AN ANALYSIS OF XENON OSCILLATIONS USING MULTI-POINT KINETICS EQUATIONS

Keisuke Kobayashi* and Shingo Tsumura
Department of Nuclear Engineering, Kyoto University
Yoshida, Sakyoku, Kyoto, Japan
kobayasi@ip.media.kyoto-u.ac.jp, tsumura@nucleng.kyoto-u.ac.jp

ABSTRACT

Using the multi-point kinetics equations derived by using the region-wise importance functions to produce fission neutrons, xenon oscillations of thermal reactors are analyzed, and a method to terminate the xenon oscillation is investigated. An advantage of the present method is that this method can be applied to any geometries of multi-dimensions by calculating kinetics parameters of the multi-point kinetics equations using conventional multi-group diffusion or transport programs for a steady state.

1. INTRODUCTION

Analytical analysis methods of a xenon spatial oscillation for thermal reactors can be classified into two categories, nodal and modal methods[1−6]. As a nodal method, two-point kinetic equations were used to analyze the xenon oscillation where Green's functions were used[6]. The advantage of this method was in that the treatment of the space variable was rigorous, however, with respect to energy variable, only the one group equation was used.

It has been shown that the rigorous multi-point kinetics equations[7] can be derived using region-wise importance functions to produce fission neutrons. It was numerically confirmed that by solving the multi-point kinetics equations derived by dividing a whole system into appropriate subregions, the exact solution could be obtained for space dependent kinetics problems[8]. In the present work, using these rigorous multi-point kinetics equations, two-point kinetics equations were derived to analyze the xenon oscillation. Making an approximation of linearization to the nonlinear two-point kinetics equations, analytical solutions were easily obtained for xenon oscillations which depended on the control rod absorber, and a timing and strength of control rod absorber were determined to terminate the xenon oscillation.

There have been many works to terminate the xenon oscillation[2,3,5,9]. For example, the axial offsets trajectory method developed by Shimazu[10] is interesting, since the oscillation can be controlled visually. However, in his method, the measured values for the variation of the

*Present status: Emeritus Professor of Kyoto University neutron flux was used. In the present method, no measured values are used, and a timing and strength of control rod absorber can be calculated in terms of kinetics parameters.

Although numerical examples are given for simplicity, for a simple one group problem of slab geometry, the present method can be applied easily to the multi-group problems in multi-dimensions.

2. THEORY

2..1 MULTI-POINT KINETICS EQUATIONS

Let us derive the multi-point kinetics equations for the xenon oscillation from time-dependent multi-group diffusion equations using region-wise importance functions for the production of fission neutrons[7]. In order to write the equations in a simple way, we define the destruction operator A and the production operators of neutrons, B and F for multi-group diffusion equations;

$$A = -\nabla D_g \nabla + \Sigma_{rg} - \sum_{g' \neq g} \Sigma_s(g \leftarrow g'), \qquad B = \chi_g F, \qquad F = \sum_{g'} \nu \Sigma_{fg'}(\boldsymbol{r}), \tag{1}$$

where D_g, Σ_{rg}, χ_g, $\nu\Sigma_{fg}$ and $\Sigma_s(g \leftarrow g')$ are the diffusion coefficient, removal cross section, fission spectrum and fission cross section multiplied by the number of fission neutrons of $g-$th group and scattering cross section from group g' to g.

We write the absorption terms by xenon and control rod as

$$\delta A_X = \sigma_{Xg} X(\boldsymbol{r}, t), \qquad \delta A_C = \delta \Sigma_g^C(\boldsymbol{r}, t), \tag{2}$$

respectively, and $A' = A + \delta A_X + \delta A_C$, where $X(\boldsymbol{r}, t)$ is the xenon number density and σ_{Xg} is the microscopic absorption cross section of xenon.

We assume that the flux changes according to the following time dependent group diffusion equation,

$$\frac{1}{v_g} \frac{\partial \phi_g(\boldsymbol{r}, t)}{\partial t} = \left(-A' + \frac{1}{k} B \right) \phi_g(\boldsymbol{r}, t), \tag{3}$$

where $\phi_g(\boldsymbol{r}, t)$ is the neutron flux of $g-$th group and k is a criticality factor to adjust the criticality in a steady state.

Using the adjoint operator $A^\dagger$ of operator A, we define the importance function to produce fission neutrons by

$$A^\dagger G_{gm}(\boldsymbol{r}) = \nu \Sigma_{fg}(\boldsymbol{r}) \delta_m(\boldsymbol{r}), \tag{4}$$

where

$$\delta_m(\boldsymbol{r}) = \begin{cases} 1, & \boldsymbol{r} \in V_m \\ 0, & \boldsymbol{r} \notin V_m. \end{cases} \tag{5}$$

We use the boundary condition that the flux and importance function vanish at the outermost boundary of the reactor. The importance function thus defined expresses the number of fission neutrons produced in region V_m by a fission neutron born at position $\boldsymbol{r}$ in a whole reactor and energy group g[7].

The number of fission neutrons produced at position $\boldsymbol{r}$ per unit time $s(\boldsymbol{r},t)$ and the number of fission neurons produced in region V_m $s_m(t)$ are given by

$$s(\boldsymbol{r},t) = \sum_g \nu\Sigma_{fg}(\boldsymbol{r})\phi_g(\boldsymbol{r},t), \qquad s_m(t) = \frac{1}{V_m}\int_{V_m} s(\boldsymbol{r},t)d\boldsymbol{r}, \tag{6}$$

respectively.

Multiplying Eq.(3) by $\int_V d\boldsymbol{r}\sum_g G_{gm}(\boldsymbol{r})$, multiplying Eq.(4) by $\int_V d\boldsymbol{r}\sum_g \phi_g(\boldsymbol{r},t)$ from the left hand side, and making a difference of the resulting two equations, we obtain

$$l_m(t)\frac{ds_m(t)}{dt} = -s_m(t) + \sum_n \left(\frac{1}{k}k_{mn}(t) - \Delta k^X_{mn}(t) - \Delta k^C_{mn}(t)\right) s_n(t), \tag{7}$$

where the time dependent coupling coefficients are defined as

$$k_{mn}(t) = \frac{\frac{1}{V_m}\int_{V_n} d\boldsymbol{r}\sum_g G_{gm}(\boldsymbol{r})\chi_g s(\boldsymbol{r},t)}{\frac{1}{V_n}\int_{V_n} d\boldsymbol{r} s(\boldsymbol{r},t)}. \tag{8}$$

Kinetics parameters of neutron generation time and the direct change of the coupling coefficients due to the change of the operators δA_X and δA_C are defined by

$$l_m(t) = \frac{\int_V d\boldsymbol{r}\sum_g G_{gm}(\boldsymbol{r})\frac{1}{v_g}\frac{\partial\phi_g(\boldsymbol{r},t)}{\partial t}}{\int_{V_m} d\boldsymbol{r}\frac{\partial s(\boldsymbol{r},t)}{\partial t}}, \tag{9}$$

$$\Delta k^X_{mn}(t) = \frac{\frac{1}{V_m}\int_{V_n} d\boldsymbol{r}\sum_g G_{gm}(\boldsymbol{r})\sigma_{Xg}X(\boldsymbol{r},t)\phi_g(\boldsymbol{r},t)}{\frac{1}{V_n}\int_{V_n} d\boldsymbol{r} s(\boldsymbol{r},t)}, \tag{10}$$

$$\Delta k^C_{mn}(t) = \frac{\frac{1}{V_m}\int_{V_n} d\boldsymbol{r}\sum_g G_{gm}(\boldsymbol{r})\delta\Sigma^C_g(\boldsymbol{r},t)\phi_g(\boldsymbol{r},t)}{\frac{1}{V_n}\int_{V_n} d\boldsymbol{r} s(\boldsymbol{r},t)}, \tag{11}$$

respectively. Equations (7) are rigorous, namely they are derived without any approximations.

2..2 APPLICATION TO XENON OSCILLATIONS

Let us apply Eqs.(7) to the xenon spatial oscillation as two point kinetics equations with some simplifying approximations. Neglecting the terms $\Delta k^c_{12}(t)$ and $\Delta k^c_{21}(t)$ for $c = X$ or C, we obtain the two point kinetics equations

$$l_m(t)\frac{ds_m(t)}{dt} = \left(\frac{1}{k}k_{mm}(t) - \Delta k^X_{mm}(t) - \Delta k^C_{mm}(t) - 1\right) s_m(t) + \frac{1}{k}k_{mn}(t)s_n(t),$$
$$\text{for} \quad m \neq n, m, n = 1, 2, \tag{12}$$

since the neglected terms are small.

We assume that the absorption by xenon and control rods are only relevant in the thermal group G, and the change of iodine and xenon concentrations is expressed by the following equations as usual,

$$\frac{dI(\boldsymbol{r},t)}{dt} = \gamma_I \Sigma_{fG}(\boldsymbol{r})\phi_G(\boldsymbol{r},t) - \lambda_I I(\boldsymbol{r},t), \tag{13}$$

$$\frac{dX(\boldsymbol{r},t)}{dt} = \gamma_X \Sigma_{fG}(\boldsymbol{r})\phi_G(\boldsymbol{r},t) + \lambda_I I(\boldsymbol{r},t) - \lambda_X X(\boldsymbol{r},t) - \sigma_{XG} X(\boldsymbol{r},t)\phi_G(\boldsymbol{r},t), \tag{14}$$

where $I(\boldsymbol{r},t)$ and Σ_{fG} are the iodine density and the fission cross section in thermal group, γ_I and γ_X are the fractions of yield per fission, and λ_I and λ_X are the decay constants for iodine and xenon, respectively.

Using the assumption that the absorption by xenon and control rods is only relevant in the thermal group, the coupling coefficients of Eqs.(10) and (11) become

$$\Delta k^X_{mm}(t) = \frac{\int_{V_m} d\boldsymbol{r} G_{Gm}(\boldsymbol{r})\sigma_{XG} X(\boldsymbol{r},t)\phi_G(\boldsymbol{r},t)}{\int_{V_m} d\boldsymbol{r} s(\boldsymbol{r},t)}, \tag{15}$$

$$\Delta k^C_{mm}(t) = \frac{\int_{V_m} d\boldsymbol{r} G_{Gm}(\boldsymbol{r})\delta\Sigma^C_G(\boldsymbol{r},t)\phi_G(\boldsymbol{r},t)}{\int_{V_m} d\boldsymbol{r} s(\boldsymbol{r},t)}. \tag{16}$$

We assume that the neutron flux, neutron production rate, xenon and the absorption cross section of control rod are expressed as the sum of the steady state values with superscript 0 and the deviation from them in the following form

$$\phi_g(\boldsymbol{r},t) = \phi^0_g(\boldsymbol{r}) + \delta\phi_g(\boldsymbol{r},t), \quad \delta\phi_g(\boldsymbol{r},t) = f^f_{gm}(\boldsymbol{r})\delta\phi_{gm}(t), \tag{17}$$

$$s(\boldsymbol{r},t) = s^0(\boldsymbol{r}) + \delta s(\boldsymbol{r},t), \quad \delta s(\boldsymbol{r},t) = f^s_m(\boldsymbol{r})\delta s_m(t), \tag{18}$$

$$X(\boldsymbol{r},t) = X^0(\boldsymbol{r}) + \delta X(\boldsymbol{r},t), \quad \delta X(\boldsymbol{r},t) = f^X_m(\boldsymbol{r})\delta X_m(t), \tag{19}$$

$$\delta\Sigma^C_G(\boldsymbol{r},t) = f^C_m(\boldsymbol{r})\delta\Sigma^C_{Gm}(t), \tag{20}$$

where the shape functions $f^c_m(\boldsymbol{r})$ are normalized as

$$\frac{1}{V_m}\int_{V_m} f^c_m(\boldsymbol{r})d\boldsymbol{r} = 1, \qquad c = f,\ s,\ X,\ \text{or}\ C. \tag{21}$$

Using Eqs.(17) to (21), we define the following integral quantities in a node;

$$\phi_{gm}(t) = \phi^0_{gm} + \delta\phi_{gm}(t),\ \phi^0_{gm} = \frac{1}{V_m}\int_{V_m}\phi^0_g(\boldsymbol{r})d\boldsymbol{r},\ \delta\phi_{gm}(t) = \frac{1}{V_m}\int_{V_m}\delta\phi_g(\boldsymbol{r},t)d\boldsymbol{r}, \tag{22}$$

$$s_m(t) = s^0_m + \delta s_m(t), \quad s^0_m = \frac{1}{V_m}\int_{V_m} s^0(\boldsymbol{r})d\boldsymbol{r}, \quad \delta s_m(t) = \frac{1}{V_m}\int_{V_m}\delta s(\boldsymbol{r},t)d\boldsymbol{r}, \tag{23}$$

$$X_m(t) = X^0_m + \delta X_m(t),\ X^0_m = \frac{1}{V_m}\int_{V_m} X^0(\boldsymbol{r})d\boldsymbol{r},\ \delta X_m(t) = \frac{1}{V_m}\int_{V_m}\delta X(\boldsymbol{r},t)d\boldsymbol{r}. \tag{24}$$

Using the assumption that the fission cross section has a non-zero value only in the thermal group, the average production rate in region V_m can be written in the form

$$s_m(t) = \frac{1}{V_m}\int_{V_m} s(\boldsymbol{r},t)d\boldsymbol{r} = \frac{1}{V_m}\int_{V_m} \nu\Sigma_{fG}(\boldsymbol{r})(\phi_G^0(\boldsymbol{r}) + f_m^f(\boldsymbol{r})\delta\phi_{Gm}(t)) = s_m^0 + \delta s_m(t), \quad (25)$$

where

$$s_m^0 = \frac{1}{V_m}\int_{V_m} d\boldsymbol{r}\nu\Sigma_{fG}(\boldsymbol{r})\phi_G^0(\boldsymbol{r}) = \frac{\frac{1}{V_m}\int_{V_m}\nu\Sigma_{fG}(\boldsymbol{r})\phi_G^0(\boldsymbol{r})d\boldsymbol{r}}{\frac{1}{V_m}\int_{V_m}\phi_G^0(\boldsymbol{r})d\boldsymbol{r}}\phi_{Gm}^0, \quad (26)$$

$$\delta s_m(t) = \frac{1}{V_m}\int_{V_m} \nu\Sigma_{fG}(\boldsymbol{r})f_m^f(\boldsymbol{r})d\boldsymbol{r}\delta\phi_{Gm}(t). \quad (27)$$

Using Eq.(23), the coupling coefficients of Eq.(8) can be written as

$$k_{mn}(t)s_n(t) = \frac{1}{V_m}\int_{V_n} d\boldsymbol{r}\sum_g G_{gm}(\boldsymbol{r})\chi_g(s^0(\boldsymbol{r}) + f_n^s(\boldsymbol{r})\delta s_n(t)) = k_{mn}^0 s_n^0 + k_{mn}^s\delta s_n(t), \quad (28)$$

where coupling coefficients k_{mn}^0 and k_{mn}^s for the steady state are defined by

$$k_{mn}^0 = \frac{\frac{1}{V_m}\int_{V_n} d\boldsymbol{r}\sum_g G_{gm}(\boldsymbol{r})\chi_g s^0(\boldsymbol{r})}{\frac{1}{V_n}\int_{V_n} d\boldsymbol{r}s^0(\boldsymbol{r})}, \qquad k_{mn}^s = \frac{1}{V_m}\int_{V_n} d\boldsymbol{r}\sum_g G_{gm}(\boldsymbol{r})\chi_g f_n^s(\boldsymbol{r}). \quad (29)$$

Similarly, using Eqs.(22) and (24) in Eq.(15), the absorption term by xenon can be written as

$$\begin{aligned}\Delta k_{mm}^X(t)s_m(t) &= \frac{1}{V_m}\int_{V_m} d\boldsymbol{r}G_{Gm}(\boldsymbol{r})\sigma_{XG}X(\boldsymbol{r},t)\phi_G(\boldsymbol{r},t)\\ &= \frac{\sigma_{XG}}{V_m}\int_{V_m} d\boldsymbol{r}G_{Gm}(\boldsymbol{r})(X^0(\boldsymbol{r}) + f_m^X(\boldsymbol{r})\delta X_m(t))(\phi_G^0(\boldsymbol{r}) + f_m^f(\boldsymbol{r})\delta\phi_{Gm}(t))\\ &= \sigma_{Xm}^{G00}X_m^0\phi_{Gm}^0 + \sigma_{Xm}^{G0f}X_m^0\delta\phi_{Gm}(t) + \sigma_{Xm}^{GX0}\phi_{Gm}^0\delta X_m(t) + \sigma_{Xm}^{GXf}\delta X_m(t)\delta\phi_{Gm}(t), \end{aligned} \quad (30)$$

where

$$\sigma_{Xm}^{G00} = \frac{\sigma_{XG}}{V_m}\frac{\int_{V_m} d\boldsymbol{r}G_{Gm}(\boldsymbol{r})X^0(\boldsymbol{r})\phi_G^0(\boldsymbol{r})}{X_m^0\phi_{Gm}}, \qquad \sigma_{Xm}^{G0f} = \frac{\sigma_{XG}}{V_m}\frac{\int_{V_m} d\boldsymbol{r}G_{Gm}(\boldsymbol{r})X^0(\boldsymbol{r})f_m^f(\boldsymbol{r})}{X_m^0}, \quad (31)$$

$$\sigma_{Xm}^{GX0} = \frac{\sigma_{XG}}{V_m}\frac{\int_{V_m} d\boldsymbol{r}G_{Gm}(\boldsymbol{r})f_m^X(\boldsymbol{r})\phi_G^0(\boldsymbol{r})}{\phi_{Gm}^0}, \qquad \sigma_{Xm}^{GXf} = \frac{\sigma_{XG}}{V_m}\int_{V_m} d\boldsymbol{r}G_{Gm}(\boldsymbol{r})f_m^X(\boldsymbol{r})f_m^f(\boldsymbol{r}). \quad (32)$$

Similarly, the absorption term of Eq.(16) by the control rod becomes

$$\begin{aligned}\Delta k_{mm}^C(t)s_m(t) &= \frac{1}{V_m}\int_{V_m} d\boldsymbol{r}G_{Gm}(\boldsymbol{r})\delta\Sigma_{Gm}^C(\boldsymbol{r},t)\phi_G(\boldsymbol{r},t)\\ &= \frac{1}{V_m}\int_{V_m} d\boldsymbol{r}G_{Gm}(\boldsymbol{r})f_m^C(\boldsymbol{r})\delta\Sigma_{Gm}^C(t)(\phi_G^0(\boldsymbol{r}) + f_{Gm}^f(\boldsymbol{r})\delta\phi_{Gm}(t))\\ &= \alpha_m^{C\phi}\delta\Sigma_{Gm}^C(t) + \alpha_m^{Cf}\delta\Sigma_{Gm}^C(t)\delta\phi_{Gm}(t), \end{aligned} \quad (33)$$

where

$$\alpha_m^{C\phi} = \frac{1}{V_m}\int_{V_m} d\boldsymbol{r} G_{Gm}(\boldsymbol{r}) f_m^C(\boldsymbol{r})\phi_G^0(\boldsymbol{r}), \qquad \alpha_m^{Cf} = \frac{1}{V_m}\int_{V_m} d\boldsymbol{r} G_{Gm}(\boldsymbol{r}) f_m^C(\boldsymbol{r}) f_{Gm}^f(\boldsymbol{r}). \tag{34}$$

We assume that the neutron flux and production rate change as a function of time in a form

$$\phi_g(\boldsymbol{r},t) = \phi_{g\omega}(\boldsymbol{r})e^{\omega t}, \qquad s(\boldsymbol{r},t) = s_\omega(\boldsymbol{r})e^{\omega t}. \tag{35}$$

Using Eq.(35) in Eq.(9) and the steady state flux of Eq.(17) for the flux $\phi_{g\omega}(\boldsymbol{r})$, the neutron generation time $l_m(t)$ can be given as

$$l_m = \frac{\int_V d\boldsymbol{r} \sum_g G_{gm}(\boldsymbol{r})\frac{1}{v_g}\phi_{g\omega}(\boldsymbol{r})}{\int_{V_m} d\boldsymbol{r} s_\omega(\boldsymbol{r})} \approx \frac{\int_V d\boldsymbol{r} \sum_g G_{gm}(\boldsymbol{r})\frac{1}{v_g}\phi_g^0(\boldsymbol{r})}{\int_{V_m} d\boldsymbol{r}\nu\Sigma_{fG}(\boldsymbol{r}) f_{Gm}^f(\boldsymbol{r})}. \tag{36}$$

Using Eqs.(28), (30), (33) and (36) in Eqs.(12), we obtaain

$$\begin{aligned} l_m\frac{d\delta s_m(t)}{dt} = &\left(\frac{1}{k}k_{mm}^0 - 1\right)s_1^0 - \sigma_{Xm}^{G00}X_m^0\phi_{Gm}^0 + \left(\frac{1}{k}k_{mm}^s - 1\right)\delta s_m(t) - \alpha_m^{C\phi}\delta\Sigma_{Gm}^C(t) \\ &-(\alpha_m^{Cf}\delta\Sigma_{Gm}^C(t) + \sigma_{Xm}^{GXf}\delta X_m(t) - \sigma_{Xm}^{G0f}X_m^0)\delta\phi_{Gm}(t) - \sigma_{Xm}^{GX0}\phi_{Gm}^0\delta X_m(t) \\ &+\frac{1}{k}(k_{mn}^0 s_n^0 + k_{mn}^s\delta s_n(t)), \quad \text{for} \;\; m \neq n, \;\; m,n = 1,2. \end{aligned} \tag{37}$$

Assuming that the time dependent quantities vanish in Eqs.(37), we obtain the steady state equations for the equilibrium state,

$$\left(\frac{1}{k}k_{11}^0 - 1\right)s_1^0 - \hat{\sigma}_{X1}^{G00}X_1^0 s_1^0 + \frac{1}{k}k_{12}^0 s_2^0 = 0, \tag{38}$$

$$\frac{1}{k}k_{21}^0 s_1^0 + \left(\frac{1}{k}k_{22}^0 - 1\right)s_2^0 - \hat{\sigma}_{X2}^{G00}X_2^0 s_2^0 = 0, \tag{39}$$

where the absorption by the control rods is assumed to vanish for the steady state, and

$$\hat{\sigma}_{Xm}^{G00} = \frac{\sigma_{Xm}^{G00}\int_{V_m} d\boldsymbol{r}\phi_G^0(\boldsymbol{r})}{\int_{V_m} d\boldsymbol{r}\nu\Sigma_{fG}(\boldsymbol{r})\phi_G^0(\boldsymbol{r})}, \qquad \hat{\sigma}_{Xm}^{G00}s_m^0 = \sigma_{Xm}^{G00}\phi_{Gm}^0. \tag{40}$$

We define similar quantities by

$$\hat{\sigma}_{Xm}^{GX0} = \frac{\sigma_{Xm}^{GX0}\int_{V_m} d\boldsymbol{r}\phi_G^0(\boldsymbol{r})}{\int_{V_m} d\boldsymbol{r}\nu\Sigma_{fG}(\boldsymbol{r})\phi_G^0(\boldsymbol{r})}, \qquad \hat{\sigma}_{Xm}^{GX0}s_m^0 = \sigma_{Xm}^{GX0}\phi_{Gm}^0, \tag{41}$$

$$\hat{\sigma}_{Xm}^{G0f} = \frac{\sigma_{Xm}^{G0f}V_m}{\int_{V_m} d\boldsymbol{r}\nu\Sigma_{fG}(\boldsymbol{r}) f_m^f(\boldsymbol{r})}, \qquad \hat{\sigma}_{Xm}^{G0f}\delta s_m(t) = \sigma_{Xm}^{G0f}\delta\phi_{Gm}(t). \tag{42}$$

Using Eqs.(38) to (42), Eqs.(37) can be rewritten

$$\begin{aligned} l_m\frac{d\delta s_m(t)}{dt} = &\left(\frac{1}{k}k_{mm}^s - \hat{\sigma}_{Xm}^{G0f}X_m^0 - 1\right)\delta s_m(t) - (\alpha_m^{Cf}\delta\Sigma_{Gm}^C(t) + \sigma_{Xm}^{GXf}\delta X_m(t))\delta\phi_{Gm}(t) \\ &-\hat{\sigma}_{Xm}^{GX0}s_m^0\delta X_m(t) - \alpha_m^{C\phi}\delta\Sigma_{Gm}^C(t) + \frac{1}{k}k_{mn}^s\delta s_n(t), \;\text{for}\; m \neq n, \;\; m,n = 1,2. \end{aligned} \tag{43}$$

Integrating Eqs.(13) and (14) over region V_m, and using Eqs.(17) to (24), we obtain

$$\frac{dI_m(t)}{dt} = \gamma_I \Sigma^{\phi}_{fm}\phi^0_{Gm} + \gamma_I \Sigma^{f}_{fm}\delta\phi_{Gm}(t) - \lambda_I I_m(t), \tag{44}$$

$$\begin{aligned}\frac{dX_m(t)}{dt} &= \gamma_X \Sigma^{\phi}_{fm}\phi^0_{Gm} + \gamma_X \Sigma^{f}_{fm}\delta\phi_{Gm}(t) + \lambda_I I_m(t) - \lambda_X X_m(t) - \sigma^{00}_{Xm}X^0_m\phi^0_{Gm} \\ &\quad -\sigma^{0f}_{Xm}X^0_m\delta\phi_{Gm}(t) - \sigma^{X0}_{Xm}\phi^0_{Gm}\delta X_m(t) - \sigma^{Xf}_{Xm}\delta X_m(t)\delta\phi_{Gm}(t),\end{aligned} \tag{45}$$

where

$$I_m(t) = \frac{1}{V_m}\int_{V_m} d\boldsymbol{r} I_m(\boldsymbol{r},t) = I^0_m + \delta I_m(t), \tag{46}$$

$$\Sigma^{\phi}_{fm} = \frac{\frac{1}{V_m}\int_{V_m} d\boldsymbol{r}\Sigma_{fG}(\boldsymbol{r})\phi^0_G(\boldsymbol{r})}{\phi^0_{Gm}}, \qquad \Sigma^{f}_{fm} = \frac{1}{V_m}\int_{V_m} d\boldsymbol{r}\Sigma_{fG}(\boldsymbol{r})f^f_m(\boldsymbol{r}), \tag{47}$$

$$\sigma^{00}_{Xm} = \frac{\sigma_{XG}}{V_m}\frac{\int_{V_m} d\boldsymbol{r}X^0(\boldsymbol{r})\phi^0_G(\boldsymbol{r})}{X^0_m\phi^0_{Gm}}, \qquad \sigma^{0f}_{Xm} = \frac{\sigma_{XG}}{V_m}\frac{\int_{V_m} d\boldsymbol{r}X^0(\boldsymbol{r})f^f_m(\boldsymbol{r})}{X^0_m}, \tag{48}$$

$$\sigma^{X0}_{Xm} = \frac{\sigma_{XG}}{V_m}\frac{\int_{V_m} d\boldsymbol{r}f^X_m(\boldsymbol{r})\phi^0_G(\boldsymbol{r})}{\phi^0_{Gm}}, \qquad \sigma^{Xf}_{Xm} = \frac{\sigma_{XG}}{V_m}\int_{V_m} d\boldsymbol{r}f^X_m(\boldsymbol{r})f^f_m(\boldsymbol{r}). \tag{49}$$

We define the following notations,

$$\hat\gamma_X = \frac{\gamma_X}{\nu}, \quad \hat\gamma_I = \frac{\gamma_I}{\nu}, \quad \hat\sigma^{0f}_{Xm} = \frac{\sigma^{0f}_{Xm}}{\nu\Sigma^{f}_{fm}}, \quad \hat\sigma^{X0}_{Xm} = \frac{\sigma^{X0}_{Xm}}{\nu\Sigma^{\phi}_{fm}}, \tag{50}$$

$$\eta_m = \frac{\hat\sigma^{X0}_{Xm}}{\lambda_X}s^0_m, \quad \beta_m = \hat\sigma^{0f}_{Xm}X^0_m, \quad \hat\sigma^{Xf}_{Xm} = \frac{\sigma^{Xf}_{Xm}}{\nu\Sigma^{\phi}_{fm}}, \quad \hat\sigma^{00}_{Xm} = \frac{\sigma^{00}_{Xm}}{\nu\Sigma^{\phi}_{fm}}. \tag{51}$$

Using Eqs.(47), Eqs.(26) and (27) can be written

$$s^0_m = \nu\Sigma^{\phi}_{fm}\phi^0_{Gm}, \qquad \delta s_m(t) = \nu\Sigma^{f}_{fm}\delta\phi_{Gm}(t). \tag{52}$$

In a steady state, from Eqs.(44) and (45), we obtain

$$\gamma_I\Sigma^{\phi}_{fm}\phi^0_{Gm} - \lambda_I I^0_m = 0, \tag{53}$$

$$\gamma_X\Sigma^{\phi}_{fm}\phi^0_{Gm} + \lambda_I I^0_m - \lambda_X X^0_m - \sigma^{00}_{Xm}X^0_m\phi^0_{Gm} = 0, \tag{54}$$

from which the equilibrium values of iodine and xenon become

$$I^0_m = \frac{\hat\gamma_I}{\lambda_I}s^0_m, \qquad X^0_m = \frac{(\hat\gamma_I + \hat\gamma_X)s^0_m}{\lambda_X + \hat\sigma^{00}_{Xm}s^0_m}. \tag{55}$$

Using Eqs.(53) and (54) in Eqs.(44) and (45), the kinetics equations for iodine and xenon become

$$\frac{d\delta I_m(t)}{dt} = \hat{\gamma}_I \delta s_m(t) - \lambda_I \delta I_m(t), \tag{56}$$

$$\frac{d\delta X_m(t)}{dt} = \left(\hat{\gamma}_X - \beta_m - \hat{\sigma}^{Xf}_{Xm}\delta X_m(t)\right)\delta s_m(t) + \lambda_I \delta I_m(t) - \lambda_X(1+\eta_m)\delta X_m(t). \tag{57}$$

For simplicity, we assume that the reactor is symmetric such that $k^0_{11} = k^0_{22}, k^0_{12} = k^0_{21}$. In this case, from Eq.(38), we obtain

$$\left(\frac{1}{k}k^0_{11} - 1 - \hat{\sigma}^{G00}_{X1}X^0_1 + \frac{1}{k}k^0_{12}\right)s^0_1 = 0. \tag{58}$$

In order to be valid for $s^0_1 \neq 0$, the term in the bracket of the above equation must vanish. Then, using Eq.(55), the criticality factor k must satisfy the following equation

$$k = \frac{(k^0_{11} + k^0_{12})}{1 + \hat{\sigma}^{G00}_{X1}X^0_1} = (k^0_{11} + k^0_{12})\left[1 + \frac{(\hat{\gamma}_X + \hat{\gamma}_I)\hat{\sigma}^{G00}_{X1}s^0_1}{\lambda_X + \hat{\sigma}^{00}_{X1}s^0_1}\right]^{-1}, \tag{59}$$

for the existence of a steady state solution.

2..4 LINEAR APPROXIMATION

In the practical case of xenon spatial oscillations in PWRs, the amplitude of flux oscillations is small and the linear approximation neglecting the nonlinear terms is known to be a good approximation[4)]. Retaining only the first order terms in Eqs.(43), we obtain the linearized equations for neutron productions as follows;

$$l_m \frac{d\delta s_m(t)}{dt} = \left(\frac{1}{k}k^s_{mm} - \hat{\sigma}^{G0f}_{Xm}X^0_m - 1\right)\delta s_m(t) + \frac{1}{k}k^s_{mn}\delta s_n(t) - \hat{\sigma}^{GX0}_{Xm}s^0_m\delta X_m(t)$$
$$-\alpha^{C\phi}_m \delta\Sigma^C_{Gm}(t), \quad \text{for} \quad m \neq n, \ m, n = 1, 2. \tag{60}$$

The linearized equations of Eqs.(57) for xenon are

$$\frac{d\delta X_m(t)}{dt} = (\hat{\gamma}_X - \beta_m)\,\delta s_m(t) + \lambda_I \delta I_m(t) - \lambda_X(1+\eta_m)\delta X_m(t). \tag{61}$$

In a steady state, Eqs.(3), (13) and (14) can be written

$$\left(A + \sigma_{XG}\delta_{gG}X^0(\boldsymbol{r})\right)\phi^0_g(\boldsymbol{r}) = \frac{1}{k}\chi_g s^0(\boldsymbol{r}), \tag{62}$$

$$\gamma_I \Sigma_{fG}(\boldsymbol{r})\phi^0_G(\boldsymbol{r}) - \lambda_I I^0(\boldsymbol{r}) = 0, \tag{63}$$

$$\gamma_X \Sigma_{fG}(\boldsymbol{r})\phi^0_G(\boldsymbol{r}) + \lambda_I I^0(\boldsymbol{r}) - \lambda_X X^0(\boldsymbol{r}) - \sigma_{XG}X^0(\boldsymbol{r})\phi^0_G(\boldsymbol{r}) = 0, \tag{64}$$

where $I^0(\boldsymbol{r})$ is the iodine density at a steady state.

Solving the multi-group diffusion equations of Eqs.(62) together with Eqs.(63) and (64), the flux for the steady state can be obtained. The importance function of Eq.(4) can be easily obtained by using a conventional multi-group diffusion program where the usual source term is replaced by the fission cross section as input quantity for the right hand side of Eq.(4). Using these flux and importance functions in Eqs.(29) to (36), the kinetics parameters used in Eqs.(56), (60) and (61) can be obtained numerically.

2..5 ANALYTICAL SOLUTION

Let us solve Eqs.(56), (60) and (61) using the Laplace transformation. Using the transformation parameter ω, the Laplace transform of $\delta s_m(t)$ is defined by

$$\delta \bar{s}_m(\omega) = \int_0^\infty \delta s_m(t) e^{-\omega t} dt. \tag{65}$$

Laplace transforms of $\delta X_m(t)$ and $\delta I_m(t)$ are defined also by similar equations. We assume as initial condition that the system is at a steady state at $t = 0$ and then the initial values of $\delta X_m(t)$ and $\delta I_m(t)$ are zero. From Eq.(56), we obtain

$$\delta \bar{I}_m(\omega) = \frac{\hat{\gamma}_I}{\omega + \lambda_I} \delta \bar{s}_m(\omega). \tag{66}$$

Substituting this equation into the Laplace transformed equation of Eq.(61), we obtain

$$\delta \bar{X}_m(\omega) = \frac{\hat{\gamma}_X - \beta_m + \frac{\hat{\gamma}_I \lambda_I}{\omega + \lambda_I}}{\omega + \lambda_X(1 + \eta_m)} \delta \bar{s}_m(\omega). \tag{67}$$

Substituting these equations into the transformed equations of Eqs.(60), we obtain

$$\left(l_1 \omega + \mathit{\Delta}_1 + \frac{\lambda_X \eta_1^G \left(\hat{\gamma}_X - \beta_1 + \frac{\hat{\gamma}_I \lambda_I}{\omega + \lambda_I} \right)}{\omega + \lambda_X(1 + \eta_1)} \right) \delta \bar{s}_1(\omega) - \mathit{\Delta}_{12} \delta \bar{s}_2(\omega) = l_1 \delta s_1(0) - \alpha_1^{C\phi} \delta \bar{\Sigma}_{G1}^C(\omega), \tag{68}$$

where

$$\eta_m^G = \frac{\hat{\sigma}_{Xm}^{GX0}}{\lambda_X} s_m^0, \qquad \mathit{\Delta}_m = 1 - \frac{1}{k} k_{mm}^s + \hat{\sigma}_{Xm}^{G0f} X_m^0, \qquad \mathit{\Delta}_{mn} = \frac{1}{k} k_{mn}^s. \tag{69}$$

Equation (59) can be written as

$$1 + \hat{\sigma}_{X1}^{G00} X_1^0 = \frac{1}{k}(k_{11}^0 + k_{12}^0). \tag{70}$$

If we use the approximations $\hat{\sigma}_{X1}^{G00} \approx \hat{\sigma}_{X1}^{G0f}, k_{11}^0 \approx k_{11}^s$ and use Eq.(70), $\mathit{\Delta}_1$ of Eq.(69) for a symmetrical system becomes

$$\mathit{\Delta}_1 \approx \frac{1}{k}(k_{11}^0 - k_{11}^s + k_{12}^0) \approx \frac{1}{k} k_{12}^0. \tag{71}$$

Similar equation as Eq.(68) can be obtained from Eq.(60), and they can be written in a form

$$\begin{pmatrix} m_1 & m_2 \\ m_2 & m_1 \end{pmatrix} \begin{pmatrix} \delta\bar{s}_1(\omega) \\ \delta\bar{s}_2(\omega) \end{pmatrix} = \begin{pmatrix} l_1\delta s_1(0) - \alpha_1^{C\phi}\delta\bar{\Sigma}_{G1}^{C}(\omega) \\ l_2\delta s_2(0) - \alpha_2^{C\phi}\delta\bar{\Sigma}_{G2}^{C}(\omega) \end{pmatrix}, \tag{72}$$

where the system is assumed to be symmetric such that $l_1 = l_2,\ \mathit{\Delta}_1 = \mathit{\Delta}_2,\ \mathit{\Delta}_{12} = \mathit{\Delta}_{21}$ for simplicity, and

$$m_1 = l_1\omega + \mathit{\Delta}_1 + \frac{\lambda_X\eta_1^G\left(\hat{\gamma}_X - \beta_1 + \frac{\hat{\gamma}_I\lambda_I}{\omega+\lambda_I}\right)}{\omega + \lambda_X(1+\eta_1)}, \qquad m_2 = -\mathit{\Delta}_{12}. \tag{73}$$

Solution of Eqs.(72) is obtained as

$$\begin{pmatrix} \delta\bar{s}_1(\omega) \\ \delta\bar{s}_2(\omega) \end{pmatrix} = \frac{1}{m_1^2 - m_2^2}\begin{pmatrix} m_1 & -m_2 \\ -m_2 & m_1 \end{pmatrix} \begin{pmatrix} l_1\delta s_1(0) - \alpha_1^{C\phi}\delta\bar{\Sigma}_{G1}^{C}(\omega) \\ l_2\delta s_2(0) - \alpha_2^{C\phi}\delta\bar{\Sigma}_{G2}^{C}(\omega) \end{pmatrix}. \tag{74}$$

Now, assuming that the control rods are moved in a steady state of $\delta s_1(0) = \delta s_2(0) = 0$, Eqs.(74) become

$$\delta\bar{s}_1(\omega) = -\frac{1}{m_1^2 - m_2^2}\left(m_1\alpha_1^{C\phi}\delta\bar{\Sigma}_{G1}^{C\phi}(\omega) - m_2\alpha_2^{C\phi}\delta\bar{\Sigma}_{G2}^{C}(\omega)\right), \tag{75}$$

$$\delta\bar{s}_2(\omega) = \frac{1}{m_1^2 - m_2^2}\left(m_2\alpha_1^{C\phi}\delta\bar{\Sigma}_{G1}^{C}(\omega) - m_1\alpha_2^{C\phi}\delta\bar{\Sigma}_{G2}^{C}(\omega)\right). \tag{76}$$

If we move the conrol rods such that $\alpha_2^{C}\delta\bar{\Sigma}_{G2}^{C}(\omega) = -\alpha_1^{C\phi}\delta\bar{\Sigma}_{G1}^{C\phi}(\omega)$, Eqs.(75) and Eq.(76) become

$$\delta\bar{s}_1(\omega) = -\frac{\alpha_1^{C\phi}}{m_1 - m_2}\delta\bar{\Sigma}_{G1}^{C}(\omega), \qquad \delta\bar{s}_2(\omega) = \frac{\alpha_1^{C\phi}}{m_1 - m_2}\delta\bar{\Sigma}_{G1}^{C}(\omega), \tag{77}$$

from which, we can deduce that the solution exists in the case of a symmetrical system such that $\delta\bar{s}_1(\omega) = -\delta\bar{s}_2(\omega)$.

In order to make an inverse transformation of Eq.(77), the roots of the denominator of the right hand side of Eq.(77) must be obtained. Namely, using Eq.(73), the roots of the following equation

$$m_1 - m_2 = l_1\omega + \mathit{\Delta}_1 + \frac{\lambda_X\eta_1^G\left(\hat{\gamma}_X - \beta_1 + \frac{\hat{\gamma}_I\lambda_I}{\omega+\lambda_I}\right)}{\omega + \lambda_X(1+\eta_1)} + \mathit{\Delta}_{12} = 0, \tag{78}$$

must be found, which is a cubic equation for ω. To obtain roots corresponding to long periods, we can put $l_1\omega \approx 0$, and Eq.(78) becomes a quadratic equation;

$$\omega^2 + \lambda_X\left(1 + \eta_1 + \frac{\lambda_I}{\lambda_X} - \frac{\eta_1^G(\beta_1 - \hat{\gamma}_X)}{\mathit{\Delta}_1 + \mathit{\Delta}_{12}}\right)\omega + \lambda_I\lambda_X\left(1 + \eta_1 + \frac{(\hat{\gamma}_I + \hat{\gamma}_X - \beta_1)\eta_1^G}{\mathit{\Delta}_1 + \mathit{\Delta}_{12}}\right) = 0, \tag{79}$$

which can be written in a form

$$\omega^2 + 2p\omega + q = 0, \tag{80}$$

where

$$p = \frac{\lambda_X}{2}\left(1 + \eta_1 + \frac{\lambda_I}{\lambda_X} - \frac{\eta_1^G(\beta_1 - \hat{\gamma}_X)}{\Delta_1 + \Delta_{12}}\right), \tag{81}$$

$$q = \lambda_I \lambda_X \left(1 + \eta_1 + \frac{(\hat{\gamma}_I + \hat{\gamma}_X - \beta_1)\eta_1^G}{\Delta_1 + \Delta_{12}}\right). \tag{82}$$

Roots of Eq.(80) are

$$\omega_1 = -p + i\sqrt{q - p^2}, \quad \omega_2 = -p - i\sqrt{q - p^2}, \tag{83}$$

and a damping time is given by $1/p$, and period T is given by

$$T = \frac{2\pi}{\sqrt{q - p^2}}. \tag{84}$$

As seen in Eq.(83) and (100), the condition for occurrence of xenon oscillations is

$$q - p^2 > 0. \tag{85}$$

Substituting Eqs.(81) and (82) into Eq.(85), this condition is written as

$$a(\Delta_1 + \Delta_{12})^2 + 2b(\Delta_1 + \Delta_{12}) + c < 0, \tag{86}$$

where

$$a = \left(1 + \eta_1 - \frac{\lambda_I}{\lambda_X}\right)^2, \quad b = -\eta_1^G\left[\frac{\lambda_I}{\lambda_X}(\hat{\gamma}_X + 2\hat{\gamma}_I - \beta_1) + (1 + \eta_1)(\beta_1 - \hat{\gamma}_X)\right],$$
$$c = (\eta_1^G)^2(\beta_1 - \hat{\gamma}_X)^2. \tag{87}$$

Defining notations

$$\Delta^{\pm} = \frac{-b \pm \sqrt{b^2 - ac}}{a}, \tag{88}$$

Eq.(86) gives the condition for the oscillation

$$\Delta^- < \Delta_1 + \Delta_{12} < \Delta^+. \tag{89}$$

The condition that this oscillation is divergent is $p < 0$, which is written, using Eq.(81) as

$$\frac{\lambda_X}{2}\left(1 + \eta_1 + \frac{\lambda_I}{\lambda_X} - \frac{\eta_1^G(\beta_1 - \hat{\gamma}_X)}{\Delta_1 + \Delta_{12}}\right) < 0, \tag{90}$$

from which we obtain

$$\Delta_1 + \Delta_{12} < \frac{\eta_1^G \lambda_X(\beta_1 - \hat{\gamma}_X)}{\lambda_I + \lambda_X(1 + \eta_1)}. \tag{91}$$

The denominator of the right hand side of Eqs.(77) is written as

$$\frac{1}{m_1 - m_2} = \frac{\omega^2 + p_1\omega + p_2}{q_0\omega^3 + q_1\omega^2 + q_2\omega + q_3} = \sum_{k=0}^{2} \frac{c_k}{\omega - \omega_k} \tag{92}$$

where

$$p_1 = \lambda_I + \lambda_X(1+\eta_1), \qquad p_2 = \lambda_I\lambda_X(1+\eta_1) \tag{93}$$

$$q_0 = l_1, \qquad q_1 = \mathit{\Delta}_1 + \mathit{\Delta}_{12} + l_1(\lambda_I + \lambda_X(1+\eta_1)) \tag{94}$$

$$q_2 = (\mathit{\Delta}_1 + \mathit{\Delta}_{12})(\lambda_I + \lambda_X(1+\eta_1)) + \lambda_X\left((\hat{\gamma}_I - \beta_1)\eta_1^G + l_1\lambda_I(1+\eta_1)\right) \tag{95}$$

$$q_3 = \lambda_I\lambda_X\left((\mathit{\Delta}_1 + \mathit{\Delta}_{12})(1+\eta_1) + (\hat{\gamma}_I + \hat{\gamma}_X - \beta_1)\eta_1^G\right), \tag{96}$$

$$c_0 = \frac{\omega_0^2 + p_1\omega_0 + p_2}{l_1(\omega_0 - \omega_1)(\omega_0 - \omega_2)}, \qquad c_1 = \frac{\omega_1^2 + p_1\omega_1 + p_2}{l_1(\omega_0 - \omega_1)(\omega_2 - \omega_1)},$$

$$c_2 = \frac{\omega_2^2 + p_1\omega_2 + p_2}{l_1(\omega_0 - \omega_2)(\omega_1 - \omega_2)}, \tag{97}$$

and $\omega_k,\ k = 0, 1, 2$ are the roots of the cubic equation of the denominator of Eq.(92),

$$q_0\omega^3 + q_1\omega^2 + q_2\omega + q_3 = 0. \tag{98}$$

The approximate roots of ω_1 and ω_2 are given by Eq.(83).

Using Eq.(92) in Eq.(77), we obtain

$$\delta\bar{s}_1(\omega) = -\alpha_1^{C\phi} \sum_{k=0}^{2} \frac{c_k}{\omega - \omega_k} \delta\bar{\mathit{\Sigma}}_{G1}^{C}(\omega), \tag{99}$$

from which we obtain a solution by the inverse transformation,

$$\delta s_1(t) = -\alpha_1^{C\phi} \sum_{k=0}^{2} c_k \int_0^t e^{\omega_k(t-t')} \delta\mathit{\Sigma}_{G1}^{C}(t')dt'. \tag{100}$$

For iodine, from Eq.(66) we obtain

$$\delta I_m(t) = \hat{\gamma}_I \int_0^t e^{-\lambda_I(t-t')} \delta s_m(t')dt', \qquad m = 1, 2. \tag{101}$$

For xenon, Eq.(67) can be written as

$$\delta\bar{X}_m(\omega) = \left(\frac{d_1}{\omega + \lambda_I} + \frac{d_2}{\omega + \lambda_X(1+\eta_m)}\right)\delta\bar{s}_m(\omega), \tag{102}$$

where

$$d_1 = -\frac{\hat{\gamma}_I\lambda_I}{\lambda_I - \lambda_X(1+\eta_1)}, \qquad d_2 = \hat{\gamma}_X + \frac{\lambda_I(\hat{\gamma}_I - \beta_1) + \lambda_X\beta_1(1+\eta_1)}{\lambda_I - \lambda_X(1+\eta_1)}. \tag{103}$$

The inverse transformation of Eq.(102) gives the solution

$$\delta X_1(t) = d_1 \int_0^t e^{-\lambda_I(t-t')} \delta s_1(t')dt' + d_2 \int_0^t e^{-\lambda_X(1+\eta_1)(t-t')} \delta s_1(t')dt'. \tag{104}$$

2..6 CONTROL OF XENON OSCILLATIONS

We consider the cases that the xenon oscillation initiated by the first movement of the control rods and is terminated by their second movement. In the present work, the control of the xenon oscillation by the simple bang-bang control method which was discussed by Shimazu[10)] is investigated, namely, the control rods are moved according to the following equation;

$$\delta\Sigma_{G1}^{C}(t) = \begin{cases} 0, & t < t_1 \\ \delta\Sigma^{C1}, & t_1 \le t \le t_2 \\ 0, & t_2 < t < t_3 \\ \delta\Sigma^{C2}, & t_3 \le t \le t_4 \\ 0, & t_4 < t \end{cases}, \qquad \delta\Sigma_{G2}^{C}(t) = -\delta\Sigma_{G1}^{C}(t). \tag{105}$$

Using Eq.(105) in Eq.(100), $\delta s_1(t)$ after the second movement of control rods at $t_4 < t$ is given by

$$\delta s_1(t) = -\alpha_1^{C\phi} \sum_{k=1}^{2} \frac{c_k}{\omega_k} e^{\omega_k t} \left\{ \left(e^{-\omega_k t_2} - e^{-\omega_k t_1}\right) \delta\Sigma^{C1} + \left(e^{-\omega_k t_4} - e^{-\omega_k t_3}\right) \delta\Sigma^{C2} \right\}, \quad \text{for } t_4 < t. \tag{106}$$

In this equation the term of ω_0 can be neglected, since this term vanishes in a short time. In order to make $\delta s_1(t) = 0$ for $t_4 < t$, it is sufficient that the following equations hold,

$$\left(e^{-\omega_k t_2} - e^{-\omega_k t_1}\right) \delta\Sigma^{C1} + \left(e^{-\omega_k t_4} - e^{-\omega_k t_3}\right) \delta\Sigma^{C2} = 0, \quad \text{for } \ k = 1 \text{ and } 2. \tag{107}$$

If the system is unstable for xenon oscillations, ω_k for $k = 1, 2$ are complex, and ω_2 is the complex conjugate of ω_1 as seen in Eq.(83). Therefore, in this case, Eq.(107) for $k = 1$ is the same as that for $k = 2$, where the real and imaginary parts gives two equations. We assume that appropriate values for $\delta\Sigma^{C1}$, t_1, and t_2 are given. If we choose an appropriate value for t_3, two values $\delta\Sigma^{C2}$ and t_4 can be obtained from Eqs.(107). Or, if we choose appropriate value for $\delta\Sigma^{C2}$, two values t_3 and t_4 can be obtained from Eqs.(107). This means that using these two values $\delta\Sigma^{C2}$ and t_4, or t_3 and t_4 determined from Eqs.(107), the xenon oscillation can be terminated completely after $t_4 < t$ by the bang-bang control.

2..7 AXIAL OFFSETS TRAJECTORY METHOD

The axial offsets trajectory method was proposed to terminate the xenon axial oscillation in PWRs by Shimazu[10)]. In this method, using the terminology of the present work, the axial offset A_{OP} for the neutron production rate is defined by

$$A_{OP} = \frac{s_1(t) - s_2(t)}{s_1(t) + s_2(t)} = \frac{\delta s_1(t) - \delta_2(t)}{s_1^0 + s_2^0}. \tag{108}$$

Using measured values of the neutron production rate $\delta s_m(t)$ in Eqs.(56) and (57), the changes of the iodine and xenon, $\delta I_m(t)$ and $\delta X_m(t)$ are calculated successively as a function of time. Substituting these iodine and xenon into the steady state equations (55), namely

$$I_m^0 + \delta I_m(t) = \frac{\hat{\gamma}_I}{\lambda_I} s_m^I(t), \qquad X_m^0 + \delta X_m(t) = \frac{(\hat{\gamma}_I + \hat{\gamma}_X) s_m^X(t)}{\lambda_X + \hat{\sigma}_{Xm}^{00} s_m^X(t)}. \tag{109}$$

fictitious production rates $s_m^I(t)$ and $s_m^X(t)$ corresponding to the iodine and xenon densities, respectively, for non-steady state, are calculated. Using these fictitious production rates, two axial offsets are defined by

$$A_{OI} = \frac{s_1^I(t) - s_2^I(t)}{s_1^I(t) + s_2^I(t)} = \frac{\delta s_1^I(t) - \delta s_2^I(t)}{s_1^0 + s_2^0}, \quad A_{OX} = \frac{s_1^X(t) - s_2^X(t)}{s_1^X(t) + s_2^X(t)} = \frac{\delta s_1^X(t) - \delta s_2^X(t)}{s_1^0 + s_2^0}. \tag{110}$$

When the xenon oscillation exists, $\delta s_m^I(t)$ and $\delta s_m^X(t)$ as well as $\delta s_1(t)$ and $\delta s_2(t)$ are not equal to zero, and the three axial offsets A_{OP}, A_{OI} and A_{OX} take in general different values. When all these three axial offsets become zero, the xenon oscillation stops and the system returns to the steady state. Making use of this fact, a trajectory is plotted as a function of time in a figure where the horizontal axis is $A_{OP} - A_{OX}$ and the vertical axis is $A_{OI} - A_{OX}$, and control rods are moved such that the state point on the trajectory moves to the origin of the coordinates. In the method by Shimazu, the kinetics equations for neutrons are not used, and only two point kinetics equations for iodine and xenon of Eqs.(56) and (57) are solved. One of the advantages of this method is that it is easy to understand the xenon oscillation and its control visually on a figure. In the present work, the kinetics equations for neutrons are solved and special future of the axial offsets trajectory method can be understood mathematically.

3. APPLICATION TO A ONE GROUP PROBLEM IN SLAB GEOMETRY

In order to investigate the applicability of the two point kinetics equations derived in the preceding section analytically, we consider a simple one group problem in slab geometry.

3..1 CALCULATION OF COUPLING COEFFICIENTS

Let us calculate the kinetics parameters, the coupling coefficients analytically. The system is assumed to be a coupled reactor of two symmetrical cores shown in Fig.1 in order to compare the result with the previous one[6]. Equation (4) for the importance function to produce fission neutrons becomes

$$\left(-D\frac{d^2}{dx^2} + \Sigma_a\right) G_m(x) = \nu \Sigma_f \delta_m(x), \tag{111}$$

where cross sections are assumed to be region-wise constant. Solution of Eq.(111) in region V_1 has a form

$$G_1(x) = \begin{cases} b_1 \sinh \kappa_r(x + a_3), & -a_3 \le x \le -a_2 \\ b_2 \cosh \kappa_c x + b_3 \sinh \kappa_c x + \dfrac{\nu\Sigma_{fc}}{\Sigma_{ac}}, & -a_2 \le x \le -a_1 \\ b_4 \cosh \kappa_r x + b_5 \sinh \kappa_r x, & -a_1 \le x \le a_1, \\ b_6 \cosh \kappa_c x + b_7 \sinh \kappa_c x, & a_1 \le x \le a_2, \\ b_8 \sinh \kappa_r(a_3 - x), & a_2 \le x \le a_3, \end{cases} \tag{112}$$

where $\kappa_i = \sqrt{\Sigma_{ai}/D_i}$, $i = c, r$ and constants b_i, $i = 1, 2, ..., 8$ are determined using the boundary conditions at the region boundary.

We assume that the shape function for the neutron flux has a form of sine curve as

$$\begin{aligned} f_1^f(x) &\propto \sin B(x + a_2 + \delta), \quad -a_2 \le x \le -a_1, \\ f_2^f(x) &\propto \sin B(a_2 - x + \delta), \qquad a_1 \le x \le a_2. \end{aligned} \tag{113}$$

For simplicity, we assume that $\phi^0_{Gm}(x) \propto f^s_m(x) = f^f_m(x)$, $f^c_m(x) = 1$, for $c = I, X, C$, $X^0(\boldsymbol{r}) =$ constant, $I^0(\boldsymbol{r}) =$ constant. Using these approximations, the coupling coefficients of Eq.(29) are obtained as

$$\begin{aligned} k^0_{11} = k^s_{11} = & \frac{B}{(B^2 + \kappa_c^2)(\cos B\delta - \cos B(a_2 - a_1 + \delta))} \\ & \times \{\cosh \kappa_c a_2 (Bb_2 \cos B\delta - \kappa_c b_3 \sin B\delta) \\ & - \cosh \kappa_c a_1 (Bb_2 \cos B(a_2 - a_1 + \delta) - \kappa_c b_3 \sin B(a_2 - a_1 + \delta)) \\ & + \sinh \kappa_c a_2 (\kappa_c b_2 \sin B\delta - Bb_3 \cos B\delta) \\ & - \sinh \kappa_c a_1 (\kappa_c b_2 \sin B(a_2 - a_1 + \delta) - Bb_3 \cos B(a_2 - a_1 + \delta))\} + \frac{\nu \Sigma_f}{\Sigma_a}, \end{aligned} \tag{114}$$

$$\begin{aligned} k^0_{12} = k^s_{12} = & \frac{B}{(B^2 + \kappa_c^2)(\cos B\delta - \cos B(a_2 - a_1 + \delta))} \\ & \times \{\cosh \kappa_c a_2 (Bb_6 \cos B\delta + \kappa_c b_7 \sin B\delta) \\ & - \cosh \kappa_c a_1 (Bb_6 \cos B(a_2 - a_1 + \delta) + \kappa_c b_7 \sin B(a_2 - a_1 + \delta)) \\ & + \sinh \kappa_c a_2 (\kappa_c b_6 \sin B\delta + Bb_7 \cos B\delta) \\ & - \sinh \kappa_c a_1 (\kappa_c b_6 \sin B(a_2 - a_1 + \delta) + Bb_7 \cos B(a_2 - a_1 + \delta))\}. \end{aligned} \tag{115}$$

Since the system is assumed to be symmetric with respect to the origin at $x = 0$, other coupling coefficients k_{22} and k_{21} are equal to k_{11} and k_{12}, respectively.

3..2 NUMERICAL EXAMPLES

Using equations derived in the preceding section, the xenon oscillation was analyzed and the control method was investigated for the coupled reactors shown in Fig.1. The thickness of outer reflectors is assumed to be $a_3 - a_2 = 30$cm for all cases. The thickness of a core is adjusted such that the system becomes just critical with $k = 1$, and the change of the strength of the coupling between cores, damping time and period were calculated for several distances between cores. Constants used are shown in Table I, which were used in reference 6. The leakage into the perpendicular direction was taken into account by using a buckling $B_\perp^2 = 7.711 \times 10^{-3}\text{cm}^{-2}$.

The importance function $G_1(x)$ of Eq.(112) is shown in Fig.1 for the case of $2a_1 = 22.5$cm. The steady state equation (62) was numerically solved together with Eqs.(63) and (64)

Table I. One Group Constants[6)]

	Moderator	Core
D (cm)	13.1	4.71
$\Sigma_a(\text{cm}^{-1})$	0.0177	0.0829
$\Sigma_f(\text{cm}^{-1})$		0.0594
ν		2.44
	Xenon	Iodine
γ	0.003	0.061
$\lambda(\text{s}^{-1})$	0.209×10^{-4}	0.287×10^{-4}
$\sigma_a(\text{cm}^2)$	0.272×10^{-17}	

by the finite difference method, and the flux and xenon densities thus obtained are shown in Fig.2 for the same case of Fig.1, where the average flux $\phi_1^0 = 5 \times 10^{13}\text{cm}^{-2}\text{sec}^{-1}$ is used. From this figure, it is known that the assumption of sine shape and constant for the flux and xenon respectively, is reasonable. The critical thickness, coupling coefficients, damping time constant p and period are shown in Table II. In the coupled reactor theory, $B = 6.92 \times 10^{-2}\text{cm}^{-1}$ and δ=3.68cm are used in Eq.(113) for all cases, which are obtained by fitting the sine function of Eq.(113) to the numerical values of the flux obtained by the finite difference method.

Table II. Critical Thickness, Coupling Coefficients, Damping Constant and Period

$2a_1$[a)]	Critical Thickness (cm)			k_{11}	k_{12}	$1/p$	Period
(cm)	FD Method[b)]	CR Theory[c)]			$(\times 10^{-2})$	(h)	(h)
10	33.99	33.87	0.37%[d)]	0.9829	3.94	3.73	—[e)]
15	35.87	35.80	0.22%	1.0016	2.11	4.78	46.1
20	36.79	36.75	0.12%	1.0109	1.21	8.73	27.4
22.5	37.08	37.04	0.12%	1.0137	0.931	21.1	24.5
25	37.29	37.25	0.09%	1.0159	0.720	−26.5	23.9

a) Distance between cores b) Finite difference method c) Coupled reactor theory
d) Difference from the finite difference method e) No oscillation

As seen in Table II, the critical thickness by the coupled reactor theory approaches those by the finite difference method, as the distance between cores becomes larger. This may be due to the fact that one core becomes more independent of the other one as the distance between cores becomes larger, and the flux shape approaches to a pure sine shape. As shown in Table II, the coupling coefficient k_{12} becomes smaller, as the distance between cores, $2a_1$, becomes larger, then the xenon instability becomes larger.

In the present approximations, $\Delta_1 + \Delta_2 = 2k_{12}^0$. The conditions of Eqs.(89) and (91) for the

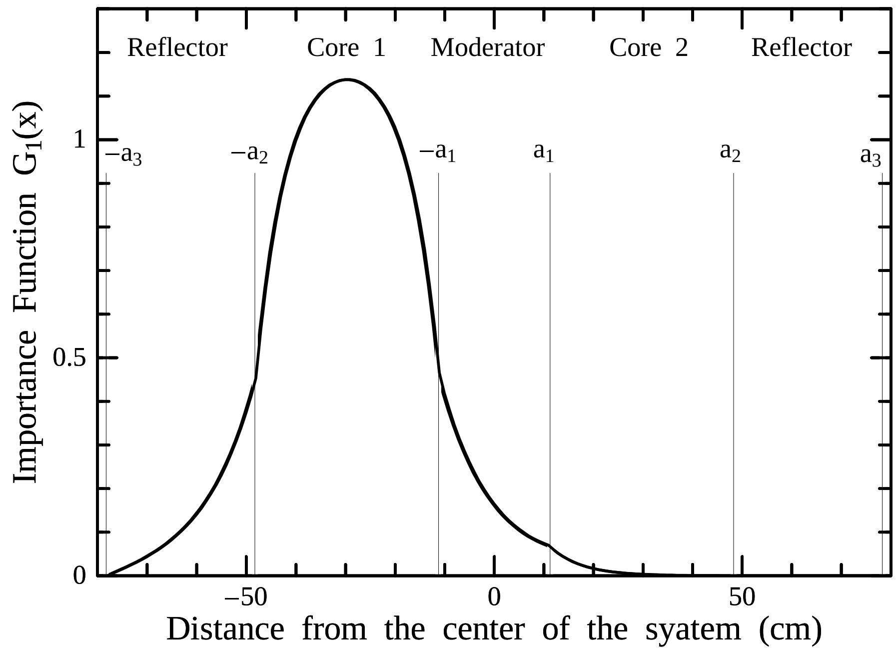

Figure 1. Importance function $G_1(x)$

oscillation and divergence become

$$0.0043 = \frac{\Delta^-}{2} < k_{12} < \frac{\Delta^+}{2} = 0.0308 \quad \text{for oscillation and,}$$
$$k_{12} < 0.0080, \quad \text{for divergence,}$$

respectively, namely, the xenon oscillation begins when $k_{12} < 0.0308$, and it will diverge when $k_{12} < 0.008$ which is confirmed in Table II.

3..3 CONTROL OF XENON OSCILLATIONS

Some numerical examples for terminating the xenon oscillation by the bang-bang control are shown for the case of the core separation $2a_1 = 22.5$cm in Figs.3 to 6. In Figs.3 and 5 are shown $A_{OP} - A_{OX}$ and $A_{OI} - A_{OX}$ calculated using Eqs.(108) and (110). In these cases, a conrol rod in region V_1 is inserted at time $t_1 = 1$h with $\delta\Sigma^{C1} = 0.00001\text{cm}^{-1}$, and withdrawn at time $t_2 = 2.5$ h to $\delta\Sigma^{C1} = 0$, by that the xenon oscillation is initiated. It is seen that the phase of $A_{OI} - A_{OX}$ is later than $A_{OP} - A_{OX}$. The reason is seen in Eq.(66) that the change of iodine density is delayed compared to the change of the production rate.

In Figs.4 and 6 are shown the trajectories of axial offsets corresponding to Figs.3 and 5, respectively, with $A_{OP} - A_{OX}$ for the horizontal axis and $A_{OI} - A_{OX}$ for the vertical axis. The following three special features are seen in Figs.4 and 6.

1. Trajectory shapes of the axial offsets look like ellipsoids which have a long axis in the first and third quadrants.

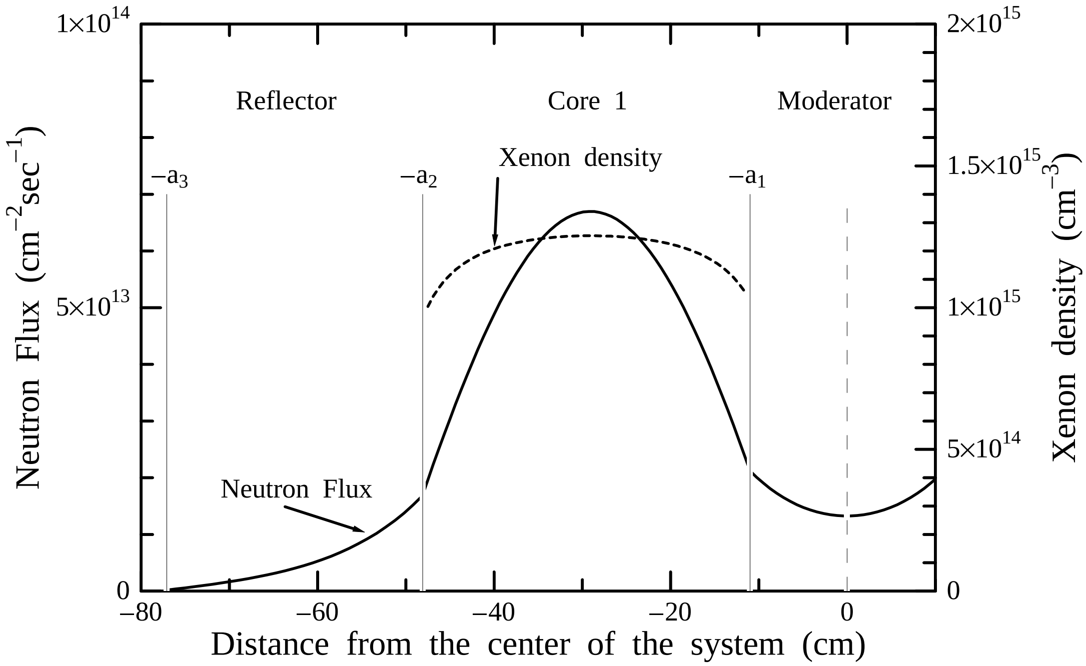

Figure 2. Neutron flux and xenon density distributions for a steady state

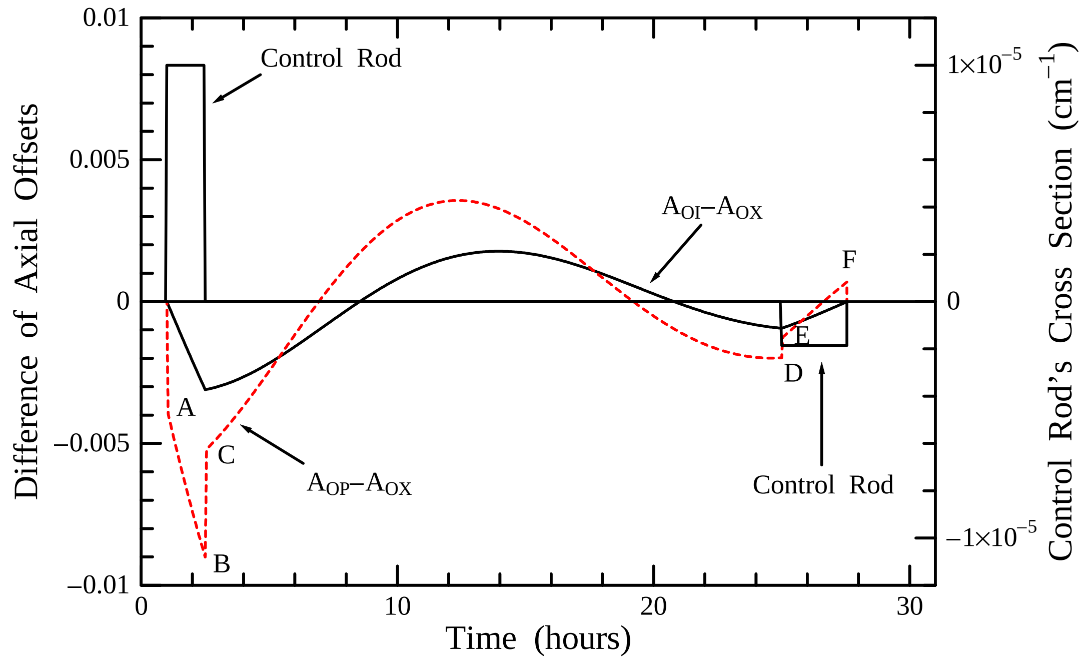

Figure 3. Time variations of $A_{OP} - A_{OX}$ and $A_{OI} - A_{OX}$ for $t_3 = 25$h

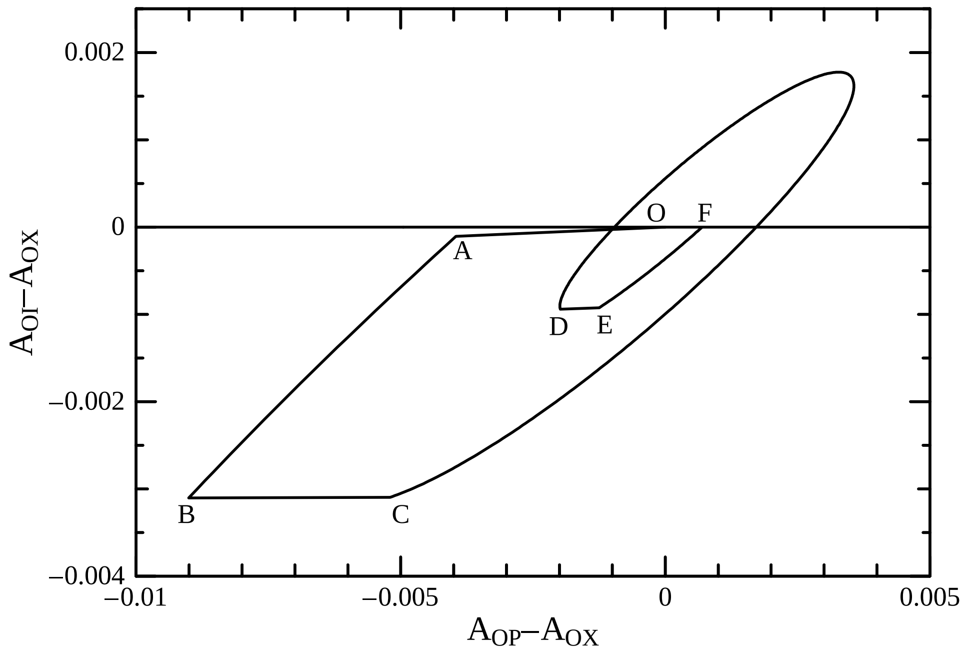

Figure 4. Trajectory of axial offset for $t_3 = 25\text{h}$

2. A point of offsets moves anticlockwise on the trajectory as a function of time.

3. A point of offsets moves quickly horizontally parallel to the horizontal axis by insertion or withdrawal of control rods.

The reason responsible for item 3 is the fact that the neutron production rate changes very quickly with time constant ω_0 as seen in Eq.(100), while iodine and xenon densities change very slowly.

In order to terminate the xenon oscillation, $t_3 = 25\text{h}$ or $t_3 = 28\text{h}$ was put in Eq.(107), and $t_4 = 27.6\text{h}$ and $\delta\Sigma^{C2} = -1.85 \times 10^{-6}\text{cm}^{-1}$ or $t_4 = 55.4\text{h}$ and $\delta\Sigma^{C2} = -3.74 \times 10^{-7}\text{cm}^{-1}$ were obtained for the cases of Figs.3 and 5, respectively. Using these values, the xenon oscillations are completely terminated as seen in Figs.3 and 5.

In Fig.4, for example, the point of offsets moved in negative direction from the origin to A almost parallel to the horizontal axis in a short time by withdrawal of a control rod, and move in positive direction quickly from point B to C by returning the control rod to the original position. At the time t_3 chosen appropriately, the rod is inserted and the point moved quickly in positive direction from point D to E, and at time t_4, the point moved quickly from point F to the origin and the oscillation is terminated. The distance from point F to the origin is nearly the same as that from point D to E. In Figs.3 and 5, the difference between the time t_3, 25h and 28h for the insertion of control rods is 3h, however, the difference of the time t_4, 27.6h and 55.4h for the withdrawal between both cases is fairly large.

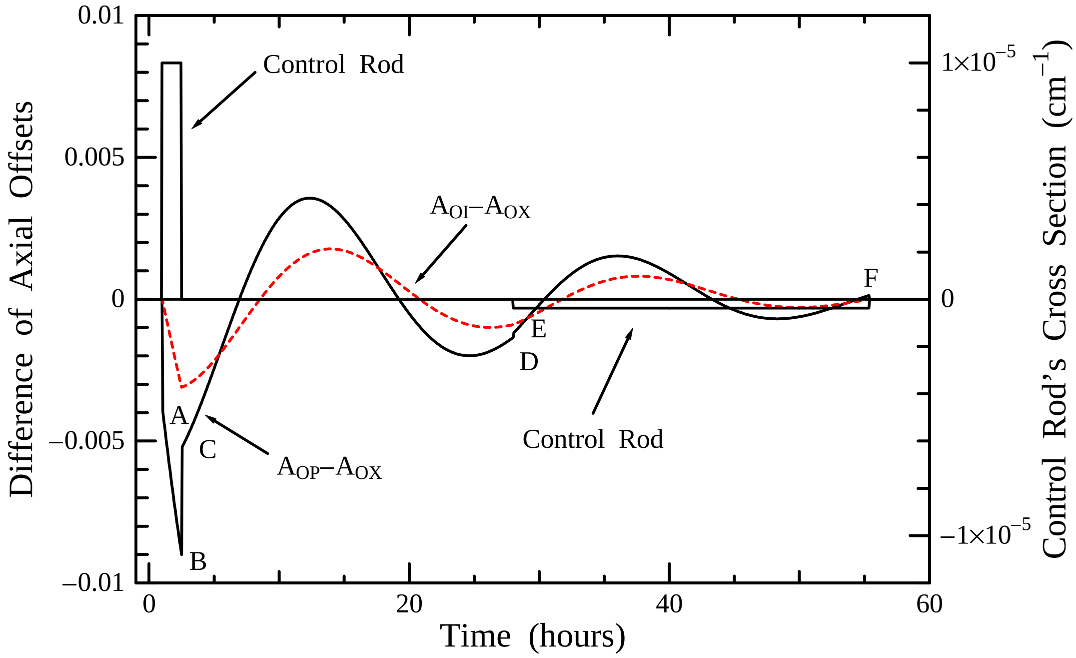

Figure 5. Time variations of $A_{OP} - A_{OX}$ and $A_{OI} - A_{OX}$ for $t_3 = 28$h

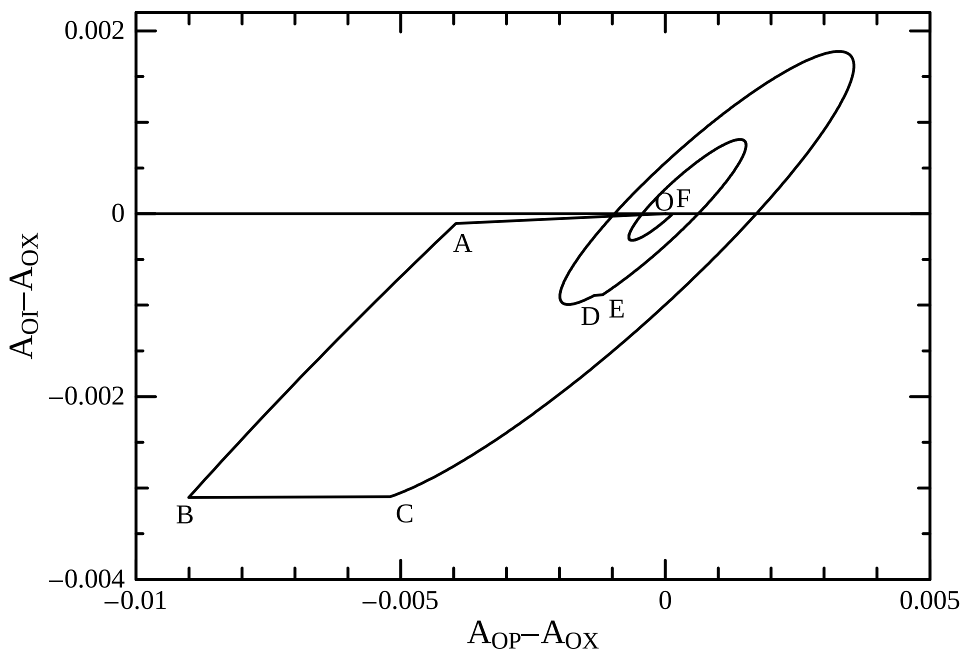

Figure 6. Trajectory of axial offsets for $t_3 = 28$h

CONCLUSIONS

It has been shown that the two-point kinetics equations which are derived using the region-wise importance functions to produce fission neutrons can be used to analyze the xenon spatial oscillation. Using these kinetics equations, the timing and magnitude for movement of control rods to terminate the xenon oscillation can be calculated in terms of kinetics parameters without using any empirical values.

Although numerical examples are given for simple one-group problems of one-dimensional slab geometry, the formulation is given for multi-group and 3 dimensional form, and there will be no difficulties for such problems in calculating kinetics parameters of two-point kinetics equations using the conventional multi-group diffusion or transport programs for a steady state.

ACKNOWLEDGEMENTS

The authors wish to express their sincere thanks to Dr. E. Kiefhaber of Forschungszentrum Karlsruhe for his many useful comments to the manuscript.

REFERENCES

1) W.M.Stacey, "Linear Analysis of Xenon Spatial Oscillation", Nucl. Sci. Eng. **30**, 453, (1967).

2) A.M. CHRISTIE and C.G. PONCELET, "On the Control of Spatial Xenon Oscillations", Nucl. Sci. Eng. **51**, 10 (1973).

3) N.Z. Chao and L.M. Grossman, "Optimal Control Spatial Oscillations in Load Follow of a Nuclear Reactor", Nucl. Sci. Eng. **83**, 136, (1983).

4) J.D. Teachman and R.J. Onega, "The Influence of Energy Group Structure and Non-linearities on the Calculation of Xenon-Induced Flux Oscillations", Nucl. Sci. Eng. **83**, 149, (1983).

5) C. Lin and Y. Lin, "Control of Spatial Xenon Oscillations in Pressurized Water Reactors Via the Kalman Filter", Nucl. Sci. Eng. **118**, 1260, (1994).

6) K. Kobayashi and M. Yoshikuni, "Analysis of Xenon Oscillation by Coupled Reactor Model", J. Nucl. Sci. Technol. **19**, 107-118 (1982).

7) K. Kobayashi, "Rigorous Derivation of Multi-Point Reactor Kinetics Equations with Explicit Dependence on Perturbation", J. Nucl. Sci. Technol., **29**, 110-120 (1992).

8) Y. Nagaya and K. Kobayashi, "Solution of 1-D Multi-Group Time Dependent Diffusion Equations Using the Coupled Reactors Theory", Ann. Nucl. Energy, **22**, 421-440 (1995).

9) Y. Shimazu, "Direct Method of Search of Optimal Xenon Oscillation Control Based on New Concept of Axial Offsets", J. Nucl. Sci. Technol., **29**, 966-971 (1992).

10) Y. Shimazu, "Continuous Guidance Procedure for Xenon Oscillation Control", J. Nucl. Sci. Technol., **32**, 95-100 (1995).