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MULTI-LEVEL METHODS FOR SOLVING MULTIGROUP TRANSPORT EIGENVALUE PROBLEMS IN 1D SLAB GEOMETRY

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ABSTRACT

A methodology for solving eigenvalue problems for the multigroup neutron transport equation in 1D slab geometry is presented. In this paper we formulate and compare different variants of nonlinear multi-level iteration methods. They are defined by means of multigroup and effective one-group low-order quasidiffusion (LOQD) equations. We analyze the effects of utilization of the effective one-group LOQD problem for estimating the eigenvalue. We present numerical results to demonstrate the performance of the iteration algorithms in different types of reactor-physics problems.

Key Words: multigroup neutron transport equation; k-eigenvalue problem; iteration methods

1. INTRODUCTION

In a variety of reactor design calculations one needs to evaluate a critical parameter of a system and its fundamental mode. To perform neutronics computations for this important class of reactor physics problems, it is necessary to solve the multigroup transport equation and determine an eigenvalue and the associated eigenfunction. The transport equation is a detailed conservation relation for neutrons in the phase space defined by the spatial, angular and energy variables. The redistribution of particles with respect to energy and direction is governed by scattering and fission processes. These physical phenomena are described in the transport equation by corresponding reaction-rate densities that form its integral operator. As a result, the equations for different groups are coupled with each other through fission, upscattering and downscattering terms. The iteration methods for solving multigroup transport problems are defined by various nested iteration schemes such as the inner iterations for within-group scattering and the outer iterations due to coupling between the energy groups. The standard method of power iterations [1] can converge very slowly in problems in which upper energy groups are strongly coupled with lower groups. The two-grid acceleration scheme [2] and P_1 synthetic acceleration method [3] have been developed to speedup iterations in fixed-source transport problems with upscattering. A methodology for solving multigroup neutron transport equation based on the quasidiffusion

(QD) method has been proposed in [4, 5]. It was applied to time-dependent and α-eigenvalue problems [6, 7]. A review of basic ideas and discussion of the QD method for multigroup problems has been presented in [1].

In this paper we formulate and compare different variants of nonlinear multi-level iteration methods for solving k-eigenvalue transport problems in 1D slab geometry. These methods use the set of multigroup low-order QD (MLOQD) equations that are derived by taking 0th and 1st angular moments of the multigroup transport equation. Then, the MLOQD equations are averaged over the energy groups to get effective one-group low-order QD (1GLOQD) equations. The whole system of equations is closed by means of exact relations. The 1GLOQD problem has the following features:

- It reproduces important details of neutron transport physics because averaging with respect to the angular and energy variables is performed without any approximations.
- It is formulated in a form of an eigenvalue problem in a projected space the solution of which depends only on the spatial variable.

These properties enable one to solve the eigenvalue problem in the low-dimensional space and get a good estimation of the eigenvalue on each outer iteration. This leads to acceleration of the multigroup transport iterations. Note that ideas of formulating one-group transport problems were also applied to develop iterative methods for solving nonlinear radiative transfer problems [8–11].

To analyze the proposed iteration methods, we use reactor-physics problems with realistic cross sections. The tests are defined to model cases that are specific for assembly-level color-set computations and full-core reactor calculations. The cross sections from the C5G7 benchmark [12] are utilized in these problems. We present numerical results which show the performance of the methods in problems with different physical properties.

The remainder of the paper is organized as follows. In Sec. 2 we present the formulation of the QD method for solving multigroup transport problems. The multi-level iteration methods for solving the QD system of equations are defined in Sec. 3. The numerical results are presented in Sec. 4. We conclude with a discussion in Sec 5.

2. THE NONLINEAR SYSTEM OF HIGH-ORDER AND LOW-ORDER EQUATIONS FOR SOLVING MULTIGROUP NEUTRON TRANSPORT K-EIGENVALUE PROBLEMS

We consider a k-eigenvalue multigroup transport problem in 1D slab geometry in case of isotropic scattering, reflective and vacuum boundary conditions:

$$\mu\frac{\partial}{\partial x}\psi^g(x,\mu)+\Sigma_t^g(x)\psi^g(x,\mu) =$$
$$\frac{1}{2}\sum_{p=1}^{N_g}\Sigma_s^{p\to g}(x)\int_{-1}^{1}\psi^p(x,\mu)d\mu+\frac{1}{2k}\chi^g(x)\sum_{p=1}^{N_g}\nu_f^p(x)\Sigma_f^p(x)\int_{-1}^{1}\psi^p(x,\mu)d\mu\,, \qquad (1)$$
$$-1\le\mu\le 1,\quad 0\le x\le X,\quad g=1,...,N_g\,,$$
$$\psi^g(0,\mu)=\psi^g(0,-\mu)\ \text{ for }\mu>0\,,\quad \psi^g(X,\mu)=\ 0\ \text{ for }\mu<0\,.$$

The QD method for solving the multigroup neutron transport k-eigenvalue problems is defined by a hierarchy of nonlinearly coupled equations which consists of the following three sets of problems [4, 5]:

1. The *high-order multigroup transport equation* for the group angular flux ψ^g

$$\mu\frac{\partial}{\partial x}\psi^g(x,\mu)+\Sigma_t^g\psi^g(x,\mu)=\frac{1}{2}\left(\overline{\Sigma}_s^g+\frac{1}{k}\chi^g\overline{\nu\Sigma_f}\right)\phi\,, \tag{2}$$

$$\psi^g(0,\mu)=\psi^g(0,-\mu)\ \text{ for } \mu>0\,,\quad \psi^g(X,\mu)=\ 0\ \text{ for } \mu<0\,,$$

where

$$\overline{\nu\Sigma_f}=\frac{\sum_{p=1}^{N_g}\nu_f^p\Sigma_f^p\phi^p}{\sum_{p=1}^{N_g}\phi^p}\,, \tag{3}$$

$$\overline{\Sigma}_s^g=\frac{\sum_{p=1}^{N_g}\Sigma_s^{p\to g}\phi^p}{\sum_{p=1}^{N_g}\phi^p}\,, \tag{4}$$

and ϕ is the total scalar flux.

2. The *multigroup low-order QD (MLOQD) equations* for the group scalar flux $\phi^g=\int_{-1}^{1}\psi^g d\mu$ and current $J^g=\int_{-1}^{1}\mu\psi^g d\mu$

$$\frac{dJ^g}{dx}+\left(\Sigma_t^g-\Sigma_s^{g\to g}\right)\phi^g=\sum_{p=1}^{g-1}\Sigma_s^{p\to g}\phi^p\ +\ \left(Q_{up}^g+\frac{1}{k}\chi^g\overline{\nu\Sigma_f}\right)\phi\,, \tag{5}$$

$$\frac{dE^g\phi^g}{dx}+\Sigma_t^g J^g=0\,, \tag{6}$$

$$J^g(0)=0\,,\quad J^g(X)=C_X^g\phi^g(X)\,, \tag{7}$$

where

$$E^g(x)=\frac{\int_{-1}^{1}\mu^2\psi^g(x,\mu)d\mu}{\int_{-1}^{1}\psi^g(x,\mu)d\mu}\,,\quad C_X^g=\frac{\int_{0}^{1}\mu\psi^g(X,\mu)d\mu}{\int_{0}^{1}\psi^g(X,\mu)d\mu}\,, \tag{8}$$

$$Q_{up}^g=\frac{\sum_{p=g+1}^{N_g}\Sigma_s^{p\to g}\phi^p}{\sum_{p=1}^{N_g}\phi^p}\,. \tag{9}$$

3. The *effective one-group low-order QD (1GLOQD) equations* for the total scalar flux $\phi=\sum_{g=1}^{N_g}\phi^g$ and current $J=\sum_{g=1}^{N_g}J^g$

$$\frac{dJ}{dx}+\overline{\Sigma}_a\phi=\frac{1}{k}\overline{\nu\Sigma_f}\phi\,, \tag{10}$$

$$\frac{d\overline{E}\phi}{dx}+\overline{\Sigma}_t J+\overline{\zeta}\phi=0\,, \tag{11}$$

$$J(0)=0\,,\quad J(X)=\overline{C}_X\phi(X)\,, \tag{12}$$

where group-averaged cross sections and factors are given by

$$\overline{E} = \frac{\sum_{g=1}^{N_g} E^g \phi^g}{\sum_{g=1}^{N_g} \phi^g}\,, \quad \overline{C}_X = \frac{\sum_{g=1}^{N_g} C_X^g \phi^g(X)}{\sum_{g=1}^{N_g} \phi^g(X)}\,, \tag{13}$$

$$\overline{\Sigma}_a = \frac{\sum_{g=1}^{N_g} (\Sigma_t^g - \Sigma_s^g)\phi^g}{\sum_{g=1}^{N_g} \phi^g}\,, \tag{14}$$

$$\overline{\Sigma}_t = \frac{\sum_{g=1}^{N_g} \Sigma_t^g |J^g|}{\sum_{g=1}^{N_g} |J^g|}\,, \quad \overline{\zeta} = \frac{\sum_{g=1}^{N_g} (\Sigma_t^g - \overline{\Sigma}_t) J^g}{\sum_{g=1}^{N_g} \phi^g}\,. \tag{15}$$

3. ITERATION METHODS

We now formulate a multi-level iteration method that is based on the MLOQD and 1GLOQD problems. The update of the angular flux is performed after the MLOQD equations are converged with the current estimation of the group QD factors. On each iteration for solving the MLOQD equations, the eigenvalue and one-group eigenfunction are evaluated from the 1GLOQD problem. Hereafter we refer to this method as the *iteration algorithm A*.

Assume that the eigenvalue $k^{(s)}$, group scalar flux $\phi^{g(s)}$, group current $J^{g(s)}$, total scalar flux $\phi^{(s)}$, total current $J^{(s)}$, and $\overline{\nu\Sigma_f}^{(s)}$ are known from the previous iteration. Here s is the iteration index. The nonlinear iteration method is defined as follows.

1. We use the group scalar flux $\phi^{g(s)}$ to compute $\overline{\Sigma}_s^{g(s)}$ (Eq. (4)).
2. Using $k^{(s)}$, $\phi^{(s)}$, $\overline{\nu\Sigma_f}^{(s)}$, $\overline{\Sigma}_s^{g(s)}$, we perform a transport sweep in each group by solving the transport equation (2) to calculate $\psi^{g(s+1/2)}$

$$\mu\frac{\partial}{\partial x}\psi^{g(s+1/2)} + \Sigma_t^g \psi^{g(s+1/2)} = \frac{1}{2}\left(\overline{\Sigma}_s^{g(s)} + \frac{\chi^g}{k^{(s)}}\overline{\nu\Sigma_f}^{(s)}\right)\phi^{(s)}\,, \quad g = 1, ..., N_g\,. \tag{16}$$

3. The updated angular flux $\psi^{g(s+1/2)}$ is used to calculate the group QD factors $E^{g(s+1/2)}$ and $C_X^{g(s+1/2)}$(Eq. 8).
4. We solve the coupled MLOQD and 1GLOQD problems

$$\frac{dJ^{g(s+1)}}{dx} + \left(\Sigma_t^g - \Sigma_s^{g\to g}\right)\phi^{g(s+1)} = \sum_{p=1}^{g-1}\Sigma_s^{p\to g}\phi^{p(s+1)} + \left(Q_{up}^{g(s+1)} + \frac{\chi^g}{k^{(s+1)}}\overline{\nu\Sigma_f}^{(s+1)}\right)\phi^{(s+1)}\,, \tag{17}$$

$$\frac{dE^{g(s+1/2)}\phi^{g(s+1)}}{dx} + \Sigma_t^g J^{g(s+1)} = 0\,, \tag{18}$$

$$J^{g(s+1)}(0) = 0\,, \quad J^{g(s+1)}(X) = C_X^{g(s+1/2)}\phi^{g(s+1)}(X)\,, \tag{19}$$

$$g = 1, ..., N_g\,,$$

$$\frac{dJ^{(s+1)}}{dx} + \overline{\Sigma}_a^{(s+1)}\phi^{(s+1)} = \frac{1}{k^{(s+1)}}\overline{\nu\Sigma_f}^{(s+1)}\phi^{(s+1)}\,, \tag{20}$$

$$\frac{d\overline{E}^{(s+1/2)}\phi^{(s+1)}}{dx} + \overline{\Sigma}_t^{(s+1)}J^{(s+1)} + \overline{\zeta}^{(s+1)}\phi^{(s+1)} = 0\,, \tag{21}$$

$$J^{(s+1)}(0) = 0\,, \quad J^{(s+1)}(X) = \overline{C}_X^{(s+1/2)}\phi^{(s+1)}(X)\,, \tag{22}$$

to determine a new estimation of the eigenvalue $k^{(s+1)}$, one-group eigenfunctions $\phi^{(s+1)}$ and $J^{(s+1)}$ as well as the group eigenfunctions $\phi^{g(s+1)}$ and $J^{g(s+1)}$ for the given $E^{g(s+1/2)}$ and $C_X^{g(s+1/2)}$. This iteration process is referred to as *s-iterations*. Note that the transport sweep in each group is performed just once on each s-iteration.

The equations (17)-(22) are solved by mean of the following nonlinear iteration scheme:

(a) Assume that the eigenvalue $k^{(s+1,n)}$, group scalar flux $\phi^{g(s+1,n)}$, group current $J^{g(s+1,n)}$, total scalar flux $\phi^{(s+1,n)}$, and total current $J^{(s+1,n)}$ are known from n-th inner iteration on the current $s+1$ outer iteration.

(b) We use the group scalar flux $\phi^{g(s+1,n)}$ to compute $\overline{\nu\Sigma_f}^{(s+1,n)}$ and $Q_{up}^{g(s+1,n)}$ (Eqs. (3) and (9)).

(c) Using $k^{(s+1,n)}$, $E^{g(s+1/2)}$, $C_X^{g(s+1/2)}$, $\overline{\nu\Sigma_f}^{(s+1,n)}$, $Q_{up}^{g(s+1,n)}$, and $\phi^{(s+1,n)}$, we solve the MLOQD equations (5)-(7) to calculate $\phi^{g(s+1,n+1)}$ and $J^{g(s+1,n+1)}$ in all groups

$$\begin{aligned}&\frac{dJ^{g(s+1,n+1)}}{dx} + \left(\Sigma_t^g - \Sigma_s^{g\to g}\right)\phi^{g(s+1,n+1)}\\ &= \sum_{p=1}^{g-1}\Sigma_s^{p\to g}\phi^{p(s+1,n+1)} + \left(Q_{up}^{g(s+1,n)} + \frac{\chi^g}{k^{(s+1,n)}}\overline{\nu\Sigma_f}^{(s+1,n)}\right)\phi^{(s+1,n)}\,,\end{aligned} \tag{23}$$

$$\frac{dE^{g(s+1/2)}\phi^{g(s+1,n+1)}}{dx} + \Sigma_t^g J^{g(s+1,n+1)} = 0\,, \tag{24}$$

$$J^{g(s+1,n+1)}(0) = 0\,, \quad J^{g(s+1,n+1)}(X) = C_X^{g(s+1/2)}\phi^{g(s+1,n+1)}(X)\,, \tag{25}$$

$$g = 1, ..., N_g\,.$$

(d) Using the updated group scalar flux $\phi^{g(s+1,n+1)}$ and current $J^{g(s+1,n+1)}$, we compute the averaged QD factors $\overline{E}^{(s+1/2,n+1)}$ and cross section data $\overline{\Sigma}_a^{(s+1,n+1)}$, $\overline{\nu\Sigma_f}^{(s+1,n+1)}$, $\overline{\Sigma}_t^{(s+1,n+1)}$, and $\overline{\zeta}^{(s+1,n+1)}$ (Eqs. (3) and (13)-(15)).

(e) Using $\overline{E}^{(s+1/2,n+1)}$, $\overline{C}_X^{(s+1/2,n+1)}$, $\overline{\nu\Sigma_f}^{(s+1,n+1)}$, $\overline{\Sigma}_a^{(s+1,n+1)}$, $\overline{\Sigma}_t^{(s+1,n+1)}$, and $\overline{\zeta}^{(s+1,n+1)}$, we solve 1GLOQD eigenvalue problem

$$\frac{dJ^{(s+1,n+1)}}{dx} + \overline{\Sigma}_a^{(s+1,n+1)}\phi^{(s+1,n+1)} = \frac{1}{k^{(s+1,n+1)}}\overline{\nu\Sigma_f}^{(s+1,n+1)}\phi^{(s+1,n+1)}\,, \tag{26}$$

$$\frac{d\overline{E}^{(s+1/2,n+1)}\phi^{(s+1,n+1)}}{dx} + \overline{\Sigma}_t^{(s+1,n+1)}J^{(s+1,n+1)} + \overline{\zeta}^{(s+1,n+1)}\phi^{(s+1,n+1)} = 0\,, \tag{27}$$

$$J^{(s+1,n+1)}(0)=0\,,\quad J^{(s+1,n+1)}(X)=\overline{C}_X^{(s+1/2,n+1)}\phi^{(s+1,n+1)}(X)\,, \tag{28}$$

to determine a new estimation of the eigenvalue $k^{(s+1,n+1)}$ and one-group eigenfunctions $\phi^{(s+1,n+1)}$ and $J^{(s+1,n+1)}$. These inner iterations for solving MLOQD equations are referred to as n-*iterations*.

The effective one-group k-eigenvalue problem (26)-(28) is solved by means of the iteration method based on the Weilandt shift approach [13]

$$\begin{aligned}\frac{dJ^{(s+1,n+1,l+1)}}{dx}+\left(\overline{\Sigma}_a^{(s+1,n+1)}-(1-\delta)\frac{1}{k^{(s+1,n+1,l)}}\overline{\nu\Sigma}_f^{(s+1,n+1)}\right)\phi^{(s+1,n+1,l+1)}\\ =\frac{\delta}{k^{(s+1,n+1,l)}}\overline{\nu\Sigma}_f^{(s+1,n+1)}\phi^{(s+1,n+1,l)}\,,\end{aligned} \tag{29}$$

$$\frac{d\overline{E}^{(s+1/2,n+1)}\phi^{(s+1,n+1,l+1)}}{dx}+\overline{\Sigma}_t^{(s+1,n+1)}J^{(s+1,n+1,l+1)}+\overline{\zeta}^{(s+1,n+1)}\phi^{(s+1,n+1,l+1)}=0\,, \tag{30}$$

$$J^{(s+1,n+1,l+1)}(0)=0\,,\quad J^{(s+1,n+1,l+1)}(X)=\overline{C}_X^{(s+1/2,n+1)}\phi^{(s+1,n+1,l+1)}(X)\,, \tag{31}$$

$$k^{(s+1,n+1,l+1)}=\frac{\int_0^X\overline{\nu\Sigma}_f^{(s+1,n+1)}\phi^{(s+1,n+1,l+1)}dx}{J^{(s+1,n+1,l+1)}(X)-J^{(s+1,n+1,l+1)}(0)+\int_0^X\overline{\Sigma}_a^{(s+1,n+1)}\phi^{(s+1,n+1,l+1)}dx}\,. \tag{32}$$

where δ is a parameter of the iteration method. This iteration process for solving 1GLOQD for determining k-eigenvalue is referred to as l-*iterations*.

We also consider a variation of the algorithm A in which we do not converge n-iterations for solving MLOQD equations, but perform just one n-iteration on every s-iteration [5]. Note that in such case the MLOQD equations are solved as many times as the multigroup transport equation. We refer to this method as the *iteration algorithm* A'.

To evaluate the effect of the 1GLOQD problem on the convergence of the multigroup transport iterations, we consider a method that uses the system of the MLOQD equations without the 1GLOQD equations. The update of the angular flux is performed after the MLOQD equations are converged with the given QD factors by means of the power iteration method and the eigenvalue is determined from the neutron conservation law. We refer to this method as the *iteration algorithm B*.

Assume that the eigenvalue $k^{(s)}$, group scalar flux $\phi^{g(s)}$, and group current $J^{g(s)}$ are known from the previous outer (s-th) iteration. The nonlinear iteration algorithm B is defined as follows.

1. Using $k^{(s)}$ and $\phi^{g(s)}$, we calculate $\psi^{g(s+1/2)}$ in all groups by solving the transport equation

$$\mu\frac{\partial}{\partial x}\psi^{g(s+1/2)}+\Sigma_t^g\psi^{g(s+1/2)}=\frac{1}{2}\sum_{p=1}^{N_g}\Sigma_s^{p\to g}\phi^{p(s)}+\frac{\chi^g}{2k^{(s)}}\sum_{p=1}^{N_g}\nu_f^p\Sigma_f^p\phi^{p(s)}\,, \tag{33}$$

$$g=1,...,N_g\,.$$

2. The updated angular flux $\psi^{g(s+1/2)}$ is used to calculate the group QD factors $E^{g(s+1/2)}$ and $C_X^{g(s+1/2)}$ (Eq. 8).

3. We solve the MLOQD k-eigenvalue problem

$$\frac{dJ^{g(s+1)}}{dx} + \Sigma_t^g \phi^{g(s+1)} = \sum_{p=1}^{N_g} \Sigma_s^{p\to g} \phi^{p(s+1)} + + \frac{\chi^g}{k^{(s+1)}} \sum_{p=1}^{N_g} \nu_f^p \Sigma_f^p \phi^{p(s+1)}\,, \tag{34}$$

$$\frac{dE^{g(s+1/2)}\phi^{g(s+1)}}{dx} + \Sigma_t^g J^{g(s+1)} = 0\,, \tag{35}$$

$$J^{g(s+1)}(0) = 0\,, \quad J^{g(s+1)}(X) = C_X^{g(s+1/2)} \phi^{g(s+1)}(X)\,, \tag{36}$$

$$g = 1, ..., N_g$$

to determine a new estimation of the eigenvalue $k^{(s+1)}$ and group eigenfunctions $\phi^{g(s+1)}$ and $J^{g(s+1)}$ for the given $E^{g(s+1/2)}$ and $C_X^{g(s+1/2)}$.

The eigenvalue problem (34)-(36) is solved by means of the power iteration method

$$\begin{aligned} &\frac{dJ^{g(s+1,n+1)}}{dx} + (\Sigma_t^g - \Sigma_s^{g\to g})\phi^{g(s+1,n+1)} \\ &= \sum_{p=1}^{g-1} \Sigma_s^{p\to g} \phi^{p(s+1,n+1)} + \sum_{p=g+1}^{N_g} \Sigma_s^{p\to g} \phi^{p(s+1,n)} + \frac{\chi^g}{k^{(s+1,n)}} \sum_{p=1}^{N_g} \nu_f^p \Sigma_f^p \phi^{p(s+1,n)}\,, \end{aligned} \tag{37}$$

$$\frac{dE^{g(s+1/2)}\phi^{g(s+1,n+1)}}{dx} + \Sigma_t^g J^{g(s+1,n+1)} = 0\,, \tag{38}$$

$$J^{g(s+1,n+1)}(0) = 0\,, \quad J^{g(s+1,n+1)}(X) = C_X^{g(s+1/2)} \phi^{g(s+1,n+1)}(X)\,, \tag{39}$$

$$g = 1, ..., N_g\,,$$

$$k^{(s+1,n+1)} = \frac{\sum_{g=1}^{N_g} \int_0^X \nu_f^g \Sigma_f^g \phi^{g(s+1,n+1)} dx}{\sum_{p=1}^{N_g} J^{g(s+1,n+1)}(X) - \sum_{p=1}^{N_g} J^{g(s+1,n+1)}(0) + \sum_{g=1}^{N_g} \int_0^X \Sigma_a^g \phi^{g(s+1,n+1)} dx}\,. \tag{40}$$

The variant of this method with only one n-iteration is referred to as the *iteration algorithm* B'.

4. NUMERICAL RESULTS

4.1. Definition of Multigroup Test Problems

We consider seven-group transport problems with realistic UO_2 and MOX assemblies. The cross sections are defined in the C5G7 MOX-Uranium benchmark [12]. Figure 1 shows the design of a fuel-pin cell. The designs of uranium and MOX assemblies are presented in Figures 2 and 3. The spatial mesh is uniform. There are 14 spatial intervals per pin cell. The width of the spatial interval is Δx=0.09 cm. A uniform angular mesh with 16 directions is used. The parameters of convergence criteria are listed in Table I. The δ-parameter of l-iterations (Eq. (29)) in the Weilandt shift method equals 10^{-3}.

Table I. Parameters of Convergence Criteria

Iterations	Criterion for Eigenvalue	Criterion for Eigenfunction
s	10^{-8}	10^{-6}
n	10^{-9}	10^{-6}
l	10^{-9}	10^{-6}

To discretize the transport equation (2), we use the step characteristic method. The MLOQD equations (5)-(7) are approximated by means of a second-order finite volume method. The 1GLOQD equations (10)-(12) are discretized consistently with the MLOQD equations.

Test 1 represents color-set calculations of MOX-UO_2 assemblies. There are two half-assemblies next to each other with reflective boundary conditions (see Figure 4). Test 2 is a 1D version of the C5G7 benchmark. The configuration of this test problem is shown in Figure 5. It consists of MOX and UO_2 assemblies with a reflector region containing a moderator. The width of the reflector region is equal to the assembly size. A reflective boundary condition at the left boundary and a vacuum boundary condition at the right one are used. Test 2 has some features that are specific for global reactor-core calculations.

Table II. Multiplication Factor

Test	1	2
k_{eff}	1.10265	1.02675

The calculated values of k_{eff} for these test problems are listed in Table II. Tables III-IV show numbers of iterations for various iterations processes in all algorithms. To see the effect of the 1GLOQD problem on the convergence, we compare the algorithm A with the algorithm B as well as A' with B'. To evaluate the performance of different methods, we compare numbers of transport sweeps (s-iterations) and how many times the MLOQD system was solved (n-iterations). One also needs to take into account that the use of the 1GLOQD equations adds an extra level of inner iterations. Each l-iteration for solving the 1GLOQD equations is very close to computational costs associated with solving single-group LOQD equations in the MLOQD problem. Thus, in a test with a particular number of energy groups, one can examine computational costs by considering the number of times a single-group LOQD system of equations was solved. Table V presents results on how many times LOQD equations were solved

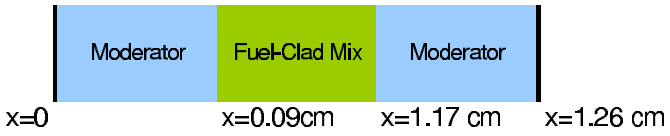

Figure 1. Pin cell design

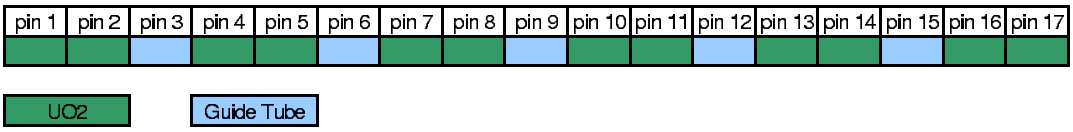

Figure 2. Design of the UO_2 assembly

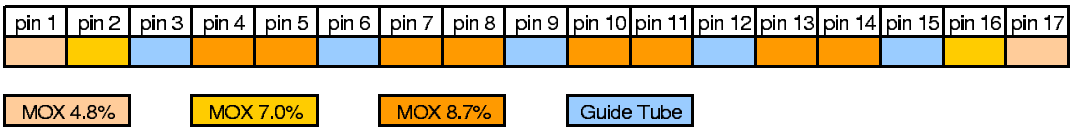

Figure 3. Design of the MOX assembly

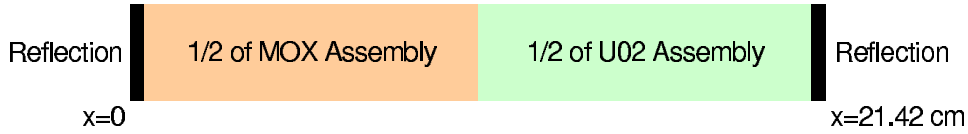

Figure 4. Test 1

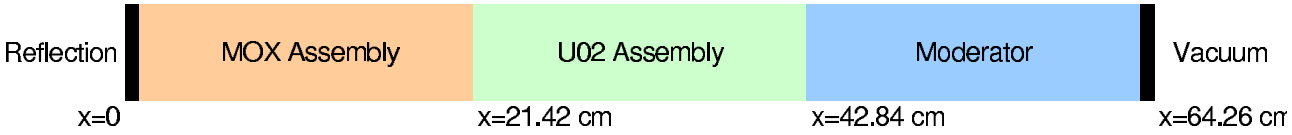

Figure 5. Test 2

per number of groups. This enables us to directly evaluate algorithms with and without 1GLOQD equations in the given tests from a different viewpoint. However, we note that the number of l-iterations required to solve 1GLOQD equations does not depend on number of energy groups in a problem. It is influenced by physical properties of a neutron transport problem.

4.2. Analysis of Test 1

In case of color-set calculations of MOX and UO_2 assemblies, the algorithm A' is far more efficient than the algorithm B' (see Table III). The number of transport sweeps per group performed by the algorithm A' is 12 (see Table V). There are 18 transport sweeps per group in case of the algorithm B'. This demonstrates the effect of using the 1GLOQD problem. The algorithms A and B have the same number of transport sweeps as the algorithm A'. The algorithm A has smaller amount of n-iterations than the algorithm B, because it uses the

Table III. Test 1. Total Number of Iterations for Each Iteration Process

Iteration Algorithm	s-iterations	n-iterations	l-iterations
A	12	81	312
A'	12	12	60
B	12	117	–
B'	18	18	–

Table IV. Test 2. Total Number of Iterations for Each Iteration Process

Iteration Algorithm	s-iterations	n-iterations	l-iterations
A	8	67	747
A'	14	14	210
B	8	273	–
B'	56	56	–

1GLOQD problem. If we compare the number of times the LOQD problem was solved per group number in this particular test (see Table V), we notice that the algorithm A needs to solve LOQD equations a little more times than the algorithm B. Overall the most efficient method in this test is the algorithm A' which uses the 1GLOQD problem and performs just one n-iteration on each s-iteration.

4.3. Analysis of Test 2

The algorithm B' results in 56 s-iterations. Thus, it requires significantly larger number of transport sweeps compared to any other iteration algorithms (see Table IV). The algorithm A' requires 14 transport sweeps. However, the algorithms A and B perform even less number of transport sweeps. They need just 8 transport sweeps per group. We also notice that the number of n-iterations in case of the algorithm A is smaller compared to the algorithm B by a factor of 4.1. This difference shows the ability of the 1GLOQD problem to accelerate the multigroup iterations. The algorithm A does perform extra calculations to execute the l-iterations for solving the 1GLOQD equations. If we compare the number of LOQD problems that were solved per group number in case of the algorithms A and B, we see that this number for the algorithm A is smaller than the one for the algorithm B by a factor of 1.6. These numerical results show that the algorithm A is the most efficient from various viewpoints in this test with features of global reactor-core calculations.

Table V. Number of Transport Sweeps and Low-Order QD Solutions per Number of Groups

Iteration Algorithm	Test 1		Test 2	
	Transport Sweeps	LOQD Solutions	Transport Sweeps	LOQD Solutions
A	12	126	8	174
A'	12	21	14	44
B	12	117	8	273
B'	18	18	56	56

5. CONCLUSIONS

We have formulated computational methods for solving the k-eigenvalue problem for the multigroup transport equation in 1D slab geometry. They are based on the quasidiffusion method and a set of nonlinearly coupled low-order problems that includes the multigroup and effective one-group low-order QD equations. The analysis showed that the effective one-group problem which is used to estimate the eigenvalue accelerates the multigroup transport iterations. Note that taking into account the type of reactor-physics problems, it is possible to further optimize the iteration methodology to gain even more efficiency.

The presented methodology can be applied to multidimensional problems and to various kind of eigenvalue problems such as the α-eigenvalue problem, calculation of critical size and critical concentration. The proposed approach of formulating an effective one-group problem can be applied to methodologies based on different methods for solving the transport equations, for example, the diffusion synthetic acceleration (DSA) and nonlinear diffusion acceleration (NDA) [1, 14].

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