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FUEL LOADING PATTERN FOR HETEROGENEOUS EPR CORE CONFIGURATION USING A DISTRIBUTED EVOLUTIONARY ALGORITHM

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ABSTRACT

In the frame of the third generation new design nuclear core, an European Pressurized Reactor (EPR) heterogeneous radial configuration loaded with fissile and fertile assemblies has been selected. The goal of this study is to establish from this configuration some acceptable loading patterns, which minimize the power peak and the so-called "draining coefficient".
To solve this multi-objectives optimization problem, an evolutionary algorithm named Vizir has been coupled with the Minos diffusion solver integrated in the new lattice-core Apollo3 system. To address this problem in an efficient way, several integer encoding and crossing operators have been developed within the evolutionary algorithm Vizir. The objective functions are evaluated by using the Minos solver. The first criterion on the power peak requires an entire core cycle calculation while the second criterion on the draining coefficient needs only one point calculation at the beginning of the core cycle. We proceed to the final validation of the selected loading pattern by computing a three dimensional diffusion calculation for an equilibrium core situation. The speed of the Minos solver associated to a distributed calculation using island's method have permit to find loading patterns proving the feasibility of such a configuration. This method still needs improvements to gain in accuracy and CPU time, but is promising for design and operation of reactors.

Key Words: Vizir evolutionary algorithm, Minos SPN solver, Island's method, Loading pattern, Radial shape factor, APOLLO3.

1. INTRODUCTION

In the frame of the third generation new design nuclear core, an EPR heterogeneous radial configuration loaded with fissile and fertile assemblies has been selected. This radial configuration associated to a square array core composed of under-moderated assemblies leads to a harder neutron spectrum inside the core and thus allows us to reach a high conversion rate. However, the presence of fertile assemblies produces a radial power peak or radial shape factor Fxy of 1.7 on some fissile assemblies. Moreover the draining coefficient (DC) defined as the reactivity discrepancy at the beginning of the cycle, between nominal core configuration and the one where accidentally it has almost completely lost its water (moderator density equal to 0.007) is worth 290 pcm. In term of safety constraints, one tries respecting the Fxy to keep margins in regard with EPR specifications [1]. Finally the Fxy must be lower than $F_{xy}^{\lim} = 1.5$, and the DC must be strictly negative. The configuration issued from the previous studies was not very satisfactory in term of these two constraints nevertheless a high sensitivity of both constraints on

the loading pattern and the repositioning map has been highlighted. Because of the lack of experiment on this core concept, and the huge number of loading patterns, we suggest in this paper to tackle this problem with a combinatorial optimization algorithm. Loading pattern could be seen as a search of an N elements permutation. Furthermore a loading pattern should respect different positioning constraints that lead to refuse some permutations. Evolutionary algorithms have been used with success on many combinatorial optimization problems; they use an internal representation of solution (chromosome) that should be adapted to the problem.

A previous study concerning fuel assembly repositioning in a PWR 900 core, has been done to reduce the radial shape factor[2]. However this study concerned a core composed only by 157 fuel assemblies. With the EPR core in a heterogeneous configuration, the number of fuel assemblies reaches now 241 fuel assemblies and some constraints have been relaxed that lead to increase the dimension of the search space.

2. FUEL LOADING PATTERN PROBLEM

As you can see in Figure 1, the studied nuclear core is filled with 241 fuel assemblies: 32 fertile assemblies composed by UO_2 materials, and the remaining of fissile assemblies with MOX materials.

About every year, the plant is stopped to restart a new cycle. Apart from the older assemblies (1/4 in our fuel management mode) is replaced by new ones. After that, all the assemblies are rearranged in order to optimize the burn-up of the core and satisfy the safety constraints.

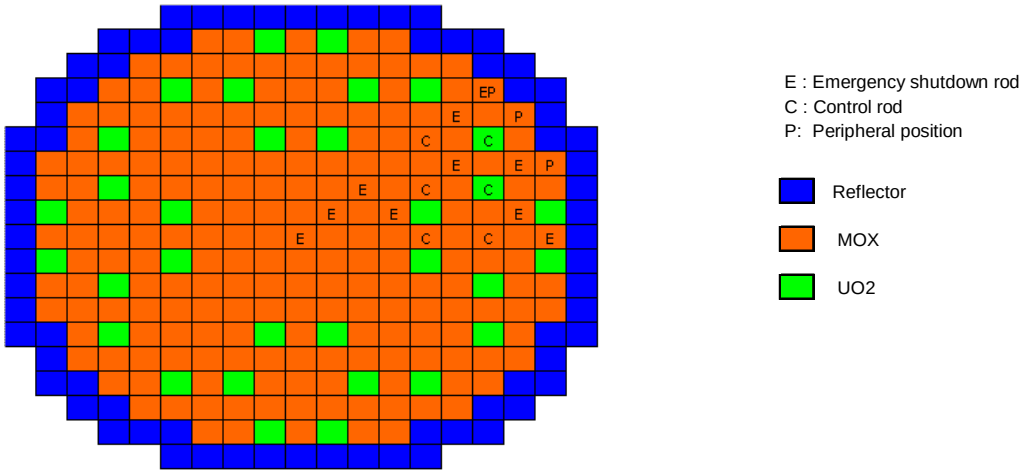

Figure 1: Reference configuration of the EPR core
(rods and peripheral zone are only indicated for the eighth of core)

In the framework of our study, the assemblies are classified according to their material and their age i.e. their cycle number (from 0 to 3 cycles, with the exception of the central assembly which is a 4 cycles MOX).

In operating conditions, the fuel loading pattern requires many constraints to respect safety and expert's rules. Two kinds of constraints are typically used: assemblies which can be forced in one position, moreover some positions are forbidden for certain classes of assemblies. In our design problem, the constraints are limited to the minimum: the central assembly is imposed; the new assemblies are forbidden in peripheral and rod positions.

This problem is an affectation problem (assemblies to positions) and so, the search space has factorial n possibilities. To limit the search space, we suppose that the configuration is subject to invariant transformations: 90° rotation and diagonal axes symmetries. In fact, the last transformation adds on to a new problem, because it creates two kinds of positions: those on the axes (vertical, horizontal and diagonal axes) which exist in four copies in the core and the others in eight. This makes it difficult to permutate assemblies between these two classes. Therefore, we opt to do away with these permutations leading to new constraints.

In a more complete fuel loading pattern problem, possible rotations of assemblies (due to invariant transformations) have to be treated, thereby increasing the search space.

3. EVOLUTIONARY ALGORITHM

Evolutionary Algorithm (EA) is a metaheuristic method used to solve different kinds of optimization problems. It is called metaheuristic because all mechanisms are subject to different variants. It is based on the metaphor of Darwin's natural selection. Its principle is simple: an initial population of subjects is randomly generated, their fitnesses depending on the evaluation of the objective function. At each generation:
- Less adapted subjects are eliminated (using the natural selection mechanism);
- The population is completed by generating new subjects (using the natural genetic mechanism). Two operators are used, a crossover operator which generates new subjects from two parents and a mutation operator which modifies a subject;
- The fitness of all new solutions is evaluated;
- At last, a convergence test is performed to eventually stop the population evolution.
EA and metaheuristics are often used in fuel cycle optimization ([3] and [4]).

Many genetic mechanisms have been proposed. The ability for the child to inherit from the useful characteristics of the parents is an important factor of efficiency. Usually, it is impossible to know which are the useful characteristics and, even if we know them, finding an adapted coding (i.e. the data representation and the associated operators) is an open problem.

3.1. Coding

The coding presented here is one among several tested. Along its accomplishment chance has contributed (as in EA searching method).

There are two characteristics that we want to deal with: to respect the affectation feature; and to satisfy constraints without having to perform an extra test. Effectively, constraints may reduce the search space in a drastic way, and if the generated solutions are not constraint safe, we might be forced to generate and run a series of tests to come up with a correct one.

Fuel assemblies are identified using structured data: the assembly class and a rank of the assembly in its class. This rank is associated to the assembly burn-up parameter. The new assemblies have all the same rank as they are very identical. The positions are identified by an integer. The chromosome of subject *i* is simply an ordered list of all assemblies $x^i_{j=1,N}$. According to eighth symmetry the dimension of that chromosome is equal to the number of assemblies in the eighth of core whose value is 38 in the case of EPR.

For the evaluation, a permutation is constructed from an initial loading pattern to obtain a new one. Several permutations are relevant, because new assemblies are identical.

They are many crossover operators that deal with permutation problems [5]. Our “mixed variant” crossover operator is adapted from the "cycle crossover". The principle is to find a subset of gene (a cell from the list) that contains the same set of value in both parents. When a subset is found, the child therefore inherits all traits from one parent which is randomly selected. This crossover operator has the following properties: for starters if both parents have the same value at the same place, the value will be maintained for the child. This will ensure that forced position assemblies will be kept. Secondly, if value is in one gene of the child, one of his parents has the same value in the same gene. This will be used to respect forbidden position for class assembly.

In our variant (color grade coding), this principle concerns only the class value but we try to preserve the rank value. Once a cycle is found, we maintain the random choice of parents, keeping the rank value, but if it is already used (inherited from the other parent) we mark the position. When all positions have been filled, for each class, we look at the marked positions and the unused assemblies, and change the rank value, minimizing the rank distance.

An example is shown in figure 2, assembly id follows the <classid>.<rankid> form, x denotes a marked position.

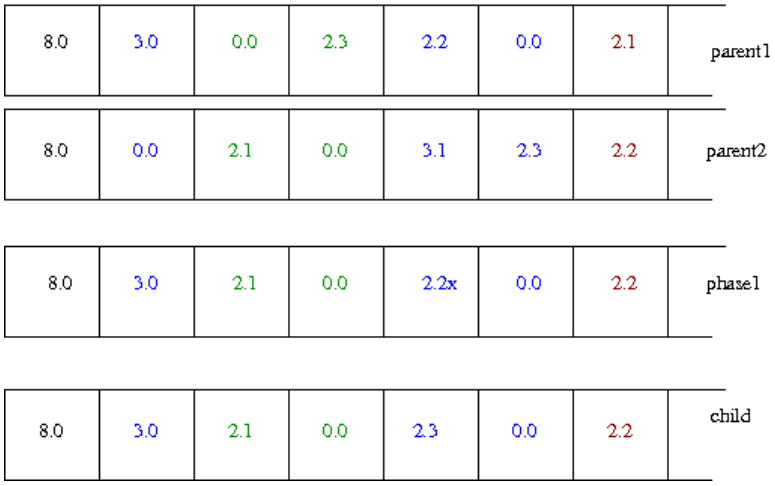

Figure 2: Crossover principle

The subset (in blue) of the 2th, 5th and 6th gene contains the same 0, 2, 3 class value. To find such a subset, cycles are identified on the class value. Here, the cycle is:
$3 \rightarrow 0$ (gene 2), $0 \rightarrow 2$ (gene 6), $2 \rightarrow 3$ (gene 5).

In figure 2 and for the blue cycle, the child inherits from the first parent. Ranks are preserved except for the 2.2 value which is already inherited from the 7th gene. During the second phase, the unused 2.3 value will be affected to the 6th gene.

Our variant has been compared to a full cycle crossover (on class and rank), and to another coding that we have already used, based on a linear affectation problem (chromosome contains the cost to put an assembly in one position [2], this second coding is still used to generate the initial population). Clearly the second coding is at a disadvantage. A more extensive test must be done to validate our initial feeling, but our variant seems to be the most efficient.

We also test a variant (named as color coding) whereby we delete the rank order to identify assembly. This suggests that many configurations are represented in one template, as all assemblies of the same class are considered as equivalent. Concerning the fitness of a template, only one configuration is evaluated (arbitrary but not randomly chosen). We expect that this template is coherent, meaning that the objectives of the templates configurations are relatively close. This simplification is interesting because it limits the search space. On the mono-objective problem, we find interesting solutions with this coding, but worse than the complete coding. On the bi-objective problem, the result is very interesting. In fact, this coding seems to act as an operator that maintains diversity [6] (which generally eliminates close solutions), and, as we are limited in term of evaluations (as well as population size), it avoids a premature convergence.

3.2. Distribution

Choosing to use a numerical code solving the diffusion equation, the distribution of evaluations is mandatory to succeed in finding some significant result in a reasonable time (we started our work on a fuel loading pattern with analytic neuron network).

It is easy to introduce parallelism in evolutionary algorithm during the population generation phase. But this natural way is not the most efficient. In the island model [7], evolution occurs in multiple parallel subpopulation (the island), each one running a local EA. After some generation, all island evolutions stop, some subjects change from one sub-population to another, and the optimization loop starts again. In our version, we developed an asynchronous migration of subjects: an island sends a subject to another island; this subject is buffered in the receiver, and will be integrated in the sub-population at the next birth, as a new generated one.

Introducing the second objective, we need an even more distributed optimization. Effectively, the second objective calls to a further costly function, and multi-objective optimization usually requires more evaluation than single one. Nonetheless reducing too much island population is irrelevant. We therefore introduce a two levels distribution: the old top level distribution using island method and asynchronous migration; a second level distribution within an island, bringing with it a series of evaluations in parallel.

When the Apollo3 system [8] will integrate the developments on domain decomposition method, a third distribution level will be added within the evaluation calculation.

4. EVALUATION OF THE OBJECTIVE FUNCTION

The objective functions are evaluated using the Minos solver integrated in the new lattice-core Apollo3 system [8]. The Minos code [9] provides a complete tool for static and transient calculations either on Cartesian or on unstructured geometries.

Minos solves the simplified transport approximation (SPN) to the transport equation. It is based on the mixed dual finite element approximation of the SPN equations. The iterative algorithm consists first in the elimination of the flux of unknowns and then to getting a linear system having as unknowns the normal current on each edge of the triangulation.

For the Cartesian geometry, the Raviart-Thomas-Nedelec elements (RTN) are used to discretize the different functional spaces. The iterative algorithm used to solve the within-group source equations can be viewed as an alternating direction sweep, performed on the different components of the current vector.

The solver has been recently extended for SPN Kinetics calculations.

In order to speed up calculations using a great number of meshes, an iterative domain decomposition method (IDD) [10] has recently been implemented in the Minos solver. This method is an iterative scheme, using non-overlapping domain decomposition and is based on the exchange of boundary values. The principle is to iterate the resolution of local problems on each sub-domain, using Robin interface conditions.

5. RESULTS

5.1. Mono criterion calculation

The present calculation concerns the search of a loading pattern for the heterogeneous EPR core previously described.
The computations are distributed on three layers of respectively 15, 30 and 15 islands. Each computing island of layer 1, 2, 3 treats respectively subpopulation of 150, 200, 300 subjects. The maximum number of evaluations per subpopulation has been fixed to 24000. The objective function depends on the radial shape factor Fxy. For each generation, the subpopulation is converged when the difference between the best and the worst evaluation of the objective function is lower than threshold accuracy. The migrations between islands of adjacent layers are triggered in asynchronous and stochastic ways.

In order to know if we are dealing with a global or a local minimum, we used two different measurements called D1 and D2. Both are based on a Euclidean norm representative of the distance from the coding $x^{i}_{j=1,N}$ of subject *i* to the coding $x^{opt}_{j=1,N}$ of the best estimate subject *opt* (reminding that N is the number of assembly in the eighth of core) :

$$D_1(i,opt) = \sqrt{\sum_{j=1,N} (X_j^i - X_j^{opt})^2} \quad \text{with} \quad \forall j \in [1\,,\,38\,] \; if \; (x_i^j \geq 4): \; X_i^j = 2 \; else \; : \; X_i^j = 0$$

$$D_2(i,opt) = \sqrt{\sum_{j=1,N} (x_j^i - x_j^{opt})^2}$$

Distance D2 provides information about permutation between 2 assemblies taking into account their class and rank while distance D1 depends only on permutation on two different material (fertile vs. fissile assembly) as defined in section 3.

In figure 3, it appears that there are many different candidates providing the same minimum around 1.33 and they are relatively close to the absolute minimum in term of rank and class. When D1 and D2 increase both there is no immediate similar behavior of the Fxy factor. There are many subjects whose Fxy is lower than 1.375 (these subjects are separated from the best estimate subject: Fxy=1.33 by a distance D_1=0, $D_1=2\sqrt{2}$, D_1=4 that corresponds respectively to zero, one and two swapping between fertile and fissile assembly). However, few of them are located above the value D1=4 (three swapping). Above D1= $2\sqrt{6}$ (four swapping which corresponds to the maximum feasible in eighth's symmetry) density of subject is clearly reduced. Besides the concentration of subjects around D1=0, and 4.5 < D2 < 5, then D1= $2\sqrt{2}$, and 5.5 < D2 < 6 not very close to the lowest values reached for D1=0, D2=0, and D1= $2\sqrt{2}$, D2= $\frac{3}{2}\sqrt{19}$, indicates that there are several local minima and that they probably have not been found. Nevertheless, many solutions meet the EPR specifications concerning the constraints on the radial shape factor (being required lower to $F_{xy}^{\lim} = 1.5$ in nominal operation [1]).

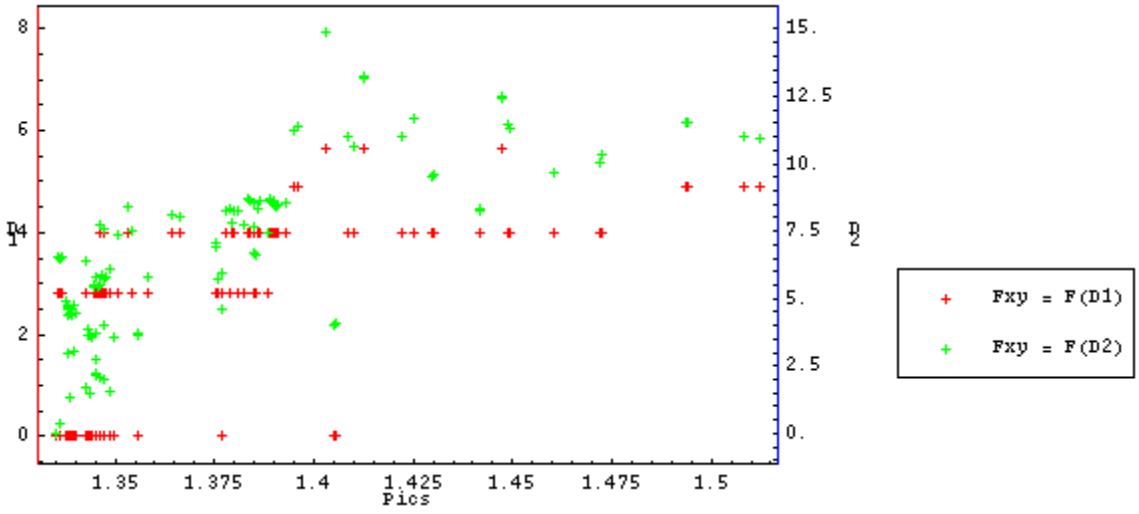

Figure 3: Distances distribution of solution subjects from the best estimate subject.

The solution subject issued from a 2-D calculation must be validated at the equilibrium cycle of the core, indeed few points have to be checked: the 2-D shape factor should be correct; some operating parameter must not get worse than the GENTR project specification (for instance the conversion factor from fertile to fissile material must remain close to 0.8); the core should respect some safety constraint not checked during optimization. One of these constraints concerns the draining coefficient DC defined as the reactivity discrepancy between nominal core configuration and the one where accidentally the core has almost completely lost its water. In this instance, the reaction must be naturally stopped and this leads to a DC that must be strictly negative.

This equilibrium has been performed by a 3-D diffusion calculation. The different radial (Fxy), axial (Fz) shape factors and hot point factor (Fq), for the reference and the optimized subject are shown in figure 4 during fuel evolution. Along the entire cycle, all factors of the optimized subject are lower than the reference ones and its Min-Max discrepancy follows the same behavior. In addition, the hot point factor Fq is slowly strictly decreasing. The maximum Fxy is reached at the beginning of the cycle. The 2-D factor Fxy is over- estimated for the 2-D calculation in comparison to the 3-D one, leading to assume that our process is conservative.

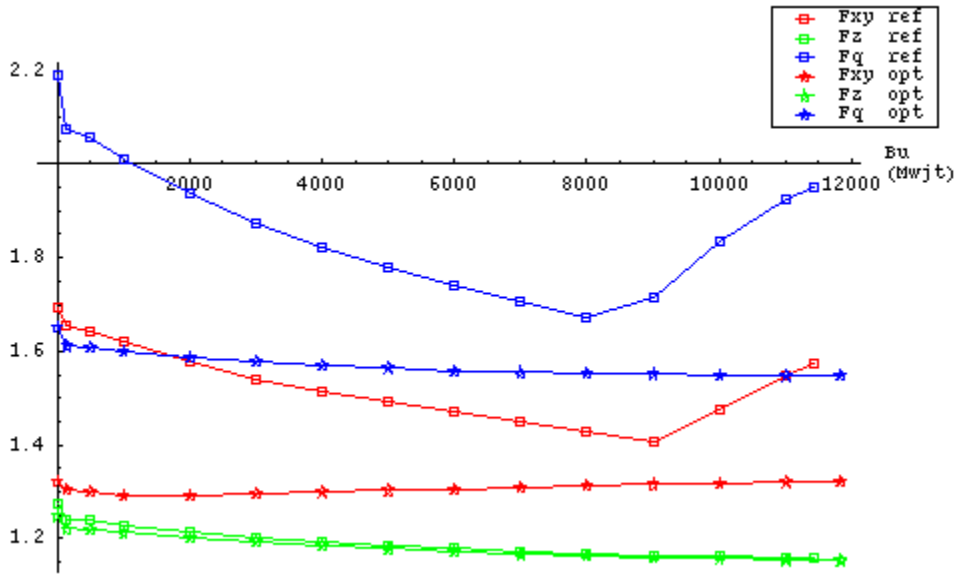

Figure 4: Evolution of the different shape factors of the optimized and the reference subject during a core cycle.

Figure 5 compares the DC of the reference performed in 2-D and 3-D to the ones of optimized subject. That comparison seems to argue for an important spreading on this measurement, according to the loading pattern. Whereas the DC of the reference is close to 6000 pcm, one of the best optimized subject is rather close to 4000 pcm. Moreover, the DCs are all largely positive making them unacceptable. This consideration leads us to take into account a new criterion: the DC must be at least negative at the beginning of the cycle where it is the highest. Considering this new criterion, should we perform a 2-D or a 3-D calculation for its estimation?

To deal with the bi-criterion problem in 3-D increases fatally the time calculation. Indeed the objective function, whose cost depends on the mesh size, is not only quite higher in 3-D but the bi-criterion problem requires more evaluations to obtain a converged population. Presently, we are also limited by the batch queue's CPU time limit on our cluster which is 24 or 48 hours according to the number of allocated processors.

In addition, the DC depends on the neutron's absorption and neutron's leakages in the core. Since we have a fast neutron spectrum in draining situation, a height group's energy diffusion computation is needed to get a good estimation of the second objective function. The difference between the 3-D and 2-D computed DC is the sum of two contributions: the absorption added with the axial leakages, and the radial leakages. The first one which is predominant may be considered as a constant while the second is lower and depends on the subject. As shown in figure 5, for two subjects, differences between 3-D and 2-D draining coefficient values are quite similar for the two subjects. This discrepancy value could be estimated to $\Delta DC = 2200$ pcm, rounded to 2000 in order not to remove interesting subjects. This value is then used to adjust the threshold of the second objective. This way tends to reach agreement between the time calculation required to allow the convergence of the EA and the physics's relevance of the 2-D computation.

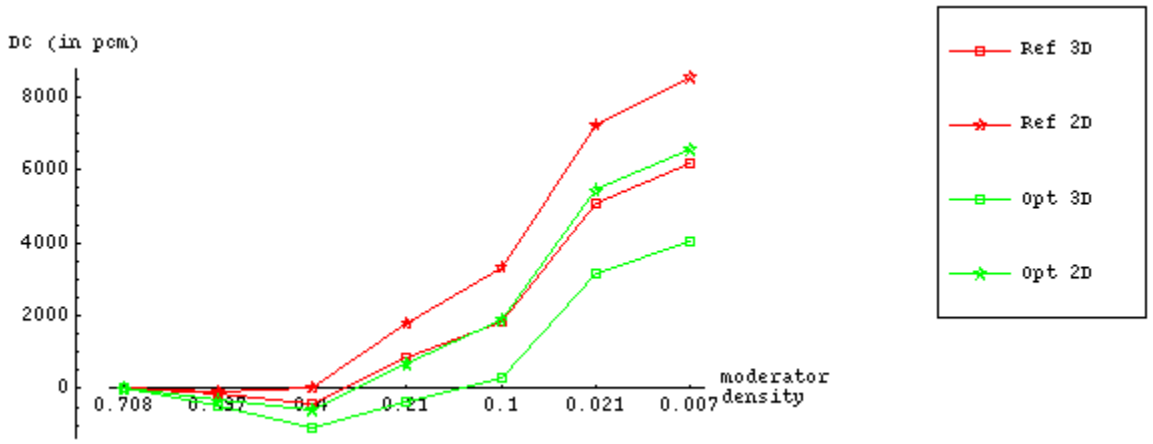

Figure 5: Comparison between 2D and 3D draining coefficient for two subjects: the case reference and the optimized one.

5.2. Bi criteria calculation

EA is often used to solve a multi-objective optimization problem but rarely for a combinatorial problem. This bi-objective problem is defined on both Fxy and DC parameters. The optimization attempts to find the Pareto's front of the problem. Two constraints are added to define feasible solutions for the 2-D evaluations: Fxy should be lower than $F_{xy}^{\lim}$ and DC lower than ΔDC. One proceeds to several calculations whose parameters are described in table I and results in figure 6. Related to islands communications, calculations are now based upon a single layer. Survival, migration and mutation rates are similar as well as the frequency of migrations.

Table I. Features of the simulations

Simulation	Number of islands	Population size	Number of processor by island	Max evaluation	Not converged island	Coding
S0	64	70	1	5000	10	Color
S1	16	140	4	16000	14	Grade
S2	32	120	8	16000	22	Grade
S3	32	150	8	16000	26	Grade
S4	16	160	16	32000	0	Grade

Adding the DC evaluation increases significantly computing time and the limits of the batch queue are reached. Thus the number of evaluations and populations must be reduced in such way that convergence may be premature. Effectively the convergence is reached when all subjects of the population are not dominated. In the case of large Pareto's front and small population, finding such a subset is quite easy even if subjects are far to the real Pareto's front. Nevertheless convergence of big populations needs ever more evaluations. Furthermore the limit of computing time on large parallel queues (number of processors greater than 64) implies using ever more processors per population to get convergence. Thus the management of large population requires lot of processors even if it means to decrease the number of islands and then to loose the supra linear effect on the parallel efficiency (with N parallel process, CPU time becomes lower than $\frac{1}{N}$ of the sequential time).

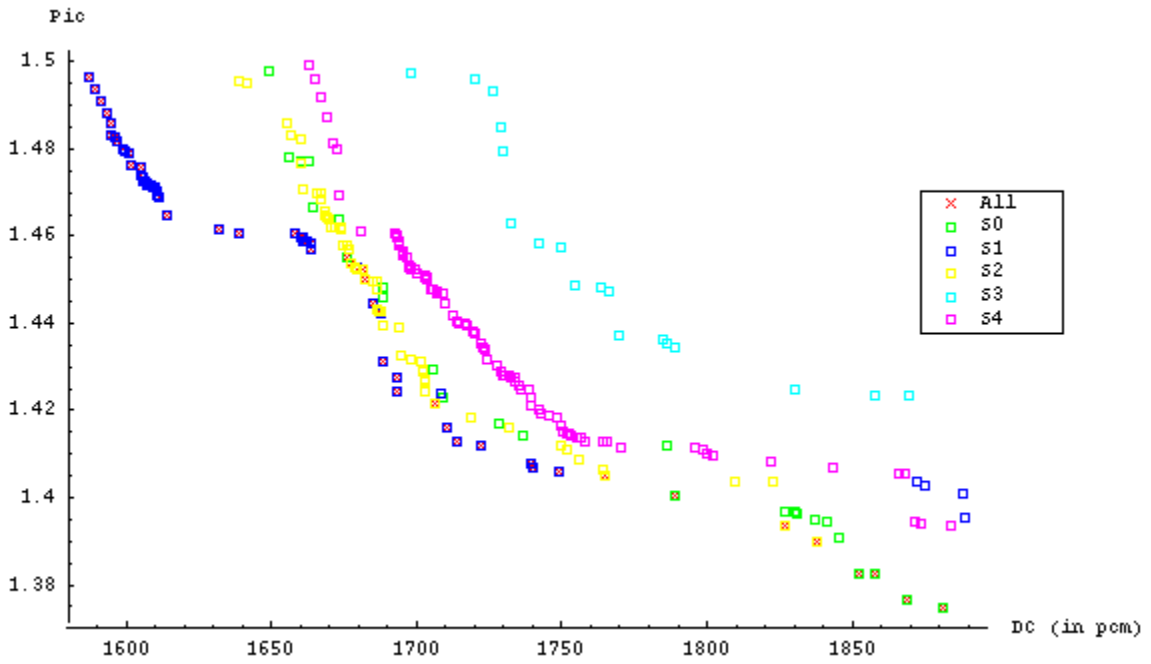

Figure 6: Pareto's front obtained from the simulation S0 to S4 and all.

The results presented in figure 6 are quite unnatural and raises few questions: why does bigger optimization (either with more islands or more subjects per island) give worse results? Why does color coding perform so efficiently? This tends to confirm our intuition that we are in an unstable zone where both population size and evaluation number are too small. This leads to either an inadequate stopping test or a non-converged population or both. Combined to the stochastic side of EA, that could explain such chaotic results. Color coding by restricting the search space may reduce such effect. Further investigations are needed to confirm that intuition and find more robust results.

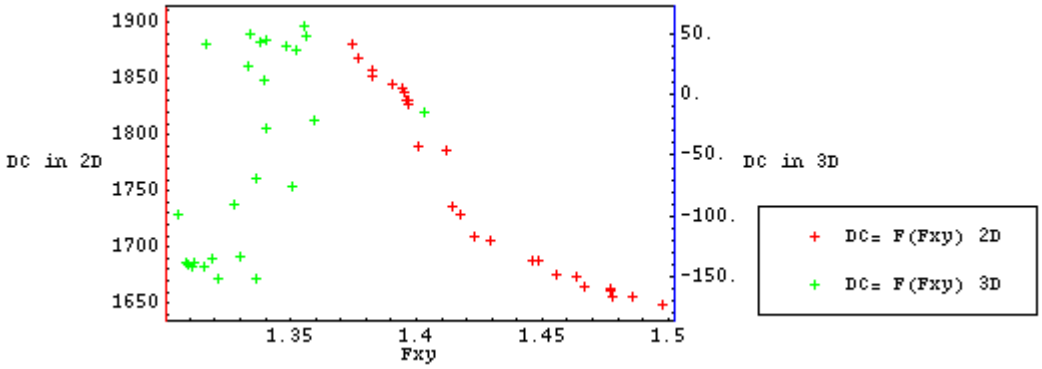

Figure 7: Pareto's front in 2D and 3D from S0 simulation

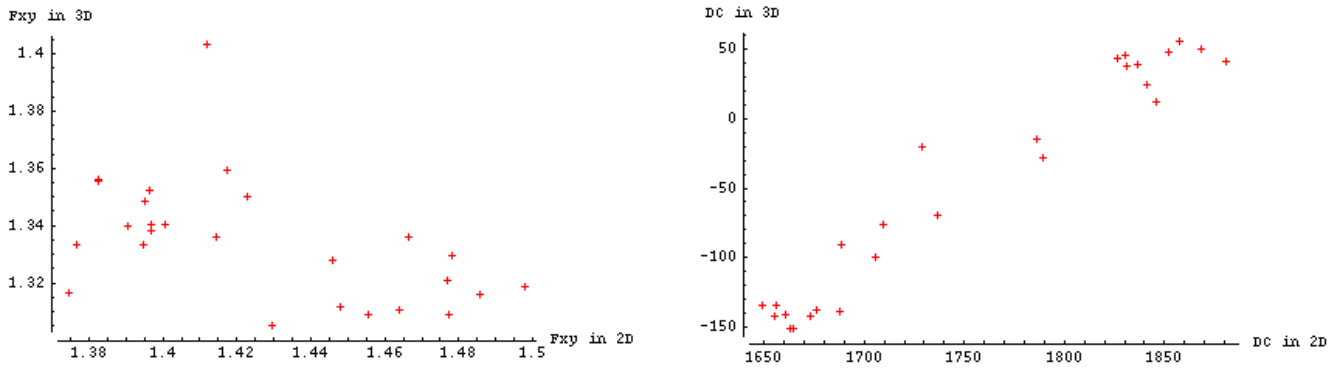

Figure 8: Evaluation in 2-D and 3-D of the objective function of the Pareto's front subjects

As all fronts arising from 2-D simulations, the front's reliability in 3-D has been tested. Figure 7 shows the fronts issued from 2-D S0 simulation and its transposition using a 3-D computation. The transposition provides a completely chaotic set. The origin of the phenomenon seems to be the Fxy as the correlations between the 2-D and the 3-D presented in figure 8 tends to assume. The mismatch on the rank level between 2-D and 3-D Fxy calculations shows that, even in the mono-objective problem, the selected LP cannot be considered as the optimal one. The 3-D validation calculation of Fxy, considering a simplified model of thermo-hydraulic feedbacks, and loop cycles to reach the equilibrium, is more accurate than the 2-D calculation. That

consideration could partly explain the weak correlation between the two set of Fxy values. Moreover the same figure confirms that the results are in good agreement with the first order approximation of the DC. There is thus an almost linear correspondence between the DC values of the subjects of the Pareto's front and their transposition to 3-D evaluation, the spreading of the values is also well preserved and remains close to 200 pcm.

6. CONCLUSION

At the present state of our investigation, the proposed bi objective optimization has reached the request fixed in the frame of GENTR project, proving the existence of LP which gets Fxy clearly lower to $F_{xy}^{\lim}$ and strictly negative DC.

Once this first objective is fulfilled, a more complete analysis, which the main goal would be to further perfect the search of the best LP, is still remaining to achieve. Improvements should be made both to the evaluation of the objective function and to the EA itself. First, the current 2-D objective function must be modified, although the results obtained are satisfactory, they are not probably the best ones. Thus, we look for a set of non dominated subjects in 3-D coming closest to the theoretical Pareto's front. For this purpose, either the Fxy objective function must be directly computed in 3-D or some fitting out should be done to the current one as setting a correction to the cross section absorption which considers the axial leakages of neutrons. Thermo-hydraulic feedbacks could be considered in both cases. Secondly, at short term, EA parameters should be refined to obtain more robust results. And, on the long view, heterogeneous islands would be tested by mixing either 2-D and 3-D Fxy calculations or color coding and ours. Another prospective way would be to use a mechanism for resuming the calculation which allows us to explore more locally interesting subjects (for instance, with constant value of D1).

Of course, if we proceed to all these fitting out, the time calculation would be higher. So, further developments related to a multi –threading distribution and the use of the domain decomposition in case of a 3-D calculation would be investigated and thus would allow us to improve the accuracy of the bi objective optimization problem with a suitable time.

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