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VERIFICATION OF GEOMETRICAL PERTURBATION CALCULATIONS FOR THE PULSED FAST REACTOR IBR-2

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ABSTRACT

The IBR-2 is a small fast reactor with plutonium oxide fuel and liquid sodium cooling. A rotating two-reflector system periodically closes and opens a non-reflected side of the reactor and leads it through the prompt super critical state in a time of about 500 microseconds. During this time a burst of fast neutrons is generated in the core of the reactor, which spreads out through moderators. Channels lead the neutrons to measurement instruments.

Certain reactivity parameters of the rotating reflector system determine essential parameters of the reactor as a pulsed neutron source. Just now a new reflector system, the so-called PO-3, has been installed. For this reason the important reactivity effects of the reflector were newly calculated with help of the MCNP-4C2 code. The contribution presents numerical results, which were obtained in different ways and compares results of calculations and measurements. The special interest consisted in studying the effect of neglecting the fission source effect, as assumed by the PERT option of the code, on the calculation results. The comparison of numerical with measurement results showed agreements in some cases but also considerable discrepancies, which could not yet be explained. Therefore, no final conclusion on the applicability of the PERT method for the calculation of total and displacement reactivity effects of the rotating reflector system of the IBR-2 reactor could be drawn.

Key Words: Pulsed fast reactor IBR-2, reactivity effect, geometrical perturbation, MCNP calculations

1 INTRODUCTION

One of the most powerful, pulsed sources of thermal and cold neutrons on the world is the pulsed fast reactor IBR-2 of the Joint Institute for Nuclear Research (JINR) Dubna, Russia [1-3]. It is a small sodium cooled reactor with plutonium oxide fuel. The core of about 22 liters produces a mean power of 1.5 MW. The neutron pulses are periodically generated with a frequency of 5 Hz by external reactivity modulation of the reactor that is accomplished by a rotating two-reflector system consisting of a main (near to the core) and an auxiliary rotor. Both reflectors rotate around the same axis but with different velocities. Figure 1 gives a schematic view of the reactor with the reflector version PO-2, which now was substituted by the new system PO-3.

The neutron pulses are produced during a time of about 500 microseconds around the reactivity maximum that is when both rotors maximally reflect the reactor at its open side. In this time the reactor goes through the prompt super-critical state and the core produces the pulse of fast neutrons. Figure 2 schematically illustrates the relationship between reactivity course and neutron pulse for the case of the PO-2 in the standard regime with rotation frequencies of 25 Hz and 5 Hz for the main and auxiliary reflector, respectively. The neutron pulse propagates outwards through the stationary reflector and through different types of moderators and feeds several channels with thermal or cold neutrons.

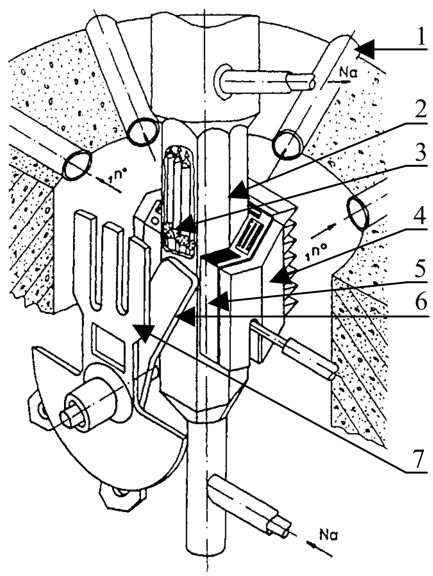

Figure 1. Schematic outline of the IBR-2 with reflector PO-2. 1 neutron channel, 2 reactor tank, 3 reactor core, 4 moderators, 5 stationary reflector with control elements, 6 main reflector, 7 auxiliary reflector.

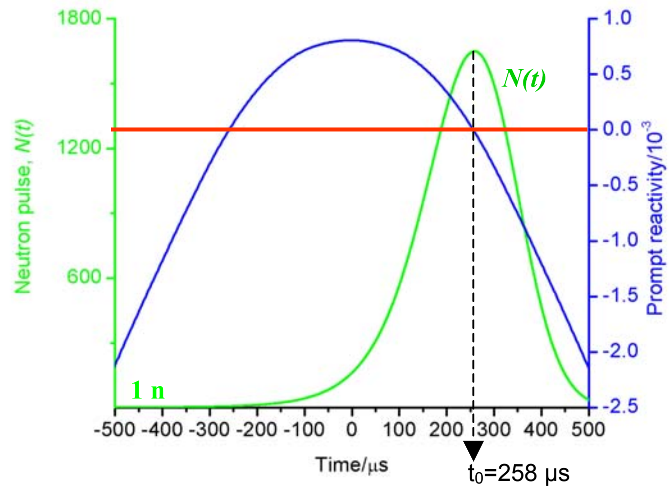

Figure 2. Courses of prompt reactivity and of fast neutron pulse (started with 1 neutron) in IBR-2 for the standard regime with the reflector system PO-2.

Two reactivity effects of the reflector system are most important: Those are the total reactivity Δk_R and the differential effect $\partial k/\partial\varphi$ with respect to the rotor rotations. In the notations used k is the effective multiplication factor of the reactor and φ is the rotation angle. The total effect determines the level of the neutron background between pulses and, therefore, for a minimal background it should be as high as possible. The reactivity deviation determines the pulse length θ of the fast neutrons, which in case of the IBR-2 follows in good approximation the relationship

$$\Theta \cong 2.35 \cdot \sqrt{\frac{\tau}{|\Gamma|}} \,. \qquad (1)$$

In this equation is τ the generation time of the fast neutrons and Γ is the time derivative of the effective multiplication factor at time t_0

$$\Gamma \cong \left.\frac{dk(t)}{dt}\right|_{t=t_0} . \tag{2}$$

The time moment t_0 is the so-called working point of the critical pulsed reactor. As illustrated in Fig. 2, in this moment the reactor comes back to the prompt sub-critical state. The working point is determined by the criticality condition of the pulsed reactor. Therefore, it depends on the course of the prompt reactivity determined by the reflectors, on the reactor parameter τ, and, furthermore, on the pulse frequency f_n and rotation frequencies f_I and f_{II} of both rotors, where the indices I and II stand for the main and auxiliary reflector, respectively. For a two-reflector system the multiplication factor depends on both rotor positions, i.e.

$$k(t) = k(\varphi_I(t), \varphi_{II}(t)) . \tag{3}$$

Therefore, it follows

$$\Gamma = \omega_I \cdot \gamma_{I_0} + \omega_{II} \cdot \gamma_{I_0} \tag{4}$$

with the partial derivatives with respect to the rotor movements just in the positions corresponding to the working point t_0

$$\gamma_{I_0} \equiv \left.\frac{\partial k}{\partial \varphi_I}\right|_{t_0} , \quad \gamma_{II_0} \equiv \left.\frac{\partial k}{\partial \varphi_{II}}\right|_{t_0} \tag{5}$$

and with the angular velocities of both rotors ω_I and ω_{II}. So, the optimization of the reflector system demands verified calculation methods both for the total reactivity effect and the partial reactivity derivatives.

A reflector optimization study was carried out in the years 1981/82 by means of experiments and calculations and resulted in a proposal for the PO-3 [4-6]. At that time, the target of the optimization was a minimal pulse length θ for given maximal rotation frequencies of the rotors. With some minor deviations from the proposed version the new reflector system was just now installed at the reactor. It will produce fast neutron pulses with nearly the same length as the PO-2, but with considerably reduced rotation velocities and, by that, allow a much longer service time. During the start-up experiments the important reactivity effects were measured. So, it was an opportunity to recalculate the reactor and the reactivity effects of the reflector with help of methods, which are offered by the actual MCNP-4C2 code [7] and to verify their applicability by comparison of measured with calculated results.

2 PERTURBATION CALCULATIONS

2.1 Total Reactivity Effects

For the MCNP calculations a new detailed geometrical model of the reactor and its near surroundings has been created. Figure 3 shows this model where only a few less important details have been merged to homogenized material zones. The outer parts of both rotors are shown in

Fig. 4. They are made of a special nickel alloy. In addition to PO-3, the total reactivity of a model of the PO-2 main reflector was also calculated. This model called 1-6H was measured previously in the experimental study EPOS-2. The measurement results are given in Ref. [4]. The model is a simple plank made of carbon steel with the dimension: thicknes=5.4 cm, width=23.4 cm, height=50 cm. The total reactivity effects were now calculated by means of MCNP-4C2 in two ways: First, as a perturbation effect with help of the PERT option and, second, as difference of two normal criticality calculations for the states “with” and “without” reflector. The results of calculations and measurements are given in Table I.

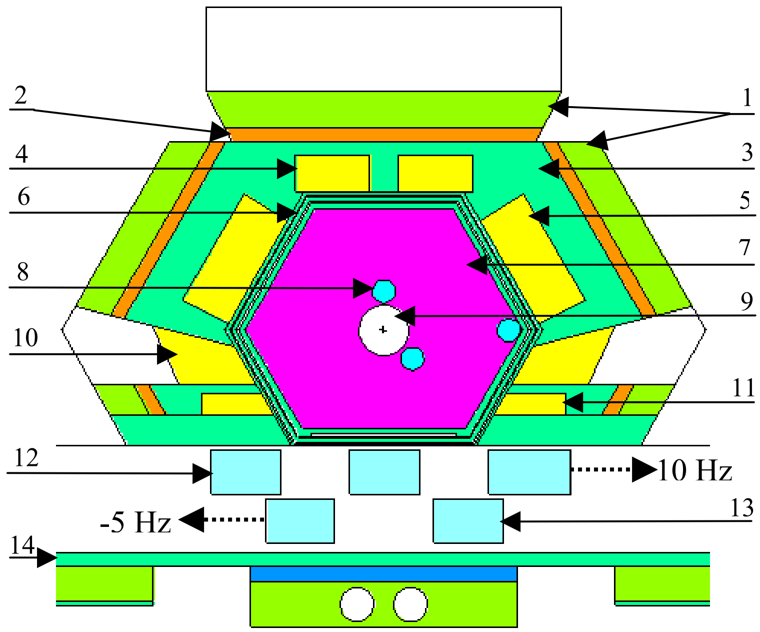

Figure 3. Horizontal cross-section of IBR-2 with the new reflector system PO-3 as used in the MCNP geometry model. 1 water moderators, 2 boron carbide, 3 stationary reflector, 4 slow shutoff element, 5 control element, 6 double wall core tank, 7 core, 8 dummy element, 9 central channel, 10 fast shutoff element, 11 operator control element, 12 main reflector (I), 13 auxiliary reflector (II), 14 case of the rotating reflector.

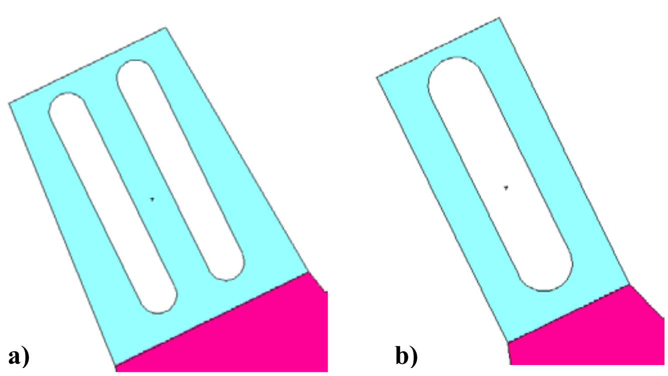

Figure 4. Outer parts of PO-3 rotors as used in the MCNP geometry model. a) Main reflector (I). b) Auxiliary reflector (II).

Table I. Measured and calculated total reactivity effects of IBR-2 reflectors

Reflector	Experiment $\Delta k_R/10^{-2}$	Calculation, $\Delta k_R/10^{-2}$ PERT	direct	C/E PERT	direct
1-6H	3.40	3.29±0.03	3.19±0.10	0.97	0.94
PO-3	2.55	2.96±0.02	3.22±0.10	1.16	1.26

2.2 Differential Reactivity Effects

The partial derivatives γ_{I_0} and γ_{II_0} determined by Eqs. 5 were approximately calculated as small, finite difference effects δk caused by small displacements in the vicinity of the rotor positions corresponding to the working point. These positions were considered to be given. At first, for the geometrically simple 1-6H reflector the displacement effect was calculated in various ways implying different approximations to clear up their influence on the numerical result. After that, the partial derivatives were calculated for the PO-3 according to one modeling scheme.

Figure 5 schematically illustrates the situation for the 1-6H reflector assuming a straight displacement δx of the reflector to the right. The total effect consists of two constituents:
(1) On the left side a material zone is voided. The reactivity effect of the voiding is negative.
(2) On the right side a void zone is filled with material. The contribution of this step is positive.

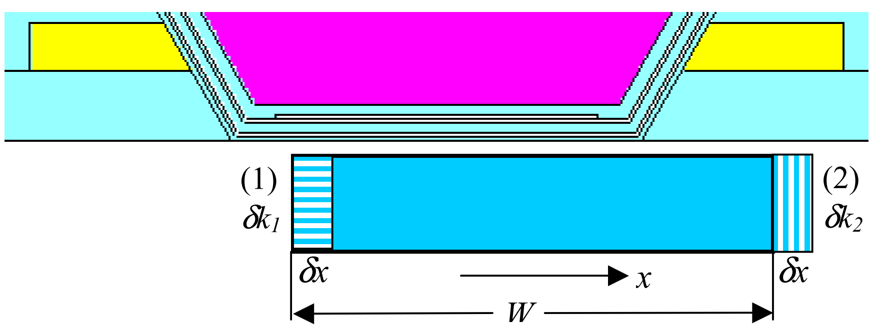

Figure 5. Schematic representation of a small reflector displacement δx in the vicinity of the working point.

The Taylor series expansion method [7] used by the PERT option of MCNP cannot calculate the effect δk_2 of materializing a void zone. For practice it is recommended to calculate the voiding effect and to take the opposite sign [8]. Considering this recipe the total displacement effect $\delta k = \delta k_1 + \delta k_2$ was calculated in following ways:

A) By two PERT runs each with one perturbation zone 1 or 2 only and giving δk_1 and $-\delta k_2$, respectively.
B) Realizing the recipe in one PERT run but comprising both perturbation zones. For this case the following two calculations were carried out:

   1. Reflector of width $W + \delta x$ in the left position, voiding both perturbation zones and calculating the effects δk_1 and δk_2 separately.
   2. The same as 1., but with a reflector of real width W.

C) As by-product of two long-time criticality calculations also the total reflector reactivity effects Δk_{s1} and Δk_{s2} in its two shifted positions $s1$ and $s2$ were calculated by means of the PERT option. From these results the displacement effect was calculated $\delta k = \Delta k_{s2} - \Delta k_{s1}$.

The two criticality calculations mentioned before under C) gave the effective multiplication factors for the system states k_{s1} and k_{s2}. From these results the displacement effect was directly calculated $\delta k = k_{s2} - k_{s1}$.

In all calculations for the 1-6H reflector the positions $x_{s1}=5\ cm$ and $x_{s2}=6\ cm$, i.e. $\delta x=1\ cm$, were used. In the PERT runs about 7 million histories were simulated, but in the criticality calculations about 600 million histories were necessary to get nearly the same statistical error for the displacement effect.

The reactivity values measured in EPOS-2 were fitted by a parabola $k(x) \cong k_0 - \alpha_{exp} \cdot x^2$ with the result $\alpha_{exp}=0.832\ 10^{-4} cm^{-2}$ [4]. The measurements showed that the parabolic dependence on the deviation x is valid up to about eight centimeters. To compare the calculated derivatives $\delta k/\delta x$ with these measurements the derivatives were also converted into an α-parameter according to the relationship $\alpha_{calc} = -(\delta k/\delta x)/(2x_0)$. Here, the derivative was assigned to the deviation from the center $x_0=5.5$ cm. Numerical results and the comparison with measurements are given in Table II.

Table II. Results of various calculations of displacement reactivity effects for the 1-6H reflector and comparison with measurements

PERT	$\delta k_1/10^{-3}$	$\delta k_2/10^{-3}$	$\alpha_{calc}/10^{-4}\ cm^{-2}$	C/E
A	-1.61 ± 0.03	0.43 ± 0.02	1.07 ± 0.04	1.29
B1	-1.56 ± 0.03	0.41 ± 0.02	1.05 ± 0.04	1.26
B2	-1.61 ± 0.03	0.51 ± 0.02	1.00 ± 0.04	1.20
C	$\Delta k_{s1}/10^{-2}$	$\Delta k_{s2}/10^{-2}$		
	3.191 ± 0.002	3.084 ± 0.002	1.30 ± 0.03	1.56
direct	k_{s1}	k_{s2}		
	0.99258 ± 0.00003	0.99118 ± 0.00003	1.27 ± 0.05	1.53

The partial derivatives for the rotors of the PO-3 were calculated for the planned counter rotation scheme of 10 Hz and –5 Hz. In the perturbation calculations small rotations in opposite directions near their working points were simulated. These positions were determined from the measurements as deviations from the physical center, i.e. from the meeting position of both rotors: $\varphi_I|_0 = 1.5^\circ$ and $\varphi_{II}|_0 = -0.75^\circ$. The system was that shown in Figs. 3 and 4 with inclined rotors. The perturbations caused by the rotor movements $\delta\varphi_I = 1°$, $\delta\varphi_{II} = -1°$ were modeled with the PERT option according to scheme B2 in one run as described above for the reflector 1-6H. The partial reactivity effects δk_I and δk_{II} were separately calculated. For good statistics 130 million particles were simulated. The curve of the total reactivity of the PO-3 was precisely measured for the rotation regime of 10:-5 Hz but, unfortunately, the partial derivatives of both rotors were not measured separately. The measured values show that in the range of rotor deviations as modeled in the calculations the reactivity curve $k(\varphi_I, \varphi_{II}(\varphi_I))$ shows already a linear dependence on φ_I with a total derivative $dk/d\varphi_I|_{exp.} = 2.10 \cdot 10^{-3}$ $degree^{-1}$. From the calculated partial derivatives the total derivative follows according to

$$\left.\frac{dk}{d\varphi_I}\right|_{I_0} = \gamma_{I_0} - 0.5 \cdot \gamma_{II_0} . \qquad (6)$$

In addition to the PERT calculations, two long-time criticality calculations were also carried out for the rotor positions $\varphi_I|_0 = 1.5°$, $\varphi_{II}|_0 = -0.75°$ giving $k_1 = 1.02412 \pm 0.00005$ and for $\varphi_I|_0 = 2.5°$, $\varphi_{II}|_0 = -1.25°$ giving $k_2 = 1.02196 \pm 0.00004$. The difference of the multiplication factors directly yields the total reactivity effect. Table III presents the results of the PO-3 calculations and the comparison with the measurement.

Table III. Total reactivity derivatives calculated for the counter rotation regime of the reflector system PO-3

Calculation	$dk/d\varphi_I\|_{I_0}$	C/E
direct	$(2.16 \pm 0.06)\ 10^{-3}$	1.03
PERT	$\gamma_{I_0} = (-1.33 \pm 0.05)\ 10^{-3}$ $\gamma_{II_0} = (+0.92 \pm 0.03)\ 10^{-3}$ $(-1.79 \pm 0.06)\ 10^{-3}$	0.85

3 DISCUSSIONS

The main numerical method used for the optimization study in the years 1981/82 was the Monte Carlo method as well. For this purpose an own code had been developed [1] and the MORSE-CG code had been specifically modified [9,10]. Both codes used the so-called perturbation source method that had been developed by Takahashi [11], Matthes [12] and others. This technique is based on an expression derived from perturbation theory for the reactivity perturbation δk as a result of material changes in certain perturbation zones. According to that the perturbation is proportional to the adjoint flux weighted difference of two scattering rates the cross-sections of which are the changes $\Delta\Sigma_{tot}$ and $\Delta\Sigma_{scat}$ of the total and scattering cross-sections in the perturbation zones, respectively. A Monte Carlo estimate of δk is obtained in the following way: Basic neutron histories are realized in the unperturbed state of the system. The (unperturbed) flux of these neutrons inside the perturbation zones generates pairs of secondary particles (perturbatons). One secondary particle is born according to a real scattering described by $\Delta\Sigma_{scat}$ and the other, representing the $\Delta\Sigma_{tot}$ term, by a δ-scattering in energy and flight direction. The subsequent random walks of the perturbatons are independently realized in the perturbed system and both particles score to the δk-tally with opposite sign. Just this feature of the perturbation source method led to unfavorable statistical behavior connected with convergence difficulties in its application to the optimization study for the reflector system of the IBR-2. Finally, to mitigate these problems, the reflector optimization was done with help of a suitably defined core-albedo instead of calculating reactivity effects directly [9]. Though the principal, statistical difficulties of the method were not removed the efficiency of the Monte Carlo calculations was considerably increased. Against the outlined historical background, the start-up of the new PO-3 reflector system of the IBR-2 was a welcome reason to test the capability of perturbation calculations offered by the up-to-date MCNP code when applied to calculate the important reactivity effects of the reflector.

The MCNP code uses a Taylor series expansion up to the second order and neglects the effect of changing the fission source distribution by the geometrical perturbation. In all our calculations we used the series expansion up to the second order. The second order term was never greater than eleven percent. Concerning the statistical error, in all cases a normal convergence ~ $N^{-1/2}$ was observed.

The reactivity effect of a reflector consists of two components: The one is the albedo that is simply the ability to reflect neutrons back into the core. The other is the effect of the local distribution of the fission source density, which determines both the criticality of the reactor without reflector and the coupling between reflector and core and its contribution to the total neutron multiplication of the reactor [13]. Therefore, the fission source effect must be expected to increase the albedo effect that is caused by a reflector perturbation. Consequently, the result of the "direct" calculation should be greater than that of the PERT calculation. The special interest in the calculations of the total reactivity effects was the question whether the approximation of the PERT option regarding the fission source can be revealed. According to the results given in Table I the expected relation is confirmed for the PO-3 but not for the 1-6H reflector. But, in the latter case the error of the "direct" result seems to be too great to allow a reliable conclusion. Comparing the PERT result of the 1-6H reflector, the statistical error of which is about one percent only, with the experimental value one could conclude that the fission source effect in this case could be merely in the order of some few percent. Considering both calculation results for the PO-3 one could draw the same conclusion. But, in this case, the comparison with the measurement reveals a great discrepancy: The experimental value is about twenty percent or even more too small. It is suspected that the experiment was not carried out under the predetermined conditions that were modeled by the calculations. Unfortunately, the actual reason of the discrepancy is not yet clearly found out and the measurement has not yet been repeated.

Similarly to the calculations of the total reactivity effects the calculations of the differential perturbations by small reflector displacements showed also normal convergence behavior. Only considerably more particle histories had to be sampled to get a statistical error of the net effect in the range of some few percent.

Four variants of the PERT option were tested. Variant A is the simulation of a two-step model, which is realized by the following two PERT runs (see Fig. 5).

1. Unperturbed system (1u): Reflector of width W is in the left position (shift position s1).
   Perturbed system (1p): Perturbation zone (1) is voided.
   The calculation result is δk_1.
2. Since the modeling restriction of the PERT option this step is simulated by changing unperturbed and perturbed system states.
   Unperturbed system (2u): Reflector of width W in the right position (shift position s2).
   Perturbed system (2p): Perturbation zone (2) is voided.
   The calculation result is $-\delta k_2$.

The total displacement effect follows $\delta k = \delta k_1 + \delta k_2$. In that case if the PERT option would exactly simulate the steps 1 and 2, i.e. considering the changes of the fission source in each step too, then δk would be the real reactivity effect $\delta k_{real} \equiv k_{s2} - k_{s1} = \delta k_{2u} - \delta k_{1u}$ of the displacement. Under this condition the system states 1p and 2p would be identically and, therefore, the net effect would be just the wanted difference between the eigenvalues of the states 2u and 1u. However, as consequence of the fission source approximation the calculated value δk is shifted from the real value, i.e. $\delta k = \delta k_{real} + \delta k_{approx}$. The additional term δk_{approx} is a difference itself, namely, between the contributions of the unperturbed part of the reflector (the intersection of states 1u and 2u) to the multiplication factor of the system in state 1u and in state 2u, respectively. For physical reasons, this term must be positive, consequently the PERT approximation results in a positive contribution and, so, to a reduction of the negative real effect.

For efficiency reasons one is interested to calculate the reflector displacement effect in one PERT run only. For that the variants B1 and B2 were tested. In B1 the unperturbed system is the unification of both rotor positions, i.e. the width of the reflector is now enlarged to $W+\delta x$. On the other hand, the perturbed system is the intersection with the width $W-\delta x$. So, even in the case that the PERT option would consider the fission source effect, the expectation value of δk would differ from δk_{real} because the actual unperturbed and perturbed system states of the reflector displacement 1u and 2u, respectively, do not appear in the calculation at all. The same situation occurs in case B2. However, in contrast to B1 the transport simulation in the unperturbed state generates the fission source for the reflector with the actual width W in the left position and the sub-effect δk_2 is displaced by δx inwards.

The fourth variant C with application of the PERT option is a two-step simulation, similarly to A. The difference consists in the introduced intermediate state, which is in C the reactor without the reflector. Of course, the statistical situation for the estimation of δk and, so, the efficiency of the calculation is much worse than in the other PERT variants. This is so, because the intersection of the reflector in both positions is involved in both calculations as perturbation zone and, therefore, they do not really contribute to the expectation value of the net displacement effect but to the fluctuations of its estimate only.

Regarding the efficiency, the direct computation of the displacement effect from the two effective multiplication factors k_{s1} and k_{s2} is the worst variant but it is the only with the exact expectation value for the displacement effect. Comparing the result of the PERT variant A with the directly obtained value one can establish an underestimation of about 15 percent the reason of which must be interpreted to be caused by the neglect of the fission source effect. However, the result of variant C is in contrast to this fact. The lower results of both variants B compared to A indicate a further increase of the distortion of the reflector displacement effect. The comparison of the numerical results with the measurement shows a substantial overestimation in the range of

20 to 50 percent by the calculations, inclusively the direct variant. The reason of this unexpectedly great discrepancy could not yet been found out.

In case of the PO-3 the agreement between numerical and experimental results is better than for the 1-6H reflector. In this case, the direct result of the total derivative well agrees with the measurement and the PERT result gives an underestimation of merely 15 percent.

4 CONCLUSIONS

The work reported on in this paper is not to be considered to be finished. In particular, considerable discrepancies observed in comparisons of some calculation and measurement results still demand an explanation in the process of which the experiments should be also included. This concerns both the total reactivity effect and the displacement effect of a reflector, which has been used for the approximate calculation of a reactivity derivative with respect to the reflector motion. Comparing numerical results obtained by means of the PERT option with those of corresponding direct calculations, one can establish that PERT results in all but one case underestimate the direct result in a range of about 5 to 15 percent. This fact could be explained as consequence of neglecting the fission source effect in the calculation of the perturbation effect with the PERT option of MCNP-4C2. This general approximation combined with the inability to calculate the perturbation of filling a void zone with material cause difficulties in the modeling of a reflector displacement in only one PERT run. They result in a distortion of the expectation value of the net effect. The obtained results indicate a possible increase of the fission source effect.

From the work carried out up to now, no final conclusions on the applicability of the PERT method for the calculation of total and displacement reactivity effects of the rotating reflector system of the IBR-2 reactor could be drawn.

5 REFERENCES

1. E. P. Shabalin, *Fast Pulsed and Burst Reactors*, Pergamon Press, New York, United States (1979).

2. V. D. Ananiev, et al., "Pulsed Reactor IBR-2 in the Nineties", *Proceeding of Int. Conference on Neutron Scattering in the Nineties,* Juelich, Germany, January 14-18, p. 63 (1985).

3. V. L. Aksenov, et al., "The IBR-2 Fast Pulsed Reactor : Present and Future", *Proceeding of ICANS-XIV Meeting*, Starved Rock Lodge, Utica, Illinois, June 14-19, Vol. 1, pp.41-45 (1998).

4. V. D. Ananiev, V. L. Lomidze, A. D. Rogov, V. S. Smirnov, E. P. Shabalin, "An optimization study of the reactivity modulator for the periodically pulsed fast reactor", *Atomenergie-Kerntechnik*, **43**, pp.253-259 (1983).

5. V. L. Lomidze, K. Noack, A. D. Rogov, E. P. Shabalin, "Experimental and numerical study of perspectivic reactivity modulator of IBR-2" (in Russian), *Sov. J. Atomic Energy*, **67**, pp. 314-320 (1989).

6. K. Noack, "Neutron-physical development of reflectors for the pulsed reactor IBR-2", *Kerntechnik*, **59**, pp.291-297 (1994).

7. H. Rief, "Generalized Monte Carlo perturbation algorithms for correlated sampling and a second-order Taylor series approach", *Ann. nucl. Energy*, **11,** pp.455-476 (1984).

8. J. F. Briesmeister, Editor, "MCNP^{TM} – A General Monte Carlo N-Particle Transport Code – Version 4C", *LA-13709-M*, LANL, USA (2000).

9. M. B. Emmett, "The MORSE Monte Carlo Radiation Transport Code", *ORNL-4972*, ORNL, USA (1975).

10. K. Noack, "Some Monte Carlo activities with the MORSE-CG code", *Kernenergie*, **29**, pp. 251-255 (1986).

11. H. Takahashi, "Monte Carlo method for geometrical perturbation and its application to the pulsed fast reactor", *Nucl. Sci. Eng.*, **41**, pp.259-270 (1970).

12. W. Matthes, "Calculation of reactivity perturbations with the Monte Carlo method", *Nucl. Sci. Eng.*, **47**, pp.234-237 (1972).

13. K. Noack, "A relation between criticality and reflector albedo", *Kernenergie*, **32**, pp.133-135 (1989).