INFLUENCE OF FLUCTUATION OF COOLANT FLOW ON THE NUCLEAR REACTOR THERMAL PARAMETERS

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ABSTRACT

The most important thermal hydraulic parameters of nuclear reactor, i.e. maximum fuel clad temperature and maximum heat flux are not directly measured but estimated based on numerous design and physical properties of the fuel and reactor cooling system. These input data are always known with some uncertainty resulting in errors of final expectation of thermal hydraulic parameters.

The type of uncertainty can be classified depending on the nature of parameter. For instance, the coolant flow fluctuation or reactor power perturbation can be assigned to a dynamic group of uncertainty, while e.g. deviation from nominal values of the dimension of fuel assembly or fissile material content, are designated as static one.

The response of maximum clad temperature and maximum heat flux of the fuel element to the coolant flow fluctuation is analyzed. The mathematical model has been proposed, where the transient heat conduction equation is solved analytically after some simplification and linearization.

Furthermore, the analysis is carried out for the hot spot, which is characterized by negligible axial temperature gradient, what allows us for using of one-dimensional approximation.

The response of temperature and heat flux to coolant flow fluctuation is expressed in terms of transfer function.

The model has been applied to MARIA research reactor [1]. The singular oscillating mode has been identified experimentally in fluctuation spectrum of MARIA reactor coolant flow. The system response both in frequency and time domain is presented.

1. Introduction

Heat generated within the nuclear reactor core has to be removed by means of cooling system with certain safety margin. In the case of high flux reactor, the heat generation density is very high. The resulting fuel clad temperatures, and heat fluxes are relatively close to their safety limits. More precisely, the attention is paid to those fuel clad points (hot spots) where the maximum temperature and heat flux appear.

Unfortunately, these two main thermal parameters i.e. fuel clad temperature and heat flux are not directly measured. Certain derived thermal factors (inlet-outlet coolant temperature, coolant flow rate and pressure, reactor power, peaking factors etc.) allow for the calculation of maximum clad temperature and heat flux. Besides the numerical errors and deviations, e.g. caused by heat transfer coefficient correlation, there exist a number of sources of errors in such a procedure. Main sources of errors and deviations are coming from departure of input data from their nominal values (dimensions, fissile material content and distribution, etc.). In addition, bias errors of measured thermal parameters (e.g. temperature, flow rate, power) may cause a significant deviation of results. All the above errors and deviations can be qualified as static ones. One can identify also another sources of uncertainty. There are global fluctuations of coolant flow and reactor power, having dynamic nature, which are manifested in fluctuation of local thermal parameters, e.g. the clad temperature and the heat flux.

The aim of this work is to analyze the response of maximum clad temperature and the heat flux to the coolant flow rate fluctuation about its mean value. It turned out that the problem could be solved analytically after some simplification and linearization. Assuming, that the fluctuating component of coolant flow and maximum clad temperature fluctuation constitute respectively the input and output to the system, one can express the system response in terms of transfer function (the same is valid for maximum heat flux as an output signal).

The model has been applied to MARIA research reactor.

Such a combination of thermodynamic and control engineering concepts, was already applied [2,3] for two-dimensional boundary problem.

The influence of other perturbed thermal factors, such as a reactor power or heat generated in fuel on the mentioned two thermal parameters will be studied separately. The possibility of examination of the influence of combination of more than one perturbed thermal factor (e.g. perturbations of both flow rate and heat generated in fuel) and their correlation is still under investigation.

2. Mathematical Model:

The temperature field in the fuel can be split onto steady state component T (space dependent) and perturbed one δT (space and time dependent), and the present model is focused on the analysis of perturbation component. The heat conduction equation for temperature perturbation:

$$\nabla^2 \delta T - \frac{1}{\psi}\frac{\partial \delta T}{\partial t} = 0 \qquad (1)$$

where, ψ – thermal diffusivity ($\lambda/\rho\, c_p$)

λ – thermal conductivity
ρ – density
c_p – constant pressure specific heat

Temperature depends on time. Unfortunately, the thermal coefficient (diffusivity) is temperature dependent, and the only assumption that temperature fluctuation amplitude is small compared with its mean value justifies an assumption that variation of the coefficient can be neglected.

To calculate the clad temperature and the heat flux on the coolant-clad boundary, the solution of equation (1) for fuel clad has to be considered.

Even with relatively high heat fluxes, the temperature varies across the clad within the range of few degrees. Thus, the thermal properties (e.g. thermal diffusivity) of clad material can be assumed constant. For example in MARIA reactor fuel the thermal diffusivity is kept constant within the range of 0.3 percent.

Due to non-uniform heat source distribution along the fuel element, having maximum value near its central part, both the fuel clad temperature and the heat flux have maximum. These maximum values can be found in multidimensional steady state calculations.

The clad region with maximum surface temperature and heat flux (hot spot) is characterized by negligible longitudinal temperature gradients in the clad. It is justified to apply for such regions only one-dimensional version of equation (1).

In the majority of research reactors, the plate type fuel elements are applied. Also in the case of tubular fuel elements (e.g. MARIA research reactor, where the ratio of clad thickness to the radius of curvature is of order of few percents) the plate approximation is allowed.

With all the above assumptions, the temperature field within the clad in the hot spot region fulfills the equation:

$$\frac{\partial^2 \delta T(x,t)}{\partial x^2} - \frac{1}{\psi}\frac{\partial \delta T(x,t)}{\partial t} = 0 \qquad (2)$$

where, the spatial variable x is shown schematically in Fig. 1.

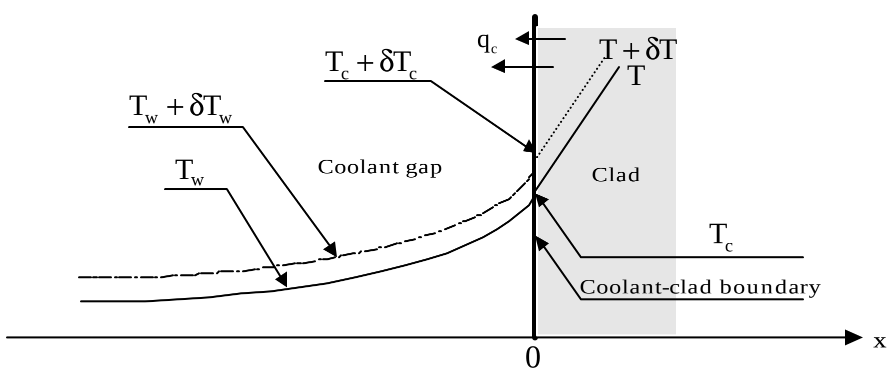


Figure 1. Coolant-clad boundary calculation scheme.

The method of separation of variables should be applied to solve (2).

$$\delta T(x,t) = X(x)\cdot \Gamma(t)$$

and the general solution has a form:

$$\delta T(x,t) = \int \left[F_1 \cdot \exp(-i\beta x) + F_2 \cdot \exp(i\beta x)\right]\cdot \exp\left(-\psi \cdot \beta^2 \cdot t\right) d\beta \qquad (3)$$

Where $i = \sqrt{-1}$, and generally variable β is a complex variable. From physical point of view the only one weighting function $F_1(\beta)$ or $F_2(\beta)$ should remain for a given β. The temperature perturbation $\delta T(x,t)$ should decreases with increasing x. Assume e.g. that imaginary part of β is greater than zero; to maintain the suppression of temperature perturbation inside the clad in this case, one has to put: $F_1(\beta) = 0$.

Boundary conditions

For a given clad point (x = 0) the following boundary condition of the third type is assumed:

$$T_c(t) = T_w(t) + \frac{q_c(t)}{\alpha(t)} \tag{4}$$

where: $T_c(t)$ – clad temperature
$T_w(t)$ – bulk water temperature
$q_c(t)$ – heat flux through the clad surface
$\alpha(t)$ – heat transfer coefficient.

Two of these parameters are directly affected by coolant flow rate i.e. the bulk water temperature and the heat transfer coefficient.

Coolant flow rate can be split onto constant (mean) value and the perturbed one.

$$Q(t) = [1 + \cdot\delta Q(t)]$$

Where δQ is interpreted as a relative flow rate perturbation. The δQ(t) function is dimensionless and assumed to be δQ << 1. The last feature will be used for linearization, which is typical in perturbation approach.

Assuming the constant power of fuel, the water temperature in a given point of coolant gap can be expressed as [4]:

$$T_w(t) = T_i + \frac{\text{rate of heat addition [W]}}{\overline{Q}(t) \cdot c_p}$$

$$T_w(t) = T_i + \frac{A}{\overline{Q}(t)}$$

where: T_i – coolant inlet water temperature
$\overline{Q}(t)$ – mean coolant mass flow rate
A – constant, defined by the equation.

Mean coolant flow rate is calculated as a time average:

$$\overline{Q}(t) = \frac{1}{t_d} \int_{t-t_d}^{t} Q(\tau)\, d\tau \tag{5}$$

where integration has to be performed for the time t_d, which is the time period necessary for inlet water to reach the clad position considered.

In the case of small perturbation of flow rate one can find that:

$$T_w + \delta T_w(t) \cong T_i + (T_w - T_i) \cdot [1 - \delta\overline{Q}(t)] \tag{6}$$

where mean relative flow rate perturbation is calculated analogous with (5):

$$\delta\overline{Q}(t) = \frac{1}{t_d} \int_{t-t_d}^{t} \delta Q(\tau)\, d\tau$$

Lineariszation can be also applied to the heat transfer coefficient, which is a monotonic function of flow rate:

$$\alpha \sim Q^{\eta}$$

where, exponent $\eta \cong 0.8$ for turbulent water flow [5]. Because in the boundary condition (4) the reciprocal of α is used, one can develop:

$$\frac{1}{\alpha(t)} \cong \frac{1}{\alpha}\left[1-\eta\cdot\delta Q(t)\right] \tag{7}$$

where α corresponds to nominal flow rate. The last variable in the boundary condition (4) i.e. the heat flux through the clad $q_c(t)$ contains mean value q_c and small perturbation:

$$q_c(t) = q_c + \lambda_c \left.\frac{\partial \delta T}{\partial x}\right|_{x=0} \tag{8}$$

where, λ_c is heat conduction of clad material.

Inserting of (6), (7), and (8) to the relation (4) one can express the boundary condition for clad temperature perturbation:

$$\delta T_c \cong -(T_w - T_i)\cdot\delta\overline{Q} - (T_c - T_w)\cdot\eta\cdot\delta Q + \frac{\lambda_c}{q_c}(T_c - T_w)\cdot\left.\frac{\partial \delta T_c}{\partial x}\right|_{x=0} \tag{9}$$

Transfer Function

The block diagram (Fig. 2) indicates coolant flow perturbation δQ as a random input variable passed through a coolant-clad interface and producing a stochastic output variation (either clad temperature or heat flux).

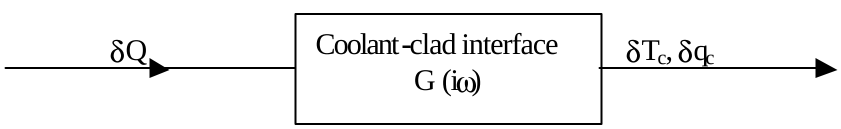


Figure 2. Block diagram for coolant flow fluctuations.

The $G(i\omega)$ transfer function means the system response to a sinusoidal excitation of unit amplitude at frequency ω. In particular:

$$\delta T_c(i\omega) = G_T(i\omega)\cdot\delta Q(i\omega)$$

and

$$\delta q(i\omega) = G_q(i\omega)\cdot\delta Q(i\omega)$$

define two transfer functions G_T and G_q for clad temperature and heat flux respectively. Three functions $\delta Q(i\omega)$, $\delta T_c(i\omega)$, and $\delta q_c(i\omega)$ in the above formulae represent the Fourier transforms of respective signals.

Applying the sinusoidal input signal (coolant flow perturbation):

$$\delta Q(t) = \delta Q \cdot e^{i\omega t} \tag{10}$$

to the system equation (2) one can directly find analytical form of transfer function. From the whole spectrum of variable β of solution (3) the only one value should remain:

$$-\psi\beta^2 = i\omega$$

or:
$$\beta = \sqrt{\frac{\omega}{2\psi}}(i-1) \qquad (11)$$

Imaginary part of β is positive and the weighting factor $F_1 = 0$. Thus, the equation (3) can be reduced to:

$$\delta T(x,t) = F_2 \cdot e^{i(\beta x + \omega t)}$$

where β fulfils the equation (11).

With, the sampling signal, given by equation (10), the average flow perturbation is described by:

$$\delta\overline{Q}(t) = \delta Q \cdot e^{i\omega t} \cdot \frac{1 - e^{-i\omega t_d}}{i\omega t_d} = \delta Q(t) \cdot \chi(\omega)$$

In fact, the integration time t_d depends on the average flow rate perturbation. The relative difference between the accurate value of t_d, and the nominal value is in order of $-\delta\overline{Q}(t)$. However, for the most important low frequency region i.e. with $\omega t_d \ll 1$, the average perturbation $\delta\overline{Q}(t)$ is nearly equal to $\delta Q(t)$:

$$\delta\overline{Q}(t) \cong \delta Q(t) \cdot \left(1 - i\frac{\omega t_d}{2}\right)$$

and depends on the integration time t_d very weakly.

Finally, to satisfy the boundary condition (9) the temperature and its gradient at the clad surface has to be rewritten explicitly as follows:

$$\delta T_c = F_2 \cdot e^{i\omega t}$$

$$\left.\frac{\partial \delta T_c}{\partial x}\right|_{x=0} = i\beta \cdot F_2 \cdot e^{i\omega t} \qquad (12)$$

From equation (11), the weighting parameter F_2 can be found as:

$$F_2 = \frac{(T_w - T_i)\cdot\chi(\omega) + (T_c - T_w)\cdot\eta}{\frac{i\beta(\omega)\cdot\lambda_c}{q_c}\cdot(T_c - T_w) - 1}\cdot\delta Q$$

and the clad temperature transfer function:

$$G_T(i\omega) = \frac{F_2}{\delta Q} = \frac{(T_w - T_i)\cdot\chi(\omega) + (T_c - T_w)\cdot\eta}{\frac{i\beta(\omega)\cdot\lambda_c}{q_c}\cdot(T_c - T_w) - 1} \qquad (13)$$

Taking into account that:

$$\delta q_c = \lambda_c \left.\frac{\partial \delta T_c}{\partial x}\right|_{x=0}$$

and using equation (12) one can find the heat flux transfer function:

$$G_q(i\omega) = \lambda_c \cdot i\beta(\omega)\cdot G_T(i\omega) \qquad (14)$$

As it can be easily found from equations (11) and (14), the phase difference between both transfer functions is equal to $\frac{5}{4}\pi$. Thus, the fluctuations of temperature and heat flux are almost in anti-phase.

Analysis in time and frequency domains:

Since the transfer function $G(i\omega)$ is the system response to a sinusoidal excitation, one can easily find output signal amplitude and phase for the input signal containing singular mode of fluctuation.

If δQ denotes the amplitude of input signal for a given frequency ω, the output temperature amplitude is:

$$\left|\delta T_c\right| = \left|G_T(i\omega)\right| \cdot \delta Q$$

and the phase shift between output and input is:

$$\varphi = \arctan\frac{\operatorname{Im} G_T(i\omega)}{\operatorname{Re} G_T(i\omega)}$$

Generally, the input signal consists of a wide spectrum of frequencies and in frequency domain can be described in terms of power spectral density (PSD).

The PSD of an input signal, defined as a Fourier transform of auto-correlation function, is a measurable signal.

Assuming, that $P_{QQ}(i\omega)$ is a measurable power spectral density of coolant flow fluctuation, the power spectral densities of clad temperature P_{TT}, and heat flux P_{qq} are given by the relations:

$$P_{TT}(i\omega) = \left|G_T(i\omega)\right|^2 \cdot P_{QQ}(i\omega)$$

$$P_{qq}(i\omega) = \left|G_q(i\omega)\right|^2 \cdot P_{QQ}(i\omega)$$

In fact, the only spectral information available from measurements arises in the form of the magnitude of the transfer function $\left|P_{QQ}\right|$. Thus, the information on output signals transfer functions is also limited to their magnitudes.

3. Coolant Flow Fluctuations in the MARIA Reactor

The model described above has been applied to the MARIA research reactor. After brief description of thermal-hydraulic features of the reactor, the explicit transfer functions G_T and G_q for the hot spot of MARIA reactor will be presented.

The experimental data on the fluctuation of coolant flow rate are then used in the analysis of resulting maximum clad temperature and heat flux perturbation. The analysis will be performed both in time and frequency domains.

MARIA reactor primary cooling system consists of two cooling circuits: fuel channel, and reactor pool cooling ones. The fuel channels are connected in parallel to the headers, situated in the reactor pool above the core. After leaving the fuel channels, cooling water flows through the return header to the heat exchangers, then to the circulation pumps and to the delivery header and is finally distributed among the fuel channels.

The schematic diagram of water flow through the individual fuel channel is depicted on Fig. 3. In addition, the hot spots on the sixth tube are indicated.

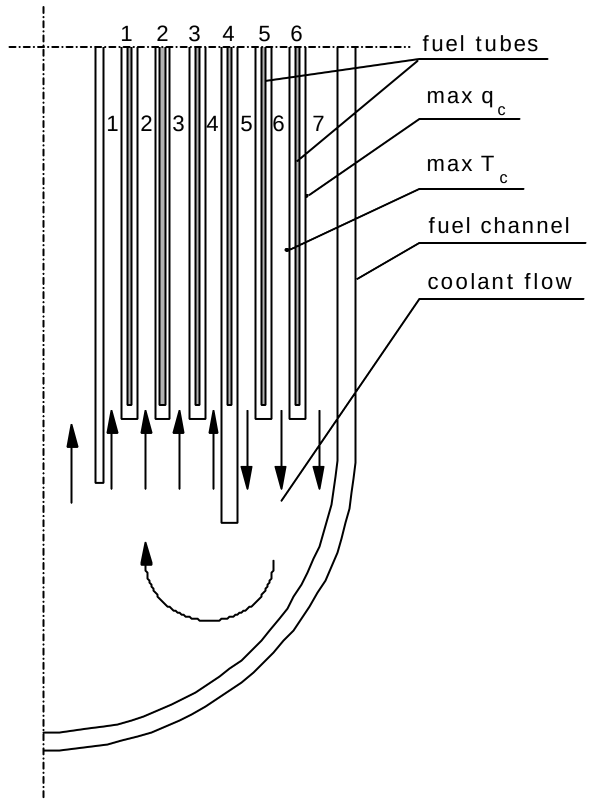


Figure 3. Schematic diagram of coolant flow through the fuel channel in MARIA reactor.

In Table 1 the thermal-hydraulic parameters for two hot spots, corresponding to maximum clad temperature and heat flux are reproduced. These calculated data are related to nominal coolant flow conditions.

Table 1. Hot spots thermal-hydraulic parameters for nominal water flow (calculated)

Parameter	Maximum T_c	Maximum q_c
T_c [^{0}C]	129	127.71
T_w [^{0}C]	74.72	61.58
q_c [W.cm^{-2}]	162.6	180.75
t_d [sec.]	0.097	0.089

Inlet water temperature was assumed T_i = 50 ^{0}C, and nominal coolant flow rate = 29 m^3.hr^{-1} per one fuel channel.

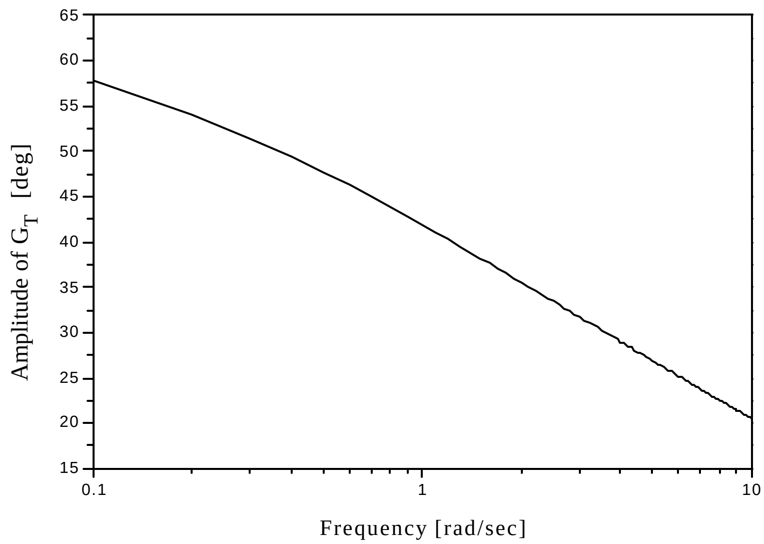
Transfer function calculation

To perform the transfer function calculation the following physical parameters were assumed:

$$\lambda_c = 2.1 \text{ [W.cm}^{-1}\text{. }^0\text{C}^{-1}\text{]}$$
$$\psi = 0.853 \text{ [cm}^2\text{.sec}^{-1}\text{]}$$

$\eta = 0.8$

Using the equations (13), and (14) it can be obtained the calculated transfer function's amplitudes and phase shifts, which is presented below on Fig. 4-a, b.



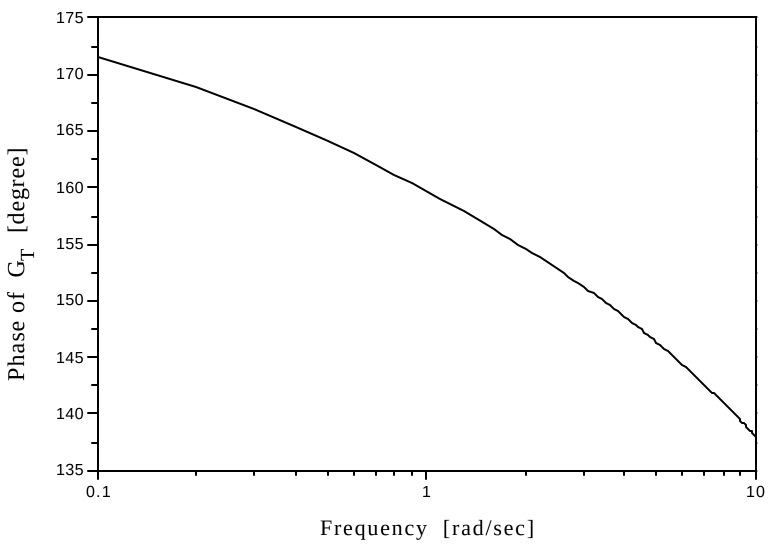
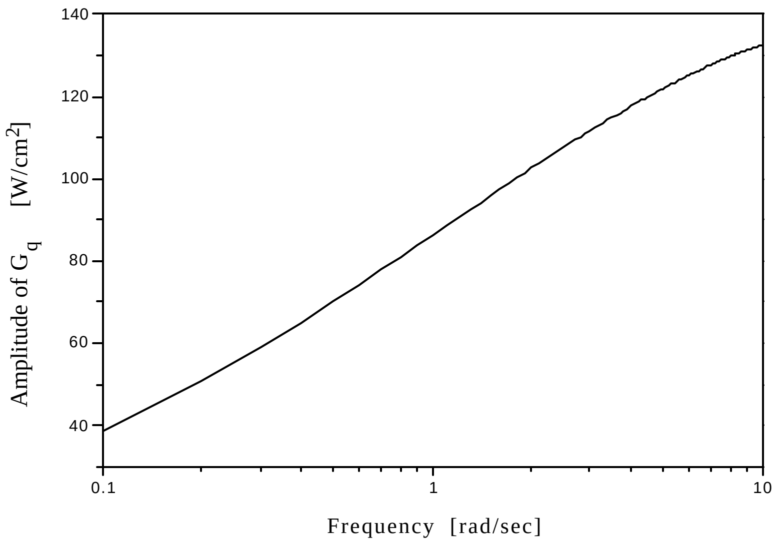


Figure 4-a Transfer function G_T



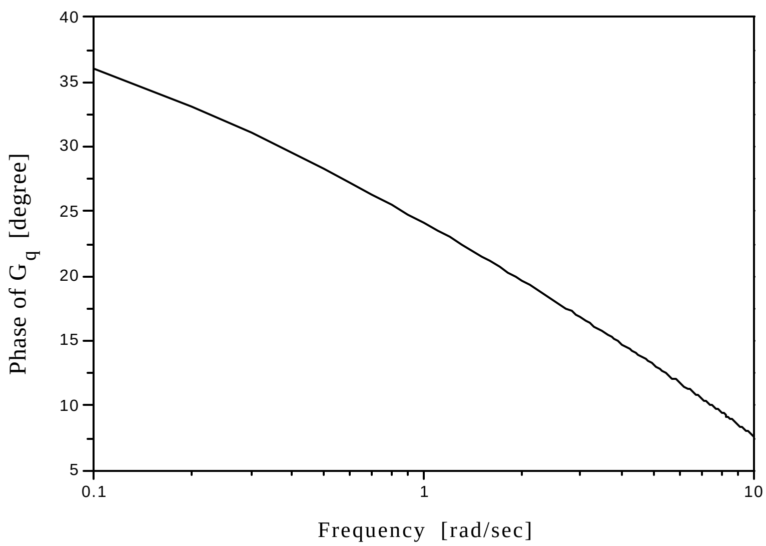


Figure 4-b. Transfer function G_q

Coolant flow fluctuation in MARIA reactor.

Water flow measurements in the primary cooling circuit were conducted by means of flow measuring orifice. The results were collected on the PC in the form of time data sequence with a time step of 7.8 msec. An example of such time record is shown on Fig. 5.

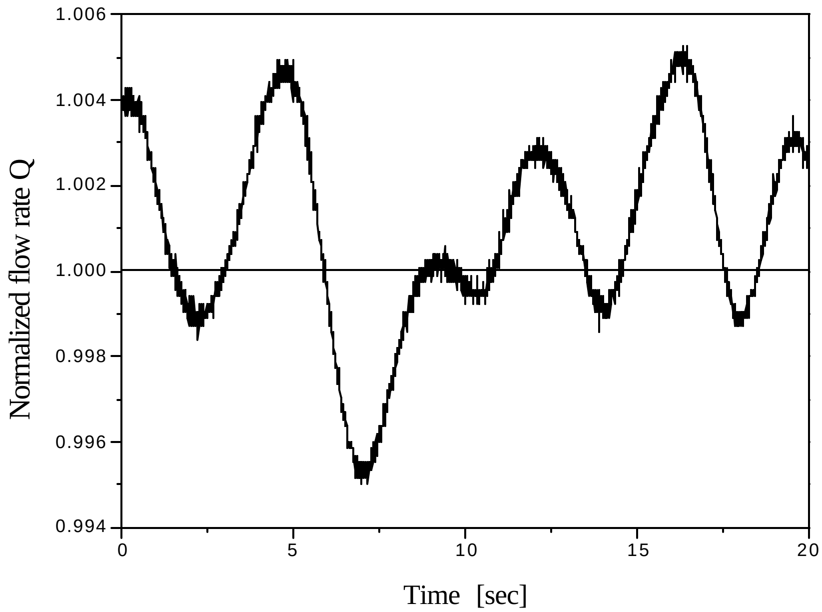


Figure 5. Time record of coolant flow fluctuations in MARIA reactor.

A very distinct oscillating mode has been observed with characteristic frequency ω_0 = 1.57 rad/sec (the origin of such low frequency fluctuation was not yet identified). The relative amplitudes δQ found were lesser than 1%:

max. $\delta Q = 0.0091$

aver. $\delta Q = 0.0028$

The existence of dominant singular oscillating mode was also confirmed by the frequency domain analysis. The Fast Fourier Transform (FFT) has been applied to the time data to calculate the power spectral density P_{QQ}.

Power spectral density of coolant flow fluctuations is presented on Fig. 6.

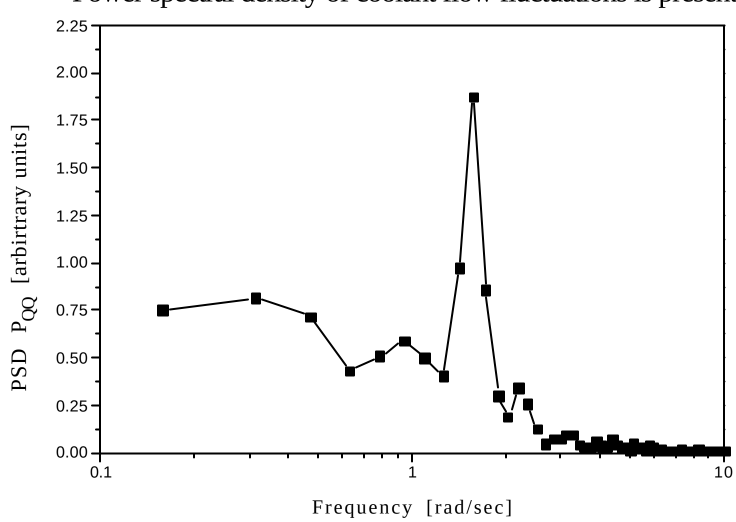


Figure 6. Measured Power spectral density P_{QQ}

Temperature and heat flux fluctuations

Using measured PSD of coolant flow fluctuations P_{QQ} and calculated transfer functions G_T and G_q, one can estimate the PSDs of the clad temperature P_{TT} and the heat flux P_{qq}, as shown in Fig. 7a-b.

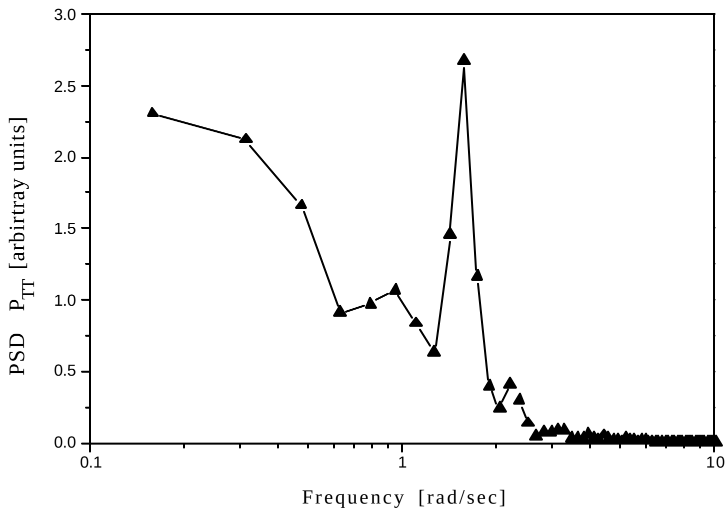


Figure 7-a. Power spectral density P_{TT}

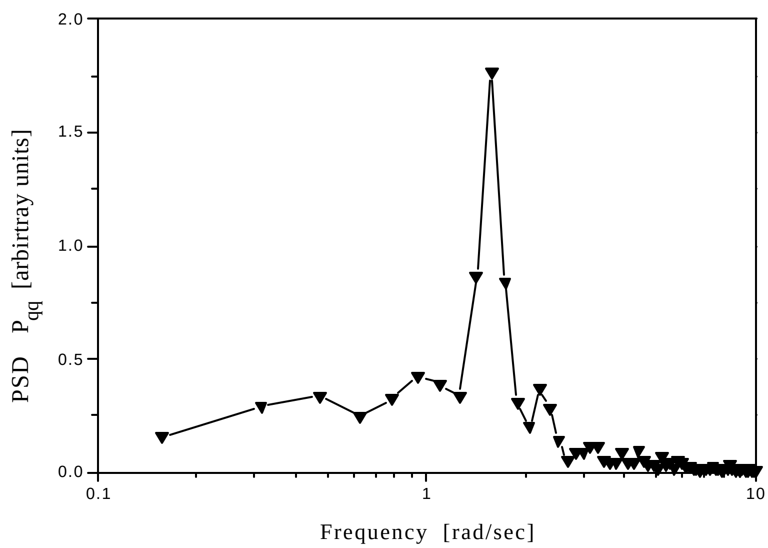


Figure 7-b. Power spectral density P_{qq}

The existence of dominant oscillating mode is clearly visible on both the clad temperature and heat flux power spectral densities.

It is justified to assume, that in time domain the coolant flow is composed with a steady state (nominal) component which is superimposed by the oscillating one with frequency ω_0 = 1.57 rad/sec. For maximum and average relative amplitudes δQ one can calculate corresponding maximum and average amplitudes of temperature and heat flux fluctuations:

max. $\delta T_c = 0.34$ [deg] max. $\delta q_c = 0.88$ [W/cm^2]
aver. $\delta T_c = 0.11$ [deg] aver. $\delta q_c = 0.27$ [W/cm^2]

The time shift t_s between the oscillations of temperature and heat flux is:

$$t_s \cong \frac{5\pi}{4\omega_0}$$

that gives for resonant frequency ω_0 value of t_s equal to 2.5 seconds, compared with 4 seconds period of oscillations.

4. CONCLUSION

The influence of fluctuations of coolant flow on the thermal limits of nuclear reactor can be analyzed using relatively simple, one-dimensional approximation of heat conduction equation with the boundary condition of the third kind on the clad-coolant boundary for transients. The advantage of the model is that, the control engineering tools, e.g. transfer functions and power spectral densities can be applied. Since the cooling system of the reactor is an inertial one, the expected region of fluctuations should be limited to very low frequencies, which have been confirmed experimentally (see Fig. 6).

In MARIA reactor example, the singular oscillating mode has been identified in spectral analysis. This gives a chance to analyze the system response also in time domain and estimates the amplitudes of fluctuations of maximum clad temperature and heat flux.

Such an analysis can be incorporated in safety margin considerations for nuclear reactors.

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