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COMBINED USE OF NEUTRON AND GAMMA MULTIPLICITIES FOR DETERMINING SAMPLE PARAMETERS

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ABSTRACT

Expressions for neutron and gamma factorial moments are known in the literature. For neutrons, these served as the basis of constructing analytic expressions for the detection rates of singles, doubles and triples, which can be used to unfold sample parameters from the measured multiplicity rates. Here we suggest the combined use of both the individual and joint neutron and gamma multiplicities and the corresponding detection rates. Counting up to third order, there are nine auto- and cross factorial moments, which are all given here explicitly. For the gamma photons, formulae are derived also for the corresponding multiplicity detection rates which, in contrast to the factorial moments, are the measured quantities and which also contain the sample fission rate explicitly.

Adding the gamma counting to the neutrons introduces new unknowns, related to gamma generation, leakage, and detection. Despite more unknowns, the total number of measurable moments exceeds the number of unknowns. On the other hand, the structure of the additional equations is substantially more complicated than the neutron moments, hence their analytical inversion is not possible.

We suggest therefore to invert the non-linear system of over-determined equations by using artificial neural networks (ANN), which can handle both the non-linearity and the redundance in the measured quantities in an effective and accurate way. The use of ANNs is demonstrated with good results on the unfolding of neutron multiplicity rates for the sample fission rate, the leakage multiplication and the α ratio. Work with using the gamma multiplicity rates is on-going and some results will be reported at the conference.

Key Words: Nuclear safeguards, neutron and gamma multiplicities, joint moments, materials control and accounting

1. INTRODUCTION

Multiplicity detection rates, based on higher order factorial moments of the neutron counts from an unknown sample, can be used to determine sample parameters [1–3]. The factorial moments here refer to those of the total number of neutrons generated in the sample by one *initial source event* (spontaneous fission or (α, n) reaction). Due to internal multiplication through induced fission, the probability distribution of the total number of generated neutrons will deviate from that by the initial source event (mostly spontaneous fission), the deviation being a function of the sample mass (via the first collision probability of the initial neutrons). This property is transferred to the measured multiplicity rates, i.e. the singles, doubles and triples, and this is corroborated by the fact that in the latter the sample fission rate occurs explicitly. This gives a possibility to determine the sample mass.

Measurement of the first three multiplicity rates enables the recovery of three unknowns, which are usually taken as the sample leakage multiplication (related to the first collision probability), the ratio α of the intensity of neutron production via (α, n) reactions to spontaneous fission, and the spontaneous fission rate, the latter being the most important parameter. This leaves the detector efficiency undetermined and it needs to be predetermined experimentally, or by using alternative approaches such as assuming the sample multiplication to be known and then the detector efficiency can be unfolded.

Recently it was suggested that in addition to neutron multiplicity counting, gamma multiplicities be also taken into account [4–6]. The motivation for using gamma counting is manifold: higher gamma multiplicity per fission, larger penetration through most of the strong neutron absorbers, and the relatively easy detection via organic scintillation detectors. The goal is still the same, i.e. to determine the above factors, plus the further unknowns introduced, such as the gamma leakage multiplication, the ratio γ of single gamma to fission gamma intensity and gamma detector efficiency. These can though be handled since three neutron and three gamma multiplicities can be measured automatically, so one has still as many unknowns as measured quantities.

However, there exists the further possibility of using the joint moments of the neutron and gamma counts, which supplies further independent measured data to determine still the same number of unknowns. Accounting also for the joint moments up to third order, there are altogether nine factorial moments. Hence the problem becomes overdetermined.

At the same time, the searched parameters are contained in a highly non-linear way in the multiplicity expressions. This is already true for the gamma moments and multiplicity rates alone. To handle the non-linearity of the problem which prevents an analytical inversion of the multiplicity rate formulas, and in addition to make maximal use of the redundant information from the measurement when also the joint moments are used, the unfolding of the parameters has to be performed by least-square type unfolding methods. Actually, there is a conceptually simple non-parametric unfolding method for such a purpose, the artificial neural networks (ANNs), whose use will be demonstrated here.

In this paper we give the definitions of the quantities used and list all nine factorial moments. To give insight into the information contained in the joint moments, the dependence of the lowest ordre joint neutron-gamma moment on the non-leakage probability will be given quantitatively. In addition to the factorial moments, the multiplicity detection rates for the gamma photos will also be given. These have not been given previously. The derivation of converting the factorial moments into multiplicity detection rates follows the method described in a recent clarifying note on the relationship between factorial moments and the single, double and triple coincidence rates [10]. This will clearly show the relation between the measurable multiplicity rates and the factorial moments, when accounting for both multiple emission (spontaneous fission) and single emission (α, n) source events. In the last Section a description and test of the unfolding procedure is given by only using neutron multiplicity rates. Extension of the unfolding with ANNs to include gamma multiplicity rates will be reported in subsequent publications.

2. DEFINITIONS

The following definitions and conventions will be used. Random variables and their moments referring to neutrons will be denoted by ν, and those for gamma photons by μ. Variables referring to spontaneous fission will have a subscript sf, and those referring to induced fission a subscript i. For the factorial moments, there will always be a second index, giving the order of the moment. Hence, $\nu_{sf,2}$ will stand for $\langle \nu(\nu-1) \rangle$ in case of spontaneous fission. In addition, we will distinguish between two sets of variables for both neutrons and photons, depending on whether they belong to a source event, or to the total number of generated neutrons, which accounts for the internal multiplication (superfission). The parameters belonging to the first set will be written with a subscript indication, such as $\nu_{s,2},\ \nu_{i3}$, whereas those of the second set will be denoted with just a numerical subscript indicating their order such as $\nu_1,\ \mu_3$.

The factorial moments corresponding to the distribution of neutrons or gammas emitted in fission, whether induced or spontaneous, are nuclear constants and are known in advance. However, as is usual in such work, it is practical to include the (unknown) contribution from generation of single neutrons and photons, such as by (α, n) processes for the neutrons, into the moments related to spontaneous fission. Inclusion of single neutron generating processes in the calculations has long been applied. However there is a need for introducing a similar correction for gamma photons, since they also can be produced either in bunches (in the spontaneous fission process) or as singular gamma photons, in the same (α, n)-reactions which lead to the emission of single neutrons. In addition, there is also the presence of a “background” type emission of single gamma photons from neutron capture processes. The need for accounting such single photon generating processes was suggested by Sanchez [7]. In principle, there is also the possibility of producing individual gamma photons not only in the source processes, but also in the induced reactions, through inelastic scattering of the neutrons. Accounting for this possibility is though deferred to later work.

To account for the presence of single neutron producing events, one introduces the statistics of the total source events as a weighted average of the two processes [1],[6]. Quantities belonging to such a generalized source event will be denoted by a subscript s. Hence, we will use

$$\nu_{s,n} = \frac{\nu_{sf,n}(1+\alpha\delta_{1,n})}{1+\alpha\nu_{sf,1}} \tag{1}$$

as source moments for neutrons. Here Q_f and Q_α are the intensities of spontaneous fission and (α, n) processes, respectively, and the factor α is defined as

$$\alpha = \frac{Q_\alpha}{Q_f \nu_{sf,1}}$$

For gamma photons produced also in connection to (α, n)-reactions, the source distribution of photons will change as:

$$f_s(n) = \frac{Q_\alpha}{Q_\alpha + Q_f}\delta_{n,1} + \frac{Q_f}{Q_\alpha + Q_f} f_{sf}(n). \tag{2}$$

This leads to the modified source moments defined as:

$$\mu_{s,n} = \frac{\mu_{sf,n} + \delta_{1,n}\alpha\nu_{sf,1}}{1+\alpha\nu_{sf,1}}. \tag{3}$$

Here, $\nu_{sf,n}$ and $\mu_{sf,n}$ are the true moments of spontaneous fission (i.e. nuclear constants), whereas $\nu_{s,n}$ and $\mu_{s,n}$ are the ones corrected for the inclusion of production of neutrons and gammas by reactions other than fission. The moments relating to induced fission remains unchanged for neutrons and photons (ν_{in} and μ_{in} respectively).

The second set of moments and random variables concerns the distribution of the total number of generated or detected neutrons and gammas due to one initial *source* event, with internal multiplication included (“superfission” in Böhnels terminology [1]). These will not be denoted by any lettered subscript, only with a single number, expressing the order of the moment. The purpose of the calculations is to express these latter type of variables with the ones given by the nuclear constants, based on the distributions from spontaneous and induced fission, and the parameter α, describing the (unknown) relative intensity of production of single neutrons.

To obtain this relationship we need even the first collision probability p. For simplicity of the description absorption will be neglected; however, detection efficiency will later be taken into account by the factors ε_n and ε_γ for the neutrons and photons, respectively. Absorption for the gamma photons can actually be included into the efficiency factor ε_γ in an exact way, due to the fact that gamma photons do not take part in the internal multiplication.

3. FACTORIAL MOMENTS

Here we only list the various single and mixed factorial moments without any details of the underlying derivation. The principles of derivation through master equations can be found in [1], [4], [5] and [6].

3.1 Neutrons

First moments (singles)

$$\nu_1 = \frac{1-\mathrm{p}}{1-\mathrm{p}\nu_{i1}}\nu_{s,1} = \mathbf{M}\nu_{s,1}. \tag{4}$$

where

$$\mathbf{M} \equiv \frac{1-\mathrm{p}}{1-\mathrm{p}\nu_{i1}}; \qquad \mathrm{p}\nu_{i1} < 1 \tag{5}$$

is called the *leakage multiplication*.

Second moments (doubles)

$$\nu_2 = \mathbf{M}^2\left\{\nu_{s,2} + \frac{\mathrm{p}}{1-\mathrm{p}\nu_{i1}}\nu_{s,1}\nu_{i2}\right\} = \mathbf{M}^2\left\{\nu_{s,2} + \frac{\mathbf{M}-1}{\nu_{i1}-1}\nu_{s,1}\,\nu_{i2}\right\}. \tag{6}$$

Third moments (triples)

$$\nu_3 = \mathbf{M}^3\left\{\nu_{s,3} + \frac{\mathbf{M}-1}{\nu_{i1}-1}(3\nu_{s,2}\nu_{i2} + \nu_{s,1}\,\nu_{i3}) + 3\left(\frac{\mathbf{M}-1}{\nu_{i1}-1}\right)^2\nu_{s,1}\,\nu_{i2}^2\right\}. \tag{7}$$

In these formulae the factorial moments of the combined source events are used. In order to be able to unfold the the sample parameters, including the unknown factor α, one has to re-write these formulae in terms of the factorial moments of spontaneous fission and α. This is easily achieved by using Eq. (1), the results being

$$\nu_1 = \frac{\mathbf{M}}{(1+\alpha\nu_{sf,1})}\nu_{sf,1}(1+\alpha) \tag{8}$$

$$\nu_2 = \frac{\mathbf{M}^2}{(1+\alpha\nu_{sf,1})}\left[\nu_{sf,2} + \left(\frac{\mathbf{M}-1}{\nu_{i1}-1}\right)\nu_{sf,1}(1+\alpha)\nu_{i2}\right] \tag{9}$$

$$\nu_3 = \frac{\mathbf{M}^3}{(1+\alpha\nu_{sf,1})}\left[\nu_{sf,3} + \left(\frac{\mathbf{M}-1}{\nu_{i1}-1}\right)[3\nu_{sf,2}\nu_{i2} + \nu_{sf,1}(1+\alpha)\nu_{i3}] + 3\left(\frac{\mathbf{M}-1}{\nu_{i1}-1}\right)^2 \nu_{sf,1}(1+\alpha)\nu_{i2}^2\right] \quad (10)$$

As discussed in [10], these formulae are incorrectly given in [8] and in several other publications. The multiplicity rates found in the literature including [8] are, however, correct.

3.2 Photons

The moments of the photons (which are more complicated due to the simple fact that photons do not self-multiply, rather they depend on the multiplication of neutrons), are given below. The detailed derivation can be found in [6].

Singles

$$\mu_1 = \mu_{s,1} + \frac{\nu_{s,1}\mathrm{p}\mu_{i1}}{1-\mathrm{p}\nu_{i1}} = \mu_{s,1} + \nu_{s,1}\mathbf{M}_\gamma. \quad (11)$$

with

$$\mathbf{M}_\gamma \equiv \frac{\mathrm{p}\mu_{i1}}{1-\mathrm{p}\nu_{i1}} \quad (12)$$

being the *gamma (leakage) multiplication per one initial neutron*.

Doubles

$$\mu_2 = \mu_{s,2} + 2\mu_{s,1}\nu_{s,1}\mathbf{M}_\gamma + \nu_{s,2}\mathbf{M}_\gamma^2 + \nu_{s,1}\, g_2. \quad (13)$$

Triples

$$\mu_3 = \mu_{s,3} + 3\mu_{s,2}\nu_{s,1}\mathbf{M}_\gamma + 3\mu_{s,1}\{\nu_{s,2}\mathbf{M}_\gamma^2 + \nu_{s,1}\, g_2\} + \nu_{s,3}\mathbf{M}_\gamma^3 + 3\nu_{s,2}g_2 + \nu_{s,1}g_3. \quad (14)$$

The moments above are those of the source distribution, given by $\nu_{s,n}$ and $\mu_{s,n}$. Again, these have to be re-written in terms of the true fission neutron and gamma photon factorial moments and the factor α by equations (1) and (3). This leads to the expressions

$$\mu_1 = \frac{\mu_{sf,1} + \alpha\nu_{sf,1}}{(1+\alpha\nu_{sf,1})} + \frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}\mathbf{M}_\gamma, \quad (15)$$

$$\mu_2 = \frac{\mu_{sf,2}}{1+\alpha\nu_{sf,1}} + 2\frac{(\mu_{sf,1}+\alpha\nu_{sf,1})}{(1+\alpha\nu_{sf,1})}\frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}\mathbf{M}_\gamma + \frac{\nu_{sf,2}}{(1+\alpha\nu_{sf,1})}\mathbf{M}_\gamma^2 + \frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}\, g_2 \quad (16)$$

and

$$\mu_3 = \frac{1}{(1+\alpha\nu_{sf,1})}\left[\mu_{sf,3} + 3\mu_{sf,2}\frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}\mathbf{M}_\gamma + 3(\mu_{sf,1}+\alpha\nu_{sf,1})\left\{\frac{\nu_{sf,2}}{(1+\alpha\nu_{sf,1})}\mathbf{M}_\gamma^2 + \frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}\, g_2\right\} + \nu_{sf,3}\mathbf{M}_\gamma^3 + 3\nu_{sf,2}g_2 + \nu_{sf,1}(1+\alpha)g_3\right]. \quad (17)$$

One can note that the (α, n) processes affect also the photon equations, in a way analogous to the neutron moment equations, due to the single photon emission processes accompanying the (α, n) reactions. The appearance of the factor α becomes also highly non-linear, due to its occurring in a multiple way in the process. This also indicates the expected fact that an analytical inversion of the gamma multiplicity expressions for the sample parameters is not possible.

The factorial moments of the photon distribution initiated by a single neutron are defined in [6] as:

$$g_n = \left. \frac{d^3 g(z)}{dz^3} \right|_{z=1} . \tag{18}$$

This leads for g_2 and g_3 to the expressions

$$g_2 = \frac{\mathbf{M}-1}{\nu_{i1}-1}\{\mu_{i2} + 2\mu_{i1}\nu_{i1}\mathbf{M}_\gamma + \nu_{i2}\mathbf{M}_\gamma^2\} \tag{19}$$

and

$$g_3 = \frac{\mathbf{M}-1}{\nu_{i1}-1}\{\mu_{i3} + 3\mu_{i2}\nu_{i1}\mathbf{M}_\gamma + 3\mu_{i1}[\nu_{i2}\mathbf{M}_\gamma^2 + \nu_{i1}g_2] + \nu_{i3}\mathbf{M}_\gamma^3 + 3\nu_{i2}\mathbf{M}_\gamma g_2\}. \tag{20}$$

With no occurrences of $\nu_{s,n}$ and $\mu_{s,n}$, these expressions do not change for a compound source compared to a pure spontaneous fission source.

3.3 Mixed moments

Instead of the first mixed moment, we give the covariance:

$$\mathbf{Cov}\{\boldsymbol{\nu},\boldsymbol{\mu}\} \equiv \mathbf{E}\{\boldsymbol{\nu}\boldsymbol{\mu}\} - \mathbf{E}\{\boldsymbol{\nu}\}\,\mathbf{E}\{\boldsymbol{\mu}\}$$

$$= \left\{(\nu_{s,2} - \nu_{s,1}^2)\mathbf{M}\mathbf{M}_\gamma + \nu_{s,1}\frac{\mathbf{M}-1}{\nu_{i1}-1}\{\mu_{i1}\nu_{i1}\mathbf{M} + \nu_{i2}\mathbf{M}\mathbf{M}_\gamma\}\right\}. \tag{21}$$

For illustration, the dependence of the coherence on the first collision probability p is shown in Fig. 1.

Higher moments:

$$\langle \boldsymbol{\nu}\,(\boldsymbol{\nu}-1)\;\boldsymbol{\mu}\rangle = \big\{\mu_{s,1}\left[\nu_{s,2}\mathbf{M}^2 + \nu_{s,1}h_2\right] + \nu_{s,3}\mathbf{M}_\gamma\,\mathbf{M}^2 +$$

$$\nu_{s,2}\left[2\mathbf{M}c_{1,1} + \mathbf{M}_\gamma\,h_2\right] + \nu_{s,1}c_{2,1}\big\} \tag{22}$$

and

$$\langle \boldsymbol{\nu}\;\;\boldsymbol{\mu}(\boldsymbol{\mu}-1)\rangle = \{\mu_{s,2}\nu_{s,2}\mathbf{M} + 2\mu_{s,1}\left[\nu_{s,2}\mathbf{M}\,\mathbf{M}_\gamma + \nu_{s,1}c_{1,1}\right] +$$

$$+\nu_{s,3}\mathbf{M}\mathbf{M}_\gamma^2 + \nu_{s,2}\left[g_2\mathbf{M} + 2\mathbf{M}_\gamma\,c_{1,1}\right] + \nu_{s,1}c_{1,2}\big\} \tag{23}$$

In the above,

$$c_{1,1} = \left\{p\frac{\mu_{i1}\nu_{i1}\mathbf{M} + \nu_{i2}\mathbf{M}\mathbf{M}_\gamma}{1-p\nu_{i1}}\right\}, \tag{24}$$

$$c_{2,1} \equiv \frac{p}{1-p\nu_{i1}}\{\mu_{r,1}[\nu_{i2}\,\mathbf{M}^2 + \nu_{i1}\,h_2] + \nu_{i3}\mathbf{M}^2\mathbf{M}_\gamma + \nu_{i2}[h_2\mathbf{M}_\gamma + 2\mathbf{M}c_{1,1}]\}, \tag{25}$$

$$c_{1,2} \equiv \frac{p}{1-p\nu_{i1}}\{\mu_{i2}\nu_{i1}\mathbf{M} + 2\mu_{i1}[\nu_{i2}\mathbf{M}\mathbf{M}_\gamma + \nu_{i1}c_{1,1}] + \nu_{i3}\mathbf{M}\mathbf{M}_\gamma^2 + \nu_{i2}[2\mathbf{M}_\gamma c_{1,1} + \mathbf{M}g_2]\}. \tag{26}$$

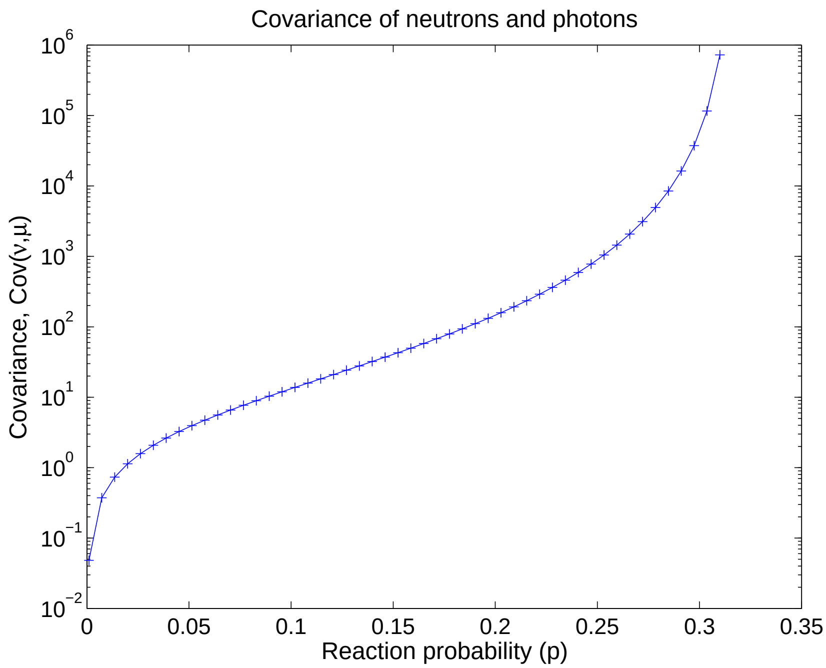

Figure 1. Covariance between neutrons and photons.

Here h_2 is the second factorial moment of the total number of neutrons generated in the sample, internal multiplication included, by *one single neutron*. This and other moments can be found in [4]:

$$h_2 = \frac{\mathbf{M}-1}{\nu_{i1}-1}\nu_{i2}\mathbf{M}^2. \tag{27}$$

While these formulae are correct for a pure spontaneous fission source, they need be expanded using Eqs. (1) and (3), to account properly also for the effect of the (α, n) process on the single neutron and gamma photon generating processes. The following expressions are then obtained:

$$\langle \boldsymbol{\nu}\ \boldsymbol{\mu}\rangle = \frac{1}{(1+\alpha\nu_{sf,1})}\left\{(\mu_{sf,1}+\alpha\nu_{sf,1})\frac{(\nu_{sf,1}(1+\alpha))}{(1+\alpha\nu_{sf,1})}\mathbf{M}+\right.$$
$$\left.+\nu_{sf,2}\mathbf{M}\mathbf{M}_\gamma+\nu_{sf,1}(1+\alpha)\frac{\mathbf{M}-1}{\nu_{i1}-1}\{\mu_{i1}\nu_{i1}\mathbf{M}+\nu_{i2}\mathbf{M}\mathbf{M}_\gamma\}\right\} \tag{28}$$

$$\langle \boldsymbol{\nu}\,(\boldsymbol{\nu}-1)\ \boldsymbol{\mu}\rangle = \frac{1}{(1+\alpha\nu_{sf,1})}\left\{(\mu_{sf,1}+\alpha\nu_{sf,1})\left[\frac{\nu_{sf,2}}{(1+\alpha\nu_{sf,1})}\mathbf{M}^2+\frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}h_2\right]+\right.$$
$$\left.\nu_{sf,3}\mathbf{M}_\gamma\,\mathbf{M}^2+\nu_{sf,2}\left[2\mathbf{M}c_{1,1}+\mathbf{M}_\gamma\,h_2\right]+\nu_{sf,1}(1+\alpha)c_{2,1}\right\} \tag{29}$$

and

$$\langle \boldsymbol{\nu}\ \boldsymbol{\mu}(\boldsymbol{\mu}-1)\rangle = \left\{\mu_{sf,2}\frac{\nu_{sf,2}}{(1+\alpha\nu_{sf,1})}\mathbf{M}+2(\mu_{s,1}+\alpha\nu_{sf,1})\left[\frac{\nu_{sf,2}}{(1+\alpha\nu_{sf,1})}\mathbf{M}\,\mathbf{M}_\gamma+\frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}c_{1,1}\right]+\right.$$
$$\left.+\nu_{sf,3}\mathbf{M}\mathbf{M}_\gamma^2+\nu_{sf,2}\left[g_2\mathbf{M}+2\mathbf{M}_\gamma\,c_{1,1}\right]+\nu_{sf,1}(1+\alpha)c_{1,2}\right\} \tag{30}$$

4. MULTIPLICITY DETECTION RATES

The measured quantities are the multiplicity rates. To convert the factorial moments of a single source event into detection rates of multiplicities, one has to account for the intensity of the source events and the detection efficiency. The effect of the finite measurement gate time in multiple coincidence measurements, quantified with the relative gate width factors as described in [8], will be omitted here.

Measurable quantities - Neutrons

To find the measurable quantities such as singles doubles etc, one need to first find the Factorial moments $\tilde{\nu}_k$ of the *detected* neutrons per one initial event. The reason this quantity is needed is that e.g. a measured doublet could be the result of detecting two particles from a higher order multiplet. This further requires the introduction of the detector efficiency and for the first few moments these factorial moments are defined as:

$$\widetilde{\nu}_1 = \varepsilon \nu_1, \tag{31}$$

$$\widetilde{\nu}_2 = \varepsilon^2 \nu_2, \tag{32}$$

$$\widetilde{\nu}_3 = \varepsilon^3 \nu_3. \tag{33}$$

Now for neutrons the detection *rates* are also related to the total source intensity. Using C_k as the notation for the k-th order multiplet (such that $C_1 = S$, $C_2 = D$ etc) and the total neutron source rate $Q_n \equiv Q_f + Q_\alpha$ it follows that

$$C_k = Q_n \left\langle \binom{\widetilde{n}}{k} \right\rangle = Q_n \sum_n \frac{n!}{k!(n-k)!} P(n) = Q_n \frac{\widetilde{\nu}_k}{k!}. \tag{34}$$

Using the α-factor the source factor can be expressed as:

$$Q_n = F + Q_\alpha = F(1 + \alpha \nu_{sf,1}). \tag{35}$$

In the case of singles for neutrons the following expression is derived:

$$S = F\varepsilon_n (1 + \alpha \nu_{sf,1}) \frac{\mathbf{M} \nu_{sf,1}(1+\alpha)}{(1 + \alpha \nu_{sf,1})} = F \varepsilon_n \mathbf{M} \nu_{sf,1}(1+\alpha). \tag{36}$$

Note how the scaling factor between the fission source and the total source intensity cancels out in the expression for the measurable singles. In a similar way doubles and triples can be derived as:

$$D = \varepsilon_n^2 C_2 = \frac{F\varepsilon_n^2 \mathbf{M}^2}{2} \left[\nu_{sf,2} + \left(\frac{\mathbf{M}-1}{\nu_{i1}-1} \right) \nu_{sf,1}(1+\alpha)\nu_{i2} \right], \tag{37}$$

$$T = \varepsilon_n^3 C_3 = \frac{F\varepsilon_n^3 \mathbf{M}^3}{6} \left\{ \nu_{sf,3} + \left(\frac{\mathbf{M}-1}{\nu_{i1}-1} \right) [3\nu_{sf,2}\nu_{i2} + \nu_{sf,1}(1+\alpha)\nu_{i3}] + 3 \left(\frac{\mathbf{M}-1}{\nu_{i1}-1} \right)^2 \nu_{sf,1}(1+\alpha)\nu_{i2}^2 \right\}. \tag{38}$$

These are the quantities one measures in multiplicity counters. It is these expressions that serve as the basis for the different approaches to find the various unknown parameters, as described in [3], Most commonly one assumes the neutron detector efficiency ε_n to be known, and solve the equations for fission rate (mass), F, leakage multiplication, $\mathbf{M}$, and α - the relative contribution from single-neutron sources.

Measurable quantities - Photons

In the case of photons, the moments are considerably more complicated due to accounting for both neutrons and photons. It is still possible to derive equations for the measurable quantities of singles, doubles and triples, in a manner similar to that of neutrons.

The modified moments accounting also for single emitted neutrons and photons, equations (1) - (3). These will lead to lengthier expressions. In addition, when accounting for the effect of all source events, for photons one has to account for the possibility of a single photon source which is not connected to the neutron chain. This can be made in a way analogous to the accounting for the (α, n) processes for neutrons. Defining γ as the ratio between the single photon source strength, Q_γ, and the neutrons source strength, Q_n, the gamma singles doubles and triples can be expressed as

$$S = \left[\gamma F(1+\alpha\nu_{sf,1}) + F(1+\alpha\nu_{sf,1})\widetilde{\mu}_1\right], \qquad (39)$$

$$D = F(1+\alpha\nu_{sf,1})\frac{\widetilde{\mu}_2}{2}, \qquad (40)$$

$$T = F(1+\alpha\nu_{sf,1})\frac{\widetilde{\mu}_3}{3!}. \qquad (41)$$

Also here we have used the tilde notation for the detected moments, i.e. accounting for a detector efficiency, ε_γ in the case of gamma photons. With the previous formulae one can list the full expressions for the singles, doubles and triples of photons. In the most simple case of singles one have:

$$S = \varepsilon_\gamma \left[\gamma F(1+\alpha\nu_{sf,1}) + F(1+\alpha\nu_{sf,1})\left\{\frac{\mu_{sf,1}+\alpha\nu_{sf,1}}{(1+\alpha\nu_{sf,1})} + \frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}\mathbf{M}_\gamma\right\}\right] =$$
$$= F\varepsilon_\gamma\left[\gamma(1+\alpha\nu_{sf,1}) + \mathbf{M}_\gamma\left\{\mu_{sf,1}+\alpha\nu_{sf,1}+\nu_{sf,1}(1+\alpha)\right\}\right]. \qquad (42)$$

For doubles and triples the expressions grow longer:

$$D = \frac{\varepsilon_\gamma^2 F}{2}\left[\mu_{sf,2} + 2(\mu_{sf,1}+\alpha\nu_{sf,1})\frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}\mathbf{M}_\gamma + \nu_{sf,2}\mathbf{M}_\gamma^2 + \nu_{sf,1}(1+\alpha)\, g_2\right] \qquad (43)$$

$$T = \frac{\varepsilon_\gamma^3 F}{6}\left[\mu_{sf,3} + 3\mu_{sf,2}\frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}\mathbf{M}_\gamma + 3(\mu_{sf,1}+\alpha\nu_{sf,1})\left\{\frac{\nu_{sf,2}}{(1+\alpha\nu_{sf,1})}\mathbf{M}_\gamma^2 + \frac{\nu_{sf,1}(1+\alpha)}{(1+\alpha\nu_{sf,1})}\, g_2\right\}\right.$$
$$\left. +\nu_{sf,3}\mathbf{M}_\gamma^3 + 3\nu_{sf,2} g_2 + \nu_{sf,1}(1+\alpha) g_3\right]. \qquad (44)$$

Our proposal is to use these equations much in the same way as those for neutron are used to find sample parameters. What is new here is the fraction γ of single photons in the source events, and the presence of the detector efficiency for photons, which could be pre-calibrated. The gamma leakage multiplication $\mathbf{M}_\gamma$ on the on other the hand is unknown much like the case of the leakage multiplication of neutrons ($\mathbf{M}$).

The unknown parameters $\mathbf{M}$, $\mathbf{M}_\gamma$, α, γ, ε_n and ε_γ can be determined from the above equations with a nonlinear least squares non-parametric fitting to the measured values. In the present model $\mathbf{M}$ and $\mathbf{M}_\gamma$ are not two independent unknowns since both contain only one unknown parameter, the first collision probability p, hence the number of unknowns is even fewer. The possibilities of the unfolding of the unknowns will be discussed below.

5. APPLICATION

As mentioned previously, the complexity of the expressions for the gamma photons prevents the possibility of using analytical inversion of the multiplicity rate expressions, which was possible for the neutron expressions. Hence we advocate the use of artificial neural network (ANN) techniques for the unfolding of sample parameters from the measured multiplicity rates.

The use of ANNs can be tested already on the known case of neutron multiplicities, which can serve also as a first test. In addition, it offers some advantages already for this relatively simple case. Namely, the analytical inversion is only possible as long as only three unknowns are attempted to be retrieved from the three multiplicity rates. This has the effect that the neutron efficiency needs to be known in advance. With ANN techniques, there is a larger flexibility, since ANNs can utilize the rich information in the non-linearity of the expressions to unfold more parameters than the number of expressions. Hence there is a chance that in addition to the usual three parameters, also the detector efficiency can be retrieved.

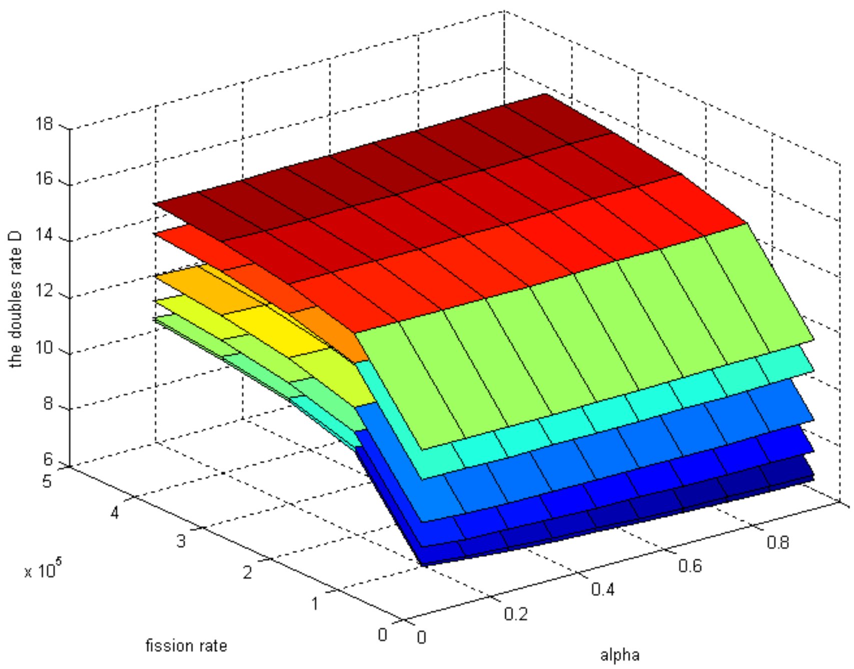

Figure 2. An example of the training data used. Here doubles are calculated depending on different values of F, p and α.

In this respect one can draw analogies between the above statement and the use of the Feynman-alpha method for determination of the reactivity. In the Feynman-alpha method, there is one singe expression giving the dependence of the relative variance of the detector counts on the measurement time length. This expression contains both the searched prompt neutron decay constant alpha, but also the unknown detector efficiency. However, due to the non-linear dependence of the formula on the measurement time, both parameters can be determined by a curve fitting method.

Some initial tests were made to unfold the three usual sample parameters from the neutron singles, doubles and triples rates by ANN methods. The analytical expressions were used, by sweeping with the parameters F, $\mathbf{M}$ and α over realistically possible values, to generate input patterns for the training of a simple feedforward backward propagation network with three inputs and three outputs. Two hidden layers were needed for successful training, but the structure of the network may be refined more in future work.

Below some results are shown, obtained after the initial training of the network was completed. The trained network was tested by further sample vectors generated the same way as the training set. The dependence of the training data on the input parameters is shown, for the case of doubles, in Fig. 2.

	fission rate (F)	α	p
max. rel. error (%)	7.9794e-001	9.8169e-001	1.5937e-001
min. rel. error (%)	-1.1729e+000	-1.5469e+000	-1.5494e-001

Table I. Preliminary training results of ANN, using the neutron equations to simulating large plutonium samples in the kg-range.

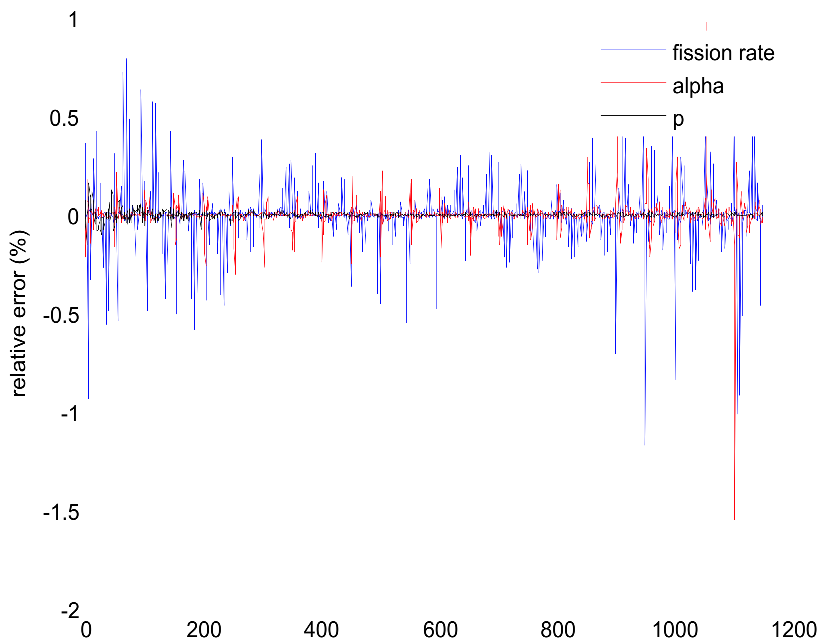

Figure 3. Relative errors after preliminary training shown for the parameters investigated: F, p and α.

Preliminary results show that the parameters F, α, p can be evaluated with the relative errors less than 1 % even after short trainings of the ANNs. These results are very promising. Figure 3 shows the variation of the error associated with the variables. The work is progressing fast with testing the possibility of determining more parameters with various combinations of neutron, photon and joint multiplicities.

6. CONCLUSIONS

The present paper shows that by taking all possible auto- and cross factorial moments of the neutron and gamma counts into account, one has nine expressions which are functions of five independent parameters. The generation of single photons by various processes, in addition to the multiple ones from fissions, was considered after a suggestion of R. Sanchez.

It is suggested that these multiplicity rates be inverted by non-linear non-parametric least squares methods, namely with the use of artificial neural networks, to which the above equations can be used to generate training data. The construction and test of the ANN is ongoing and further results will be shown during the conference presentation and future communications. The preliminary results show are very promising and of good accuracy. While the training and performance of ANN's using all moments for both neutrons and photons will be computationally more demanding, but still within manageable range. Another advantage could be that when using Monte-Carlo simulations to generate training data, the network might also adapt to the statistical uncertainties in the input data, which are reconstructed by the simulations in a realistic manner. Such effects cannot be accounted for by analytical inversion methods.

7. ACKNOWLEDGEMENTS

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