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COMPARISON OF PROBABILISTIC AND DETERMINISTIC ERROR PROPAGATION CALCULATIONS IN DRAGON

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ABSTRACT

One of the major goal of lattice calculations is to evaluate cell homogenized and few group condensed cross sections for finite reactor calculations. However, there is no general provision in most lattice codes to take into account the impact of lattice property uncertainties (enrichment, temperature, density) on the final cross sections. Here we propose two different approaches to resolve this problem. The first approach is probabilistic in nature and relies on probability distribution functions to generate perturbations in the cell properties that can then be analyzed using the lattice code. The uncertainties in the cross section are then inferred from these calculations using a statistical analysis. The perturbative calculations can be performed using two different techniques: the direct technique where a new transport solution is obtained for each perturbation and the generalized perturbation theory (GPT) technique where the perturbed cross sections are evaluated approximately using perturbation theory methods. The second approach we will consider is deterministic and uses the GPT method to evaluate the sensitivity coefficients required for error propagation calculations.

These two approaches are compared to assess their relative performance for error propagation calculations. Here fuel and coolant temperature perturbations for a simple PWR fuel pin are considered. Our analysis shows that the results, obtained using the deterministic approach, are very similar to the reference probabilistic results but with a considerable CPU gain. Using an approximate rather than an exact flux solution for the probabilistic approach has only a very small impact on the error predictions and has much smaller impact on the CPU requirements.

Key Words: Error propagation, Perturbation theory, Sensitivity coefficients

1. INTRODUCTION

Lattice codes, such as APOLLO-2 [1], HELIOS [2] and DRAGON [3], are used to solve the transport equation inside a cell or an assembly to determine the k_{eff} and the multigroup neutron flux distribution for a given problem. Using this flux distribution they then produce region homogenized and few groups condensed cross sections for finite reactor calculations. Fixed lattice properties are generally considered for such calculations. Accordingly, there is no provision to take into account the impact of uncertainties in the lattice properties such as enrichment, temperature and density, on the final cross section results.

Here we propose two different techniques to resolve this problem. The first technique is probabilistic in nature and relies on using a Gaussian probability distribution function to generate a large number of perturbations on the cell properties representative of the uncertainties in the lattice parameters. The results of a series of transport calculations for each perturbation can then be analyzed and uncertainties in k_{eff} and the few groups homogenized cross sections derived. [4]

One alternative to this direct approach consists of using a generalized perturbation theory (GPT) solver such as the one programmed in DRAGON [5–8] to evaluate the effect of each perturbation on the final cross sections. The second technique is deterministic and relies on the use of GPT to evaluate the sensitivity coefficients required for error propagation calculations. [6, 9] These sensitivity coefficients can then be used to evaluate the uncertainties in cell reactivity and cross sections therefore bypassing the repetitive transport calculations implied by the probabilistic approach.

Here we will apply these techniques to a simple PWR fuel cell. Errors in k_{eff} and the few group cell averaged absorption cross sections resulting from uncertainties in the fuel and coolant temperatures will be evaluated and compared for precision and CPU requirements.

In Section 2 we discuss our two probabilistic approaches (direct and GPT) followed in Section 3 by a presentation of the deterministic approach. Section 4 presents the problem to be analyzed as well as the results obtained using the different techniques proposed. Finally, in Section 5 we present our conclusions concerning the efficiency and precision of the GPT (probabilistic and deterministic) based approaches.

2. PROBABILISTIC METHOD

Here we will assume that the uncertainties on a given set of lattice properties $\{x\}$ are statistical in nature (due to measurements for example) and can be sampled from a normally distributed probability density function

$$p(x_n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_n - \bar{x}_n}{\sigma}\right)^2} \tag{1}$$

where σ is the standard deviation (the uncertainty) and $\bar{x}_n$ the measured (reference) value of parameter $x_n \in \{x\}$. Now consider a functional G that depends on $\{x\}$. The value of G evaluated at the reference parameters will be denoted $G[\{\bar{x}\}]$.

For a probabilistic evaluation of the standard deviation σ_G associated with $G[\{x\}]$, the following procedure can be selected:

1. Using a random number generator, drawn one value for each parameter $x_n \in \{x\}$ using the normal distribution described in Eq. 1. Repeat this procedure as many time as required (K times) the successive sets of parameters being denoted $\{x\}_k$.
2. Evaluate $G[\{x\}_k]$ for each k.
3. Compute $\bar{G}[\{x\}]$, the mean of $G[\{x\}]$, using the relation

$$\bar{G}[\{x\}] = \frac{1}{K}\sum_{i=1}^{K} G[\{x\}_k] \tag{2}$$

and compare with $G[\{\bar{x}\}]$. One would expect that for K sufficiently large $\bar{G}[\{x\}] \to G[\{\bar{x}\}]$.

4. Evaluate the uncertainty σ_G using

$$\sigma_G = \sqrt{\frac{1}{K-1}\sum_{i=1}^{K}\left(G\left[\{x\}_k\right] - \bar{G}\left[\{x\}\right]\right)^2} \tag{3}$$

In lattice calculations the functionals $G[\{x\}]$ of interest are $k_{\mathit{eff}}[\{x\}]$ the eigenvalue of the transport equation or $\Sigma^G_{y,H}[\{x\}]$ the cross section of type y homogenized over region H and condensed over the macrogroup G:

$$\Sigma^G_{y,H}[\{x\}] = \frac{\sum_{g\in G}\sum_{i\in H} V_i \Sigma^g_{y,i}[\{x\}]\phi^g_i[\{x\}]}{\sum_{g\in G}\sum_{i\in H} V_i \phi^g_i[\{x\}]} \tag{4}$$

Here $\phi^g_i[\{x\}]$ is the neutron flux, solution to the transport equation, and $\Sigma^g_{y,i}[\{x\}]$ the multigroup absorption cross section in region i of volume V_i. In lattice codes, evaluating $k_{\mathit{eff}}[\{x\}]$ and $\Sigma^G_{y,H}[\{x\}]$ is not straight forward but generally involves many steps. For a DRAGON calculation based on the collision probability method this implies

1. Evaluating the macroscopic cross section associated with each region (without resonance self shielding) using

$$\tilde{\Sigma}^g_{y,i}[\{x\}] = \sum_{l=1}^{L} N_{i,l}[\{x\}]\sigma^g_{y,l}[\{x\}] \tag{5}$$

   where $N_{i,l}[\{x\}]$ represents the concentration of isotope l in mixture i and $\sigma^g_{y,l}[\{x\}]$ is the infinite dilution temperature interpolated microscopic cross section associated with isotope l.

2. Correcting for the resonance self-shielding effect using

$$\Sigma^g_{y,i}[\{x\}] = \tilde{\Sigma}^g_{y,i}[\{x\}] + \sum_{l=1}^{L} N_{i,l}[\{x\}]\delta\sigma^g_{y,l}[\{x\}] \tag{6}$$

   where $\delta\sigma^g_{y,l}[\{x\}]$ is the resonance self-shielding correction for isotope i. The computation of $\delta\sigma^g_{y,l}[\{x\}]$ involves a non linear iterative process where at each step the transport equation is effectively solved in the resonance groups.

3. Solving the multigroup neutron transport equation for the flux and k_{eff} using $\Sigma^g_{y,i}[\{x\}]$. Here, two steps are considered:

   (a) Compute the collision probabilities $p_{i,j}(\Sigma^g_{y,i}[\{x\}]) = p^g_{i,j}[\{x\}]$.

   (b) Solve the linear system

$$\phi^g_i[\{x\}] = \sum_j p_{i,j}[\{x\}]\left(S^g_j[\{x\}] + \frac{1}{k_{\mathit{eff}}[\{x\}]}F^g_j[\{x\}]\right) \tag{7}$$

   where $S^g_j[\{x\}]$ and $F^g_j[\{x\}]$ are respectively the scattering and fission sources.

4. Homogenizing and condensing the cross sections using Eq. 4.

This procedure is called the direct statistical approach and will be selected as our reference.

In DRAGON, one can also approximate steps 3 and 4 above using a generalized perturbation theory method. In this case, one first computes $k_{\mathit{eff}}[\{\bar{x}\}]$ and $\Sigma^G_{y,H}[\{\bar{x}\}]$ using Eq. 4. Then in addition to solving Eq. 7 for the flux, one also solves the adjoint and generalized adjoint equations for $\phi^{g,*}_i[\{\bar{x}\}]$ and $\Gamma^{g,*}_{y,i}[\{\bar{x}\}]$ (see Reference [6]). Using a first order perturbation method one then obtains

$$\Sigma^{\text{GPT},G}_{y,H}[\{x\}] = \Sigma^G_{y,H}[\{\bar{x}\}] + \delta\Sigma^G_{y,H}[\{x\}] \tag{8}$$

$$k^{\text{GPT}}_{\mathit{eff}}[\{x\}] = k_{\mathit{eff}}[\{\bar{x}\}] + \delta k_{\mathit{eff}}[\{x\}] \tag{9}$$

where

$$\frac{1}{\delta k_{\mathit{eff}}[\{x\}]} \approx \frac{\sum_{g\in G}\sum_{i\in H} V_i \phi^{g,*}_i[\{\bar{x}\}]\delta\mathbf{M}[\{x\}]\phi^g_i[\{\bar{x}\}]}{\sum_{g\in G}\sum_{i\in H} V_i \phi^{g,*}_i[\{\bar{x}\}]\mathbf{F}[\{\bar{x}\}]\phi^g_i[\{\bar{x}\}]} \tag{10}$$

$$\delta\Sigma^G_{y,H}[\{x\}] \approx \frac{\sum_{g\in G}\sum_{i\in H} V_i \delta\Sigma^g_{y,i}[\{x\}]\phi^g_i[\{\bar{x}\}]}{\sum_{g\in G}\sum_{i\in H} V_i \phi^g_i[\{\bar{x}\}]} + \sum_{g\in G}\sum_{i\in H} \Gamma^{g,*}_i[\{\bar{x}\}]\delta\mathbf{M}[\{x\}]\phi^g_i[\{\bar{x}\}] \tag{11}$$

with $\mathbf{F}[\{\bar{x}\}]$, the fission operator and

$$\delta\mathbf{M}[\{x\}] = \left(\frac{1}{k_{\mathit{eff}}[\{\bar{x}\}]}\delta\mathbf{F}[\{x\}] - \delta\mathbf{A}[\{x\}]\right)$$

Here $\delta\Sigma^g_{y,i}[\{x\}]$, $\delta\mathbf{F}[\{x\}]$ and $\delta\mathbf{A}[\{x\}]$ are respectively the perturbations in $\Sigma^g_{y,i}$, the fission operator and the transport operator resulting from the change $[\{\bar{x}\}] \rightarrow [\{x\}]$. This procedure is called the GPT statistical approach. It should be much faster than the direct statistical approach since :

- It implies only a single CP evaluation (very CPU intensive) since both the adjoint and generalized adjoints equations rely on the same collision probabilities matrix as the direct equation.
- The number of flux solutions required is proportional to the number of homogenized/condensed cross section to be generated and independent of the number of perturbations considered.
- Evaluating $k^{\text{GPT}}_{\mathit{eff}}[\{x\}_k]$ and $\Sigma^{\text{GPT},G}_{y,H}[\{x\}_k]$ for the k^{th} perturbation only consist of performing a series of generalized scalar products (see Eq. 10 and 11).

Once k_{eff} and $\Sigma^G_{y,H}$ have been evaluated for the set of parameters $\{x\}_k$, the means and standard deviations are computed. One expects the mean of each distribution to be identical to the reference value of the functional while the standard deviation represents the uncertainty associated with the perturbed parameters.

3. DETERMINISTIC METHOD

The deterministic evaluation of the standard deviation is based on the following relation: [6,9]

$$\frac{\sigma_G}{G[\{\bar{x}\}]} = \sqrt{\sum_n \left(\frac{\sigma_n S_G^n}{x_n}\right)^2 + \sum_{\substack{n,m \\ n\neq m}} c_{n,m} \left(\frac{\sigma_n S_G^n}{x_n}\right)\left(\frac{\sigma_m S_G^m}{x_m}\right)} \tag{12}$$

where σ_n is the variance associated with parameter $x_n \in \{x\}$, $c_{n,m}$ the covariance factor between x_n and x_m and S_G^n the sensitivity coefficient of the functional $G[\{x\}]$ with respect to parameter x_n. These sensitivity coefficients can be computed using the following relations: [6]

$$S^n_{k_{\mathit{eff}}} = -\frac{x_n}{k_{\mathit{eff}}[\{\bar{x}\}]}\left[\frac{\sum_{g\in G}\sum_{i\in H} V_i \phi^{g,*}_{,i}[\{\bar{x}\}]\partial_n \mathbf{M}\phi^g_i[\{\bar{x}\}]}{\sum_{g\in G}\sum_{i\in H} V_i \phi^{g,*}_i[\{\bar{x}\}]\mathbf{F}[\{\bar{x}\}]\phi^g_i[\{\bar{x}\}]}\right] \tag{13}$$

$$S^n_{\Sigma^G_{y,H}} = \frac{x_n}{\Sigma^G_{y,H}[\{\bar{x}\}]}\left[\frac{\sum_{g\in G}\sum_{i\in H} V_i \partial_n \Sigma^g_{y,i}\phi^g_i[\{\bar{x}\}]}{\sum_{g\in G}\sum_{i\in H} V_i \phi^g_i[\{\bar{x}\}]} + \sum_{g\in G}\sum_{i\in H}\Gamma^{g,*}_i[\{\bar{x}\}]\partial_n \mathbf{M}\phi^g_i[\{\bar{x}\}]\right] \tag{14}$$

where

$$\partial_n \mathbf{M} = \left(\frac{1}{k_{\mathit{eff}}[\{\bar{x}\}]}\frac{\partial \mathbf{F}[\{\bar{x}\}]}{\partial x_n} - \frac{\partial \mathbf{A}[\{\bar{x}\}]}{\partial x_n}\right)$$

$$\partial_n \Sigma^g_{y,i} = \frac{\partial \Sigma^g_{y,i}[\{\bar{x}\}]}{\partial x_n}$$

One can evaluate easily these sensitivity coefficients in the cases where the explicit dependencies of the fission and transport operators F and A as well as the cross section $\Sigma^g_{y,i}$ on x_n are known. If this is not the case, the partial derivatives can be approximated using

$$\partial_n \Sigma^g_{y,i} \approx \frac{\Sigma^g_{y,i}[\{\bar{x}\} + \{\delta x_n\}] - \Sigma^g_{y,i}[\{\bar{x}\}]}{\delta x_n} \tag{15}$$

assuming $\{\delta x_n\} = \{0, 0, \cdots, \delta x_n, \cdots, 0\}$.

This approach is called the deterministic GPT method. It should be faster than the statistical approach since the sensitivity coefficients are computed once and for all, avoiding the repetitive evaluation of $k^{\mathrm{GPT}}_{\mathit{eff}}[\{x\}]$ and $\Sigma^{\mathrm{GPT},G}_{y,H}[\{x\}]$ for a series of perturbation. In addition, these coefficients do not dependent on the explicit choice of the probability distribution function nor on the width of this distribution.

4. RESULTS

The problem we have considered represents a PWR pin cell fueled with 3.9 % enriched uranium oxide. [10] The fuel and the coolant are respectively at a temperature of 600 K and 500 K. Here our transport calculation were performed in DRAGON using the collision probability method on a cell subdivided into 5 regions (3 for the fuel, one for the cladding and one for the coolant) and a 69 group library. The flux distributions resulting from these calculations were then used to

generate cell averaged and 2 group cross sections. The upper energy limit of the thermal group (group 2) was set at 0.625 eV.

Here we will study the effect of uncertainties of 1, 5, 10 and 50 K in both the fuel and coolant temperatures on k_{eff} and the two group cell averaged absorption cross sections. In fact two cases will be analyzed here:

1. These uncertainties only affect the microscopic cross sections.
2. The uncertainties in the coolant temperature also affects its density (the fuel density is not strongly affected by the fuel temperature). Here, we will assume that the coolant density varies linearly with temperature.

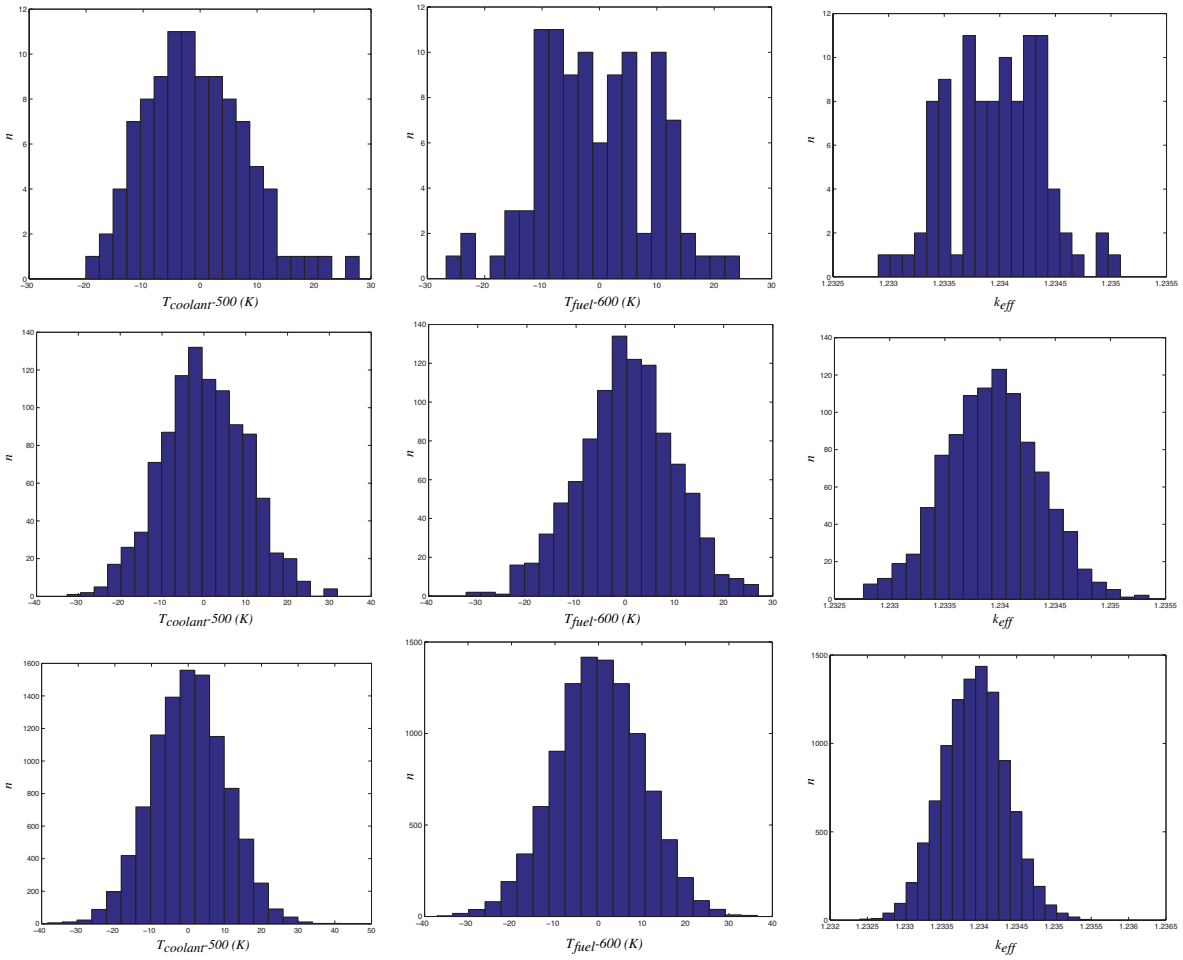

Figure 1. Coolant temperature, fuel temperature and k_{eff} distribution for K=100 (top), 1000 (center) and 10000 (bottom) for the direct statistical method with σ_T=10 K.

We will also compare the performance of the three procedures described in Sections 2 and 3 taking the probabilistic evaluation using the direct approach with a very large number of

perturbations as the reference.

However, before going into these comparisons, let us first analyze the impact of K, the number of perturbation, on the computed uncertainties. Here the calculations will be performed assuming a value for the measured experimental error for both fuel and coolant temperatures of σ_T =1 and 10 K. The uncertainty propagation calculations will be performed using the direct statistical method with K =100, 1000 and 10000 transport calculations which should produce statistical deviations in the uncertainties around 10, 3.2 and 1 %. One can find in Figure 1 histograms for the coolant and fuel temperatures distributions generated for this study when σ_T=10 K. The random number generator we used is that provided in MATLAB. [11]. As one can see, when the number of random numbers is too small, the distribution in temperature is far from being Gaussian. As expected, for K =1000 and 10000 much more uniform distributions are observed. The right hand graphics on each line represents the k_{eff} distributions obtained from these distributions. The same behavior as that observed for the temperature distributions is observed, namely for K =1000 and above, a uniform distribution is observed.

Table I. k_{eff} statistics for different values of K (direct method).

σ_T (K)	K	k_{eff}^{avg}	$\sigma_{k_{eff}}$ (%)	t_r
1	100	1.23394	0.003	75
	1000	1.23394	0.003	761
	10000	1.23394	0.003	9781
10	100	1.23397	0.034	72
	1000	1.23393	0.035	737
	10000	1.23394	0.035	9372

A statistical analysis of these distributions is presented in Table I for σ_T =1 and 10 K. Here k_{eff}^{avg} and $\sigma_{k_{eff}}$ are respectively the average value of k_{eff} and the standard deviation obtained from the distributions generated using the direct statistical method. The relative computation time t_r is the ratio of the CPU for the full calculation compared to CPU time required to obtain the reference solution only. Note that k_{eff}=1.23394 is the reference value for this cell. The first observation is that all the k_{eff}^{avg} are within 0.004 % of the reference value, the largest difference being observed when K = 100 and σ_T=10 K as expected. The results for $\sigma_{k_{eff}}$ are also nearly independent of K (less than 3 % differences) which shows that even if the distribution in temperature and k_{eff} is relatively far from being a Gaussian (see Figure 1), the statistical analysis we performed is reliable. Finally, the CPU time for the various cases increases nearly linearly with K for small values if one takes into account the DRAGON initialization overhead that appears only once for both the multiple transport and the reference calculation. For K=10000, the calculations are much less efficient because of the size of the files that need to be processed.

The same statistical analysis was also repeated for the fast and thermal absorption cross sections. For Σ_a^1, the averaged value extracted from the distribution is again identical to the reference value

Table II. Σ_a^1 and Σ_a^2 statistics for different values of K (direct method).

		Σ_a^1		Σ_a^2	
σ_T (K)	K	Value (cm^{-1})	σ (%)	Value (cm^{-1})	σ (%)
1	100	0.01029	0.007	0.10739	0.04
	1000	0.01029	0.007	0.10739	0.05
	10000	0.01029	0.007	0.10739	0.05
10	100	0.01029	0.068	0.10742	0.41
	1000	0.01029	0.067	0.10734	0.45
	10000	0.01029	0.067	0.10737	0.46

while the standard deviations remain nearly independent of K. For Σ_a^2, the averaged value extracted from the distribution with σ_T=1 K is also identical to the reference value. For σ_T=10 K the averaged value leads to a slight overestimation of Σ_a^2 when $K = 100$ while for K=1000 and 10000, an average cross section that is slightly lower than the reference is obtained. This effect is

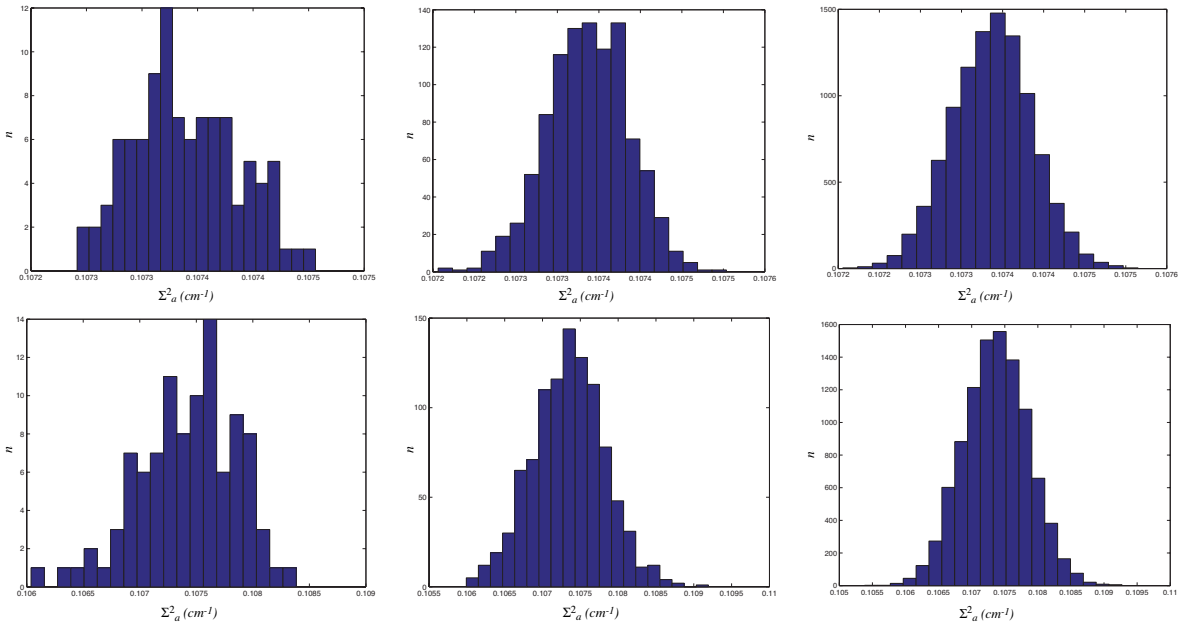

Figure 2. Thermal absorption cross section distribution for K=100 (left), 1000 (center) and 10000 (right). The top line is for σ_T= 1 K and the bottom line for σ_T= 10 K (direct statistical method).

mainly due to the displacement of the thermal absorption distribution towards lower values as

illustrated in Figure 2. Finally, selecting a value of K =100 can lead to a 20 % underestimation of the standard deviation for the thermal absorption cross sections.

Table III. Results for $\sigma_{k_{eff}}$ using GPT calculations.

σ_T (K)	K	k_{eff}^{avg}	$\sigma_{k_{eff}}$ (%)	t_r
1	100	1.23394	0.003	28
	1000	1.23394	0.003	263
	10000	1.23394	0.003	3337
	Deterministic	1.23394	0.003	3
10	100	1.23393	0.033	28
	1000	1.23397	0.033	255
	10000	1.23394	0.033	3180
	Deterministic	1.23394	0.033	3

The results obtained using GPT (both statistical and deterministic methods) are illustrated in Tables III and IV. As one can see the statistical GPT approach gives results which are consistent with the direct statistical approach. However, the evaluation in this case is nearly 3 times as fast as for the direct method. For the deterministic approach, we assumed that the sensitivity

Table IV. Results for Σ_a^1 and Σ_a^2 using GPT calculations.

		Σ_a^1		Σ_a^2	
σ_T (K)	K	Value (cm^{-1})	σ (%)	Value (cm^{-1})	σ (%)
1	100	0.01029	0.007	0.10739	0.04
	1000	0.01029	0.007	0.10739	0.05
	10000	0.01029	0.007	0.10739	0.05
	Deterministic	0.01029	0.007	0.10739	0.05
10	100	0.01029	0.067	0.10742	0.41
	1000	0.01029	0.068	0.10734	0.45
	10000	0.01029	0.068	0.10736	0.46
	Deterministic	0.01029	0.068	0.10739	0.47

coefficients could be computed assuming that the cross sections had a linear dependence on temperature. For all our calculations, we used Eq. 15 with $\delta T = \sigma_T$. Note that in the case of the deterministic approach, the average values for k_{eff} and the absorption cross sections are identical to the reference values. As one can see this approach is also consistent with the direct approach

with $K \geq 1000$ for k_{eff} and Σ_a^1 while it leads to a slight overestimation of the uncertainties for Σ_a^2.

Table V. Uncertainties as a function of σ_T.

	Statistical with K=1000						Deterministic		
	Direct			GPT			GPT		
σ_T (K)	k_{eff}	Σ_a^1	Σ_a^2	k_{eff}	Σ_a^1	Σ_a^2	k_{eff}	Σ_a^1	Σ_a^2
1	0.003	0.006	0.05	0.003	0.006	0.05	0.003	0.007	0.05
5	0.017	0.034	0.22	0.016	0.034	0.23	0.016	0.034	0.23
10	0.035	0.067	0.45	0.033	0.068	0.46	0.033	0.067	0.47
50	0.169	0.330	2.34	0.162	0.330	2.33	0.164	0.332	2.33

The uncertainties as a function of σ_T for the the three different techniques are illustrated in Table V. As one can see the deterministic method is very reliable even for uncertainties in temperature reaching 50 K with standard deviations in the thermal absorption cross section larger than 2 %.

Table VI. Uncertainties using statistical approach with coolant density effects.

	Direct			GPT		
σ_T (K)	k_{eff}	Σ_a^1	Σ_a^2	k_{eff}	Σ_a^1	Σ_a^2
1	0.01	0.02	0.09	0.01	0.02	0.09
5	0.04	0.11	046	0.04	0.11	0.46
10	0.08	0.22	0.90	0.08	0.22	0.90
50	0.48	1.17	4.77	0.41	1.15	4.75

We have repeated the same set of simulations but now taking into account the fact that the density of the coolant is a function of temperature. The results are presented in Table VI when the statistical method (direct and GPT) is used with K =1000. For small uncertainties in the temperatures ($\sigma_T \leq$10 K), the results obtained using both the direct and GPT approaches are identical. For σ_T=50 K, the GPT approach leads to an underestimation of the uncertainties in k_{eff} (15 %) while the prediction of the uncertainties in the absorptions cross sections are still good (2 % error).

For the deterministic approach, two different options were examined. First, we considered the case where the sensitivity coefficient with respect to the coolant temperature includes the dependence of the density on temperature (Combined (T_c, ρ_c) sensitivity). As a result only two parameters are considered in Eq. 12 (fuel and coolant temperature), these two parameters being

uncorrelated. In the second case, three sensitivity coefficients were considered, namely one for the fuel temperature, one for the coolant temperature and one for the coolant density. The covariance factor between the fuel temperature and the other two parameters vanish while we assumed a full correlation ($c_{T_c,\rho_c} = 1$) between the coolant temperature and density (Independent (T_c, ρ_c) sensitivities). Here, the results are presented in Table VII.

Table VII. Uncertainties using deterministic approach with coolant density effects.

	Combined (T_c, ρ_c) sensitivity			Independent (T_c, ρ_c) sensitivities		
σ_T (K)	k_{eff}	Σ_a^1	Σ_a^2	k_{eff}	Σ_a^1	Σ_a^2
1	0.01	0.02	0.09	0.01	0.02	0.09
5	0.04	0.11	0.47	0.04	0.11	0.47
10	0.08	0.22	0.93	0.08	0.22	0.93
50	0.41	1.10	4.49	0.41	1.10	4.67

As one can see, both deterministic evaluations yield again identical results when $\sigma_T \leq$10 K, the results being consistent with the statistical method. For the case where $\sigma_T =$50 K, the uncertainties in k_{eff} are similar to those obtained using the GPT statistical approach (under predicted by 15 %) while the uncertainties in the absorption are now underestimated by up to 7 %. Finally, the use of independent rather than combined (T_c, ρ_c) sensitivities improve the evaluation of the uncertainty on the thermal absorption cross section but has no impact on the fast absorption cross section or k_{eff}.

5. CONCLUSIONS

We have shown that the use of the DRAGON GPT approach can accelerate by a factor of 3 (probabilistic evaluation) the process of evaluation the standard deviations in two groups cell averaged properties resulting from uncertainties in the lattice parameters. The precision of the probabilistic GPT approach is very close to that of the direct approach.

The deterministic approach provides results that are nearly as good as the statistical GPT method but with gain in computation time that are extremely interesting (factor of nearly $K/10$). In fact only a slight under evaluation of the standard deviations is apparent when the effect of temperature on the coolant density is neglected. When the coolant density effect is taken into account and small uncertainties in the temperature are considered this statement remains true independent of the calculation option selected. However, for the case with large uncertainties, evaluating independently the sensitivity coefficients and using covariance factors leads to more reliable predictions.

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