A SUBGROUP METHOD FOR CALCULATING ESCAPE CROSS SECTIONS AND INTERFERENCE FACTORS

Miodrag J. Miloševi‰
Vin…a Institute of Nuclear Sciences, Nuclear Engineering Laboratory
P.O.Box 522, NET-150, 11001 Belgrade, Yugoslavia
Email: mmilos@rt270.vin.bg.ac.yu

ABSTRACT

Resonance self-shielding methods that rely on the equivalence theory between a heterogeneous and a homogeneous geometry are widely used in many multigroup lattice physics codes. Among these, the generalised theory, based on the analytic representation for the elastic slowing-down operator and the principle in which the equivalent homogeneous media is chosen to preserve the absorption rate, can be applied to multidimensional domains without loss of accuracy. In the current work, a rigorous numerical scheme is used to examine the correctness of equivalence theory based on the narrow resonance approximation, and a subgroup procedure is proposed to reduce the problem of resolved resonance calculation in heterogeneous media containing arbitrary mixtures of resonant nuclei to the escape cross section and interference factor calculations. The accuracy of these methods is compared with reference results obtained from a fine-group slowing-down calculation. Example application is given for a heterogeneous lattice consisting of recycled UO_2-PuO_2 pins in light water moderator.

1. INTRODUCTION

In multigroup neutronic calculations it is necessary to obtain mixture dependent cross sections for various burnup conditions and resonant zones of different characteristics. Most of the sophisticated methods[1-3] assume that the flux spectrum is determined entirely by particular nuclide whose cross sections is being averaged, and neglect perturbations due to other resonant nuclei. The multigroup self-shielded cross sections for a single resonant nuclide are obtained by invoking equivalence theory between a heterogeneous and a homogeneous geometry. These methods can give acceptable results only for mixtures composed of a main nuclide (such as ^{238}U) and very small amounts of secondary resonant nuclei, and they are still using for the simple geometries, including pin cell arrays and fuel clusters.

Therefore, a new generation of improved resonance self-shielding methods has appeared. One class of these methods is based on generalised equivalence theory that preserves the resonance integrals calculated with an approximate model for the spatial interference between resonant mixtures.[4,5] Another class is the subgroup (or multiband) treatment where the detailed energy-dependent cross section behaviour is replaced by its probability density representation.[6] This representation is subsequently used to replace difficult Reimann integral over the group energy interval by the simpler Lebesque-Stieltjes summation,[7,8] or to calculate the reaction rates by using integral transport equation derived within the neutron conservation in the zone-subgroup domain.[9,10]

Since the more detailed modelling, used in improved resonance self-shielding methods, increases the running time, a simple model is proposed to decrease the CPU costs down to an acceptable value for design calculations in cases where many resonant nuclei are mixed together. In this model, the subgroup concept is used to reduce the problem of space and energy dependence of resonance

absorption in a heterogeneous lattice, to the escape cross section and interference factor calculations for an equivalent homogeneous medium. The proposed procedure is implemented in a design oriented lattice physics code and compared with exact values obtained using a fine-group elastic slowing-down calculation. Results are presented for a PWR cell containing a recycled uranium-plutonium oxide.

2. GENERALISED EQUIVALENCE THEORY

In general case with any number of resonant nuclei (0), non-resonant nuclei (1), and zones (a zone is a volume small enough for flat-flux approximation), the collision-probability approach leads to the neutron slowing-down equation,

$$\Phi_i(E) = \sum_j P_{ij}(E) \frac{R_{0j}\Phi_j(E) + R_{1j}\Phi_j(E) + Q_j(E)}{\Sigma_{0j}(E) + \Sigma_{1j}(E)}, \quad (1)$$

where it is assumed that the medium is perfectly reflected and the scattering is isotropic. Here Φ_i is the energy-dependent flux in zone i, P_{ij} is the first-flight collision probability from zone i to zone j, R_j is the elastic slowing-down operator, Σ_j is the total cross section and Q_j is the neutron source in zone j.

To obtain a very detailed solution versus the neutron energy, the multigroup slowing-down module CESD from the VEGA2 code[11] is used. This module solves the slowing-down equation in the multigroup approximation and with collision probability method for a heterogeneous geometry. The energy mesh was chosen to obtain a very good representation of the heavy nuclide cross sections. This is why the module CESD is considered as a reference. Since the resonance interference effects were studied only for resolved resonances that cause the largest effect for thermal reactors (i.e., for E<367.26 eV), the neutron source in Eq. (1) is approximated as a lethargy-independent source between 2.25 keV and 3.35 keV. Also, it is assumed that the neutrons are uniformly and isotropically emitted in the moderator materials.

However, the rapidly varying cross sections for heavy nuclei require a large number of energy groups and make the reference calculation unsuitable for most practical applications. Thus, the effective cross section concept and self-shielding formalism, which authorise a broad energy mesh, must be used. The initial assumption in the widely-used methods,[1-5] based on the equivalence theory, may be described as the approximation in which the flux is factorised as the product of a resonant fine structure function φ_j and a macroscopic distribution ψ_j, i.e., $\Phi_j(E) = \varphi_j(E)\psi_j(E)$. Since the slowing-down makes the neutron distribution spatially uniform, the approximation $\psi_j = \psi$ $(\forall j)$ is usually adopted. The assumption that the non-resonant nuclei produce the slowing-down source of the form 1/E, gives $R_{1j}\Phi_j(E) = \Sigma_{1j}\psi(E)/E$. Second assumption, based on the fact that the slowing-down operator for the heavy nuclide (k) contributes over a short lethargy range, leads to $R^k_{0j}\Phi_j(E) = R^k_{0j}\varphi_j(E)\psi(E)$. With these assumptions, the flux is given by

$$\varphi_j(E) = \sum_j \frac{P_{ij}(E)}{\Sigma_{0j}(E) + \Sigma_{1j}} \left[\sum_k R^k_{0j}\varphi_j(E) + \frac{\Sigma_{1j}}{E}\right]. \quad (2)$$

To simplify the problem, the zones can be grouped into regions. A resonant region (represented by subscripts I and J) is a collection of one or more zones containing the same mixture of light and heavy nuclei. The conservation property of collision probabilities leads to

$$\sum_J \frac{P_{IJ}(E)}{\Sigma_{0J}(E)+\Sigma_{1J}} \sum_k [\frac{\Sigma_{0J}^k(E)}{E} - R_{0J}^k \varphi_J(E)] = \frac{1}{E} - \varphi_I(E). \quad (3)$$

Equation (3) is the fundamental relation for describing the resonance interference effects in the general case. This equation is more complicated, because of the greater number of unknowns. Therefore, some approximate procedures for the preserving of absorption rate have been considered.

An approximate Livolant-Jeanpierre formalism, which allows the treatment of resonance shielding in a heterogeneous media containing arbitrary mixtures of resonant nuclei, has been proposed by Sanchez and Mondot.[4] This formalism (also called "PIC approximation") artificially forces the diagonalisation of system (3) by writing $R_{0J}^k \varphi_J(E) = R_{0I}^k \varphi_I(E)$ for each self-shielding region I≠J. New generalisation of the Livolant-Jeanpierre theory[5] (called "background matrix formalism") takes exactly into account the spatial interference between resonant mixtures when all the resonant mixtures are similar. In this case, the system (3) is diagonalised by using $R_{0I}^k \approx R_0$, $\sigma_{0I}^k \approx \sigma_0$, $\forall I$. As in the classical Livolant-Jeanpiere approach, the equivalent homogeneous mixtures are constructed so that the resonance integrals are preserved.

In this paper, a consistent procedure based on the conservation of the self-shielded cross sections, derived primary as a diagnostic tool, is considered. The equivalent homogeneous medium in each resonant region, for each energy group and for each resonant nuclide is evaluated in the exact heterogeneous geometry by combining the narrow resonance (NR) approximation of Eq. (3)

$$\varphi_I(E) = \frac{1}{E} - \sum_J \frac{P_{IJ}(E)}{\Sigma_{0J}(E)+\Sigma_{1J}} \sum_k \frac{\Sigma_{0J}^k(E) - \Sigma_{pJ}^k}{E}, \quad (4)$$

with the Reimann integration. This integration is carried out by using the point cross section data pretabulated on a consistent energy grid. The equivalent neutron flux $\overline{\varphi}_I^k(E)$ in each resonant region (I) and for each resonant nuclide (k) is approximated with

$$\overline{\varphi}_I^k(E) = \frac{\Sigma_{pI} + \overline{\Sigma}_{eI}^k}{[\Sigma_{0I}(E) + \overline{\Sigma}_{eI}^k]E}. \quad (5)$$

An iterative strategy, in which the self-shielded cross section in energy group g, in resonant region I, and for resonant nuclide k, is defined for equivalent neutron flux $\overline{\varphi}_I^k(E)$,

$$\overline{\varphi}_I^k(E) = \frac{\frac{\Sigma_{pI}}{E} + \overline{\Sigma}_{eI}^k [\frac{1}{E} - \overline{\varphi}_I^k(E)]}{\Sigma_{0I}(E)}, \quad (6)$$

and for the escape (dilution) cross section $\overline{\Sigma}_{eI}^k$,

$$\int_{E_{g+1}}^{E_g} \sigma_a^k(E) \frac{\Sigma_{pI}}{\Sigma_{0I}(E)} \frac{dE}{E} + \int_{E_{g+1}}^{E_g} \sigma_a^k(E) \frac{\frac{1}{E} - \overline{\varphi}_I^k(E)}{\Sigma_{0I}(E)} dE =$$

$$= \sigma_{a,gI}^{k}\left[\int_{E_{g+1}}^{E_g} \frac{\Sigma_{pI}}{\Sigma_{0I}(E)}\frac{dE}{E} + \bar{\Sigma}_{eI}^{k}\int_{E_{g+1}}^{E_g} \frac{\frac{1}{E}-\bar{\varphi}_I^k(E)}{\Sigma_{0I}(E)}dE\right]. \quad (7)$$

Here $\sigma_{a,gI}^{k}$ is the multigroup value of absorption cross section, weighted with neutron flux that is obtained by solving the Eq. (4) in exact heterogeneous geometry,

$$\sigma_{a,gI}^{k} = \int_{E_{g+1}}^{E_g} \sigma_a^k(E)\varphi_I(E)dE \Big/ \int_{E_{g+1}}^{E_g} \varphi_I(E)dE\,. \quad (8)$$

In order to validate the adequacy of this theory, the remaining equation in equivalent homogeneous medium

$$[\sigma_t^k(E) + \frac{\sum_{l\neq k} N_I^l \sigma_t^l(E) + \bar{\Sigma}_{eI}^k}{N_I^k}]\bar{\varphi}_I^k(E) = r_{0I}^k \bar{\varphi}_I^k(E) + \frac{\sum_{l\neq k} N_I^l R_{0I}^l \bar{\varphi}_I^k(E) + \frac{\bar{\Sigma}_{eI}^k}{E}}{N_I^k}, \quad (9)$$

is solved with a fine-group slowing-down calculation. This equivalence procedure is implemented in the GLJ-CESD code, which represents primarily a diagnostic tool and is not intended for design computations.

3. PROPOSED SUBGROUP METHOD

The application of the subgroup concept in the generalised equivalence theory is based on the idea that is possible to replace the Reimann integration with the Lebesgue-Stieltjes integration technique. For this purpose, the subgroup parameters are evaluated by using simultaneous approximation of the self-shielded total $\bar{\sigma}_t(\sigma_0)$ and absorption cross section $\bar{\sigma}_a(\sigma_0)$ as functions of dilution cross section σ_0 by rational functions with identical denominator:

$$\frac{1}{\ln(E_g/E_{g+1})}\int_{E_{g+1}}^{E_g} \frac{1}{\sigma_t(E)+\sigma_0}\frac{dE}{E} = \frac{1}{\bar{\sigma}_t(\sigma_0)+\sigma_0} = \sum_{h=1}^{H} \frac{a_h^{NR}}{\sigma_{th}+\sigma_0}, \quad (10)$$

$$\frac{1}{\ln(E_g/E_{g+1})}\int_{E_{g+1}}^{E_g} \frac{\sigma_a(E)}{\sigma_t(E)+\sigma_0}\frac{dE}{E} = \frac{\bar{\sigma}_a^{NR}(\sigma_0)}{\bar{\sigma}_t(\sigma_0)+\sigma_0} = \sum_{h=1}^{H} \frac{a_h^{NR}\sigma_{ah}^{NR}}{\sigma_{th}+\sigma_0}, \quad (11)$$

where H is number of subgroups in the energy interval (E_{g+1}, E_g) of the g'th group, a_h^{NR} is lethargy fraction of subgroup h in group g, and σ_{th} and σ_{ah}^{NR} and are subgroup values of cross sections "t" and "a" for subgroup h. In this evaluation, the self-shielded total and absorption cross sections are obtained by treating the slowing-down in the NR approximation,

$$\overline{\sigma}_x(\sigma_0)=\frac{\int_{E_{g+1}}^{E_g}\frac{\sigma_x(E)}{\sigma_t(E)+\sigma_0}\frac{dE}{E}}{\int_{E_{g+1}}^{E_g}\frac{1}{\sigma_t(E)+\sigma_0}\frac{dE}{E}},\ x=t,a. \tag{12}$$

The subgroup parameters a_h^{NR} ($a_h^{NR}>0$, $a_h^{NR}=1-\sum_{h'=1}^{H-1}a_{h'}^{NR}$) and σ_{xh} [x=t,a, $\min(\sigma_{xh})>\min[\sigma_x(E)]$, $\max(\sigma_{xh})<\max[\sigma_x(E)]$] are calculated by solving the system containing Eq. (10) for M1 and Eq. (11) for M2 chosen values of σ_0 (where M1+M2=3H-1). In order to account for resonance interference between resonant nuclei in the Lebesgue-Stieltjes integration, the lethargy boundaries for each subgroup are included in the set of NR subgroup parameters. They are selected from the point cross section data pretabulated on a consistent energy grid (microgroups), by adding to the current subgroup h the lethargy widths of microgroups sorted from the highest to the lowest total cross section values, until the lethargy width $a_h^{NR}\Delta u_g$ of subgroup h is filled. The result is sets of intervals $[\Delta u_n^{NR},\ n\in N_h^{NR}]$ that define lethargy boundaries for each subgroup h, where N_h^{NR} denotes a set of indices for microgroups that belong to the subgroup h.

The subgroup form of Eq. (4) for resonant nuclide k in exact heterogeneous geometry is

$$\varphi_{hI}^k=a_h^{NR,k}\Delta u_g\left[1-\sum_J\frac{P_{h,IJ}^k(\Sigma_{0,hJ}^k+\Sigma_{1J})}{\Sigma_{0,hJ}^k+\Sigma_{1J}}[N_J^k(\sigma_{th}^k-\sigma_p^k)+\sum_{l\neq k}(\Sigma_{t,hJ}^l-\Sigma_{pJ}^l)]\right]. \tag{13}$$

The term $\Sigma_{0,hI}^k$ is the subgroup total cross section, defined by

$$\Sigma_{0,hI}^k=N_I^k\sigma_{th}^k+\sum_{l\neq k}\Sigma_{t,hI}^l=N_I^k\sigma_{th}^k+\sum_{l\neq k}N_I^l\sum_{h'=1}^{H_l}\alpha_{h'h}^{NR,lk}\sigma_{th'}^l, \tag{14}$$

where the correlation parameter $\alpha_{h'h}^{NR,lk}$ represents the relative fraction of subgroup h' for nuclide l that belong to subgroup h of nuclide k,

$$\alpha_{h'h}^{NR,lk}=\frac{[\Delta u_{n'}^{NR,l},n'\in N_{h'}^{NR,l}]\cap[\Delta u_n^{NR,k},n\in N_h^{NR,k}]}{a_h^{NR,k}\Delta u_g}. \tag{15}$$

The quantity to be preserved $\overline{\Sigma}_{eI}^k$, obey the equation

$$\sum_{h=1}^{H_k}\sigma_{ah}^{NR,k}\frac{\Sigma_{pI}}{\Sigma_{0,hI}^k}a_h^{NR,k}\Delta u_g+\overline{\Sigma}_{eI}^k\sum_{h=1}^{H_k}\sigma_{ah}^{NR,k}\frac{1}{\Sigma_{0,hI}^k}[a_h^{NR,k}\Delta u_g-\overline{\varphi}_{hI}^k]=$$

$$= \sigma_{a,gI}^{NR,k}\left[\sum_{h=1}^{H_k}\frac{\Sigma_{pI}}{\Sigma_{0,hI}^{k}}a_h^{NR,k}\Delta u_g + \bar{\Sigma}_{eI}^{k}\sum_{h=1}^{H_k}\frac{1}{\Sigma_{0,hI}^{k}}(a_h^{NR,k}\Delta u_g - \bar{\varphi}_{hI}^{k})\right], \tag{16}$$

where the equivalent neutron flux $\bar{\varphi}_{hI}^{k}$ is defined as

$$\bar{\varphi}_{hI}^{k} = \frac{\Sigma_{pI} + \bar{\Sigma}_{eI}^{k}}{\Sigma_{0,hI}^{k} + \bar{\Sigma}_{eI}^{k}}a_h^{NR,k}\Delta u_g, \tag{17}$$

and multigroup value of absorption cross section $\sigma_{a,gI}^{NR,k}$ is weighted with subgroup flux φ_{hI}^{k},

$$\sigma_{a,gI}^{NR,k} = \sum_{h=1}^{H_k}\sigma_{ah}^{NR,k}\varphi_{hI}^{k} \Big/ \sum_{h=1}^{H_k}\sigma_{ah}^{NR,k}. \tag{18}$$

In this stage, the dependence of spatial resonance interference effects on the geometrical configuration is reduced to the set of the escape cross sections characterised each resonant nuclide in the problem.

Next, the escape cross sections $\bar{\Sigma}_{eI}^{k}$ and independent resonance weighting (IRW) approximation of Eq. (9), which states that the flux spectrum for particular resonant nuclide is not affected by resonance reactions in the other nuclei in the mixture, are employed to calculate the dilution cross sections. Dilution cross sections σ_{0I}^{k} for resonant nuclide k with the IR parameter[12] λ_k, defined as

$$\sigma_{0I}^{k} = \frac{1}{N_I^k}\left(\sum_{l\neq k}N_I^l\lambda_l\sigma_p^l + \bar{\Sigma}_{eI}^{k}\right), \tag{19}$$

are used to interpolate the computed self-shielded factors f_{xg}^{k}. Finally, the Williams analytic representation[13] for the flux weighting function, which treats the slowing-down by the wide resonance (WR) approximation, and Lebesgue-Stieltjes integration are applied to calculate resonance interference factors (RIFs). These factors account for the effects that resonances of one nuclide have on the weighting functions used in averaging the cross sections of other resonant nuclei.

As in the procedure of escape cross sections determination, the subgroup parameters a_h^{WR}, σ_{ah}^{WR}, σ_{fh} and $[\Delta u_n^{WR},\ n\in N_h^{WR}]$ used for the RIF treatment are evaluated by using simultaneous approximation of the self-shielded fission and capture cross sections by rational functions with identical denominators,

$$\frac{1}{\ln(E_g/E_{g+1})}\int_{E_{g+1}}^{E_g}\frac{1}{\sigma_a(E)+\sigma_0}\frac{dE}{E} = \frac{1}{\bar{\sigma}_a(\sigma_0)+\sigma_0} = \sum_{h=1}^{H}\frac{a_h^{WR}}{\sigma_{ah}^{WR}+\sigma_0}, \tag{20}$$

$$\frac{1}{\ln(E_g/E_{g+1})}\int_{E_{g+1}}^{E_g}\frac{\sigma_f(E)}{\sigma_a(E)+\sigma_0}\frac{dE}{E} = \frac{\bar{\sigma}_f(\sigma_0)}{\bar{\sigma}_a(\sigma_0)+\sigma_0} = \sum_{h=1}^{H}\frac{a_h^{WR}\sigma_{fh}}{\sigma_{ah}^{WR}+\sigma_0}. \tag{21}$$

In this case, the self-shielding fission and absorption cross sections are calculated as functions of dilution cross section by treating the slowing-down in the WR approximation,

$$\overline{\sigma}_x(\sigma_0) = \frac{\int_{E_{g+1}}^{E_g} \frac{\sigma_x(E)}{\sigma_a(E)+\sigma_0}\frac{dE}{E}}{\int_{E_{g+1}}^{E_g} \frac{1}{\sigma_a(E)+\sigma_0}\frac{dE}{E}}, \quad x=a,f. \tag{22}$$

Assuming that all resonant nuclei, appearing in Eq. (9) can be approximated as WR scatters, the subgroup form for the RIFs is

$$f^k_{xgI,RIF} = \frac{\sum_{h=1}^{H_k} \frac{a_h^{WR,k}\sigma^k_{xh}}{\Sigma^k_{a,hI}+N^k_I\sigma^k_{0I}} \sum_{h=1}^{H_k} \frac{a_h^{WR,k}}{\sigma^{WR,k}_{ah}+\sigma^k_{0I}}}{\sum_{h=1}^{H_k} \frac{a_h^{WR,k}}{\Sigma^k_{a,hI}+N^k_I\sigma^k_{0I}} \sum_{h=1}^{H_k} \frac{a_h^{WR,k}\sigma^k_{xh}}{\sigma^{WR,k}_{ah}+\sigma^k_{0I}}}, \quad x=a,f. \tag{23}$$

Here $\Sigma^k_{a,hI}$ is the subgroup absorption cross section,

$$\Sigma^k_{a,hI} = N^k_I\sigma^{WR,k}_{ah} + \sum_{l\neq k} N^l_I \sum_{h'=1}^{H_l} \alpha^{WR,lk}_{h'h}\sigma^{WR,l}_{ah'}, \tag{24}$$

and $a^{WR,k}_{h'h}$ is correlation parameter determined by using the intervals $[\Delta u^{WR}_n,\ n\in N^{WR}_h]$ in Eq. (15). The RIFs values are then used to correct the calculated self-shielded factors for resonance interference effects.

The energy group and temperature parameters are not explicitly included in the current development. They should be recognised as being implicitly treated, since the parameters $a^{NR,k}_h$, $\sigma^{NR,k}_{th}$, $\sigma^{NR,k}_{ah}$, $[\Delta u^{NR,k}_n,\ n\in N^{NR,k}_h]$, $f^k_{xg}(\sigma_0,T)$, $a^{WR,k}_h$, $\sigma^{WR,k}_{ah}$, σ^k_{fh} and $[\Delta u^{WR,k}_n,\ n\in N^{WR,k}_h]$ are stored for each energy group g of resonant nuclide k and for a few values of temperature. The proposed subgroup procedure is implemented in the GLJ-SUBGR code, which is intended for design applications.

4. RESULTS

The correctness of generalised equivalence theory and proposed subgroup procedure was analysed by using an identical database, i.e., the point cross sections generated from latest version of the ENDF/B evaluation (ENDF/B-VI Release 6) with PREPRO96 codes[14] (LINEAR, RECENT and SIGMA1). In the GLJ-CESD code, the equivalence based on the NR approximation, Reimann integration technique and slowing-down solution for the evaluated homogeneous medium, are performed by using the auxiliary cross section data library with a 16.000 point energy grid. The resonance interference effects were considered only between resolved resonances that cause the largest effect for thermal reactors.

For these reactors, the major resonant nuclei are ^{235}U, ^{238}U, ^{239}Pu and ^{240}Pu, and the major resonance interference effects are caused by ^{238}U absorption. In the VEGA2 55-group structure there are seven groups between 4.0 eV and 367.26 eV (Table I) for which the resonance absorption was calculated.

TABLE I. Energy Boundaries for VEGA2 55-Group Structure

VEGA2 Group	Upper Energy (eV)
17	367.260
18	148.728
19	75.501
20	48.052
21	27.700
22	15.968
23	9.877
	4.000

Accuracy of the GLJ-CESD and GLJ-SUBGR codes was tested for a heterogeneous lattice[4] consisting of recycled UO_2-PuO_2 pins (0.819 cm diameter) placed on a square pitch of 1.25 cm in light water moderator. The composition of recycled (33000 MWd/t) UO_2-PuO_2 fuel is given in Table II.

TABLE II. Composition of Recycled UO_2-PuO_2 Mixture (33000 MWd/t)

Nuclide	Atom Density
^{235}U	$2.250 \cdot 10^{-4}$
^{238}U	$2.080 \cdot 10^{-2}$
^{239}Pu	$1.000 \cdot 10^{-3}$
^{240}Pu	$3.900 \cdot 10^{-4}$

The comparison of the GLJ-CESD and GLJ-SUBGR with the VEGA2/CESD results for ^{235}U, ^{238}U, ^{239}Pu and ^{240}Pu is shown in Tables III, IV, V and VI.

TABLE III. Effective Shielding Factor for ^{235}U Fission in the Lattice with Uranium-Plutonium Mixture

Group	VEGA2/CESD $\sigma_{mix} / \sigma_{\infty}$	Percent error from VEGA2/CESD	
		GLJ-CESD	GLJ-SUBGR
17	1.0018	-0.12	0.26
18	0.9830	-0.09	0.65
19	1.0262	-0.12	0.14
20	1.0091	-0.60	-1.06
21	0.9788	-0.15	-0.20
22	1.0219	-0.19	-0.32
23	1.0233	-0.12	-0.34

TABLE IV. Effective Shielding Factor for ^{238}U Absorption in the Lattice with Uranium-Plutonium Mixture

Group	VEGA2/CESD $\sigma_{mix}/\sigma_{\infty}$	Percent error from VEGA2/CESD	
		GLJ-CESD	GLJ-SUBGR
17	0.2126	0.28	1.36
18	0.1521	0.64	1.02
19	0.0929	0.21	-2.54
20	0.0682	1.09	1.38
21	0.0570	0.58	1.28
22	1.0194	-1.24	-1.28
23	0.0619	0.22	0.56

TABLE V. Effective Shielding Factor for ^{239}Pu Fission in the Lattice with Uranium-Plutonium Mixture

Group	VEGA2/CESD $\sigma_{mix}/\sigma_{\infty}$	Percent error from VEGA2/CESD	
		GLJ-CESD	GLJ-SUBGR
17	0.9653	-0.14	-0.27
18	0.9516	-0.09	-0.90
19	0.8076	0.17	-1.70
20	0.8799	-0.12	-0.12
21	0.8363	0.03	-0.34
22	0.7962	0.08	0.16
23	0.7904	0.14	0.25

TABLE VI. Effective Shielding Factor for ^{240}Pu Absorption in the Lattice with Uranium-Plutonium Mixture

Group	VEGA2/CESD $\sigma_{mix}/\sigma_{\infty}$	Percent error from VEGA2/CESD	
		GLJ-CESD	GLJ-SUBGR
17	0.9793	0.02	-4.20
18	0.7753	-0.17	0.49
19	0.5405	0.03	-0.68
20	0.5837	0.14	-4.36
21	0.6047	2.50	-10.82
22	1.0000	0.00	0.00
23	1.0486	0.00	-0.10

It can be seen that the proposed subgroup procedure (GLJ-SUBGR code) consistently improves agreement with more accurate equivalence theory (GLJ-CESD code) and a fine-group slowing-down calculation (VEGA2/CESD code) for all groups that exhibit resonance interference effects.

5. CONCLUSION

A generalised equivalence theory, and subgroup procedure for calculating the escape cross sections and interference factors have been developed and tested. It was shown that accurate results could be achieved with a consistent equivalence theory, based on the NR approximation, Reimann integration technique and rigorous solution for the evaluated homogeneous medium. The presented results constitute a set of reference values that can be used to validate the theory of reducing the problem of resolved resonance absorption to the escape cross section and interference factor calculations. Also, it was shown that proposed subgroup procedure for calculating the escape cross sections and interference factors could improve agreement with more accurate slowing-down calculation.

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