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CONVERGENCE CRITERION OF FUNDAMENTAL MODE FISSION SOURCE DISTRIBUTION IN MONTE CARLO CALCULATIONS

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ABSTRACT

The Monte Carlo (MC) eigenvalue calculation for a nuclear system starts with inactive cycle MC runs to determine the fundamental-mode fission source distribution (FSD) in a nuclear system. The issue here is when, and what criteria to use, to halt the inactive cycle MC runs. This paper aims at deriving new convergence criteria of the FSD that can be applicable to ordinary MC runs with and without incorporating the fission matrix method (FMM) and validating their qualification. The new criteria are derived on a notion that the FSD is regarded as converged to the fundamental mode when two FSDs a certain number of cycles called a correlation length L (>1) apart are statistically close to each other. Unlike the existing criteria, the new ones have characteristics that they require comparing the relative difference between two FSDs a correlation length L apart instead of those of the consecutive two cycles, and that they are region-dependent and free from empiricism. The qualification and effectiveness of new criteria are discussed in terms of a continuous MC neutronics analysis for a pressurized water reactor (PWR).

Key Words: inactive cycle, convergence criterion, FSD, FMM, correlation length

1 INTRODUCTION

The Monte Carlo (MC) calculations for the criticality and the power distribution in a nuclear system require knowledge of stationary or fundamental-mode FSD in the system. Because it is a priori unknown, so-called inactive cycle MC runs are performed to determine it. Like active cycle MC runs to be followed, each inactive cycle MC run involves generating particle histories corresponding to a preset fixed number of source neutron starters from the FSD either from an initial guess or from the previous cycle MC run, locating fission sites in the course of particle history generations, and determining the new FSD normalized to the fixed number of starters for the next cycle run. The inactive cycle MC runs should be continued until the FSD converges to the stationary FSD. The questions here are when to halt inactive cycle MC runs and what criterion to be used to do so. Obviously, if one stops them prematurely, the follow-up active cycles may be run with the non-stationary FSD. Conversely, if one performs the inactive cycle MC runs more than necessary, one is apt to waste computing time because inactive cycle MC runs are used to elicit the fundamental-mode FSD only.

As a way to save the computation time for inactive cycle MC calculations, Carter and McCormick [1] proposed early the FMM. They showed that the FMM approach requires fewer MC iterations than the usual non-matrix method. But they did not elaborate on when, and what criterion to use, to halt inactive cycle MC runs. Recently, Kitada and Takeda [2] demonstrated again effectiveness of the FMM in accelerating the source convergence. They introduced a convergence criterion that must be met by converged eigenvectors of two successive-cycle fission matrices. Besides, they suggested that, to ensure convergence of the FSD, a few more ordinary MC cycle runs be conducted even after the fission matrix eigenvector satisfies their convergence criterion. We observe that their convergence criterion does not always guarantee convergence of the FSD. In addition, it contains an empirical parameter.

The purpose of this paper is to present convergence criteria that are self-contained without empiricism and applicable for termination of inactive cycle MC calculations with and without incorporating the FMM and to establish qualification of the new criteria. To do so, we will perform continuous energy MC calculations for nuclear systems such as a single fuel pin, a fuel assembly, and a small pressurized water reactor (PWR) designated SMART [3], which involve 400 cycles far into stationary cycles, establish the conditions that should be met by the converged FSD or fission source vectors, compare those conditions with the new criteria, and present the effectiveness of the new criteria in terms of computer time saving and the degree of the fission source convergence.

2 DERIVATION OF CONVERGENCE CRITERIA

2.1 Convergence Criterion for MC Simulation

The MC simulation for determining criticality of and power density distribution in a nuclear system is based on the fission source iteration described by

$$S^t(\mathbf{r}) = \frac{1}{k^{t-1}}\int d^3r' K(\mathbf{r}' \to \mathbf{r}) S^{t-1}(\mathbf{r}') \text{; t=1,2,...} \quad (1)$$

The superscript t stands for the cycle index. $K(\mathbf{r}' \to \mathbf{r})$ is the number of first-generation fission neutrons born per unit volume about $\mathbf{r}$, due to a parent neutron born at $\mathbf{r}'$ with fission energy spectrum. $\mathrm{S}^{\mathrm{n}}(\mathbf{r}')$ (n=t-1,t) is the source density of neutrons born at any energy at $\mathbf{r}'$ at cycle n. Equation (1) is used for both inactive and active cycle MC simulations.

The inactive MC simulation is designed for determining the stationary FSD that will be used for the follow-up active cycle MC simulation. It starts with an initial guess on the FSD, $\mathrm{S}^0(\mathbf{r})$, and determines $\mathrm{S}^1(\mathbf{r})$ by Eq. (1), which in turn is used to determine $\mathrm{S}^2(\mathbf{r})$, and so forth. The cycle-by-cycle MC simulation of obtaining $\mathrm{S}^{\mathrm{t}}(\mathbf{r})$ from $\mathrm{S}^{\mathrm{t-1}}(\mathbf{r})$ (t>0) by Eq. (1) continues until $\mathrm{S}^{\mathrm{t}}(\mathbf{r})$ converges. The convergence here implies that the relative difference between two FSDs that are L (>0) cycles apart, i.e., $\left|\mathrm{S}^{\mathrm{t}}(\mathbf{r}) - \mathrm{S}^{\mathrm{t-L}}(\mathbf{r})\right| / \left|\mathrm{S}^{\mathrm{t-L}}(\mathbf{r})\right|$ falls within a certain specified limiting value throughout the region $\mathbf{r}$. Because the FSD from MC simulation is not continuous but discrete function of $\mathbf{r}$ due to a finite number of particle history generations, the point-wise relative difference of two FSDs can not be an accurate and practical measure of source convergence. In practice, therefore, one divides the whole region into smaller zones with spatial volume V_{m}, defines the zone-wise fission source intensities by

$$\mathrm{S}_{\mathrm{m}}^{\mathrm{n(MC)}} = \int_{V_m} d^3\mathbf{r}\mathrm{S}^n(\mathbf{r}) \text{ : (n=t, t-L)}$$

and determines source convergence based on relative difference of the zone-wise source intensities L cycles apart, i.e., $\left|\frac{S_m^{t(MC)} - S_m^{t\text{-}L(MC)}}{S_m^{t\text{-}L(MC)}}\right|$.

From the statistical nature of the MC calculation, the zone-wise fission source intensities and their differences carry statistical uncertainties with them. Therefore, one may well claim the source convergence when two FSDs are statistically close to each other, namely, when the zone-wise relative difference of two FSDs are within the statistical uncertainties of desired confidence. In mathematical terms, this can be expressed by

$$\left|\frac{S_m^{t(MC)} - S_m^{t\text{-}L(MC)}}{S_m^{t\text{-}L(MC)}}\right| \le \varepsilon_m \tag{2}$$

where

$$\varepsilon_m = \kappa\sigma\left[\frac{S_m^{t(MC)} - S_m^{t\text{-}L(MC)}}{S_m^{t\text{-}L(MC)}}\right] \ (\kappa = 2, 3) \tag{3}$$

σ is the standard deviation (SD) of the bracketed quantity. In terms of SDs associated with $S_m^{t(MC)}$ and $S_m^{t\text{-}L(MC)}$, it can be approximated by

$$\begin{aligned}\sigma\left[\frac{S_m^{t(MC)} - S_m^{t\text{-}L(MC)}}{S_m^{t\text{-}L(MC)}}\right] &\simeq \sqrt{\frac{\sigma^2[S_m^{t(MC)}] + \sigma^2[S_m^{t\text{-}L(MC)}]}{[S_m^{t\text{-}L(MC)}]^2}} \\ &= \sqrt{\frac{\sigma_0^2[S_m^{t(MC)}] + \sigma_{\Delta S}^2[S_m^{t(MC)}] + \sigma_0^2[S_m^{t\text{-}L(MC)}] + \sigma_{\Delta S}^2[S_m^{t\text{-}L(MC)}]}{[S_m^{t\text{-}L(MC)}]^2}} \\ &= \sqrt{1+\alpha^2} \bullet \sqrt{\frac{\sigma_0^2[S_m^{t(MC)}] + \sigma_0^2[S_m^{t\text{-}L(MC)}]}{[S_m^{t\text{-}L(MC)}]^2}}\end{aligned} \tag{4}$$

where

$$\alpha = \sqrt{\frac{\sigma_{\Delta S}^2[S_m^{t(MC)}] + \sigma_{\Delta S}^2[S_m^{t\text{-}L(MC)}]}{\sigma_0^2[S_m^{t(MC)}] + \sigma_0^2[S_m^{t\text{-}L(MC)}]}}$$

σ_0^2 is the variance of the bracketed quantity while $\sigma_{\Delta S}^2$ that of the fluctuating source component about the average source distribution at the designated cycles t or t-L. σ_0^2 can be estimated by treating each particle history from a starter sampled from $S^{t\text{-}1}(r)$ as an independent experiment producing $S_m^{t(MC)}$. On the other hand, $\sigma_{\Delta S}^2$ can be hardly known in practice because the MC simulation from the given $S^{t\text{-}1}(r)$ is performed just once to determine $S^t(r)$. The parameter α in Eq. (4) accounts for the unknown $\sigma_{\Delta S}^2$.

Equation (2) implies that, when the FSD converges, the absolute value of the zone-wise relative difference between $S_m^{t(MC)}$ and $S_m^{t-L(MC)}$ must fall within $\kappa\,\sigma$ with the confidence level of about 95.4 % (when κ=2) or 99.7 % (when κ=3) . Note that the source convergence criterion, Eq. (2), is region-dependent. Note also that Eq. (2) calls for knowledge of two FSDs L cycle apart. L=1 implies that one is comparing the FSDs of two consecutive cycles. As will be shown later, L=1 may lead to premature termination of the MC simulation before it reaches the stationary FSD. This is due to the fact that FSDs of two consecutive cycles tend to be similar to each other in MC simulation because source particles generally do not move far from their respective birth sites. Therefore, the choice of proper L affects efficiency of the source convergence, which will be shown through numerical calculations.

2.2 Fission Matrix Method

The FMM is designed to speed up convergence of the FSD to the fundamental mode by the ordinary inactive MC simulation by Eq. (1) alone. The method consists of dividing the system into a number of spatial zones and constructing a matrix eigenvalue equation [1]

$$S_m^t = \frac{1}{k^t}\sum_{m'} R_{mm'}^t S_{m'}^t \quad \text{or} \quad \mathbf{S}^t = \frac{1}{k^t}\mathbf{R}^t\mathbf{S}^t \,, \tag{5}$$

where

$$R_{mm'}^t = \int_{V_m} d^3\mathbf{r}\int_{V_{m'}} d^3\mathbf{r}'K(\mathbf{r}'\rightarrow\mathbf{r})S^{t-1}(\mathbf{r}')/\int_{V_{m'}} d^3\mathbf{r}'S^{t-1}(\mathbf{r}') \tag{6}$$

R^t and k^t are the fission matrix and the multiplication factor at cycle t, respectively. The element of fission matrix $R_{mm'}^t$ is the number of fission neutrons produced at zone m by one neutron born at zone m' at cycle t. The FMM calls for setting the fission matrix R^t with the known source distribution $S^{t-1}(r)$ either from the previous cycle MC run (t>1) or from an initial guess (t=1), solving Eq. (2) for S^t, and using it to renormalize the new source distribution for the next inactive cycle MC simulation, $S^t(r)$. References [1] and [2] offer some of source renormalization methods that are different according to the way to use the source vectors for the source renormalization. For example, Kitada et al. [2] make the source renormalization by correcting weights of fission source starters with the source vector as follows:

$$\tilde{\omega}_{m,l}^t = \frac{S_m^{t(FM)}}{S_m^{t(MC)}}\omega_{m,l}^t \tag{7}$$

$S_m^{t(MC)}$ and $\omega_{m,l}^t$ are the relative source intensity of region m and the weight of the fission source at site l within the region m that are obtained from the ordinary MC run by Eq. (1). $S_m^{t(FM)}$ is the relative source intensity of region m determined from the solution to fission matrix eigenvalue equation (3). The FMM provides the corrected weight of each fission site $\tilde{\omega}_{m,l}^t$ for the next cycle MC run. This is continued until $S_m^{t(FM)}$ satisfies the following convergence condition for all region m;

$$\left|\frac{S_m^{t(FM)} - S_m^{t\text{-}1(FM)}}{S_m^{t(FM)}}\right| < \varepsilon \quad \text{for all m} \tag{8}$$

ε is a preset positive constant.

As will be shown in the next section, the above convergence criterion does not always guarantee convergence of the FSD. We observe that uncertainties associated with relative source intensities $S_m^{t(FM)}$ are region dependent because frequency of fission in a region is dependent on the fission-to-absorption cross section ratio of the region. In a region where this ratio is larger, fissions would occur more frequently and thus the uncertainty of the relative source intensity of the region tends to be smaller, while the reverse is the case in a region where this ratio is smaller. Therefore, use of constant ε may cause uneven convergence of the source distribution. It is apt to be overly strict criterion for one region and rather loose one for some other region, if one imposes region-independent constant ε. As the case may be, therefore, it may cause premature termination of inactive MC simulation. Moreover, Equation (8) alone does not tell what value should be used for ε in a given problem.

In order to overcome the first deficiency and ensure source convergence at all time, we suggest conducting ordinary inactive MC runs even after convergence of $S_m^{t(FM)}$ by Eq. (8) until one gets the FSD satisfying the source convergence criterion, Eq. (2), of the MC simulation.

To derive an empiricism-free source convergence criterion for the FMM, we note that the matrix $\mathbf{R^t}$ consists of two components; the mean and the deviation about its mean. Without loss of generality, therefore, it can be given by

$$\mathbf{R^t} = \mathbf{R_o^t} + \Delta\mathbf{R^t} \tag{9}$$

Because of this, the eigenvector of $\mathbf{R^t}$, $\mathbf{S^{t(FM)}}$, has the SD about its mean value. Let us denote it by $\sigma[\mathbf{S^{t(FM)}}]$ or $\sigma[\mathbf{S_m^{t(FM)}}]$ in terms of the elements of $\mathbf{S^{t(FM)}}$. Then the SD of $\mathbf{S}_m^{\mathbf{t(FM)}} - \mathbf{S}_m^{\mathbf{t-1(FM)}}$ becomes

$$\begin{aligned}
\sigma\left[\frac{\mathbf{S}_m^{\mathbf{t(FM)}} - \mathbf{S}_m^{\mathbf{t-1(FM)}}}{\mathbf{S}_m^{\mathbf{t(FM)}}}\right] &\simeq \sqrt{\frac{\sigma^2[\mathbf{S}_m^{\mathbf{t(FM)}}] + \sigma^2[\mathbf{S}_m^{\mathbf{t-1(FM)}}]}{[\mathbf{S}_m^{\mathbf{t(FM)}}]^2}} \\
&\simeq \sqrt{\frac{\sigma_0^2[\mathbf{S}_m^{\mathbf{t(FM)}}] + \Delta\sigma^2[\mathbf{S}_m^{\mathbf{t(FM)}}] + \sigma_0^2[\mathbf{S}_m^{\mathbf{t-1(FM)}}] + \Delta\sigma^2[\mathbf{S}_m^{\mathbf{t-1(FM)}}]}{[\mathbf{S}_m^{\mathbf{t(FM)}}]^2}} \\
&\simeq \sqrt{\frac{\sigma_0^2[\mathbf{S}_m^{\mathbf{t(FM)}}] + \sigma_0^2[\mathbf{S}_m^{\mathbf{t-1(FM)}}]}{[\mathbf{S}_m^{\mathbf{t(FM)}}]^2}}
\end{aligned} \tag{10}$$

$\Delta\sigma^2[\mathbf{S}_m^{\mathbf{t(FM)}}]$ in Eq. (10) denotes the effect of the fluctuation about the mean value of $\sigma[\mathbf{S_m^{t(FM)}}]$ designated by $\sigma_0^2[\mathbf{S}_m^{\mathbf{t(FM)}}]$. It is difficult to estimate $\Delta\sigma^2[\mathbf{S}_m^{\mathbf{t(FM)}}]$ because only a single cycle MC run from $S^{t\text{-}1}(r)$ is conducted to determine $\mathbf{R^t}$. In practice, therefore, we assume that

$\Delta\sigma^2[\mathbf{S}_m^{\mathbf{t(FM)}}]$ is negligibly small in comparison with $\sigma_0^2[\mathbf{S}_m^{\mathbf{t(FM)}}]$, hoping that the fluctuation of $\mathbf{R^t}$ about its mean from the multiple cycle MC runs starting from $\mathrm{S}^{\text{t-1}}(r)$ might be small. On the other hand, one can estimate $\sigma_0^2[\mathbf{S}_m^{\mathbf{t(FM)}}]$ by assuming a normal distribution function for $\mathbf{R^t}$ with the mean $\mathbf{R_0^t}$ and its standard deviation $\Delta\mathbf{R^t}$, sampling a number of different $\mathbf{R^t}$ from it, solving the fission matrix equations for $\mathrm{S}_\mathrm{m}^{\mathrm{t(FM)}}$ and evaluating the standard deviations of the mean value of $\mathrm{S}_\mathrm{m}^{\mathrm{t(FM)}}$. This is then approximated as $\sigma_0^2[\mathbf{S}_m^{\mathbf{t(FM)}}]$.

Because of uncertainties associated with $\mathbf{S^{t(FM)}}$ and $\mathbf{S^{t-1(FM)}}$, one may presume that the source vectors are converged when the estimation of $\mathbf{S}_m^{\mathbf{t(FM)}} - \mathbf{S}_m^{\mathbf{t-1(FM)}}$ falls within 2-3 times the standard deviation about its mean. As a convergence criterion for the FMM, therefore, one may impose

$$\left|\frac{\mathrm{S}_\mathrm{m}^{\mathrm{t(FM)}} - \mathrm{S}_\mathrm{m}^{\mathrm{t\text{-}1(FM)}}}{\mathrm{S}_\mathrm{m}^{\mathrm{t(FM)}}}\right| \le e_m \qquad (11)$$

where

$$\mathrm{e}_m = \kappa\sigma\left[\frac{\mathrm{S}_\mathrm{m}^{\mathrm{t(FM)}} - \mathrm{S}_\mathrm{m}^{\mathrm{t\text{-}1(FM)}}}{\mathrm{S}_\mathrm{m}^{\mathrm{t(FM)}}}\right] \simeq \kappa\sqrt{\frac{\sigma_0^2[\mathbf{S}_m^{\mathbf{t(FM)}}] + \sigma_0^2[\mathbf{S}_m^{\mathbf{t-1(FM)}}]}{[\mathbf{S}_m^{\mathbf{t(FM)}}]^2}} \ (\kappa\text{=2 or 3}) \qquad (12)$$

Unlike ε in Eq. (8), e_m is region dependent and may be estimated by Eqs. (10) and (12).

3 NUMERICAL RESULTS AND DISCUSSION

In order to validate qualification and effectiveness of these criteria, we performed continuous energy MC eigenvalue calculations for a single fuel pin in a small-size pressurized water reactor (PWR) SMART [3](cf. Fig. 1). The convergence behavior of the FSD solely by ordinary MC runs and then by the MC runs combined with the FMM are examined.

3.1 Qualification of MC Source Convergence Criterion, Equation (2)

The 200-centimter-long fuel pin is divided into 10 equal-length zones. The zone is numbered from (I) to (X) in the ascending order from the bottom to the top of the fuel pin. In order to determine the source convergence condition in the fuel pin by a numerical experiment, we performed ordinary MC runs to 400 cycles on 10,000 histories per cycle. Confirming that FSD is fully converged for cycles greater than 200, we estimated the values of $\left|(\mathrm{S}_\mathrm{m}^{\mathrm{t(MC)}} - \mathrm{S}_\mathrm{m}^{\mathrm{t\text{-}L(MC)}})/\mathrm{S}_\mathrm{m}^{\mathrm{t\text{-}L(MC)}}\right|$ averaged over cycles from t =201 to 400 as a function of L for t>200. They are designated as $\varepsilon_m(\exp)$ in Table I. As noted in Table I, $\varepsilon_m(\exp)$ is region dependent. The larger the fission rate of a region is, the smaller it is. It increases with increasing L and saturates roughly to constant value after L$\ge$ 20.

Because $\varepsilon_m(\exp)$ is derived from the fully converged FSD, it can serve as ε_m in Eq. (2) for the convergence test of the FSD in the fuel pin problem. We tested how effective it is in the ordinary MC simulation. N_{stop} in Table I denotes the cycle number at which the inactive MC calculation terminates. It is shown that the inactive MC run terminates much earlier than the 200th cycle. It is also shown that, the larger L, the longer N_{stop} is. But one must note that convergence of the FSD actually occur at the 22nd or 23rd cycle, considering the fact that two FSDs L cycles apart agree with each other by the convergence criterion, Eq. (2). Figure 2 displays the converged FSD with $\varepsilon_m(\exp)$ in Eq. (2) in comparison with the fully converged reference FSD that is obtained by averaging the FSDs over cycles 201-400. Except for the L=1 case, the agreement between two FSDs is generally good. It appears that the larger L is, the better the agreement is.

In practice, there is no knowing $\varepsilon_m(\exp)$ without running inactive MC calculation far beyond the stationary cycles. Instead of $\varepsilon_m(\exp)$, therefore, we obtained an approximate estimation for ε_m using Eqs. (3) and (4) with $\alpha = 0$ and $\kappa = 2$. Table I shows ε_m at the cycle when inactive MC runs terminate. It is interesting to note that ε_m is region-dependent like $\varepsilon_m(\exp)$, that it is weakly dependent on L, and that, except for L=1, it is less than $\varepsilon_m(\exp)$ for all m. The last row shows N_{stop} with the use of ε_m. The use of ε_m instead of $\varepsilon_m(\exp)$ makes it necessary to perform more cycle MC runs for L>5 in obtaining the converged FSD, because ε_m here is more stringent limiting parameter than $\varepsilon_m(\exp)$. Figure 3 shows a comparison of the converged FSDs from the use of ε_m with the fully converged reference FSD. Agreements between the two distributions are similar to those noted in Fig. 2 obtained with the use of $\varepsilon_m(\exp)$.

3.2 Efficiency Enhancement by FMM and its Stopping Criterion

The FMM can speed up convergence of the FSD to the fundamental mode by the inactive MC simulation if one combines it with ordinary MC calculations. The question of interest now is qualification of Eq. (12) as a way to estimate the limiting parameter e_m in the convergence criterion of the FMM, Eq. (11). We examine this question in terms of the fuel pin MC neutronics problem discussed above.

The 200-cm-long fuel pin is now divided into 5 equal-length coarse zones for an application of the FMM. The 10,000 5x5 fission matrices every cycle are constructed by random sampling from R^t, which is assumed to have the normal probability density distribution function with the mean R_0^t and the standard deviation of ΔR^t. The eigenvectors of randomly sampled 10,000 fission matrices allow one to determine $S_m^{t(FM)}$ and $\sigma_0^2[S_m^{t(MM)}]$ each cycle. By using these two values at two consecutive cycles, say, t and t-1 cycles, one can determine e_m by Eqs. (10) and (12) and also test convergence of the FSD at the current cycle t by Eq. (11). Table II shows the cycle number at which the FSD converges. For the sake of comparison, it also shows $e_m(\exp)$ which is obtained by averaging e_m for the presumed stationary cycles 201-400. Note that $e_m(\exp)$ and e_m are roughly the same. Note also that the FSD by the FMM calculation converges at the same cycle number 5, regardless of the use of either e_m (exp) or e_m in Eq. (11). Figure 4 compares the FSD converged from the FMM with the fully converged reference one. The FSD from FMM appears close but not so well converged to the reference one. As pointed out in Ref. [2], this is why one needs to perform additional ordinary MC runs a few more cycles even after having attained the convergence by the FMM. We made additional MC runs to obtain the desired FSD converged to the reference in the same way as we did before, starting with the FSD from the FMM. Table III lists the limiting parameter ε_m of the MC convergence criterion and the total cycle number at which the MC simulation terminates by Eq. (2). The total cycle number here includes the number of cycles used already for the FMM. Note that the values of ε_m here are roughly similar to the ones in Table I. In particular, they are almost the same irrespective of the correlation length L. This may indicate that the converged FSD out of the FMM is near the stationary distribution. Note also that the total cycle number of the FSD convergence here is much smaller than that required by the MC run only (cf. Table I). Needless to mention, this is the speed-up effect of the FMM on source convergence**.** Figure 5 displays the converged FSD from the combined FMM and MC calculations. Except for the L=1 case, the MC simulation after the FMM calculation produces the FSD converged well to the reference one.

4 CONCLUSION.

The advantageous feature of the new criteria for convergence of the FSD to the fundamental mode is self-contained and free from empiricism. Unlike the previous criterion for the ordinary MC calculations, the new MC convergence criteria call for comparing two fission source intensities a certain correlation length L (>1) apart instead of those of two consecutive cycles. The correlation L depends on the system to be analyzed.

The fuel pin results demonstrate these features clearly. They also show that the new criteria are well qualified as the stopping criteria of inactive MC cycle calculations toward the fission source convergence to the desired stationary FSD in the Monte Carlo neutronics calculations. To consolidate this further, we performed both fuel assembly (FA) and core analysis. They are not included here because of the page limitation, yet the new criteria are shown to work well for the FA and core problems, too.

5 REFERENCES

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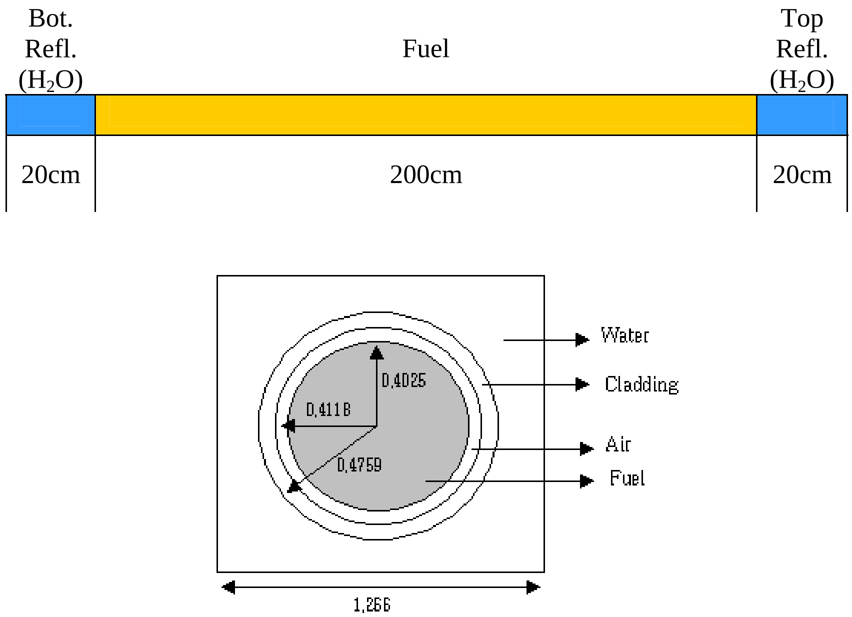

Figure 1. Axial and radial UO_2 fuel pin geometry

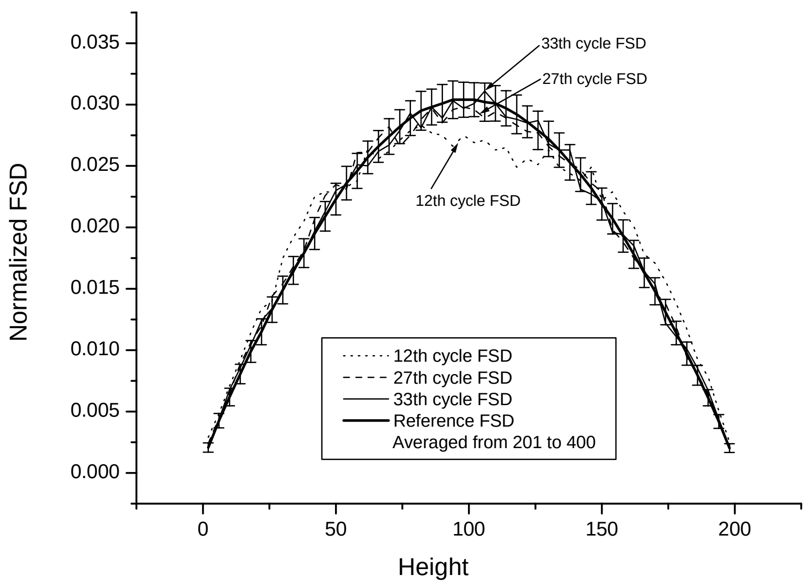

Figure 2. Converged fission source distributions in fuel pin with $\varepsilon_m(\exp)$ in the convergence criterion, Eq. (2)

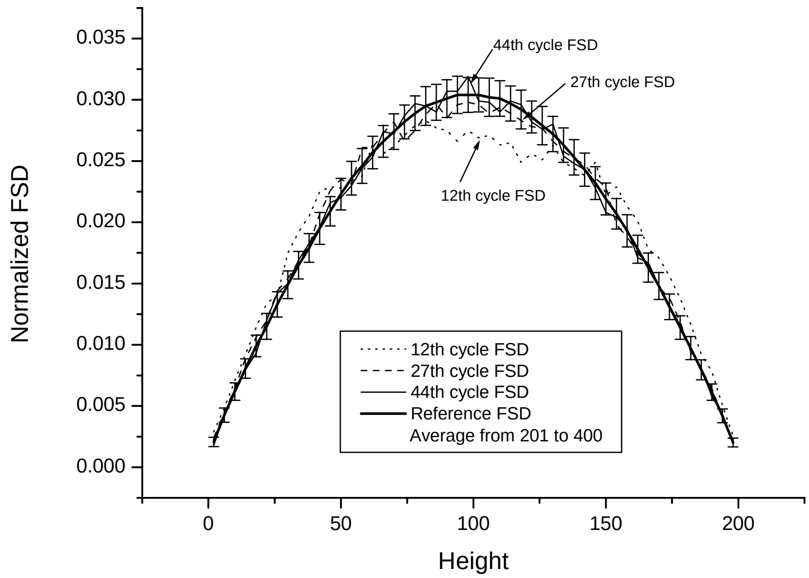

Figure 3. Converged fission source distributions in fuel pin with ε_m of Eq. (3) in the use of convergence criterion, Eq. (2)

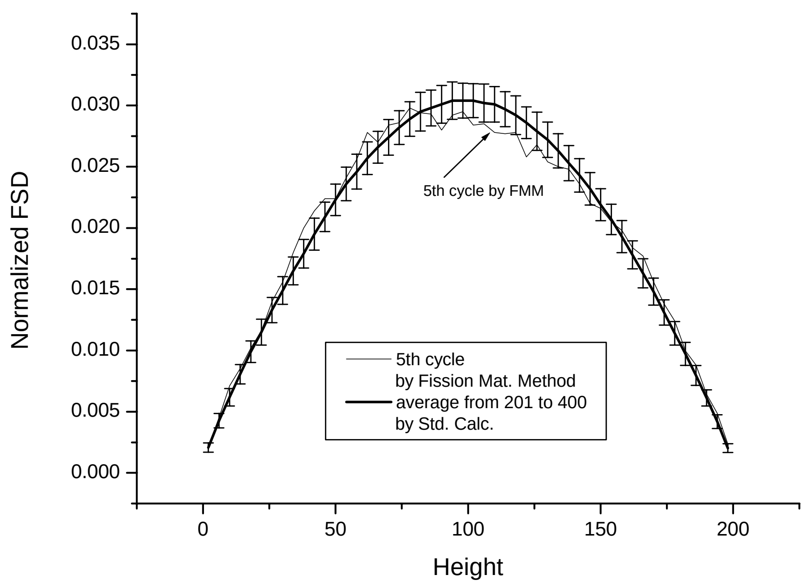

Figure 4. Converged fission source distributions in fuel pin from the fission matrix method

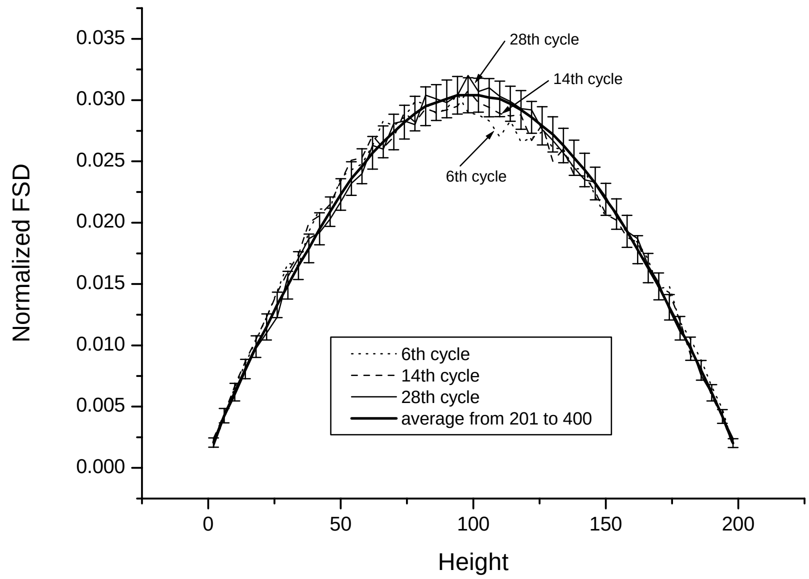

Figure 5. Converged fission source distributions in fuel pin from the combined use of FMM and MC

Table I. Comparison of two source convergence limiting parameters for fuel pin analysis; ε_m(exp) and ε_m from Eq. (4)

LP	L / Region No.	1	5	10	20	30
ε_m(exp) *	(I)	0.0536	0.0802	0.0861	0.1057	0.1088
	(II)	0.0332	0.0438	0.0560	0.0706	0.0762
	(III)	0.0255	0.0380	0.0463	0.0562	0.0597
	(IV)	0.0229	0.0306	0.0332	0.0341	0.0386
	(V)	0.0228	0.0339	0.0393	0.0387	0.0417
	(VI)	0.0214	0.0297	0.0309	0.0346	0.0368
	(VII)	0.0222	0.0316	0.0371	0.0434	0.0495
	(VIII)	0.0260	0.0351	0.0419	0.0532	0.0589
	(IX)	0.0339	0.0459	0.0550	0.0661	0.0708
	(X)	0.0546	0.0682	0.0792	0.0922	0.1017
	N_{STOP}** No.	12	27	33	43	53
ε_m ($\alpha = 0$, $\kappa = 2$)	(I)	0.0601	0.0628	0.0661	0.0633	0.0648
	(II)	0.0388	0.0400	0.0413	0.0406	0.0416
	(III)	0.0333	0.0337	0.0337	0.0343	0.0340
	(IV)	0.0313	0.0307	0.0307	0.0304	0.0303
	(V)	0.0310	0.0290	0.0289	0.0294	0.0288
	(VI)	0.0313	0.0297	0.0290	0.0290	0.0286
	(VII)	0.0316	0.0308	0.0305	0.0307	0.0306
	(VIII)	0.0329	0.0337	0.0342	0.0339	0.0342
	(IX)	0.0387	0.0411	0.0417	0.0418	0.0424
	(X)	0.0577	0.0639	0.0636	0.0655	0.0649
	N_{STOP}** No.	12	27	44	52	65

* $\varepsilon_m(\exp) = \sqrt{\frac{1}{200}\sum_{t=201}^{400}\left(\frac{S_m^{t(MC)} - S_m^{t-L(MC)}}{S_m^{t-L(MC)}}\right)^2}$

** N_{STOP} = the cycle number at which inactive MC runs terminate.

Table II. The limiting parameter of the convergence criterion, e_m for FMM application to fuel pin

Region No.	$e_m(\kappa = 2)$	$e_m(\exp)$	$e_m / e_m(\exp)$
(1)	0.1830	0.2042	0.8962
(2)	0.1186	0.1241	0.9557
(3)	0.0604	0.0508	1.1881
(4)	0.1220	0.1182	1.0318
(5)	0.2019	0.1990	1.0144
N_{con} *	5	5	-

* The cycle number at which the converged FSD is obtained by the FMM.

Table III. Limiting parameter ε_m of MC source convergence criterion for fuel pin

	Region No. \ L	1	5	10	20	30
ε_m	(Ⅰ)	0.064	0.063	0.064	0.063	0.064
	(Ⅱ)	0.042	0.041	0.041	0.041	0.042
	(Ⅲ)	0.034	0.033	0.033	0.033	0.033
	(Ⅳ)	0.031	0.030	0.030	0.030	0.031
	(Ⅴ)	0.029	0.029	0.029	0.029	0.029
	(Ⅵ)	0.030	0.030	0.029	0.029	0.029
	(Ⅶ)	0.031	0.031	0.031	0.031	0.031
	(Ⅷ)	0.034	0.034	0.035	0.035	0.035
	(Ⅸ)	0.040	0.042	0.042	0.042	0.042
	(Ⅹ)	0.062	0.065	0.064	0.066	0.065
	Converged Cycle	6	14	28	36	47