EVALUATION OF PERTURBATION EFFECT DUE TO FISSION-SOURCE CHANGE IN EIGENVALUE PROBLEMS BY MONTE CARLO METHODS

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ABSTRACT

The formulae to evaluate the fission-source change by the correlated and differential operator sampling methods have been derived. The perturbation effect due to the fission-source change has been well evaluated and the obtained change in an effective multiplication factor are improved. The perturbation procedures based on the formulae have been incorporated into a continuous-energy Monte Carlo code MVP and verification calculations have been performed for both fast and thermal systems.

1. INTRODUCTION

It is very difficult to evaluate a small perturbation effect for neutronic parameters by multiple Monte Carlo calculations. Thus some special techniques such as the correlated sampling method or the differential operator sampling method[1] have been employed for perturbation calculations. The availability of these methods has been well established for fixed-source problems but not for eigenvalue problems. One of the additive difficulties lies in evaluation of the perturbation effect due to the perturbed fission-source distribution (perturbed eigenfunction). This makes it quite difficult to evaluate a change in an effective multiplication factor, which is of great interest in the nuclear reactor analysis.

In order to avoid the difficulty for the eigenvalue problems, the fission matrix method[2] has been employed with the correlated sampling method[3,4,5]. This method has a distinct advantage that only the fission matrices in the unperturbed and perturbed system are required regardless of the disturbed fission-source distribution. The fission matrices represent probabilities that a source particle in a region will generate fission neutrons in another region and can be obtained in a single run by the correlated sampling method. Then the eigenvalue (effective multiplication factor) and the corresponding eigenvector (fission source distribution) are straightforwardly calculated by the numerical solution for both the systems. Rief also proposed the fission matrix method with the differential operator sampling method[6].

The fission matrix method is quite effective for eigenvalue problems. However, we have to divide fissionable region into some subregions and it causes additional problems. Firstly, we have to investigate the number of subregions and geometry of the subregions for the perturbation effect to be evaluated enough accurately. Secondly, the fission-source distribution is assumed to be constant in the subregions and thus many subregions may be required for locally large perturbations such as insertion of control rods. For this problem, Rief proposed the source adjustment technique which provided an approximation to the correct fission-source distribution in the different subregions[4].

The other methods with the correlated sampling technique are the identical flight path method[7] and the similar flight path method[8]. Nakagawa *et al.* extended the methods to evaluate the first-order perturbation effect due to the fission-source change and demonstrated that changes in effective multiplication factors were improved[9].

Recently, Kitada *et al.* proposed the exact expression for the correlated sampling method[10]. In this expression, source particles in the perturbed system must have initial weights different from the unperturbed system to represent the fission-source change. The propagation of the particle weights through generations causes an unacceptable fluctuation of the weights in some cases and the variance might be unbounded. In order to suppress the fluctuation, they divided the system into many subregions and employed the averaged weight for source particles in each subregion. However, it causes the same problems as the fission matrix method.

In the present work, we have derived formulae to evaluate the fission-source change by the correlated and differential operator sampling methods and have verified them for a one-energy-group benchmark problem. Furthermore, we have incorporated the procedure into a general-purpose continuous-energy Monte Carlo code MVP[11] to solve general perturbation problems. Verification calculations have been also performed for fast and thermal systems and the perturbation effect due to the fission-source change has been investigated.

2. FORMULATION OF MONTE CARLO PERTURBATION METHODS

2.1 DEFINITION OF EFFECTIVE MULTIPLICATION FACTOR

We begin with the following outcoming collision density equation[13] without extraneous sources:

$$q(P) = \int dP' K_s(P;P') q(P') + S_f(P) \tag{1}$$

$$S_f(P) = \int dP' K_f(P;P') q(P'), \tag{2}$$

where P denotes the spatial coordinates, energy, direction of flight of a particle in the 6-dimentional phase space, $q(P)$ the outcoming collision density which is the number of particles coming out of a collision at P, $S_f(P)$ fission source density, and K_s, K_f scattering, fission transport kernels, respectively. Furthermore, $K_x(x = s \text{ or } f)$ is defined as the product of the collision and transition kernels:

$$K_x(P;P') = C_x(P;P'')T(P'';P'). \tag{3}$$

One can easily find that Eq.(1) is a homogeneous equation. In order to obtain the nonzero solution and solve the equation by successive iteration, we introduce an eigenvalue and the concept of generations. Then the outcoming collision density in the i-th generation is defined as

$$q_1(P) = S_{f,1}(P) \qquad (i = 1) \tag{4}$$

$$q_i(P) = \int dP' K_s(P;P') q_i(P') + S_{f,i}(P) \qquad (i \geq 2) \tag{5}$$

$$S_{f,i}(P) = \frac{1}{k_{i-1}} \int dP' K_f(P;P') q_{i-1}(P'), \tag{6}$$

where k_{i-1} is the eigenvalue in the $(i-1)$-th generation. This procedure corresponds to the source or outer iteration in the deterministic method and we can obtain $q_i(P)$ for the given $S_{f,i}(P)$ that is evaluated from the random walk in the previous generation.

The eigenvalue k_i is also called the effective multiplication factor in the reactor physics and is usually defined as the ratio of the number of generated particles to the number of source particles in the i-th generation:

$$k_i = \frac{\int dP \int dP' K_f(P;P') q_i(P')}{\int dP' S_{f,i}(P')}. \tag{7}$$

Eq.(7) can be rewritten by using the Neumann-series solution of Eq.(5):

$$k_i = \frac{\int dP \int dP' K_F(P;P') S_{f,i}(P')}{\int dP S_{f,i}(P)}, \tag{8}$$

where

$$K_F(P;P') = \int dP'' K_f(P;P'') \sum_{m=0}^{\infty} K_{s,m}(P'';P') \tag{9}$$

$$K_{s,m}(P;P') = \int dP_1 \cdots \int dP_{m-1} K_s(P;P_{m-1}) K_s(P_{m-1};P_{m-2}) \cdots K_s(P_1;P'). \tag{10}$$

2.2 CORRELATED SAMPLING METHOD

In the correlated sampling method, a change in an effective multiplication factor is expressed as follows[10]:

$$\begin{aligned} \Delta k &= k^* - k \\ &= \frac{1}{\int S_f(P')dP'} \int dP \int dP' \left[\frac{K_F^*(P;P') \dfrac{S_f^*(P')}{\int S_f^*(P'')dP''}}{K_F(P;P') \dfrac{S_f(P')}{\int S_f(P'')dP''}} - 1 \right] \\ &\quad \times K_F(P;P') S_f(P'), \end{aligned} \tag{11}$$

where Δk represents a difference of the effective multiplication factor between the unperturbed and perturbed systems, the parameters with the asterisk indicate those in the perturbed system and the generation index i is omitted for simplicity. We rewrite the equation as

$$\begin{aligned} \Delta k &= \frac{1}{\int S_f(P')dP'} \int dP \int dP' \left[\frac{K_F^*(P;P')}{K_F(P;P')} - 1 \right] K_F(P;P') S_f(P') \\ &\quad + \frac{1}{\int S_f(P')dP'} \int dP \int dP' \left[\frac{K_F^*(P;P')}{K_F(P;P')} \left(\frac{\dfrac{S_f^*(P')}{\int S_f^*(P'')dP''}}{\dfrac{S_f(P')}{\int S_f(P'')dP''}} - 1 \right) \right] \\ &\quad \times K_F(P;P') S_f(P'). \end{aligned} \tag{12}$$

The first and second terms of the RHS of Eq.(12) represent the change in the effective multiplication factor without and only with the effect due to the fission-source change, respectively.

If the perturbation is small, the perturbed fission-source distribution can be written as

$$S_f^*(P) \approx S_f(P) + \Delta S_f(P). \tag{13}$$

Substituting it into Eq.(12) and neglecting the second-order terms, Eq.(12) becomes

$$\begin{aligned}\Delta k \ &\approx \ \frac{1}{\int S_f(P')dP'}\int dP \int dP' \left[\frac{K_F^*(P;P')}{K_F(P;P')} - 1\right] K_F(P;P')S_f(P') \\ &\quad + \frac{\int dP \int dP' K_F^*(P;P')\Delta S_f(P')}{\int S_f(P')dP'} \\ &\quad - \frac{\int \Delta S_f(P')dP'}{\int S_f(P')dP'}\frac{\int dP \int dP' K_F^*(P;P')S_f(P')}{\int S_f(P')dP'}. \end{aligned} \tag{14}$$

This is the first order approximation to Eq.(12) in regard to the fission-source change for the correlated sampling method[12]. The identical and similar flight path methods are based on this equation. Only the first term of the RHS of Eq.(14) is often evaluated and there are a few cases where Δk is evaluated by the first order expresstion[9].

In the present work, we evaluate the perturbation effect due to the fission-source change by Eq.(12). The ratio of the fission source between the perturbed and unperturbed system in the i-th generation is evaluated by the following equation;

$$\frac{\dfrac{S_f^*(P')}{\int S_f^*(P'')dP''}}{\dfrac{S_f(P')}{\int S_f(P'')dP''}} = \frac{\dfrac{\int dP' K_F^*(P;P')S_{f,i-1}^*(P')}{\int dP \int dP' K^*(P;P')S_{f,i-1}^*(P')}}{\dfrac{\int dP' K_F(P;P')S_{f,i-1}(P')}{\int dP \int dP' K(P;P')S_{f,i-1}(P')}}. \tag{15}$$

This equation can be derived from Eq.(6) which normalizes the fission source distribution and Eq.(8) of the definition of k_i.

2.3 DIFFERENTIAL OPERATOR SAMPLING METHOD

The change in the effective multiplication factor can be expressed by the Taylor series expansion as

$$\Delta k = \frac{\partial k}{\partial a}\Delta a + \frac{1}{2}\frac{\partial^2 k}{\partial a^2}(\Delta a)^2 + \cdots + \frac{1}{n!}\frac{\partial^n k}{\partial a^n}(\Delta a)^n + \cdots, \tag{16}$$

where a is a perturbation parameter such as material density, number density for some nuclide or temperature. In the differential operator sampling method, each differential term is evaluated separately and the first-order or second-order approximation is usually employed for an estimate of the change. By differentiating Eq.(8) with a, we obtain the first-order differential form,

$$\begin{aligned}\frac{\partial k}{\partial a} &= \frac{1}{\int S_f dP_0}\sum_m \int dP_m \cdots \int dP_0 \left[\frac{1}{C_{f,m}}\frac{\partial C_{f,m}}{\partial a} + \frac{1}{T_m}\frac{\partial T_m}{\partial a}\right. \\ &\quad \left. + \frac{1}{C_{s,m-1}}\frac{\partial C_{s,m-1}}{\partial a} + \frac{1}{T_{m-1}}\frac{\partial T_{m-1}}{\partial a} \cdots + \frac{1}{T_1}\frac{\partial T_1}{\partial a} + \frac{1}{S_f}\frac{\partial S_f}{\partial a}\right] \\ &\quad \times C_{f,m}T_m C_{s,m-1}T_{m-1}\cdots C_{s,1}T_1 S_f, \end{aligned} \tag{17}$$

where T_m is the transition kernel of the m-th flight, $C_{x,m}$ the collision kernel of the m-th collision. The last term in the bracket represents the first-order perturbation effect due to the fission-source change. We evaluate it by the following equation:

$$\begin{aligned}\frac{\partial}{\partial a}S_{f,i}(P) &= \frac{\int S_{f,i-1}dP'}{\int dP \int dP' K_F S_{f,i-1}}\left(\int dP' \frac{\partial}{\partial a}\left[K_F S_{f,i-1}\right]\right. \\ &\quad \left. - \int dP \int dP' \frac{\partial}{\partial a}\left[K_F S_{f,i-1}\right] \frac{\int dP' K_F S_{f,i-1}}{\int dP \int dP' K_F S_{f,i-1}}\right). \end{aligned} \tag{18}$$

This equation can be derived from Eq.(6) and Eq.(8) as in the correlated sampling method.

Similarly, we can obtain the second-order differential form for k,

$$\begin{aligned}\frac{\partial^2 k}{\partial a^2} &= \frac{1}{\int S_f dP_0}\sum_m \int dP_m \cdots \int dP_0 \left\{\left[\frac{1}{C_{f,m}}\frac{\partial C_{f,m}}{\partial a} + \frac{1}{T_m}\frac{\partial T_m}{\partial a}\right.\right. \\ &\quad \left. + \frac{1}{C_{s,m-1}}\frac{\partial C_{s,m-1}}{\partial a} + \frac{1}{T_{m-1}}\frac{\partial T_{m-1}}{\partial a} \cdots + \frac{1}{T_1}\frac{\partial T_1}{\partial a} + \frac{1}{S_f}\frac{\partial S_f}{\partial a}\right]^2 \\ &\quad + \frac{1}{C_{f,m}}\frac{\partial^2 C_{f,m}}{\partial a^2} + \frac{1}{T_m}\frac{\partial^2 T_m}{\partial a^2} + \frac{1}{C_{s,m-1}}\frac{\partial^2 C_{s,m-1}}{\partial a^2} + \frac{1}{T_{m-1}}\frac{\partial^2 T_{m-1}}{\partial a^2} \\ &\quad + \cdots + \frac{1}{T_1}\frac{\partial^2 T_1}{\partial a^2} + \frac{1}{S_f}\frac{\partial^2 S_f}{\partial a^2} - \left[\left(\frac{1}{C_{f,m}}\frac{\partial C_{f,m}}{\partial a}\right)^2 + \left(\frac{1}{T_m}\frac{\partial T_m}{\partial a}\right)^2\right. \\ &\quad + \left(\frac{1}{C_{s,m-1}}\frac{\partial C_{s,m-1}}{\partial a}\right)^2 + \left(\frac{1}{T_{m-1}}\frac{\partial T_{m-1}}{\partial a}\right)^2 + \cdots + \left(\frac{1}{T_1}\frac{\partial T_1}{\partial a}\right)^2 \\ &\quad \left.\left. + \left(\frac{1}{S_f}\frac{\partial S_f}{\partial a}\right)^2\right]\right\} C_{f,m}T_m C_{s,m-1}T_{m-1}\cdots C_{s,1}T_1 S_f. \end{aligned} \tag{19}$$

The second-order perturbation effect due to the fission-source change $\partial^2 S_f/\partial a^2$ can be evaluated in a similar form to the first-order one but we neglect the second-order effect in this work.

3. CALCULATED RESULTS

3.1 VERIFICATION BY ONE-GROUP CALCULATIONS

In order to verify the above formulation, benchmark calculations were performed for a one-group, one-dimensional slab geometry problem. The geometry consists of 3 homogeneous regions (0.5, 2, 0.5 cm in thickness) where total, absorption and production cross sections are 1.0, 0.2 and 0.36 cm^{-1}, respectively. The multiplication factor is almost unity in the unperturbed system. The perturbation is introduced by changing the density in the central region.

Table I. Change in the effective multiplication factor for the one-dimensional slab geometry in the case of a density change of -0.1%

Method		Δk	1σ
ANISN	(error criterion = 1.0E-6)	-4.05E-4	
2 Independent M. C. Runs		0.659E-4	4.258E-4
Correlated Sampling (Without any approximation)	Without perturbation effect due to fission-source change	-3.345E-4	0.007E-4
	Only with perturbation effect due to fission-source change	-0.712E-4	0.048E-4
	Total	-4.055E-4	0.048E-4
2nd-Order Differential Operator Sampling	Without perturbation effect due to fission-source change	-3.345E-4	0.007E-4
	With perturbation effect due to fission-source change	-4.055E-4	0.048E-4

* Number of total histories = 4.4 million; batch size = 20,000; Fujitsu VPP500

Tables I and II show comparison of the change in the effective multiplication factor between 3 Monte Carlo schemes for the density decrease by 0.1% and 10%, respectively. These schemes are classified as follows:

(a) Direct calculation by 2 independent Monte Carlo runs,

(b) Correlated sampling method by Eq.(12),

(c) Differential operator sampling method in the present work.

Table II. Change in the effective multiplication factor for the one-dimensional slab geometry in the case of a density change of -10%

Method		Δk	1σ
ANISN	(error criterion = 1.0E-6)	-4.204E-2	
2 Independent M. C. Runs		-4.195E-2	0.046E-2
Correlated Sampling (Without any approximation)	Without perturbation effect due to fission-source change	-3.481E-2	0.007E-2
	Only with perturbation effect due to fission-source change	-0.637E-2	0.127E-2
	Total	-4.118E-2	0.127E-2
2nd-Order Differential Operator Sampling	Without perturbation effect due to fission-source change	-3.477E-2	0.008E-2
	With perturbation effect due to fission-source change	-4.168E-2	0.051E-2

* Number of total histories = 4.4 million; batch size = 20,000; Fujitsu VPP500

The deterministic result obtained by the ANISN code[14] is also shown in the tables. All the Monte Carlo calculations were done for 4.4 million histories with the collision estimator.

In the case of the small perturbation (-0.1%), the result obtained by 2 independent Monte Carlo runs is unreliable but the other results obtained by the correlated and differential operator sampling methods are in very good agreement with the ANISN result. As found from these results, the perturbation effect due to the fission-source change is about 20% of the total change in the effective multiplication factor and the fission-source change must be taken into account.

On the other hand, the result of 2 independent Monte Carlo runs is obtained with a sufficiently small statistical uncertainty for the relatively large perturbation (-10%) and is in very good agreement with the ANISN result. The result obtained by the correlated sampling method agrees with the ANISN one within 1 standard deviation but the variance is relatively large. The result by the differential operator sampling method is in good agreement and the variance is much smaller than that by the correlated sampling method.

3.2 VERIFICATION BY CONTINUOUS-ENERGY MONTE CARLO CODE MVP

The procedure has been incorporated into a general-purpose continuous-energy Monte Carlo code MVP[11] to solve general perturbation problems. Firstly we calculated the effects of a density perturbation to a nominal Godiva assembly[15]. The geometry is a bare uranium sphere with a radius of 8.741 cm as illustrated in Fig 1. The density is 18.74 g/cm^3 and the nominal composition is 94.73 wt% ^{235}U and 5.27 wt% ^{238}U. All the MVP calculations were

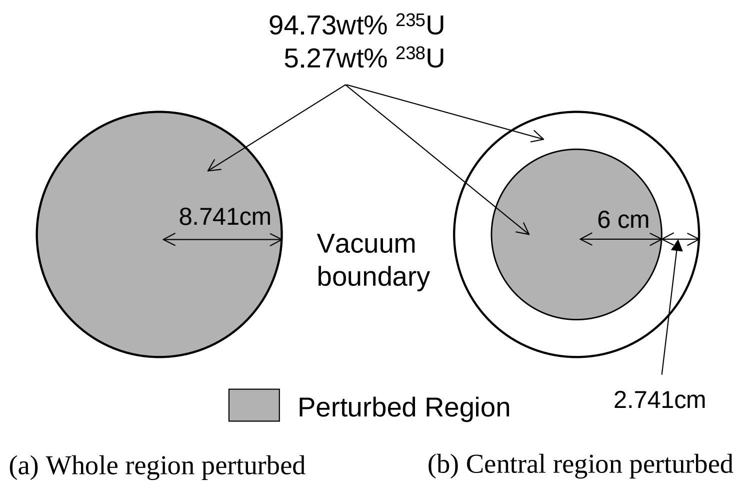

Figure 1. Geometry for the Godiva assembly

performed with a nuclear data library JENDL-3.2[16] in this work. The effective multiplication factor is 1.00402 ($1\sigma = 0.0097\%$) in the unperturbed system.

A density perturbation was introduced by increasing the density from 18.74 to 20.00 g/cm^3. The effective multiplication factor is then 1.05860 ($1\sigma = 0.0098\%$). Table III shows the results for the Godiva assembly with the MCNP4C result[17]. The reference result was obtained

Table III. Change in the effective multiplication factor for Godiva in the case of the whole region perturbation

Method		Δk	1σ
2 Independent M. C. Runs*		5.458E-2	0.014E-2
MCNP4C		5.282E-2	0.015E-2
MVP Correlated Sampling** (Without any approximation)	Without perturbation effect due to fission-source change	5.252E-2	0.006E-2
	Only with perturbation effect due to fission-source change	0.196E-2	0.030E-2
	Total	5.448E-2	0.031E-2
MVP 2nd-Order Differential** Operator Sampling	Without perturbation effect due to fission-source change	5.252E-2	0.006E-2
	With perturbation effect due to fission-source change	5.485E-2	0.028E-2

* Number of total histories = 40 million: batch size = 20,000: Fujitsu VPP500

** Number of total histories = 6 million: batch size = 40,000: Linux/Alpha by the 2 independent Monte Carlo runs with the large number of histories (40 million histories). All the results without the perturbation effect due to the fission-source change underestimate the reference one by about 3% including the MCNP4C result. Taking into account the fission-source change, the Δk values obtained by both the correlated sampling and second-order differential operator methods are well improved and are in very good agreement with the reference one. However, the statistical uncertainties (1σ) are larger than those without the perturbation effect due to fission-source change by a factor of 5. As found in Table III, the large statistical uncertainties are ascribed to the perturbation effect due to fission-source change.

We also investigated a 2-region perturbation problem for Godiva where only the central region was perturbed to verify the present procedure for the relatively large effect due to the fission-source change. The radius of the perturbed region is 6 cm and the isotropic composition in both the regions is the same as the nominal Godiva one. The density change for the introduced perturbation is also the same as in the case of the whole region perturbation.

Table IV. Change in the effective multiplication factor for Godiva in the case of only the central region perturbed

Method		Δk	1σ
2 Independent M. C. Runs*		2.980E-2	0.014E-2
MVP Correlated Sampling** (Without any approximation)	Without perturbation effect due to fission-source change	1.869E-2	0.005E-2
	Only with perturbation effect due to fission-source change	1.085E-2	0.035E-2
	Total	2.954E-2	0.035E-2
MVP 2nd-Order Differential** Operator Sampling	Without perturbation effect due to fission-source change	1.869E-2	0.005E-2
	With perturbation effect due to fission-source change	2.988E-2	0.028E-2

* Number of total histories = 40 million: batch size = 20,000: Fujitsu VPP500

** Number of total histories = 6 million: batch size = 40,000: Linux/Alpha

Table IV shows the Δk values for Godiva in the case of the central region perturbed. The reference solution was obtained by the 2 independent Monte Carlo runs with 40 million histories. In this case, the perturbation calculation without the effect due to the fission-source change gives a significant discrepancy, while the Δk values are in very good agreement with the reference one by both the correlated and second-order differential sampling method with the changed fission-source effect. Thus the perturbation effect due to the fission-source change is well estimated by the present procedure for the relatively large fission-source change effect of about 37%. The statistical uncertainties also become larger by taking into account the changed fission-source effect as in the case of the whole region perturbation.

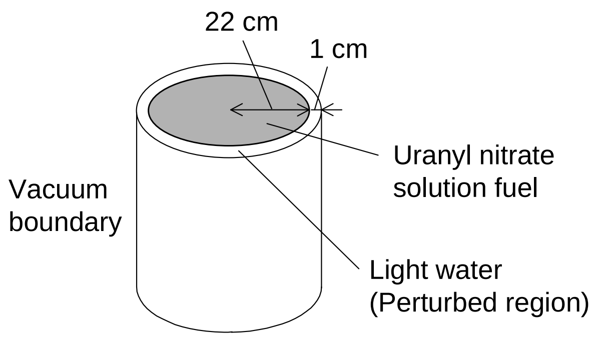

Figure 2. Geometry for the simplified STACY core

Table V. Atomic number density for the STACY fuel

	Nuclide	Density (atoms/barn/cm)
Fuel	^{235}U	7.92122E-5
	^{238}U	7.06258E-4
	^{1}H	5.69525E-2
	^{14}N	2.87772E-3
	^{16}O	3.80270E-2
Reflector	^{1}H	6.66566E-2
	^{16}O	3.33283E-2

In order to investigate the perturbation effect for a thermal system, we analyzed an infinite cylindrical core containing a uranyl nitrate solution fuel of the STACY facility at the Nuclear Fuel Cycle Safety Engineering Research Facility (NUCEF)[18]. The actual STACY geometry is a cylindrical tank 60 cm in diameter with a water reflector. However, we simplified and modified it as an infinite cylindrical model with a radius of 22 cm and a 1cm thick reflector for a benchmark problem as illustrated in Fig. 2. The uranyl nitrate solution fuel has a ^{235}U enrichment of 9.97 wt% and an isotropic composition for the uranium concentration 310.1 g/ℓ and the acidity 2.17 mol/ℓ. The atomic number density used is listed in Table V. A perturbation was introduced by decreasing the density in the reflector region by 10%.

Table VI. Change in the effective multiplication factor for the simplified STACY model

Method		Δk	1σ
ANISN	(Error criterion = 1.0E-6)	-1.698E-3	
2 Independent M. C. Runs*		-1.690E-3	0.145E-3
MCNP4B**		-3.642E-3	0.042E-3
MVP Correlated Sampling*** (Without any approximation)	Without perturbation effect due to fission-source change	-3.658E-3	0.025E-3
	Only with perturbation effect due to fission-source change	2.121E-3	0.226E-3
	Total	-1.537E-3	0.229E-3
MVP 2nd-Order Differential*** Operator Sampling	Without perturbation effect due to fission-source change	-3.655E-3	0.028E-3
	With perturbation effect due to fission-source change	-1.607E-3	0.261E-3

* Number of total histories = 40 million: batch size = 20,000: Fujitsu VPP500

** Number of total histories = 2.5 million: batch size = 5,000: Fujitsu AP3000

*** Number of total histories = 10 million: batch size = 20,000: Linux/Alpha

Table VI shows the results of the perturbation calculations for the simplified STACY model. Since the 2 independent Monte Carlo result involves a statistical uncertainty of about 9%, the reference Δk value was obtained by ANISN implemented in the SRAC code[19] with 107 energy groups, S_8 angular quadrature and P_1 scattering cross sections. We also performed the perturbation calculation by MCNP4B[20] because no fissionable region was perturbed. The large discrepancies can be found for the results obtained without the perturbation effect due to the fission-source change including the MCNP4B result. The MVP results with the changed fission-source effect are in good agreement with the ANISN one but the statistical uncertainties are still large. The large uncertainty is obviously caused by the changed fission-source effect. The perturbation effects without and only with the fission-source change tend to cancel out each other and thus the total Δk results in the small value, which makes the statistical uncertainty relatively larger.

CONCLUSIONS

We have proposed the formulae to evaluate the perturbation effect due to the fission-source change for the correlated and differential operator sampling methods in this work. It has been shown that the change in the effective multiplication factor including the changed fission-source effect can be evaluated by the formulae and the estimate of the change is improved.

The statistical uncertainties of the k_{eff} change become larger in taking into account the perturbation effect due to the fission-source change: a factor of $5 \sim 7$ for the Godiva assembly, an order of about one for the simplified STACY model. The dominant part of the uncertainties arises from evaluation of the perturbation effect due to the fission-source change Further study is required to improve the uncertainties by introducing an approximation method.

The statistical uncertainties in the differential operator sampling method tend to be smaller than those in the correlated sampling method as the perturbation becomes large. This may be caused by truncation of the higher order fission-source change in the differential operator sampling method, and quantitative and theoretical study would be necessary.

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