test case. The layout of the positions of the detectors is also shown. The detector in the source region measures the neutron flux at very high energies, two detectors record neutron thermal fluxes in different positions (thermal region) and a detector in the fast booster is sensitive to fast neutrons.

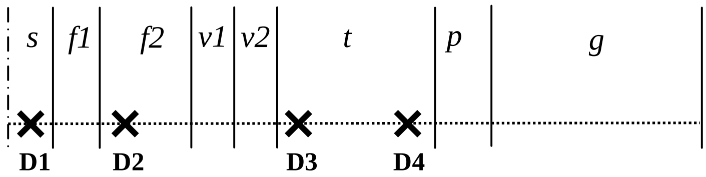

Figure 2: Sketch of a subcritical system and detector positions. Legend: *s*: source; *f1* and *f2*: fast regions; *v1* and *v2*: valve regions; *t*: thermal region; *p*: polyethylene; *g*: graphite. Detector D1 measures the neutron flux in the source energy group, D2 in the fast group and D3-D4 at the thermal energy.

The time-dependent behavior of the reactor is modeled by adopting a multigroup diffusion approximation, considering also the contribution of delayed emission:

$$\begin{cases} \frac{1}{v_g}\frac{\partial}{\partial t}\Phi_g(x,t) = \frac{\partial}{\partial x}D_g(x)\frac{\partial}{\partial x}\Phi_g(x,t) - \Sigma_{r,g}(x)\Phi_g(x,t) + \chi_{p,g}\sum_{g'}\upsilon\Sigma_{f,g'}(x)\Phi_{g'}(x,t) \\ \qquad + \sum_{g'}\Sigma_{g'\to g}(x)\Phi_{g'}(x,t) + \sum_j \lambda_j C_j(x,t) + S(x,E_g,t) \\ \frac{\partial}{\partial t}C_j(x,t) = -\lambda_j C_j(x,t) + \chi_{d,j}\sum_{g'}\upsilon\Sigma_{f,g'}(x)\Phi_{g'}(x,t), \end{cases} \tag{1}$$

with $g = 1,...,3$ and $j = 1,...,6$. More accurate studies are foreseen using the neutron Boltzmann equation in order to properly consider transport effects of the source propagation. The solution of the diffusion problem is obtained by spatially discretizing Eqs. (1) in finite volumes and iterating for solving the energy problem with the classical outer iteration procedure. The time derivative is approximated with a finite difference and the Implicit Euler scheme is adopted. The training set is obtained by preliminary solving the steady-state critical problem to obtain the desired value of k_{eff}. Then the system of equations in (1) is solved for 500 values of k_{eff}. A 1 ms source pulse is considered and 100 sampling times are considered.

Figure 3 represents the simulation of the power for various values of subcriticality, whereas Figure 4 reports the corresponding curves for the detector measurements along the source transient.

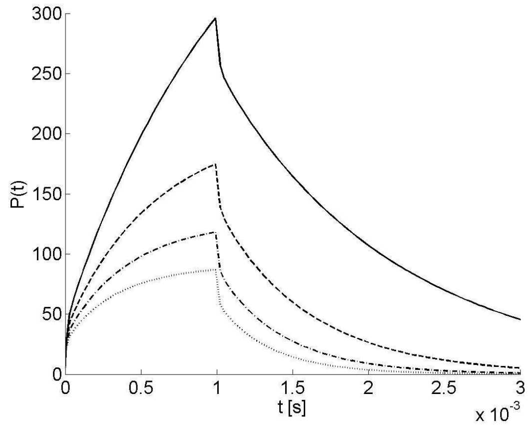

Figure 3: Power evolutions in a source pulse transient. Dotted curve: k_{eff} = 0.93000; dashed: 0.95000; dash-dotted: 0.97000; bold line: 0.99000.

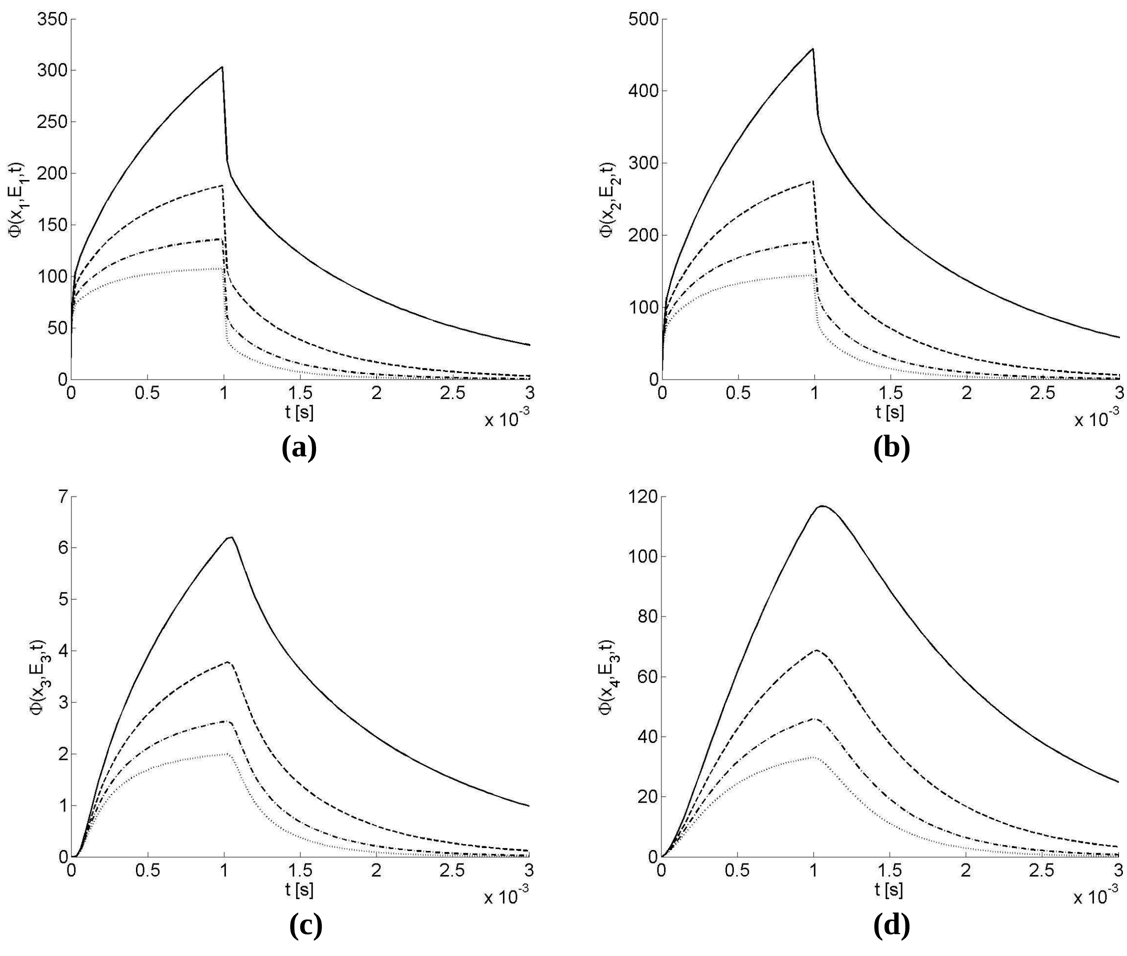

Figure 4: Detector evolutions in a source pulse transient. Dotted curve: k_{eff} = 0.93000; dashed: 0.95000; dash-dotted: 0.97000; bold line: 0.99000.

Figures 4-5 underline that none of the detectors can accurately approximate the power evolution. In fact, for any value of the reactivity, detector signals in fast groups are far more peaked than the thermal power, while measures in the thermal group exhibit a delay due to space distance from the source and slowing down. Additionally, neither power evolution computed from the space-energy solution nor detector signals are accurately described by a point model. Indeed detector and power shapes are very different for detector 3 and 4, but even in the case of a fast detector close to the source, the presence of more time constants is easily noticeable. Only asymptotically the system approaches a point-like behavior. For these reason, we consider important the attempt to develop an inversion method for the time-dependent neutron balance equation.

The architecture of the network used is a two-layer net, with 120 neurons in the hidden layer. Logarithmic sigmoid function is implemented as transfer function. The training set is composed by 500 cases (i.e. values of k_{eff}), each of which is composed by 400 input data (4 detectors and 100 sampling times). The training time for the ANN using OSS optimization algorithm is around 5 minutes. Figure 5 reports the behavior of the performance parameter (connected to the cost function) as a function of the number of epochs. Three curves are reported in Fig. 5, since part of the data from the training set is not actually used to train the net, but to monitor the training process [6]. In the training phase, it is common not to change the net parameters after each training step, but averaging the value of the weight modifications over the entire set of data, operating in the so-called “batch mode”. Each epoch corresponds to a step in the direction obtained using of the whole training set. The initial values of the net parameters are normally randomly chosen and therefore, even using the same training set, several trials are suggested in order to avoid local minima and choose the most performing NN.

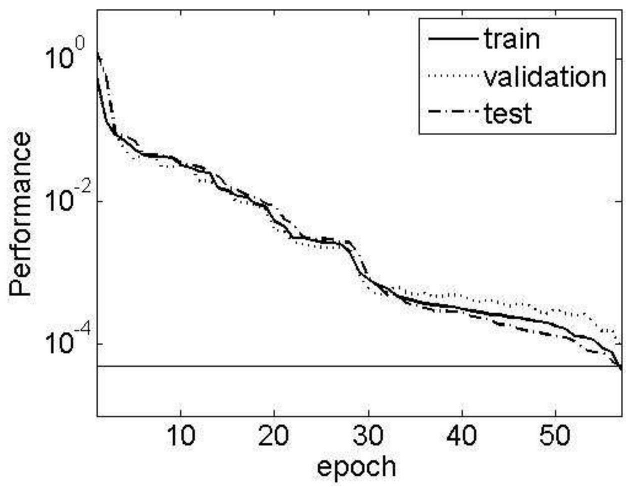

Figure 5: Training of the NN on simulations.

Tables I and II test the generalization capabilities of the same network architecture trained with two training sets. In the tables, the results of the ANN inversion for ten new simulations are compared with direct simulations. For Table I, training is performed with k_{eff} ranging from 0.93000 to 0.99000. The results are quite encouraging since the maximum error for the 10 reported cases is about 150 pcm. Additionally, a more detailed analysis performed over 100 new cases shows that the root mean squared error (RMSE) is around 80 pcm. The accuracy of the results can be improved by enlarging the dimension of the training set (i.e. additional transient simulations) and consequently increasing also the number of neuron in the network. As a result, the computational time and memory requirement for the training of the larger net is greatly increased.

Table I. Generalization on a new set of data of the neural network, trained with k_{eff} spanning from 0.93000 to 0.99000.

	k_{eff}		
test case #	**direct solution**	**ANN**	**deviation [pcm]**
1	0.94214	0.94193	21
2	0.97857	0.98000	143
3	0.97379	0.97435	56
4	0.97935	0.98068	134
5	0.98789	0.98838	49
6	0.95914	0.95969	55
7	0.95446	0.95554	108
8	0.93209	0.93265	57
9	0.96879	0.96982	103
10	0.94088	0.94061	27

An alternative strategy to considering more points in the training set is to start searching the k_{eff} in a large range (e.g. 7000 pcm in Table I) and successively refining the analysis to a smaller interval, taking advantage of the preliminary results. With this approach, the training set dimension remains unchanged as well as the network architecture. For instance, supposing that from a preliminary computation it is found that the reactivity is around -4000 pcm, then the subcriticality search could be limited to -4000 to -2000 pcm. Table II shows the performance of the NN inversion of detector signals in case of a refined search for the multiplication constant (over 1000 pcm). The same training set dimension is kept equal to that in Table I, i.e. 400×500. In this case, the maximum error decreases to about 50 pcm and RMSE over to 40 pcm.

It is worth remarking that non-neural based algorithms might be employed to infer the level of reactivity from the multi-detector responses. For instance, the least squared error was adopted in [11] for PK inversion and in [12] for the determination of reactivity from a static calculation. The applications of these ideas to full space-energy kinetics could in principle be exploited. In practice, the ANN approach represents a simple and fast computational tool to perform this optimization. Furthermore, the method proposed in this work is chosen since many extensions are foreseen in future works which would result in a heavy computational effort for LSM, such as the introduction of the uncertainties on geometric data and cross sections [12].

Table II. Generalization on a new set of data of the neural network, trained with k_{eff} spanning from 0.96000 to 0.98000.

	k_{eff}		
test case #	**direct solution**	**ANN**	**deviation [pcm]**
1	0.96240	0.96291	51
2	0.97699	0.97647	52
3	0.96471	0.96508	37
4	0.96135	0.96192	57
5	0.96650	0.96671	21
6	0.97911	0.97867	44
7	0.97287	0.97238	49
8	0.97438	0.97385	52
9	0.96496	0.96531	35
10	0.96201	0.96254	53

4. CONCLUSIONS

This work presents a new methodology for approaching the problem of the reactivity determination through the interpretation of pulsed-source experiments. After a brief review of the limitation of current techniques, the neural-based inversion strategy is introduced by explaining the different steps in the network design and in the training. Some results are reported in order to show its performances. The ANN technique proves to be interesting in subcriticality evaluation and very powerful. Being based directly on the time-dependent neutron balance, the classical problems of point-like based inversion are overcome and more realistic results for the estimate of k_{eff} ~~the~~ are possible. Future efforts are foreseen in order to consider the noise superposed to the detector signals and the uncertainties on the input nuclear data.

REFERENCES

1. G. I. Bell, S. Glasstone, *Nuclear Reactor Theory*, Van Nostrand Reinhold, New York, 1970.
2. J. F. Lebrat, G. Aliberti, A. D'Angelo, A. Billebaud, R. Brissot, H. Brockmann, M. Carta, C. Destouches, F. Gabrielli, E. Gonzalez, A. Hogenbirk, R. Klein-Meulenkamp, C. Le Brun, E. Liatard, F. Mellier, N. Messaoudi, V. Peluso, M. Plaschy, M. Thomas, D. Villamarín, J. Vollaire, Global Results from Deterministic and Stochastic Analysis of the MUSE-4 Experiments on the Neutronics of Accelerator-Driven Systems, *Nuclear Science and Engineering*, **158**, 49-67, 2008.
3. S. Canepa, S. Dulla, P. Ravetto, Comparative assessment of methods for the reactivity measurement in subcritical systems by pulsed experiments, *Kerntechnik*, **72**, 38-43, 2007.

4. F. Gabrielli, M. Carta, A. D'Angelo, W. Maschek, A. Rineski, Inferring reactivity in acceleration driven systems: corrective spatial factors for source-jerk and area method, *Progress in Nuclear Energy*, **50**, 370-376, 2008.
5. P. Picca, R. Furfaro, B. D. Ganapol, S. Dulla, P. Ravetto, Inverse point kinetics with neural network, *Transactions of the American Nuclear Society*, **99**, 348-349, 2008.
6. M. T. Hagan, H. B. Demuth, M. H. Beale, *Neural Network Design*, PWS Publishing, Boston. 1996.
7. S. Dulla, P. Picca, D. Tomatis, P. Ravetto, M. Carta, The problem of the formulation of integral parameters in source-driven systems: a critical evaluation, *International Conference PHYSOR2008*, Interlaken, 2008.
8. V. Bournos, Y. Fokov, H. Kiyavitzkaya, B. Martsynkevich, C. Routkovskaia, Y. Gohar, C.M. Persson, W. Gudowski, YALINA-Booster benchmark specifications, *IAEA report*, 2007.
9. R. Furfaro, B. D. Ganapol, L. F. Johnson, S. R. Herwitz, Neural Network Algorithm for Coffee Ripeness Evaluation Using Airborne Images, Applied Eng. in Agriculture, **23**, 1-9, 2007.
10. H. Taninaka, K. Hashimoto, Determination of subcriticality and effective source strength by source drop and jerk experiments, *International Conference PHYSOR2008*, Interlaken, 2008.
11. X. Jiang, S. Wei, S. Zhang, A. Ma, A feasibility study of sub-criticality monitoring for ADS via static flux distribution measurement, *International Conference PHYSOR2008*, Interlaken, 2008.
12. C. Bacour, F. Baret, D. Bèa, M. Weiss, K. Pavageau, "Neural Network Estimation of LAI, fAPAR, fCover, and LAIxCab from top of canopy MERIS Reflectance data: Principles and Validation", *Remote Sensing and Environment*, **105**, 313-325 (2006).