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EXTENDING THE BERTINI CASCADE MODEL TO KAONS

Dennis H. Wright
Stanford Linear Accelerator Center
2575 Sand Hill Road
Menlo Park, California, USA 94025
dwright@slac.stanford.edu

ABSTRACT

The Bertini Cascade model in Geant4 currently simulates the hadronic interaction of protons, neutrons and pions with surrounding materials. It is typically valid for incident kinetic energies up to about 10 GeV. This energy range is important for hadron calorimetry in large, high energy detectors such as those being constructed at the LHC and those planned for the next linear collider. While protons, neutrons and pions account for the bulk of the interactions, incident kaons represent a significant part and should be treated with the detail offered by this model.

In Geant4 there is currently only one model that treats incident kaons in this energy range. The Low Energy Parameterized (LEP) model, as its name implies, describes kaon-induced reactions based on parameterizations of data over large ranges of energy and target masses. This model does not attempt to generate final states which conserve quantum numbers on an event-by-event basis, but which instead conserve them on average. The Bertini Cascade model relies much less on parameterization and actually simulates the intra-nuclear cascade using the measured free particle cross sections as input. It has already been demonstrated that this model is more successful than the LEP for protons, neutrons and pions. It is therefore being extended to incident K^+, K^-, K^0_L and K^0_S.

This extension represents a straightforward application of the methods currently used for pions, with the further complication that many more free particle cross sections are required, including for example (K^0, p) and (Λ, n). Many of these channels have very little or no data, which requires that reasonable guesses be made for the cross sections. In this case, isospin invariance and detailed-balance are used.

The final state spectra predicted by the extended model will be compared to those predicted by the LEP model, and to data where it is available.

Key Words: Geant4, Bertini cascade, kaon scattering

1 INTRODUCTION

Simulating the propagation of low energy kaons (0 - 5 GeV) through materials is becoming an important part of the design of new high energy detectors and the validation of results from existing detectors. The Geant4 toolkit [1] currently provides three hadronic models which apply to pions, protons and neutrons in this energy range. These are the Low Energy Parameterized (LEP), Binary Cascade, and Bertini Cascade. Only one of these, the LEP model, may also be applied to incident kaons. However, the LEP model is not especially suited for these energies and is known to perform poorly for kaons. The Binary Cascade model may in the future deal with strange particles but this must await further theoretical developments. The remaining candidate, the Bertini Cascade model, is the most likely to be extended to incident kaons; it is relatively easy to extend to strange particles and is known to work reasonably well in this energy range.

2 EXTENDING THE BERTINI CASCADE

The Bertini Cascade model [2] propagates incident particles through the nucleus in much the same way as Geant4 propagates particles through a material: an interaction cross section is found, the interaction length is calculated, an interaction occurs and a final state is generated. The final state particles are in turn propagated until they either interact or leave the nucleus.

2.1 Cross Sections

The Bertini cascade assumes that particle-particle interaction cross sections and final state branching ratios within the nucleus are given by their free-space counterparts. So, in order to extend the model to include kaons, K^+p, K^-p, K^+n, and K^-n cross section measurements are required. An extensive list of these cross sections and branching ratios is provided in one of the CERN particle reaction catalogs [3]. Above an incident momentum of about 15 GeV/c the data begin to thin out significantly, setting an upper bound on the applicability of the extended model. Incident K^0_L and K^0_S must also be included in the model, thus requiring the K^0 and $\overline{K^0}$ cross sections for intra-nuclear propagation. After the initial interaction with a nucleon, hyperons may be produced and subsequently interact with other nucleons before leaving the nucleus. Hence, Λ-, Σ- and Ξ-nucleon cross sections are also needed. Many of these were taken from another CERN catalog [4]. Ω^{--} reactions were not included due to the small cross section values at these energies.

Of course many of the required cross sections have never been measured, and estimates or guesses are required to fill in the missing information. Where there are gaps in the energies at which measurements were made, a simple linear interpolation of the cross sections is employed in the extended model. If there are unmeasured final states from a given reaction, the partial cross sections for those final states are filled in by using the total cross section measurements as a constraint. As much as possible, missing cross sections were estimated by using isospin and strangeness conservation. For example, it was assumed that

$$\sigma_{K^0 p} = \sigma_{K^+ n} \quad (1)$$

and

$$\sigma_{\overline{K^0} n} = \sigma_{K^- p}. \quad (2)$$

2.2 Final State Generation

For each particle-particle interaction type, the model keeps a list of final state channels and particle types. For incident pions, protons and neutrons, the existing model keeps track of one-through six-body final states up to 10 GeV. In the extended model the number of particle types was increased to include kaons and the lowest mass hyperons. No resonances were included. The possible final state multiplicity was also increased (for incident strange particles) from six to seven. This reflects the fact that the sum of seven-body final states represents a significant fraction of the total cross section.

The final state momenta are sampled from angular and momentum distributions parameterized from data. The extended model currently uses the same distributions for strange particle channels as those used for pion, proton and neutron channels. Coming improvements of the extended model will include angular and momentum distributions developed specifically for incident kaons. Some angular distribution data exists for incident kaons with energies up to 3 GeV, which can be used to parameterize the new distributions. Beyond 3 GeV however, phase space calculations will be necessary.

2.3 Intra-nuclear Propagation

In the Bertini model, particles are propagated through a detailed, three-dimensional model of the target nucleus. The nuclear field is approximated by real-valued potentials for protons, neutrons and pions. In the extended model, strange particles are propagated through the nucleus using a very simple hard-sphere potential which is 7 MeV deep. This is identical to that used for propagating pions. This will be improved in a future version of the model since it is known that the real part of the optical potential for kaons and hyperons must be deeper. Different values of the potential depth will be used for kaons and hyperons.

Other features of the Bertini model include Pauli blocking and nucleon-nucleon correlations. Because Pauli blocking will not affect strange particles propagating through the nucleus, it can be ignored in the model extensions. The existing model incorporates nucleon-nucleon correlations through pion absorption on quasi-deuterons. Kaon absorption on quasi-deuterons is not yet included in the extended model, but sufficient data exists such that this process may be parameterized and included.

3 LOW ENERGY PARAMETERIZED MODEL

The Low Energy Parameterized Model [1] was designed to be a fast, versatile simulator of particle-nucleus interactions. The projectile may be any long-lived meson or baryon with incident energies less than about 40 GeV. The nuclear model is very simple and the intra-nuclear cascade is not directly simulated. The most energetic final state particles are generated by the fragmentation of the highly excited hadrons formed in the initial projectile-nucleon collision. Their momenta are obtained by sampling from a two-parameter exponential distribution. The less energetic final state particles come from nuclear de-excitation, whose distributions are also parameterized.

These parameterizations were performed over large ranges of energy and target masses. This model does not attempt to generate final states which conserve quantum numbers on an event-by-event basis, but instead attempts to conserve them on average. This average treatment leads to undesirable features such as the outright disappearance of strangeness or, for example, the conversion of K^+ into K^0_L, K^0_S or pions.

4 PRELIMINARY RESULTS

Four targets, lead, calcium, carbon and deuterium, were tested using both the extended Bertini cascade and LEP models. Comparisons were made to the inelastic K^+ data of Kormanyos et al. [5]. The data feature a study of the quasielastic peak in the above four nuclei at scattering angles of approximately 24 and 43 degrees, at an incident K^+ momentum of 0.705 GeV/c. These data provide a useful test of the intra-nuclear kinematics in the models. Preliminary results for the above targets are shown in Figs. 1, 2, 3 and 4.

In all cases, the LEP model produced no K^+ in the quasielastic peak at either angle. In fact, no K^+ appeared in the spectra at any value of the nuclear excitation energy. Instead the model converted the K^+ into K_L^0, K_S^0 and pions. The incident K^+ momentum had to be raised from 0.705 GeV/c to to 2 GeV/c before any K^+ appeared in the final spectra.

In contrast, the extended Bertini model reproduces several features of the data. For all four target nuclei, as the 43 ^{o} data indicate, the model gets the energy of the quasielastic peak more or less correct. For lead, calcium and carbon, the width of the peak is underestimated. This may be related to the nuclear potential which is known to be too shallow for kaons. For lead, calcium, and carbon the overall normalization is low by about 30%. The data have a systematic error of 11%, so most of the difference is likely due to the model or the value of the total inelastic cross section. For deuterium the disagreements with data are larger. Because there is essentially no intra-nuclear cascade in this case, the elementary kaon interactions are likely at fault.

At first glance, the predictions at 24^o appear to disagree significantly with data. However, the model does not simulate the elastic or low-lying collective states of the nucleus which show up prominantly in the data. Also, at this angle the quasielastic peak begins to merge with the low-lying states, making it difficult to separate the two features. Taking this into account, it can be

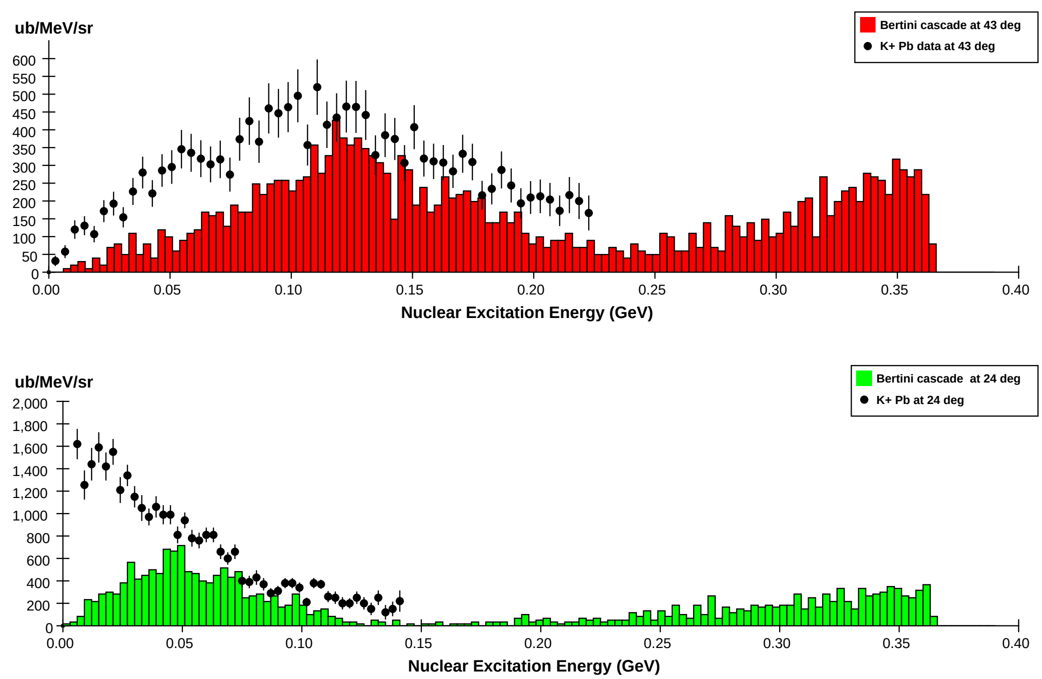

Figure 1: Quasielastic K^+ scattering from lead at 43 and 24 degrees (top and bottom, respectively). The incident K^+ momentum was 0.705 GeV/c. The horizontal axis is the nuclear excitation energy in GeV, so that elastic scattering would appear at 0. The vertical axis is the double differential cross section, $d^2\sigma/dE/d\Omega$, in μb/MeV/sr.

said that the extended Bertini model still reproduces the location of the quasielastic peak for all four nuclei. Although it is difficult to tell, the width of the quasielastic peak appears to be underestimated for lead, calcium and carbon, as it was at 43^o. For all four nuclei the overall normalization is again low by about 30%.

5 CONCLUSIONS

The extension of the Bertini cascade model to include kaons is still in its early stages, but preliminary results already show an obvious improvement over the LEP model, at least in the 1 GeV region. Comparison of extended Bertini cascade predictions with K^+ quasielastic scattering data show reasonable agreement over a wide mass range of nuclear targets. The LEP model does not produce any final state K^+ at these energies. Validation of the extended model will be continued at higher energies when data become available, and comparisons to K^-, K_L^0 and K_S^0 data need to be made.

As mentioned above, several improvements to the extended Bertini model will be made:

1. better nuclear potentials for K^+, K^- and Λ
2. nucleon-nucleon correlations due to quasi-deuterons

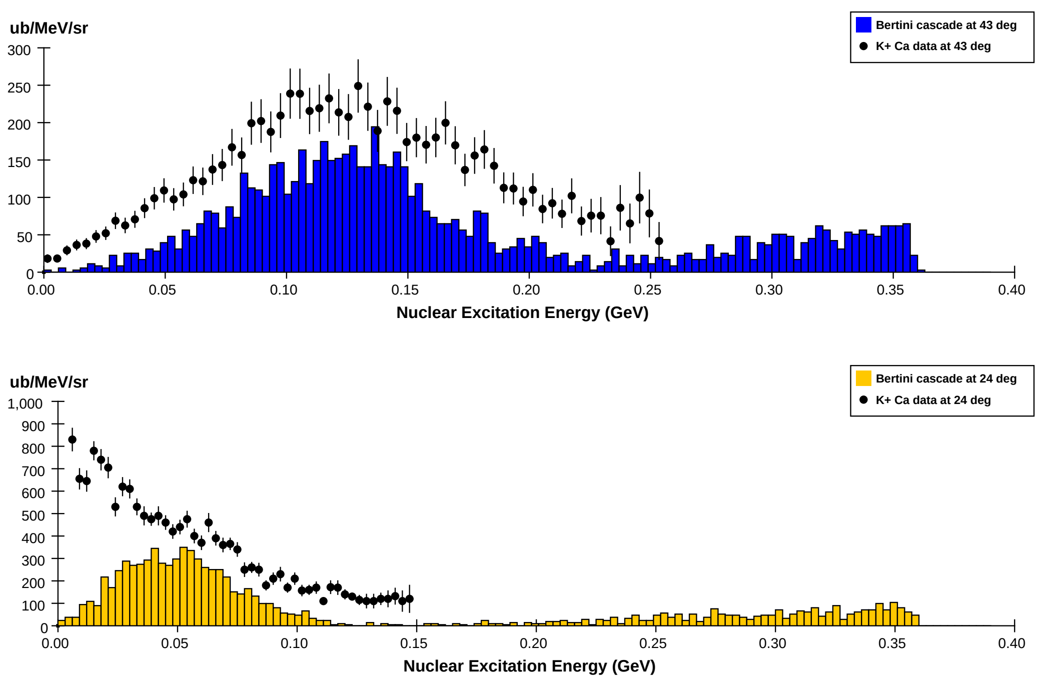

Figure 2: Quasielastic K^+ scattering from calcium at 43 and 24 degrees (top and bottom, respectively). The incident K^+ momentum was 0.705 GeV/c. The horizontal axis is the nuclear excitation energy in GeV, so that elastic scattering would appear at 0. The vertical axis is the double differential cross section, $d^2\sigma/dE/d\Omega$, in μb/MeV/sr.

3. parameterized angular and momentum distributions for strange particle final states.

While it was not mentioned above, pion-, proton- and neutron-induced strange particle production can also be added relatively easily, now that strange particles can be propagated.

It may also be possible to extend the Bertini model to include incident Λs and other hyperons. Data with which to validate the extensions is not likely to appear any time soon, but because the model conserves quantum numbers on an event-by-event basis, there will be clear qualitative improvements over the LEP model.

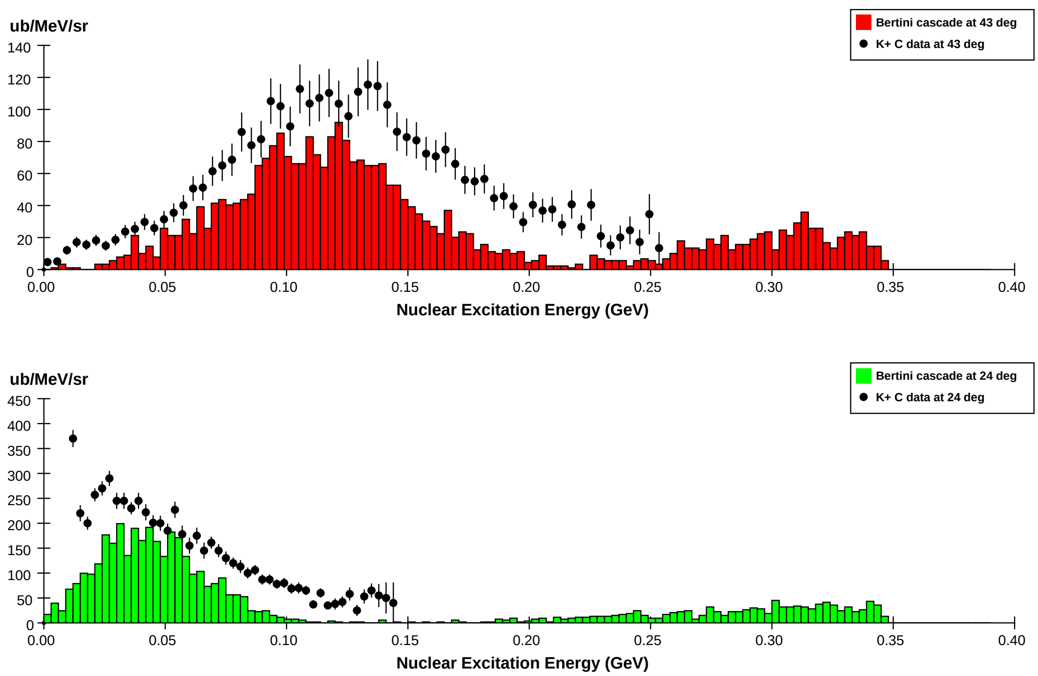

Figure 3: Quasielastic K^+ scattering from carbon at 43 and 24 degrees (top and bottom, respectively). The incident K^+ momentum was 0.705 GeV/c. The horizontal axis is the nuclear excitation energy in GeV, so that elastic scattering would appear at 0. The vertical axis is the double differential cross section, $d^2\sigma/dE/d\Omega$, in μb/MeV/sr.

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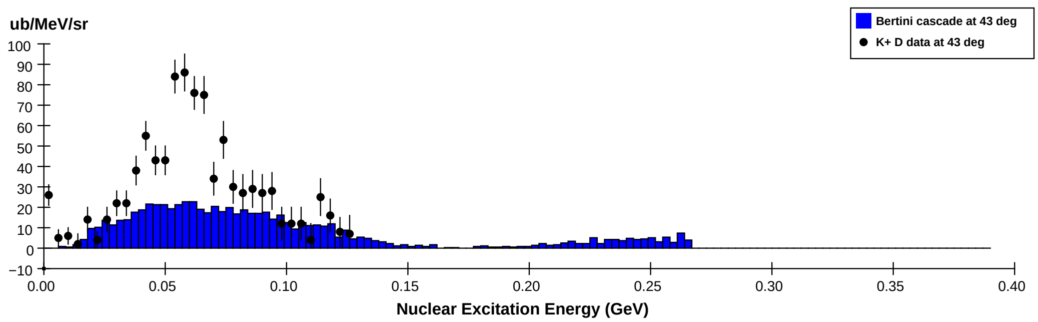
7 REFERENCES

1. S. Agostinelli et al., “Geant4 - A Simulation Toolkit”, *Nuclear Instruments and Methods*, **A 506**, 250 (2003).

2. A. Heikkinen, N. Stepanov and H-P. Wellisch, “Bertini intra-nuclear cascade implementation in Geant4”, *Proceedings of the 2003 Conference for Computing in High Energy and Nuclear Physics*, La Jolla, CA, SLAC-R-636 (2003).
3. V. Flaminio, W.G. Moorhead, D.R.O. Morrison and N. Rivoire, “Compilation of Cross-Sections II: K+ and K- Induced Reactions”, CERN-HERA 83-02 (1983).
4. S.I. Alekhin et al., “Compilation of Cross-Sections IV: γ, ν, Λ, Σ, Ξ and K_L^0 Induced Reactions”, CERN-HERA 87-01 (1987).
5. C.M. Kormanyos et al., *Physical Review*, **C51**, 669 (1995).



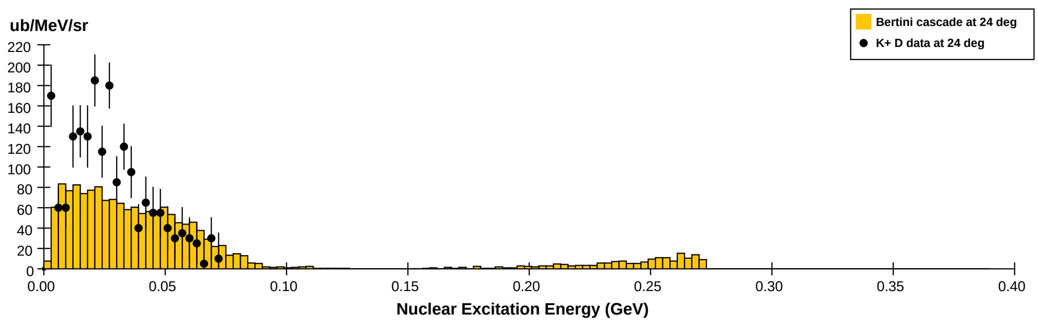


Figure 4: Inelastic K^+ scattering from deuterium at 43 and 24 degrees (top and bottom, respectively). The incident K^+ momentum was 0.705 GeV/c. The horizontal axis is the nuclear excitation energy in GeV, so that elastic scattering would appear at 0. The vertical axis is the double differential cross section, $d^2\sigma/dE/d\Omega$, in μb/MeV/sr.