A STUDY ON EFFECTS OF PIN CELL HOMOGENIZATION IN AN ACTUAL REACTOR CORE GEOMETRY

Masahiro Tatsumi[†] and Akio Yamamoto
Software Engineering Group, PWR Fuel Division
Nuclear Fuel Industries, Ltd. (NFI)
950 Ohaza Noda, Kumatori-cho, Sennan-gun, Osaka 590-0481, JAPAN
tatsumi@nfi.co.jp ; a-yama@nfi.co.jp

Shin-ya Kosaka and Etsuro Saji
In-Core Fuel Management System Dept.
Toden Software, Inc. (TSI)
6-19-15 Shimbashi, Minato-ku, Tokyo 105-0004, JAPAN
kosaka@tsi.co.jp ; saji@tsi.co.jp

ABSTRACT

In this paper, the effects of homogenization inside a pin cell in core calculation were firstly investigated. The homogenization effects, sometimes reffered as heterogeneiety effect, were well studied in single or multi-assembly geometry. This is the first study of the effect in a core geometry.

It is found that no remarkable effects were observed for UO_2 and MOX fuel assemblies. However significant power variation due to underestimation of reactivity of Gd fuel assembly was observed. Therefore correction of spatially homogenized cross section has to be incorporated.

The feasibility to reproduce calculation results by heterogeneous calculation within the framework of pin-by-pin homogeneous calculation has been investigated.
The cell-average group constants were prepared by the SPH method so that reaction rates can be preserved between precise heterogeneous model and simple homogeneous model. It was found that assembly-wise power distributions in 3-loop type PWR cores obtained by pin-by-pin homogeneous SP3 calculations agreed with those by reference heterogeneous calculations with ~1% of R.M.S. error.

1. INTRODUCTION

[†] Also a Ph. D. candidate at the Graduate School of Osaka University.

Design calculations needed for in-core fuel management are becoming more complicated because of higher heterogeneity in the cores and needs on pursuit of economics. To accurately estimate pin power distribution becomes difficult within the framework of reconstruction method in two-group core calculations with strong heterogeneity, such as the interface between UO_2 and MOX assemblies[1]. One possible approach to this situation will be incorporation of the very detail model such as multi-group transport calculations in three-dimensional fine mesh geometry against a whole reactor core. Although it will require extremely large amount of computation, this kind of calculation is becoming realistic[2-4] thanks to rapid development of computer systems. Even in three-dimensional fine mesh geometry, it is still difficult to explicitly treat heterogeneity inside pin cell in core calculations from the viewpoint of computing time and requiring storage size. Consequently, the scheme of pin cell homogenization have to be incorporated when the effective cross sections are prepared, and its effects on characteristics of the core should be examined.

Effects of pin cell homogenization in assembly geometry have been well studied by many researchers. The *superhomogé né isation* (SPH) method developed by Kavenoky5 had been extended by H*é b*ert and Benoist[5], and Mondot[6] . The SPH method adjusts group constants so that reaction rates can be preserved between heterogeneous and homogeneous calculations. Other contributions for the basis of the discontinuity factor method were made by Koebke[7] and by Henry, Worley, and Morshed[8], Aragnoes and Ahnert[9]. However fundamental problem remains: discontinuity factor calculated with boundary condition of zero net current cannot be utilized directly to core calculations.

What in nature is the reason for core calculations to give discrepancy with true but hard-to-execute heterogeneous calculation after homogenization? In principal, if the pin cell average is done correctly, the use of average pin cell cross sections should have no effect on reactivity or power distributions. However it is physically impossible because there exist neutron currents in core calculation that cannot be accurately predicted in cell and/or assembly calculations. Consequently this is a reflexive problem: Which come first, discontinuity factors calculated with accurate neutron currents or neutron currents calculated with discontinuity factors? If iterative procedures are required to answer this question, the merit of this method would vanish all at once. This is the motivation for researchers to develop leakage-corrected homogenization techniques[10-12].

The purpose of this study is to estimate and quantify the effects of homogenization inside the pin cell in a reactor core geometry. Changes of core characteristics such as multiplication factor and power distribution were directly and consistently examined by precise core calculation model. Furthermore the correction methods were discussed to obtain calculation results that are consistent with heterogeneous model within the framework of pin-by-pin homogeneous calculation. Effects of pin cell homogenization are studied in Section 2. Correction methods of the effect are investigated in Section 3. Finally conclusions are given in Section 4.

2. ESTIMATION OF PIN CELL HOMOGENIZATION EFFECTS

In this section, we try to estimate and quantify the effects of pin cell homogenization in large-scale geometry. With the very precise calculation methodology, the effects can be directly estimated via comparisons of heterogeneous/homogeneous calculation results for criticality and power distributions. In the following subsections, estimation methods and numerical results are given.

2.1 METHODS FOR ESTIMATION

Effects by pin cell homogenization, which is also referred as heterogeneity effect, can be estimated by a precise deterministic method such as method of characteristics on condition with detail space discretization. Solution by the method of characteristics is very accurate when calculation domains under the flat-source approximation are partitioned enough small and when neutron path lines are drawn at very fine intervals. The method of characteristics is the most suitable method for estimation of pin cell homogenization effects since minimum assumptions are required on geometry handling and no statistical error that is definitely involved in stochastic methods. This is the reason why the CHAPLET[13] code has been employed throughout the study. CHAPLET can perform very efficient transport calculation by the method of characteristics with the feature of direct neutron path linking (DNPL) that enable very precise large-scale calculation in parallel. The capability of scalable parallel execution within CHAPLET greatly reduced computation time. Typical computation time for a 9-group heterogeneous transport calculation in steady state within 1/8 core of 3-loop type PWR core geometry is about 1600 seconds with 8 processors of SUN Enterprise 4000 (250MHz).

Effects by pin cell homogenization are estimated by comparison between two-dimensional heterogeneous and homogeneous calculations. In the heterogeneous calculation, all the elements inside a cell such as fuel pin, air gap, cladding, and moderator with annular and plus-type divisions were explicitly modeled and different effective cross sections for each region were utilized Therefore, known and unknown parameters are explicitly treated for each sub-divided domain in a pin cell. On the other hand, in homogeneous calculation, only the representative cross section, which was prepared by simple flux weighting method in the previous stage, were utilized for the cell. Therefore the difference between heterogeneous and homogeneous calculations is whether distribution of material and associated flux distributions inside a pin cell are explicitly treated or not.

Effective cross sections were prepared by the CASMO-4[14] code, which is yet another code based on the method of characteristics. The effective cross sections in the heterogeneous geometry are directly transferred and utilized within CHAPLET calculations. Thus no corrections were performed on effective cross sections in the case of heterogeneous calculation. Homogenized effective cross sections are also calculated within CASMO-4 by the flux-weighting method where fluxes were calculated by the KRAM[15] module in CASMO-4.

Figure 1 shows loading patterns of a 1/8 core of 3-loop type PWR. Heterogeneous calculation and homogeneous calculations were performed with the CHAPLET code for each core. In both case, baffle reflector are placed in the peripheral region of the core between fuel assemblies and reflector region. Figure 2 illustrates two kinds of pin cell geometry utilized in heterogeneous and homogeneous calculation. In heterogeneous calculation, moderator region is divided into inner

annular and outer box to treat steep flux gradient near fuel pellets and partitioned into octants for each moderator domain, while the other regions were divided into quadrants. Example of heterogeneous geometry is shown in Fig. 3. Note that reflector cells are divided with the same manner as in homogeneous model.

Discretization parameter for space and angle used in both of heterogeneous and homogeneous calculations are given in Table I. Validity of calculation results by CHAPLET with those discretization parameters was confirmed by inter-comparison among couple of calculation codes such as GMVP[16], a multi-group Monte-Carlo calculation codes, and TWODANT[17], a discrete ordinate transport calculation code. Heterogeneous and homogeneous calculations were performed with GMVP with 30 million of neutron histories, which leads ~0.01%$\Delta k/k$ and ~1% of standard deviation on multiplication factor and assembly-wise power distribution, respectively. In TWODANT calculations, a homogenized pin cell was divided into two-by-two to reduce meshing error. T order of S_N calculation was 8.

2.2 RESULTS OF CORE CALCULATIONS

Table II summarize the calculation results of multiplication factors (k_{eff}'s) for the core configurations shown in figure 1. Results in two series of heterogeneous calculations by CHAPLET-Hetero and GMVP-Hetero agreed well, by comparing the 1st row and 2nd row of Table II. Results in the other three series of homogeneous calculations by CHAPLET-Homo, GMVP-Homo and TWODANT also agreed well each other. Those Consequently it is confirmed that those calculation codes solved neutron transport properly within the configurations.

One can see the effects by pin cell homogenization on multiplication factor, in other word core reactivity, within comparisons between CHAPLET-Hetero and CHAPLET-Homo calculations in Table II. Comparisons of assembly-wise power distributions between heterogeneous and homogeneous calculations summarized in Table III. Difference of assembly-wise power distribution in the core map between CHAPLET-Hetero calculation and CHAPLET-Homo are given in Fig. 4.

Let us review the calculation results in detail.

Core-1: 4.95 wt% UO_2 with 10wt% Gadolinia / 2.0wt% UO_2

- Core reactivity in homogeneous model was underestimated ~0.4%$\Delta k/k$ due to Gd assemblies.
- Neutron balance between inside and outside of the core changed in two models: heterogeneous and homogeneous models. Heterogeneous model gave higher assembly power in the center of the core and lower power in the peripheral region of the core since most of Gd fuel assemblies were loaded in the outside of the core. This is because that reactivity of the Gd fuel assemblies was underestimated in the homogeneous model.
- No specific trend of power difference on UO_2 and MOX fuel assemblies were found between heterogeneous and homogeneous models.

Core-2: 10.7wt% Put MOX / 2.0wt% UO_2

- Error on core reactivity seems to be canceled with overestimated reactivity of UO2 assemblies and underestimated reactivity of MOX assemblies.
- As Same as the Core-1 case, though not so large, neutron balance between inside and outside of the core changed in two models. This is because that, in homogeneous model, reactivity of MOX assembly was slightly underestimated and that of UO2 assembly was slightly underestimated.
- No specific trend of power difference on UO_2 and MOX fuel assemblies were found between heterogeneous and homogeneous models.

Core-3: 10.7wt% Put MOX / 4.95 wt% UO_2 with 10wt% Gadolinia

- Core reactivity in homogeneous model was underestimated ~0.4%$\Delta k/k$ due to Gd assemblies.
- As Same as the Core-1 case and the Core-2 case, neutron balance between inside and outside of the core changed in two models. This is because of the existence of Gd assemblies. In the homogeneous model, reactivity of Gd assembly was greatly underestimated.
- No specific trend of power difference on UO_2 and MOX fuel assemblies were found between heterogeneous and homogeneous models.

Core-4: 4.95 wt% UO_2 with 10wt% Gadolinia

- Core reactivity in homogeneous model was underestimated ~1.0% $\Delta k/k$ due to Gd assemblies.
- No specific difference was found between heterogeneous and homogeneous models. This is because that no reactivity mismatch among assemblies existed since fuel assemblies of monotype (Gd assemblies) were was loaded.

Core-5: 2.0wt% UO_2

- Core reactivity in homogeneous model was slightly overestimated but could be neglected.
- No specific difference was found between heterogeneous and homogeneous models. This is because that the core consists of only the UO_2 assembly which definitely has very little of homogenization effects.

Core-6: 10.7wt% Put MOX / 2.0wt% UO_2

- Core reactivity in homogeneous model was slightly underestimated but could be neglected.
- No specific difference was found between heterogeneous and homogeneous models. MOX assembly has very little of homogenization effects.

2.3 SUMMARY

Pin cell homogenization effects in an actual core geometry were firstly investigated with precise transport calculation method in two-dimensional system by the method of characteristics. The level of the pin cell homogenization effect depends on fuel configuration and loading patterns. It is found, however, that UO_2 and MOX fuel assemblies did not produce notable heterogeneity effect with simple homogenization technique. Thus no remarkable pin cell homogenization effects were found in configurations of the UO_2 core, the MOX core and the UO_2-MOX mixed core. However, with strong absorber, the simple homogenization model did not give consistent results with heterogeneous model. Reactivity of the Gd fuel assembly was remarkably underestimated, which lead to power variation between heterogeneous and homogeneous models.

There were no specific trends of power variation, such as location dependence, due to homogenization. It rather depends on the type of fuel assembly. This fact leads a possibility to eliminate the variation by adjusting reaction rates for each type of assembly. In the next section, correction methods are studied to obtain calculation results with heterogeneous model within the framework of homogeneous model.

3. CORRECTION OF THE EFFECTS

3.1 OBJECTIVE

The calculation of three dimensional power distributions is now quite usual. Normally, to obtain the pin power distribution we use reconstruction methods. This process gives good results, although it becomes less efficient for a core calculation with strong heterogeneities, such as the interface between MOX and UO_2 assemblies in the new core management schemes. The alternative solution is a fully discretized pin-by-pin calculation in three-dimensional system[1]. With recent innovation in computer technology, it is becoming possible to perform this kind of calculations at realistic cost for design calculations.

It is still difficult, however, to explicitly treat the heterogeneity inside pin cells in three-dimensional pin-by-pin calculations even with the very innovative computing environment. Therefore, homogenization inside pin cell has to be incorporated to formulate the multi-group finite difference diffusion or transport equations. As described in the previous sections, the effect of pin cell homogenization for strong absorber is considerably large, which requires introduction of correction methods.

3.2 CORRECTION METHODS

What kinds of correction methods can be utilized to correct the heterogeneity effect in fine-mesh calculations? From the historical point of view, the following two methods are major possibilities to be applied. One is the interface discontinuity factor method and the other is so-called equivalence method. The former methods provide correction factor called discontinuity factor on

the cell surface to preserve total reaction rate within a heterogeneous cell by adjusting surface flux in homogeneous calculations. However normally discontinuity factor depends on partial current on a surface and difficult to utilize in core calculations. The latter method adjusts effective cross sections for homogeneous calculations so that reaction rates can be preserved between heterogeneous calculations and homogeneous calculations. This method, also referred as the *superhomogé né isation* (SPH) method[5,6,18], is prospective since correction factors of group constants less sensitive to neutron currents compared to the case of discontinuity factors. Therefore calculation results with heterogeneous model may be reconstructed by calculation within homogeneous model.

In the next subsections, we briefly review the methodology of the SPH methods and its applicability to large-scale pin-by-pin calculations within a whole PWR geometry.

3.3 SUPERHOMOGÉNÉISATION (SPH) METHOD

It is desired to obtain calculation results with heterogeneous model by performing homogeneous pin-by-pin calculations. The basic strategy is to adjust cell-averaged group constants for homogeneous calculations to reproduce reaction rates in heterogeneous calculations within the framework of homogeneous pin-by-pin calculations[18]. Actual procedure is described as follows.

We perform two sets of calculations within single assembly geometry. The number of energy groups is identical in both calculations. One is reference heterogeneous transport calculation (heterogeneous model) and the other is pin-by-pin diffusion or transport calculation with homogenized but different macroscopic cross sections assigned to each pin cell (homogeneous model).

A heterogeneous assembly calculation is performed to obtain few group heterogeneous macroscopic cross sections and associated flux distribution. Inside a pin cell at the position of (*r*, *c*) in heterogeneous system, an integrated reaction rate of type *x* for energy group *g* is calculated by Eq. (1),

$$\tilde{R}_x^g(r,c) = \sum_{i \in (r,c)} \Sigma_{i,x}^g \phi_i^g V_i \tag{1}$$

where

$\Sigma_{i,x}^g$ = macroscopic cross section within region *i* of group *g*

ϕ_i^g = neutron flux within fine flux distribution of group *g*

V_i = volume of region *i* .

With an integrated flux inside the cell, as described by Eq. (2), a cell-averaged group constant is calculated by Eq. (3),

$$\widetilde{F}^{g}(r,c)=\sum_{i\in(r,c)}\phi_i^g V_i \tag{2}$$

$$\widetilde{\Sigma}_x^g(r,c)=\frac{\widetilde{R}_x^g(r,c)}{\widetilde{F}^g(r,c)} \tag{3}$$

Those cell-averaged group constants are corrected so that the reaction rates can be preserved between two models.

$$\overline{\Sigma}_x^g(r,c)=\mu^g(r,c)\widetilde{\Sigma}_x^g(r,c) \tag{4}$$

$$\overline{D}^g(r,c)=\mu^g(r,c)\widetilde{D}^g(r,c) \qquad or \qquad \overline{\Sigma}_{tr}^g(r,c)=\frac{1}{\mu^g(r,c)}\widetilde{\Sigma}_{tr}^g(r,c) \tag{5}$$

The SPH correction factor $\mu^g(r,c)$ can be determined by iterative procedure. At the stating point, the factor is initialized.

$$\mu^g(r,c)=1 \tag{6}$$

With the corrected constants, pin-by-pin diffusion and/or transport calculation is performed in pin-by-pin homogeneous system. The flux distribution calculated in a pin-by-pin homogeneous system is normalized by Eq. (7) to be connected with calculation results in heterogeneous model.

$$\overline{F}^{*g}(r,c)=\overline{F}^g(r,c)\times\frac{\sum_{r,c\in\,assembly}\widetilde{F}^g(r,c)}{\sum_{r,c\in\,assembly}\overline{F}^g(r,c)} \tag{7}$$

With the normalized flux distribution, reaction rates in homogeneous system are calculated.

$$\overline{R}_x^g(r,c)=\overline{\Sigma}_x^g(r,c)\overline{F}^{*g}(r,c)V(r,c) \tag{8}$$

At the beginning of this iterative process, at the position of pin cell, the reaction rate in heterogeneous system and that in homogeneous system disaccord each other. By correcting group constants for homogeneous calculation by Eq. (4), (5) and (8), homogeneous calculation in assembly system is performed again.

$$\mu^g(r,c)=\frac{\overline{R}_x^g(r,c)}{\widetilde{R}_x^g(r,c)} \tag{9}$$

This iterative procedure is continued to meet the convergence criterion:

$$\left|\frac{\mu^{g,(k)}-\mu^{g,(k-1)}}{\mu^{g,(k)}}\right|<10^{-5} \tag{10}$$

3.4 NUMERICAL RESULTS

Two-dimensional pin-by-pin diffusion/transport calculations for 1/8 3-loop type PWR cores shown in Fig. 1 were performed by the SCOPE[4,19,20] code. The SCOPE code can perform multi-dimensional multi-group diffusion and/or transport calculations based on the object-oriented approach on parallel/distributed environment. Macroscopic cross sections were prepared through the SPH method to preserve reaction rates in assembly calculations within the framework of heterogeneous transport calculations by CHAPLET. The SPH factors were calculated through iterative calculations described above and converged properly.

The effectiveness of the correction is confirmed by direct comparison between calculation results by CHAPLET and SCOPE. The former was obtained by the method of characteristics with heterogeneous model while the latter by the finite difference diffusion (FDD) or SP3 transport calculations.

Summaries of calculation results for each core configuration are given from Table IV to Table IX. Although it is impossible to obtain identical results with heterogeneous calculation by SPH-corrected homogeneous calculations, errors due to homogenization inside pin cells were remarkably dissolved. Calculation results by SP3 transport calculations with half mesh size of standard finite difference scheme, in which a pin cell is divided by 2x2, agreed very much with the reference solution. By dividing outermost fuel pin cells of a single assembly into 3x3 where the others divided by 2x2, denoted as " 2x2-3" in Table IV etc., the degree of agreement improved because of precise proper treatment of flux gradient at the interface of assemblies.

From the engineering point of view, SPH-corrected pin-by-pin SP3 calculations showed good performance and most cost effective. Comparisons on assembly-wise power distribution between pin-by-pin SP3 calculation and reference are given in Fig. 5. Observe that the 2% of maximum difference on assembly-wise power distribution would not trouble because of small power at the peripheral assemblies. It is true, however, power variation on MOX fuel in the Core 3 exceeded our criterion: up to ~1%. This variation is due to meshing error, therefore is could be fixed by introducing the improved coarse mesh method[21].

4. CONCLUSIONS

The effects of pin cell homogenization in an actual core geometry are systematically estimated through comparisons between heterogeneous and homogeneous models by the method of characteristics. No remarkable effects were observed for UO_2 and MOX fuel assemblies. However significant power variation due to underestimation of reactivity of Gd fuel assembly was observed.

The feasibility to reproduce calculation results by heterogeneous calculation within the framework of pin-by-pin homogeneous calculation has been investigated. The cell-average group constants were prepared by the SPH method so that reaction rates can be preserved between precise heterogeneous model and simple homogeneous model. It was found that assembly-wise power

distributions in 3-loop type PWR cores obtained by pin-by-pin homogeneous SP3 calculations agreed with those by reference heterogeneous calculations with ~1% of R.M.S. error.

The knowledge obtained through this study is of great use for further development of the fine-mesh three-dimensional calculation scheme.

ACKNOWLEDGEMENTS

The authors wish to acknowledge Prof. Toshikazu Takeda of Graduate School of Osaka University for invaluable discussions.

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Table I. Discretization Parameter in CHAPLET core calculations.

Energy Domain	Horizontal Division	Polar Division	Neutron Path Intervals (cm)
Fast	128	3	0.23
Resonance	32	2	0.2
Thermal	32	2	0.1

Table II. Calculation Results of Effective Multiplication Factor.

	Core 1 Gd-UO_2	Core 2 MOX-UO_2	Core 3 MOX-Gd	Core 4 Gd	Core 5 UO_2	Core 6 MOX
CHAPLET Hetero	1.13583	1.12607	1.13641	1.15694	1.11443	1.12912
GMVP Hetero	1.13515 (-0.06%)	1.12600 (-0.01%)	1.13708 (+0.06%)	1.15674 (-0.02%)	1.11379 (-0.06%)	1.12991 (+0.07%)
CHAPLET Homo	1.13152 (-0.38%)	1.12585 (-0.02%)	1.13173 (-0.41%)	1.14589 (-0.96%)	1.11503 (+0.05%)	1.12772 (-0.12%)
GMVP Homo	1.13175 (-0.36%)	1.12579 (-0.02%)	1.13208 (-0.38%)	1.14665 (-0.89%)	1.11501 (+0.05%)	1.12784 (-0.11%)
TWODANT	1.13155 (-0.38%)	1.12568 (-0.03%)	1.13164 (-0.42%)	1.14602 (-0.94%)	1.11494 (+0.05%)	1.12752 (-0.14%)

(Hetero/Homo : Heterogeneous/Homogeneous Configuration)

Table III. Summary of Difference in Assembly-wise Power Distribution
(Upper: Maximum Difference / Lower: R.M.S Difference)

Calculation Codes	Core Configurations					
	Core1 (Gd-UO_2)	Core2 (MOX-UO_2)	Core3 (MOX-Gd)	Core4 (Gd)	Core5 (UO_2)	Core6 (MOX)
GMVP(Hetero) vs CHAPLET(Hetero)	1.0 0.5	1.2 0.6	1.3 0.4	1.7 0.9	0.9 0.4	2.5 1.3
GMVP(Homo) Vs CHAPLET(Homo)	2.1 0.6	1.5 0.7	2.8 1.3	2.0 0.9	1.1 0.3	1.4 0.6
TWODANT Vs CHAPLET(Homo)	0.3 0.1	0.3 0.1	0.5 0.2	0.4 0.1	0.4 0.1	0.5 0.2
CHAPLET(Homo) Vs CHAPLET(Hetero)	8.9 4.3	1.4 0.7	7.8 4.1	-0.4 0.1	-0.5 0.1	-0.2 0.1

Table IV. Effectiveness of the SPH method (Core 1)

Code	Case	Mesh in a Cell	k_{eff} (Dif.)		Dif. in Pow. Dist.		Relative Time
					Max	R.M.S.	
CHAPLET	Hetero	Exact	1.13583	---	---	---	---
SCOPE	FDD	1x1	1.13598	(0.01%)	-1.43%	0.72%	1.00
	FDD	2x2	1.13640	(0.05%)	-1.87%	0.83%	4.12
	SP3	1x1	1.13600	(0.01%)	-1.51%	0.62%	1.87
	SP3	2x2	1.13649	(0.06%)	-0.50%	0.27%	8.24
	SP3	2x2-3	1.13658	(0.07%)	-0.36%	0.22%	7.93

Table V. Effectiveness of the SPH method (Core 2)

Code	Case	Mesh in a Cell	k_{eff} (Dif.)		Dif. in Pow. Dist.		Relative Time
					Max	R.M.S.	
CHAPLET	Hetero	Exact	1.12607	---	---	---	---
SCOPE	FDD	1x1	1.12605	(0.00%)	-2.20%	1.17%	1.00
	FDD	2x2	1.12635	(0.02%)	-2.22%	1.03%	3.90
	SP3	1x1	1.12647	(0.04%)	-1.99%	1.09%	1.81
	SP3	2x2	1.12690	(0.07%)	-0.45%	0.31%	7.76
	SP3	2x2-3	1.12692	0.08%	-0.42%	0.23%	7.22

Table VI. Effectiveness of the SPH method (Core 3)

Code	Case	Mesh in a Cell	k_{eff} (Dif.)		Dif. in Pow. Dist.		Relative Time
					Max	R.M.S.	
CHAPLET	Hetero	Exact	1.13641	---	---	---	---
SCOPE	FDD	1x1	1.13662	(0.02%)	-1.81%	0.66%	1.00
	FDD	2x2	1.13641	(0.00%)	-1.73%	0.85%	4.78
	SP3	1x1	1.13734	(0.08%)	-1.46%	0.90%	1.98
	SP3	2x2	1.13719	(0.07%)	-0.82%	0.41%	8.95
	SP3	2x2-3	1.13711	0.06%	-0.61%	0.32%	8.71

Table VII. Effectiveness of the SPH method (Core 4)

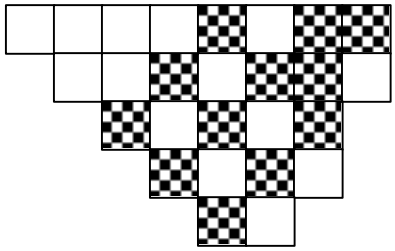
Code	Case	Mesh in a Cell	k_{eff} (Dif.)		Dif. in Pow. Dist.		Relative Time
					Max	R.M.S.	
CHAPLET	Hetero	Exact	1.15694	---	---	---	---
SCOPE	FDD	1x1	1.15713	0.02%	-1.09%	0.41%	1.00
	FDD	2x2	1.15722	0.02%	-1.50%	0.64%	4.81
	SP3	1x1	1.15736	0.04%	-1.17%	0.40%	2.19
	SP3	2x2	1.15749	0.05%	-0.51%	0.16%	8.93
	SP3	2x2-3	1.15750	0.05%	-0.41%	0.13%	8.15

Table VIII. Effectiveness of the SPH method (Core 5)

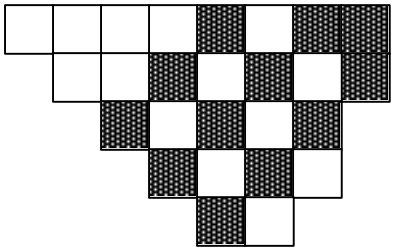
Code	Case	Mesh in a Cell	k_{eff} (Dif.)		Dif. in Pow. Dist.		Relative Time
					Max	R.M.S.	
CHAPLET	Hetero	Exact	1.11443	---	---	---	---
SCOPE	FDD	1x1	1.11455	0.01%	-1.01%	0.36%	1.00
	FDD	2x2	1.11467	0.02%	-1.80%	0.65%	4.05
	SP3	1x1	1.11478	0.03%	-1.23%	0.45%	1.83
	SP3	2x2	1.11498	0.05%	-0.44%	0.14%	8.42
	SP3	2x2-3	1.11457	0.01%	-0.31%	0.10%	7.99

Table IX. Effectiveness of the SPH method (Core 6)

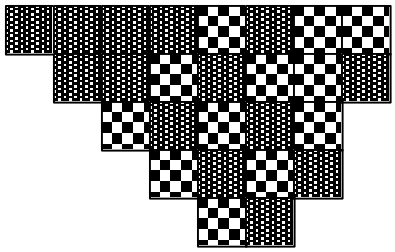
Code	Case	Mesh in a Cell	k_{eff} (Dif.)		Dif. in Pow. Dist.		Relative Time
					Max	R.M.S.	
CHAPLET	Hetero	Exact	1.12912	---	---	---	---
SCOPE	FDD	1x1	1.12914	0.00%	-1.54%	0.67%	1.00
	FDD	2x2	1.12924	0.01%	-1.78%	0.78%	4.38
	SP3	1x1	1.12937	0.02%	-0.76%	0.24%	2.02
	SP3	2x2	1.12952	0.04%	-0.38%	0.10%	8.99
	SP3	2x2-3	1.12953	0.04%	-0.38%	0.10%	7.99



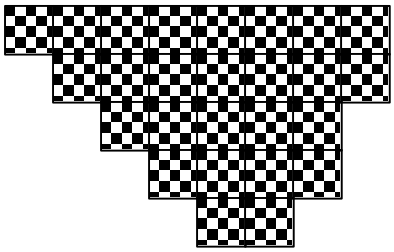
Core 1: 4.95wt%Gd-2.0wt%UO2



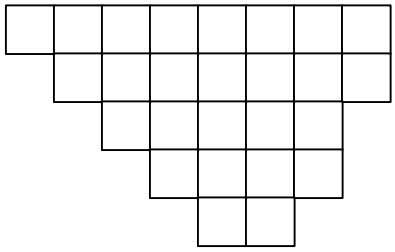
Core 2: 10.7wt%PutMOX-2.0wt%UO2



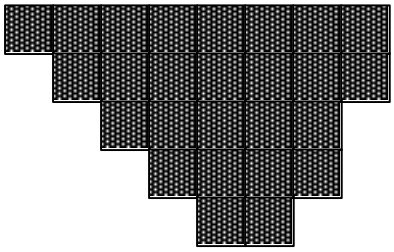
Core 3: 10.7wt%PutMOX-4.95wt%Gd



Core 4: 4.95wt% Gd



Core 5: 2.0wt% UO2



Core 6: 10.7wt%PutMOX

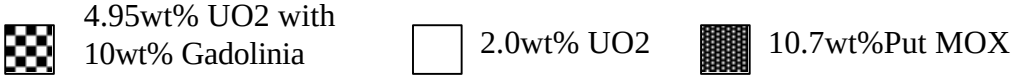


Figure 1. Loading Patterns of 3-loop type PWR 1/8 core

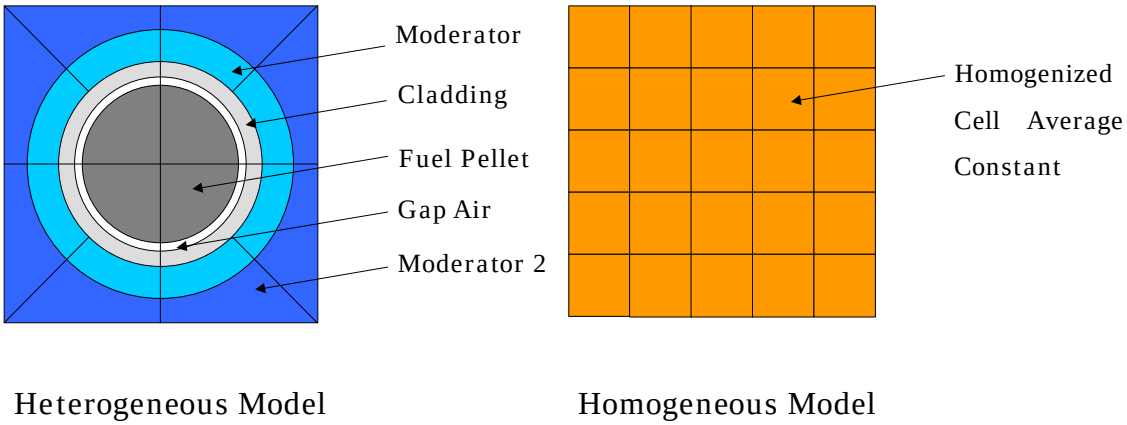


Figure 2. Geometry Inside A Pin Cell For Heterogeneous and Homogeneous Calculations.

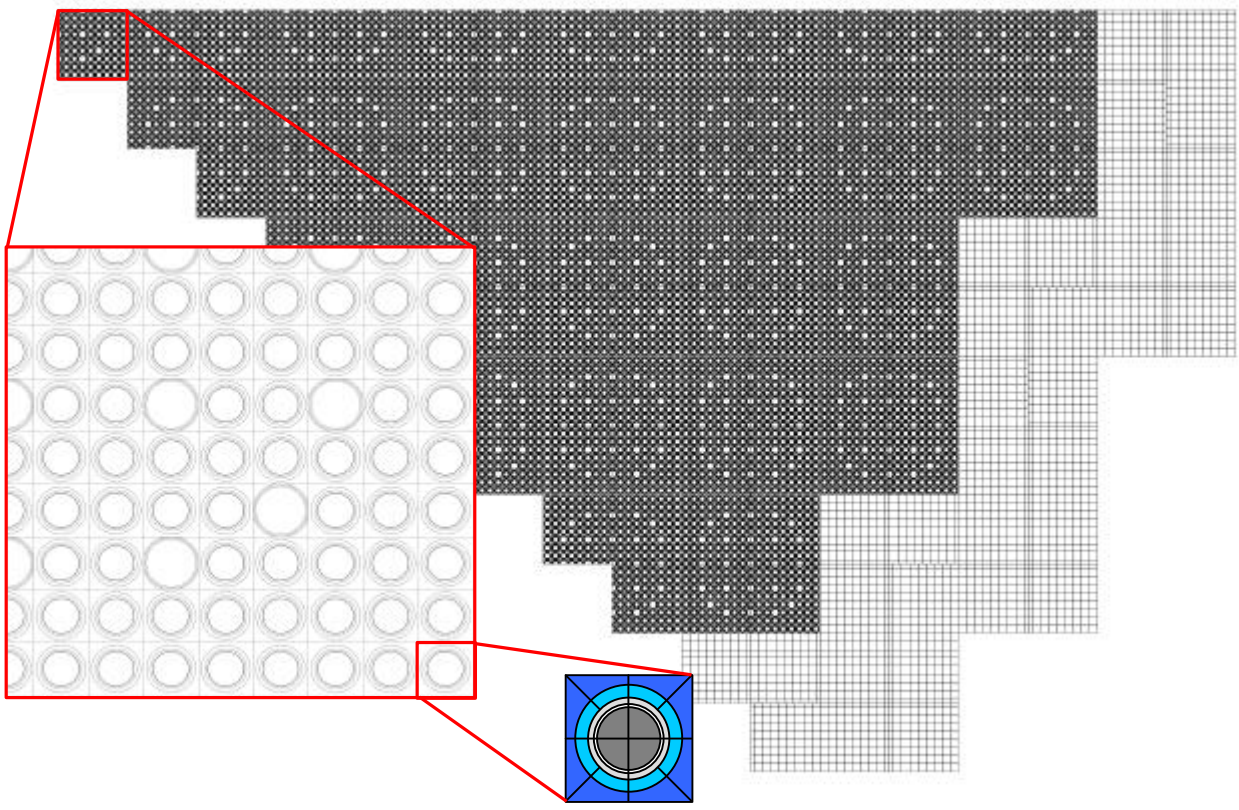
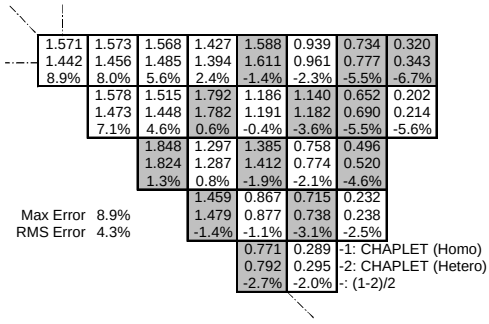
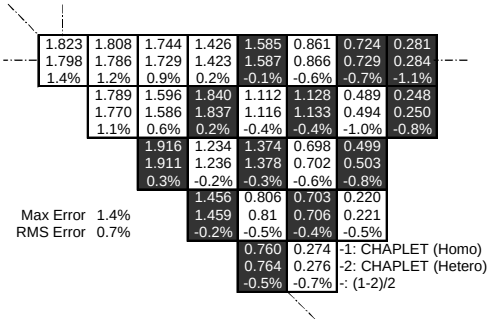


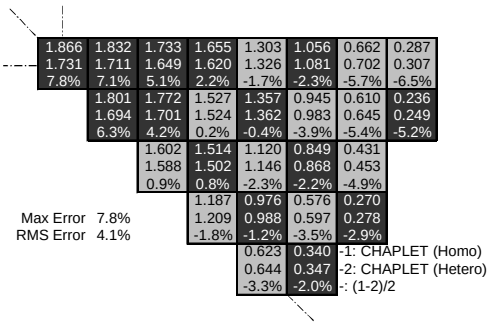
Figure 3. Sample Geometry for Heterogeneous Calculation with CHAPLET (Core 5)



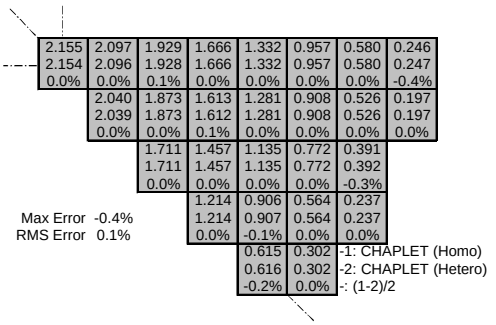
Core-1(UO_2-Gd)



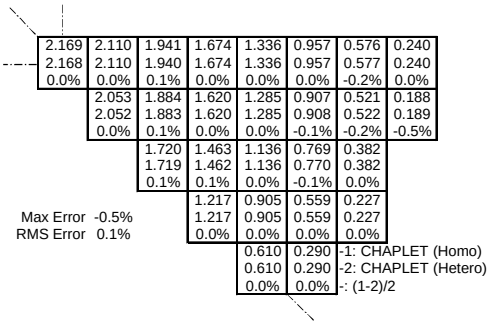
Core-2(MOX- UO_2)



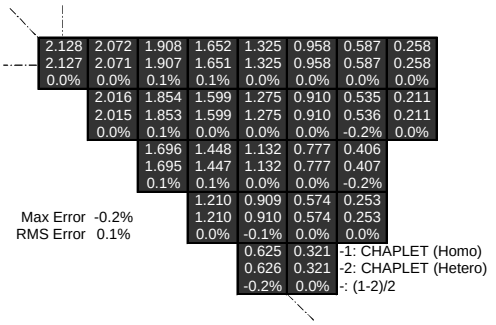
Core-3(MOX-Gd)



Core-4(Gd)

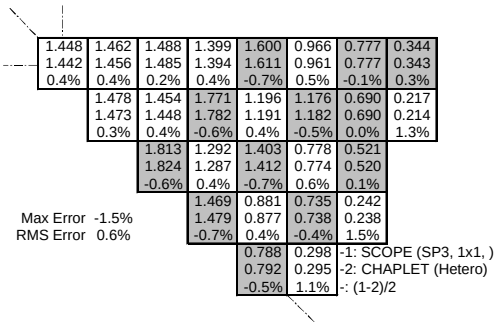


Core-5(UO_2)

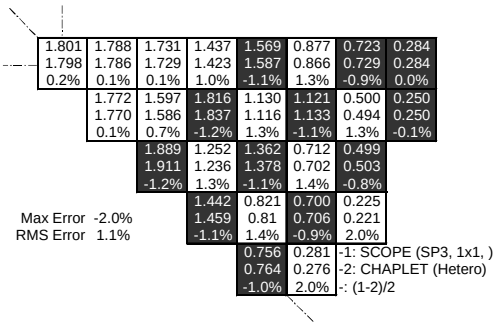


Core-6(MOX)

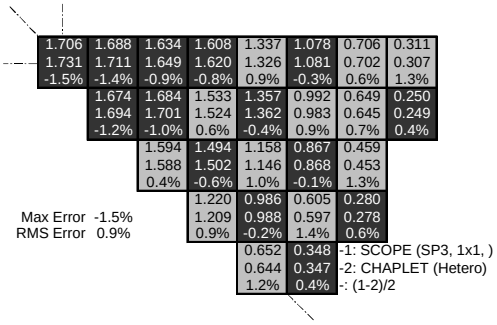
Figure 4. Discrepancy between Heterogeneous Model and Homogeneous Model.



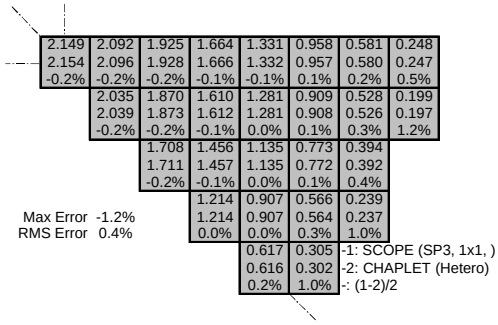
Core-1(UO_2-Gd)



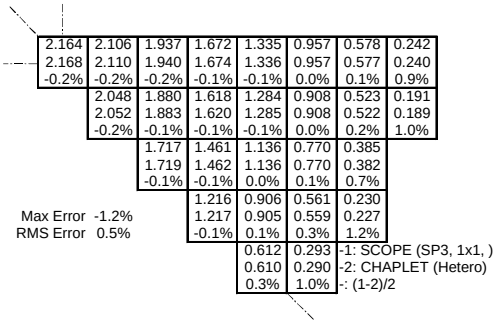
Core-2(MOX- UO_2)



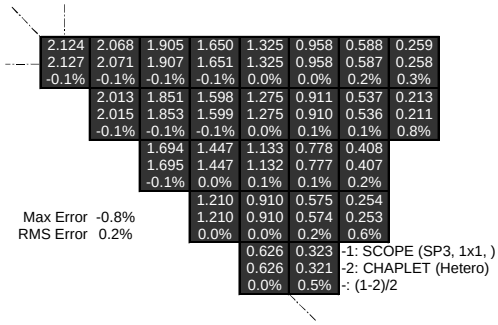
Core-3(MOX-Gd)



Core-4(Gd)



Core-5(UO_2)



Core-6(MOX)

Figure 5. Effectiveness of SPH-Corrected SP3 Pin-by-Pin Calculation.