International Conference on Mathematics, Computational Methods & Reactor Physics (M&C 2009)
Saratoga Springs, New York, May 3-7, 2009, on CD-ROM, American Nuclear Society, LaGrange Park, IL (2009)

UNCERTAINTY AND SENSITIVITY ANALYSIS IN THE NEUTRONIC PARAMETERS GENERATION OVER THE COUPLED THERMALHYDRAULIC-NEUTRONIC NUCLEAR POWER PLANT SIMULATIONS

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ABSTRACT

This paper presents a study of the influence of the uncertainty in the macroscopic neutronic information that describes a three-dimensional BWR core model on the most relevant results of the simulation of a Reactivity Induced Accident (RIA). The analysis of the BWR-RIA has been carried out with a three-dimensional thermal-hydraulic and neutronic model for the coupled system TRACE-PARCS. The cross section information has been generated by the SIMTAB methodology based on the joint use of CASMO-SIMULATE. The Best Estimate analysis consists of a coupled thermal-hydraulic and neutronic description of the nuclear system´s behaviour, uncertainties from both aspects should be included and jointly propagated. The statistically based methodology performs a Monte-Carlo kind of sampling of the uncertainty in the macroscopic cross sections. The size of the sampling is determined by the characteristics of the tolerance intervals by applying the Noether-Wilks formulas. A number of simulations equal to the sample size is carried out in which the cross sections used by PARCS are directly modified with uncertainty, and non-parametric statistical methods are applied to the resulting sample of the values of the output variables to determine their intervals of tolerance.

Key Words: Uncertainty, Sensitivity, Coupled TH-3DNK codes, Wilks Methodology

1. INTRODUCTION

Best-estimate computer programs make use of the best physical models and numerical solution methods available to simulate the behaviour of nuclear power plants. It is well known that their results are affected by the uncertainty in the methods and the models, and in order to draw proper

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conclusions from them, it is necessary to apply methodologies for the propagation of uncertainty so that it can be quantified. When the Best Estimate analysis consists of a coupled thermal-hydraulic and neutronic description of the nuclear system´s behaviour, uncertainties from both aspects should be included and jointly propagated. This paper presents a study of the influence of the uncertainty in the macroscopic neutronic information that describes a three-dimensional BWR core model on the most relevant results of the simulation of a Reactivity Induced Accident (RIA).

The analysis of the BWR-RIA has been carried out with a three-dimensional thermal-hydraulic and neutronic model for the coupled system TRACE-PARCS. The cross section information has been generated by the SIMTAB methodology based on the joint use of CASMO-SIMULATE, which parameterizes the neutronic information in the form of multidimensional tables as a function of fuel and moderator temperatures, and void fraction. The propagation of their uncertainty in the coupled calculation has been carried out based on the determination of tolerance intervals with a certain probability content and confidence level for the output variables of interest. The statistically based methodology performs a Monte-Carlo kind of sampling of the uncertainty in the macroscopic cross sections. The size of the sampling is determined by the characteristics of the tolerance intervals by applying the Noether-Wilks formulas. A number of simulations equal to the sample size is carried out in which the cross sections used by PARCS are ***directly modified with uncertainty***, and non-parametric statistical methods are applied to the resulting sample of the values of the output variables to determine their intervals of tolerance.

The systematic study has quantified the uncertainty in the cross section data by assuming it can be represented normal and uniform probability density functions (*pdf*), each one with three different variances of 1%, 5% and 10%. The purpose is to identify the influence of the type of pdf and of the variances in the uncertainty of the output variables considered, and to perform a sensitivity analysis to identify those cross sections that most influence those variables.

The simulated control rod withdrawal accident at zero power (HZP) is driven by the central control rod withdrawal from a core position with high reactivity worth, starting at criticality with a very low power level (10^{-9} times nominal value).The control rod bank are withdrawn, at 1 m/s. The evolution consists of a continuous reactivity insertion. The main factor limiting the consequences of the accident is the void feedback reactivity and the Doppler Effect. The peak power occurs while important power distribution changes take place in the core and also while the rod extraction continues. In this accident, the maximum power is less important than its time integral. If the reactivity insertion rate is low, the heating of the fuel may be sufficient to have Doppler antireactivity, balancing the inserted reactivity while the power level is still under the trip level.

From a subcritical initial state at zero power (28.9 W), the control rod bank 3 is withdrawn in 0.916 seconds. Basically the evolution consists of a continuous reactivity insertion. With a higher reactivity insertion rate, the transient produces a fast power burns. If the reactivity insertion rate is low, the heating of the fuel may be sufficient to have Doppler antireactivity effect, balancing the inserted reactivity. The consequence of this transient is a rapid reactivity insertion along with an adverse core power distribution. If this transient takes place, a fuel rod thermal transient that could cause departure from nucleate boiling could also occur, together with limited fuel damage. The transient is terminated by the Doppler effect caused by the increased fuel temperature but this occurs before conditions are reached that can be dangerous for the nuclear power plant safety

2. DESCRIPTION OF TRACE/PARCS MODEL

The initial steady state is a HZP where the moderator temperature is 563.71 K and the initial density is 731.15 kg/m^3 and the reactor power is 28.9 W. The fixed thermal-hydraulic variables should be equally distributed through the whole core. The transient is started by the fall of the central control rod. There are 177 control rods that can be grouped according to their initial insertion degree. Fig. 1 shows the initial position of the 3 assigned groups. Groups 2 and 3 are fully inserted at the initial time step, and group 1 (central control rod) is fully extracted. During the transient, group 3 is extracted 12 notches at 1 m/s [1].

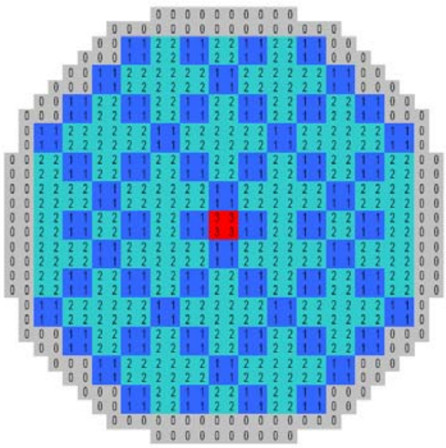

Figure 1: Control rod groups assignment.

The simulations are made by a coupled 3-D kinetics/core thermal-hydraulic boundary conditions model. The developed TRACE model was built based on different thermal-hydraulic components. The core is modeled by 14 channels, bottom and top boundary conditions are specified in this model using the FILL and BREAK components, Fig. 2. The 14 thermal-hydraulic channels are coupled to the neutronic model according to the radial distribution shown in Fig. 3.

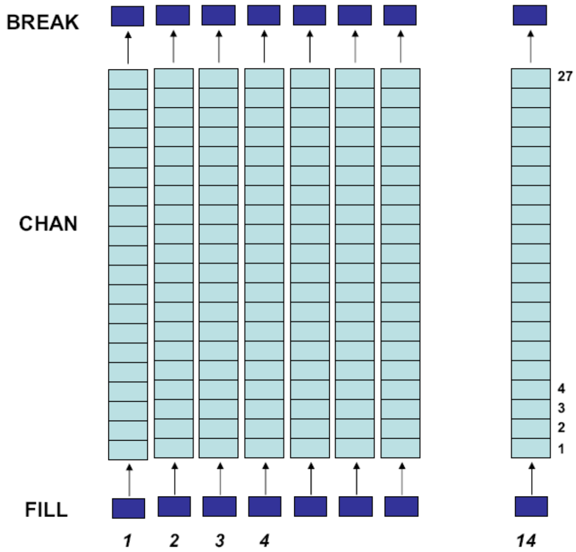

Figure 2: TRACE model.

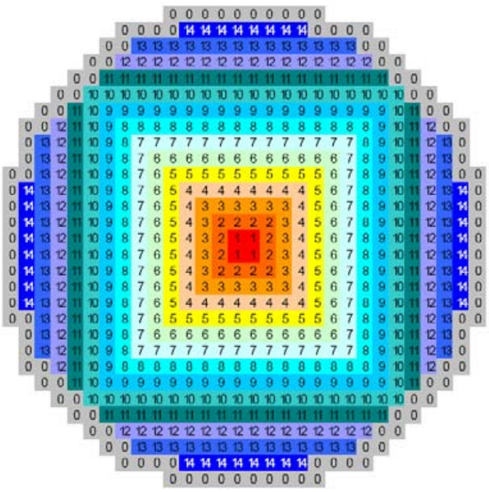

Figure 3: Radial channel mapping.

Radially, the core is divided into cells 15.24 cm, each corresponding to one fuel assembly plus a radial reflector. There are 624 fuel assemblies and 116 reflector assemblies. Axially, the core is divided into 27 layers (25 fuel layers plus top and bottom reflector) with a height of 15.24 cm each, being the total active core height 381 cm. There are 76 different assembly types. For the neutronic model, fuel assemblies are considered homogeneous having 1878 different compositions. The set of 3D cross sections are obtained from tables generated with the translator SIMTAB from SIMULATE to PARCS code.

3. BEST ESTIMATE CALCULATIONS AND UNCERTAINTY

A source of uncertainties, related with simplifications, assumptions, nodalization schemes and neutronic parameters generation may affect the behavior of the code physical models and the solution procedures.

The methodology used in this work is based on the use of statistical techniques to calculate sensitivity and uncertainty information from the results of a computer simulation. Input variables like the neutronic parameters are all considered as sources of uncertainty, which is then propagated to the code results.

The statistical techniques are based on well-established concepts and tools from probability and statistics. One important step is the assignment of the so-called Subjective Probability Density Functions (SPDF) to quantify the uncertainty of the input neutronic parameters. The selection of these functions is the most subjective part of the methodology. The SPDFs reflect how well the uncertainty in input parameters and code models is known. This knowledge may range from detailed data on mean values and other statistical parameters to a simple notion of the expected range of variation of a certain parameter, i.e. maximum and minimum values. In addition, dependencies between parameters and models must also be accounted for during this phase, by means of appropriate correlations or joint probability distributions, provided they are known or can be reasonably derived. The rest of the development of the methodology is conditioned by the quantification of the uncertainty in the results of the code simulations of NPP transients. The statistical part is objective and based on probability theory and statistical methods. The process starts with the generation of a random sample of the input parameters and of the parameters that quantify the uncertainty in the code models, according to the SPDFs assigned to them. After executing the code for all the elements of the sample, relevant statistically based uncertainty and sensitivity information can be extracted from the code results. The minimum number of code calculations is given by the Wilks' formula [2], [3], and according to the degree of precision desired for the uncertainty measures. Thus, the number of required calculations does not depend on the number of input parameters or on any assumption about the probability distribution of the results [4].

During the assessment phase, the uncertainty information computed from the analysis of the code results is usually in the form of uncertainty limits.

Computer codes are deterministic by nature, that is, for a given set of input values, they yield a unique output. However, if the input parameters are treated as random variables, then the stochastic (random) nature of the inputs translates into a stochastic nature of the outputs. The justification of the use of statistical methods to computer codes is based on this premise.

Randomness in the input variables can be due to various factors, e.g., statistical variation, unknown values, etc. Uncertainty is related to the random nature of these variables and is commonly quantified by means of Probability Density Functions (PDFs). This uncertainty is propagated through a deterministic computer model to the results. The uncertainty in the code results is measured in the form of tolerance bands, standard deviations, or even a PDF if possible.

4. QUANTIFICATION OF UNCERTAINTIES

Once the sample of code output results, $(Y)_N$, has been obtained by executing the code, for the sample of input values $(X)_N$, there remains the task of extracting statistically meaningful information from the sample of outputs, $(Y)_N$. This is accomplished by applying appropriate techniques that use this information to generate uncertainty estimations from the code results.

As previously discussed, as a result of describing the uncertainties in the input variables with probability distribution functions, the code output results are also random variables. The PDFs of the code results contain all the information needed to compute their uncertainty. The problem is that such functions are usually unknown. Therefore, in order to quantify exactly the uncertainty one should generate the PDFs from the sampled output values. Thus, the only remaining alternative is to obtain as much information as possible about the PDFs properties and main parameters from empirical distribution functions and estimators.

One of the most useful estimators is the quantiles. A quantile is a point in the sample space of a random variable such that the probability of the variable being less than or equal to the quantile is a given value $p < 1$. In mathematical terms, a $p = 0.95$ quantile for a random variable X is expressed as:

$$P(X \leq x_{0.95}) = 0.95 \tag{1}$$

and, in general, the p quantile, χ_p is:

$$P(X \leq x_p) = p \tag{2}$$

When the probabilities are expressed in percentages, the quantiles are known as percentiles. In the case of the 95% percentile, (equal to the 0.95 quantile) there is a 95% probability that the value of the random variable X will be less than or equal to 95%.

If X is a time dependent random variable $X(t)$, e.g. the output of a transient code, then the quantiles vary with time and can be used to quantify uncertainty for the duration of the transient. The use of quantiles in the analysis is important when the sampling size is big.

Another important concept is the tolerance interval, which is a more qualitative and conservative measure of uncertainty [5]. In order to understand its application, it is necessary to make the distinction between confidence and probability content of a random variable. Thus, a tolerance interval *(L,U)* is an estimate of a random variable that contains a specified fraction of the variable's probability, p, with a prescribed level of confidence, γ [6]. Tolerance intervals are constructed from sampled data so as to enclose $p\%$ of the population of a random variable X with a given confidence γ. They show where most of the population of the random variable X can be expected to lie.

If a random sample of output values, $((Y)_1,\ldots, (Y)_n)$, has a normal PDF, it is possible to compute tolerance intervals from the sample mean, my, and sample standard deviation, s_y as:

$$(L,U) = (m_y - Ks_y, m_y + Ks_y) \tag{3}$$

where K is the so-called tolerance factor, whose values depend on the sample size, probability coverage, p, and confidence level, γ. For example, for a sample of N = 100, a coverage of 90% probability and a confidence of 95%, K is equal to 1.874 [5]. The values for K are tabulated for different p, N, and γ in standard statistical tables [6], [7]. In the case of the output values are not normally distributed we will use non-parametric estimations [8].

5. QUANTIFICATION OF SENSITIVITY

One of the advantages of the methodology described in this Section is that it is not necessary to perform in a priori selection of important input variables and code models. From the information contained in the sample of the input variables and in the sample of the code output results, statistical measures of correlation and importance can be computed.

The purpose of a sensitivity analysis is to quantify the influence of the input variables on the code results. Sensitivity measures can assign a numerical value to this influence, and thus, be useful for an a posteriori ranking of the importance of each of the input variables with respect to the output variable of interest.

The most detailed sensitivity measures are local in nature, that is, they are calculated for variations in the neighborhood of a point in the sample space of the input variable. Usually, the point of interest for the sensitivity is the nominal value, x_o, (nominal input values) and the local sensitivity measure is a vector, s, of partial first order derivatives of the code output variable of interest Y with respect to each of the code input variables X_i

$$s(x_0) = \left[\frac{\partial Y}{\partial X_1}, \frac{\partial Y}{\partial X_2}\right]_{x_o} \tag{4}$$

whose value is a function of the vector of input variables x_o.

Global measures statistically quantify the variability of the code results with respect to the entire sample space of the input variables. They are less precise than the local measures, because the influence of a given input variable is quantified by a single number covering the entire range of variation of the variable. However, they are much less computer intensive and provide valuable information for the ranking of input variables by importance. They are especially efficient when the effect of the variation of an input variable on the code results is not dramatically different at two distinct locations within the interval of variation of the variable.

Most of the global sensitivity measures are related to regression analysis. Some of them are useful to detect linear relationships, and some others, like the so-called Rank Correlations are useful to quantify relations between variables that behave monotonically with respect to each other (e.g., smooth variations of one variable correspond to smooth variations of the other one.) Comparison between these two types of measures applied to the same set of data can detect non-linearity in the behavior of the computer code.

Examples of linear measures are the Simple Correlation Coefficient SCC or Pearson's moment product, and the Partial Correlation Coefficient (PCC). Their rigorous definition can be found in [7] and [5]. The most important advantage of the PCC is that it eliminates the linear influence of the remaining input variables on the output, leaving only that of the input variable whose sensitivity is being calculated.

In order to deal with models which are not clearly linear, Simple (SRCC) or Partial Rank Correlation (PRCC) coefficients can be used. To calculate these two measures, the sample values of the input and output whose relationship is to be determined are separately 'ranked', i.e. forming two separate, ordered lists in decreasing or increasing order, and a rank (ordinal) assigned to each value. If the two 'un-ranked' original series of values are related monotonously (see above), then the ordered series are linearly related. This is true even if the relationship between the unordered series is not linear. Thus the absolute values of SRCC and PRCC will quantify the degree of relationship between the given input and the output of interest. The closer the values of these coefficients to one, the more influence the input will have on the behavior of the output. Several authors have proposed formulas to compute the SRCC and PRCC from the sampled values. The most used in sensitivity analysis is the Spearman's coefficients [7].

In [5] a comparison between the interpretation of the meaning of a regression coefficient and a correlation coefficient sheds light on their application to sensitivity analysis. While they are mathematically related, a correlation coefficient measures the degree of strength of the relation between the input and the output. The normalization of these coefficients by the sample standard deviations makes them non-dimensional. On the other hand, the regression coefficients measure the intensity of this relation, i.e. how much the output changes per unit of change in the input. As an example, in a time dependent case, an input and output variables can be 'strongly' correlated during the whole calculation, but the intensity of this relation may vary with time, e.g. take the linear model $y = a + b(t)\cdot x$, there is a complete linear relationship (correlation coeff. = 1) between y and x, but the intensity of this relationship varies with time as given by the regression coefficient $b(t)$.

From the discussion above, it is clear that the selection of the sensitivity measure depends on the information desired from the analysis:

If a ranking of input parameters according to their global importance for a given output is sought, then a correlation coefficient should be selected.

When detailed knowledge of the influence of an input variable on a given output for the entire range of variation the parameter is desired, then regression coefficients are needed.

Finally, if local, detailed sensitivity information in a neighborhood about the nominal values of the input variables is required, a local measure based on partial derivatives about the nominal point in the input space should then be computed.

6. RESULTS

The uncertainty and sensitivity analysis is based on a sample of 100 simulations, sufficient to guarantee double tolerance limits for the output variables covering 95% of their uncertainty with 95% of statistical confidence. Intervals, as a function of time, have been produced for maximum power, maximum fuel temperature and the reactivity values: total, Doppler, void and control rod. A sensitivity analysis for the seven macroscopic neutronic parameters with assigned uncertainty (1:D_F, 2:D_{Th}, 3:Σ_s, 4:$\Sigma_{a,F}$, 5:$\Sigma_{a,Th}$, 6:$\nu\Sigma_{f,F}$, 7:$\nu\Sigma_{f,Th}$) has also been carried out for the same output variables as a function of time (time-dependent sensitivity) and for maximum power, time to maximum power and maximum and minimum reactivity values at the time they were reached (scalar sensitivity).

The uncertainty analysis has shown that variations about 1% have little influence in the output variables of interest, e.g. similar peak power and time at which it is reached. Comparison of the type of *pdf* used has shown that a normal *pdf* results in more “conservative” results from the point of view of maximum peak value and spread of the time of the power transient, if compared to an uncertainty quantified with a uniform *pdf*. Similar conclusions can also be extracted for the uncertainty in the reactivities.

In figures 4 and 5 we show the extreme results for the 10% normal transients.

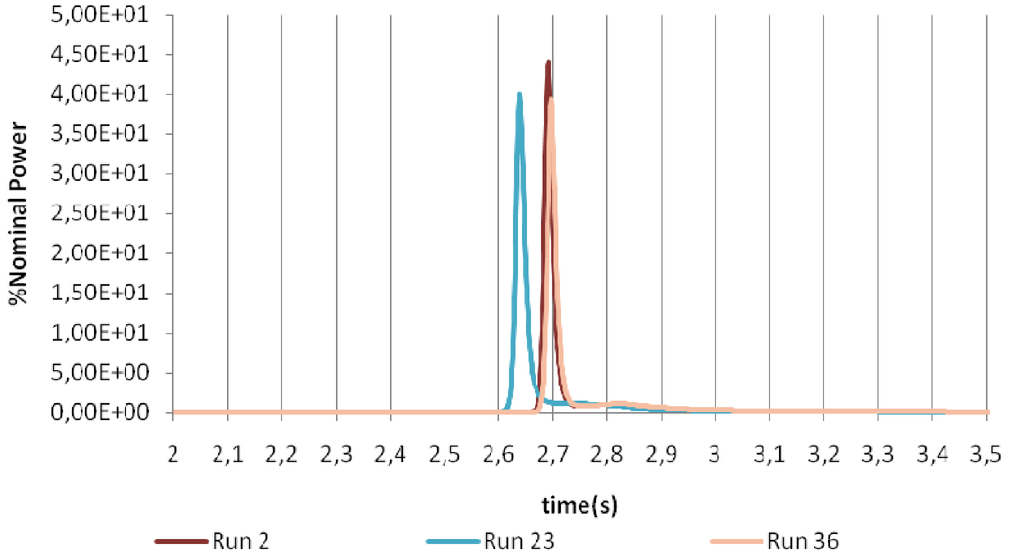

Figure 4. Cases with power peaks above the 40% of nominal power.

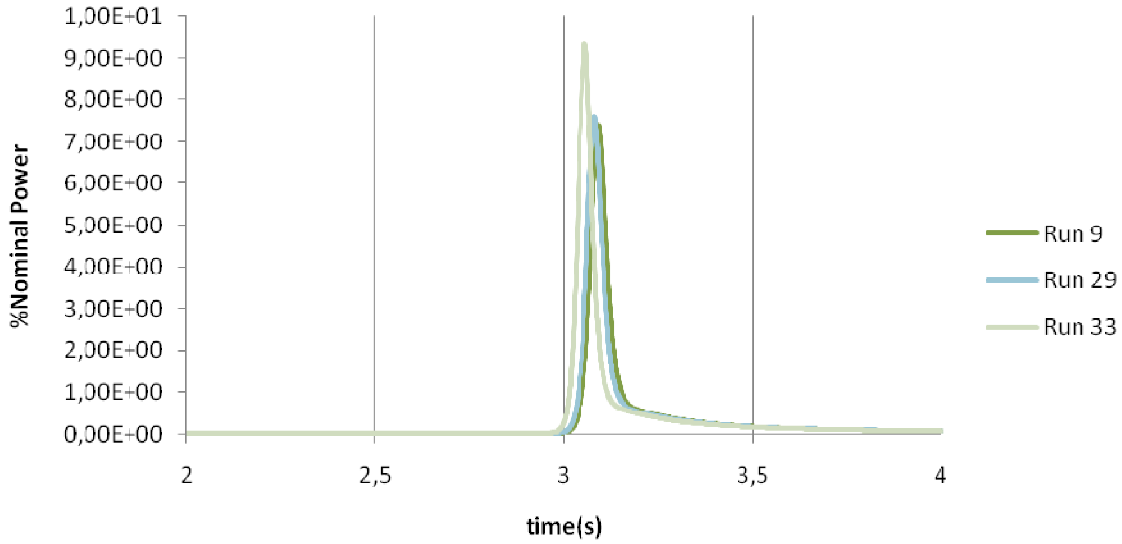

Figure 5. Cases with power peaks under the 10% of nominal power.

Increasing the uncertainty, there is a spread in the time at which the power rises and falls, but the maximum value is not greatly changed, even for uncertainty as large as 10% (see Fig. 6).

The sensitivity analysis (see Fig. 7) has shown that the most influential uncertainties correspond to the fast diffusion coefficient (1), which determines the leakage, the scattering cross section (3), which determines the moderation, and both fission cross sections, which determine the rate of fission power release. The uncertainty in the absorption cross sections appear to have very little influence in the output variables considered.

Finally, an analysis of the type of uncertainty of the output variables has shown that, none of them can be assumed to have a normally distributed uncertainty, even when the uncertainty in the neutronic data is assumed normally distributed. This is justified by the non-linear character of the feedback mechanisms driving the BWR system response during the transient, which distorts the normality as the uncertainties are propagated to the results.

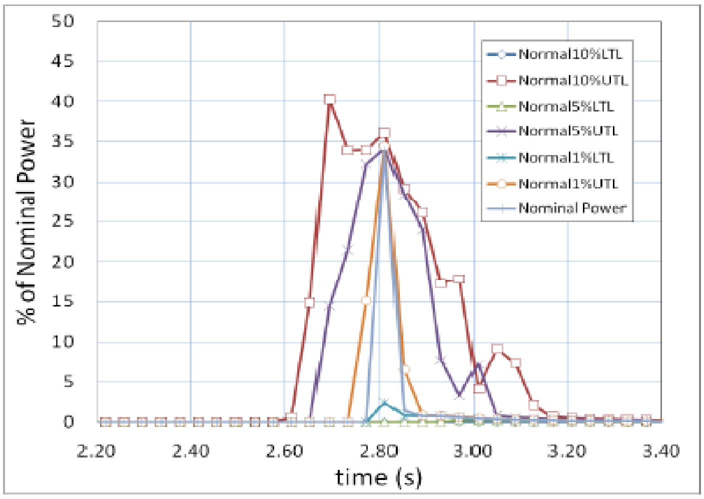

Figure 6: Tolerance Intervals (95%,95%) for Normal Uncertainty in Cross-sections (*LTL: Low Tolerance Limit, UTL: Upper Tolerance Limit*).

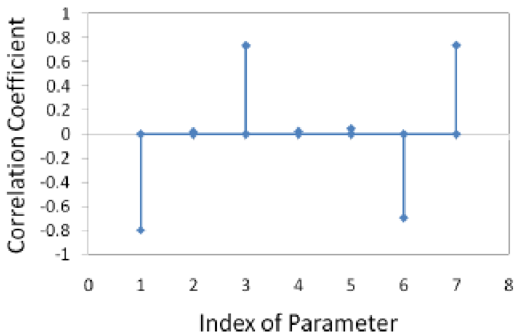

Figure 7: Partial Product-Moment Correlation Coefficient for Maximum Power. R^2=0.85. (1:D_F, 2:D_{Th}, 3:Σ_s, 4:$\Sigma_{a,F}$, 5:$\Sigma_{a,Th}$, 6:$\nu\Sigma_{f,F}$, 7:$\nu\Sigma_{f,Th}$).

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