# ON THE QUALIFICATION OF BOILING WATER REACTOR STABILITY MARGIN INDICATORS USING LINEAR STABILITY ANALYSIS

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## ABSTRACT

This paper adresses some important issues concerning BoilingWater Reactor (BWR) stability. The first issue is the relation between the Decay Ratio (DR) and the operational stability margins. It is shown that it can be misleading to use solely the value of the DR in BWR stability monitoring as an indicator for stability. The second part of the paper examines the stability characteristics of a natural circulation BWR via the root-loci of the complex poles of the characteristics equation of the system. The different instability mechanisms of the thermalhydraulic subsystem of the reactor cannot be attributed to one pole pair. For the thermohydraulic subsystem of a BWR, always the same pole pair dominates and determines the stability. However, the root loci of the reactor system show that two different pole pairs can be interchanged in dominating the system stability, depending on the strength of the void reactivity feedback.

## 1. INTRODUCTION

For the quantitative description of BWR stability, the Decay Ratio (DR) is a widely used and well-accepted parameter.[1] The DR is defined as the ratio between two consecutive maxima of the impulse response of the system. The DR is believed to be a good indicator of BWR stability and it is also used by core stability monitoring systems. Currently, several commercial BWR plants are equipped with such a safety system, which determines the DR through noise analysis of measured neutron flux signals.

The DR proves to be a correct measure of the linear stability of the system. However, relying exclusively on the value of the DR can be misleading since the DR does not give any information about the operational margin to unstable system behavior. It is demonstrated here that a reactor operating at a point described by a certain DR can be closer to instability than at an operating point described by a larger DR. This fact is very important from a practical point of view.

Van der Hagen *et al.*[2] have performed a series of stability measurements on the Dutch Dodewaard natural circulation BWR. They proved that a very slight change (less than a few percents) in the operational conditions might cause a dramatic increase in the DR (from 0.7 to 1.02 in their experiments). This means that the reactor was much closer to the stability boundary (SB) in operational sense than one would expect based on the value (0.7) of the DR. For reactor operators it would be useful to have also the margins to unstable reactor behavior expressed in terms of operational parameters (power, system pressure etc.), not only the DR.

To gain more insight into this problem we have investigated it using a low-dimensional (so-called reduced-order) analytical BWR model. The geometry and parameters of the Dodewaard reactor are used in this study.

The second part of the paper examines the stability characteristic of a natural circulation BWR via the root loci of the complex poles of the system. The relation of the poles to the different physical phenomena driving the instabilities is examined there.

## 2. THE RELATION BETWEEN DECAY RATIO AND OPERATIONAL STABILITY MARGINS

In a natural circulation BWR the power, the system pressure, and the temperature of the feedwater are the independent operating variables. We have investigated the relation between the DR and the operational margins to instability expressed in these three variables.

The reduced-order BWR dynamics model, developed at the Interfaculty Reactor Institute[3], enables a fast determination of the linear stability by frequency domain analysis. Neutron point-kinetics and a first-order fuel heat transfer process are applied in the model. For the description of the channel thermalhydraulics the homogeneous equilibrium mixture model is used. Flashing, subcooled boiling, and carry-under are not taken into account. The model uses the so-called Zuber number ($N_{Zu}$) and Subcooling number ($N_{sub}$) to characterize the reactor operating conditions. The former is proportional to the ratio between power and core inlet mass flow rate; the latter is proportional to the core inlet subcooling. Assuming a stationary feedwater system, a simple relationship exists which determines typical trajectories of possible reactor operating conditions in the $N_{Zu}$-$N_{sub}$ plane.

The solid lines in figure 1a show the linear stability boundary (SB) and lines of operating points with equal DR (equi-DR lines) in the $N_{Zu}$-$N_{sub}$ plane. In an operating point on the right side of the SB the reactor is linearly unstable (the DR is larger than unity). Using the $N_{Zu}$-$N_{sub}$ plane has the advantage that the SB and the equi-DR lines are practically independent of the system pressure. Of course, the positions of the different operating points of the system in the $N_{Zu}$-$N_{sub}$-plane depend on the pressure.

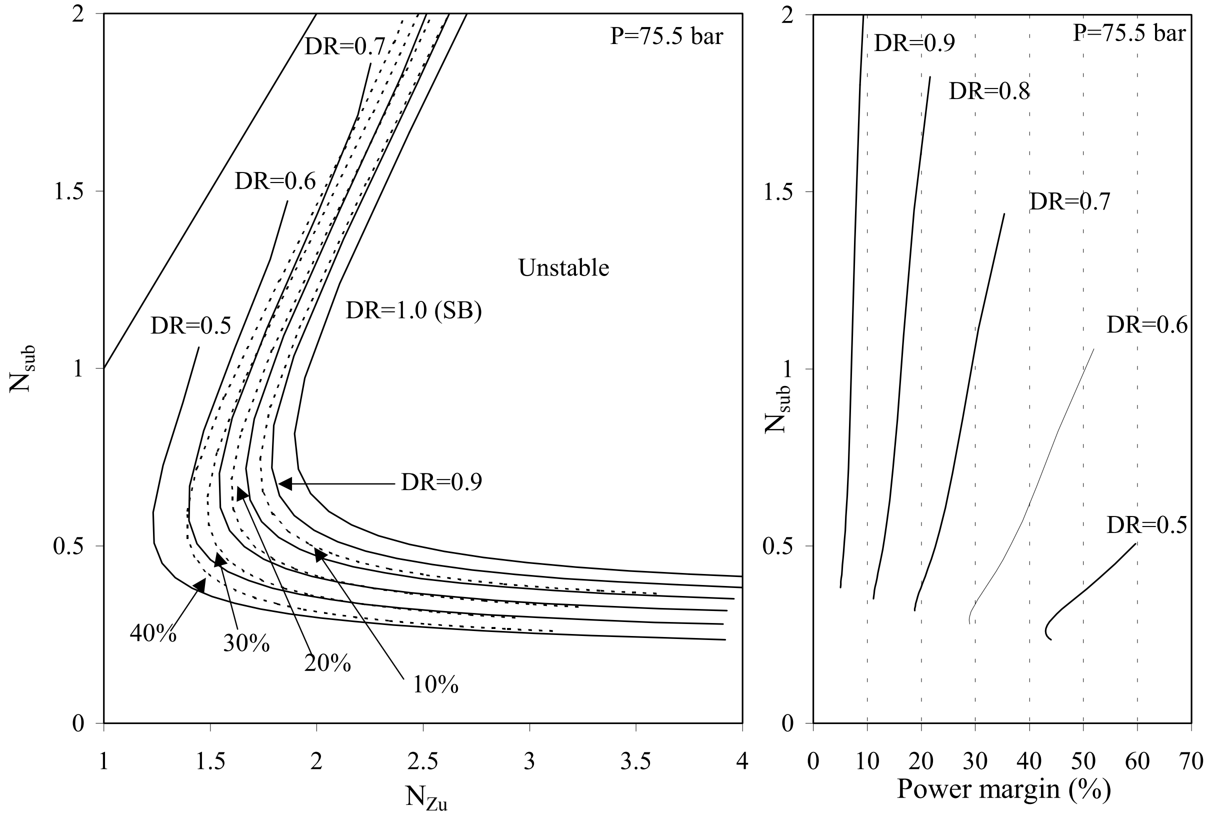



Figure 1. Equi-DR lines (solid) and power equi-stability margin lines (dashed) intersect in the $N_{Zu}$-$N_{sub}$ plane (left: Figure 1a). This means that the power stability margin changes along the same equi-DR line (right: Figure 1b).

The operational margin to instability in terms of the reactor power is determined by changing the power until the stability boundary is reached, meanwhile keeping the other independent variables constant. This is done for the operating points along the different equi-DR lines. The results are shown in figure 1b. It is obvious that operating points with the same DR can have quite different power margins to instability. Moreover, the curves show that an operating point with a certain DR may have a larger margin to instability than an operating point with a smaller DR (consider for example the DR=0.7 and DR=0.6 curves around 25-35% power margin).

Similar to figure 1b, the operational margins for the other two independent variables are given in figure 2. The curves are given only in the low subcooling region; since in practice operating conditions in the high subcooling region cannot be reached.

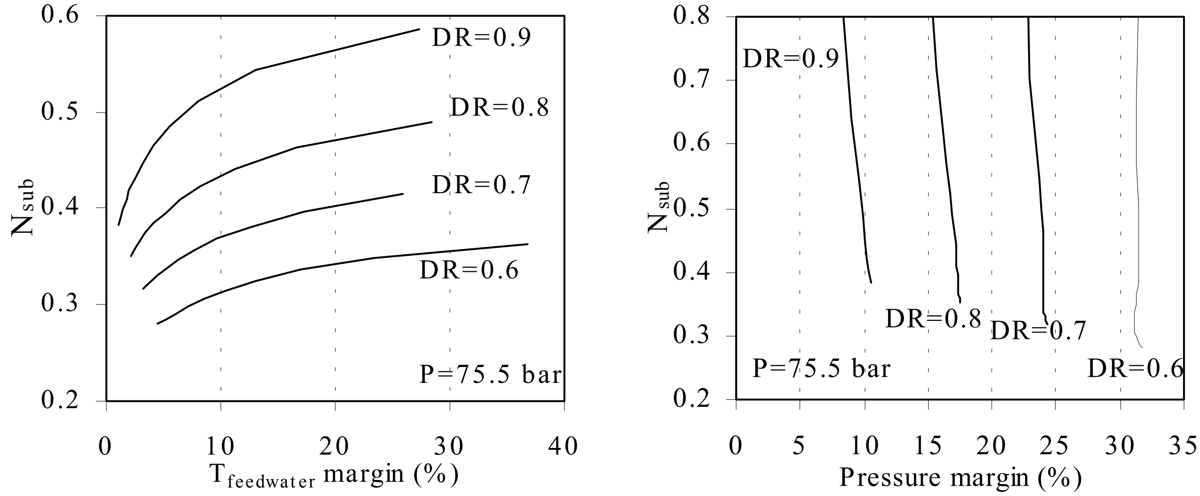



Figure 2. Operational stability margins in terms of the feedwater temperature (left figure) and the system pressure (right figure) along different equi-DR lines.

The stability margin in terms of the feedwater temperature changes a lot along an equi-DR line. A remarkable result is that an operating point with DR=0.9 might have a feedwater temperature margin as high as 25%, while an other point with DR=0.6 has a margin of only 5%! This reconfirms our concern about the applicability of the DR as a sole indicator for stability.

The pressure margin changes only slightly as a function of operating point along the equi-DR lines. It indicates that for operational changes in the pressure the decay ratio can be used as a reliable indicator for the margin to instability.

The above reported operational stability margins and equi-stability margin lines have been evaluated at the nominal system pressure of the Dodewaard reactor (75.5 bar). However, since the equi-DR lines are independent of pressure but the positions of the operating points in the $N_{Zu}$-$N_{sub}$ plane do depend on the system pressure, the operational stability margins also depend on the pressure. The power and the feedwater temperature equi-margin lines at nominal pressure are compared with the same equi-margin lines at 35 bar in figure 3. The curves are given in the power-$T_{feed}$ plane, which is more useful from practical point of view to use since these are the independent variables of the system. The difference is quite significant especially for the feedwater temperature margin.

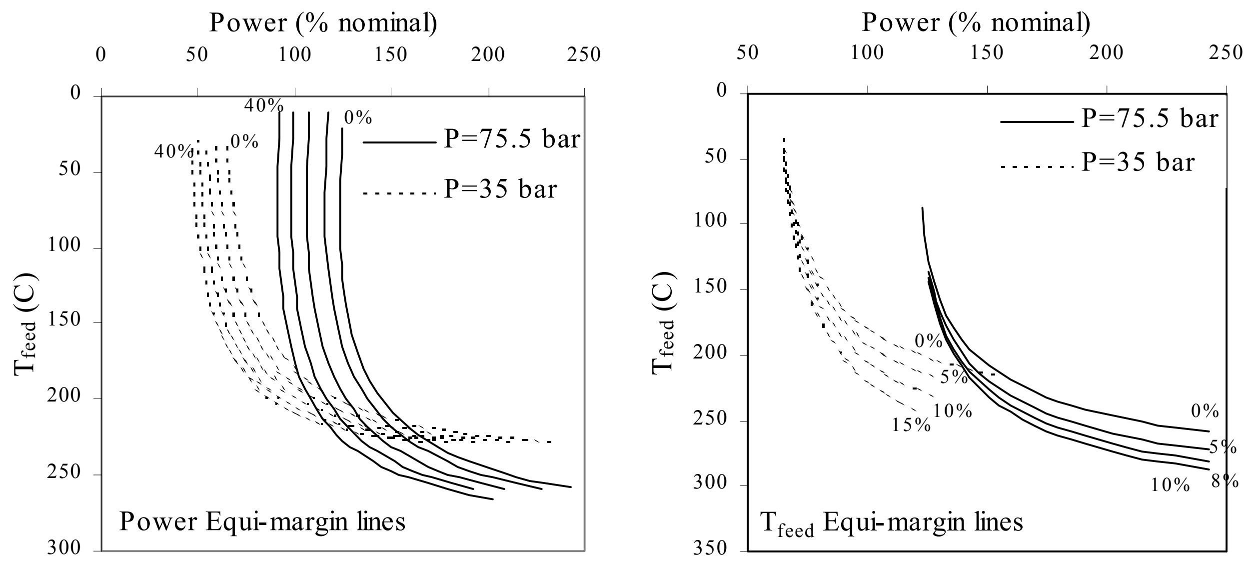



Figure 3. The power stability margins lines (0%, 10%, 20%, 30%, 40% lines from right to left, left figure) and the feedwater temperature margin lines (right figure) change with the system pressure. The curves are shown in the Power – $T_{feed}$ plane.

## 3. LINEAR BWR STABILITY ANALYSIS OBTAINED VIA ROOT-LOCI

### 3.1 INTRODUCTION

We have examined above the relation between DR and stability margins, which is determined by the stability characteristic of the BWR system. In this section, the stability characteristic of a BWR is examined further by considering the poles of the dynamic (reactor) system using linear stability analysis. A BWR is a higher-order dynamical system using the terminology of linear stability analysis. The stability characteristics are determined by the position of the poles of the characteristic equation of the system in the complex domain. A higher-order system has several poles, which are real and/or form complex conjugate pole pairs. The complex conjugate pole pair at the Laplace variable $s = \sigma \pm i\omega$ contributes to the impulse response of the system as:

$$x(t) \sim \sin(\omega t) \cdot e^{\sigma t}, \qquad (1)$$

and the DR associated with this pole pair is DR = $\exp(2\pi\sigma/\omega)$.

To simplify the treatment and make the analysis more transparent, we investigate first the thermalhydraulic subsystem of a BWR separately. The dynamic instabilities of the thermalhydraulic subsystem occur when the total channel pressure drop response to inlet mass flux perturbations is zero at a certain, finite frequency (Fukuda Kobori[6], Rao et al.[7]). This can be expressed in the form of the characteristic equation of the system in the complex- (Laplace-) domain:

$$\frac{\partial \Delta P_{total}}{\partial M_{C,i}}(s) = 0 \,, \tag{2}$$

where $M_{C,i}$ is the channel inlet mass flux density and $s = i\omega$. This characteristic equation can be split into physical pressure drop derivative terms, which are the transfer functions from inlet mass flux to the different type of pressure drops over the channel. We distinguish the following pressure drop terms: inertial, one-phase frictional, two-phase frictional, gravitational and accelerational pressure drop.

We examine how the poles of the thermalhydraulic subsystem correspond to the different pressure drops present in the boiling channel and what the relation of the poles is to the physical instability mechanism involved. Thereafter, we analyze the coupled (reactor) system using the parameters of a natural circulation BWR.

## 3.2 ANALYSIS OF THE THERMALHYDRAULIC SUBSYSTEM

In this section, the root loci of the poles are studied as the operating conditions change, and the individual contribution of pressure drop terms in the characteristic equation is studied. The pure thermalhydraulic instabilities form the basis of the stability characteristics of a BWR. The coupling between the thermalhydraulics and neutronics, the void reactivity feedback, can further stabilize or destabilize the coupled system. First, the thermalhydraulic subsystem of a BWR is examined. Our reduced-order BWR dynamics model is capable of predicting the instabilities of the thermalhydraulic subsystem. As in the previous section, the data and geometry of the Dodewaard reactor are used.

### 3.2.1. FORCED CIRCULATION SYSTEM

We start the investigation studying a forced circulation boiling system, which has the dimensions of one average coolant channel. Figure 4a shows the SB’s of the system.

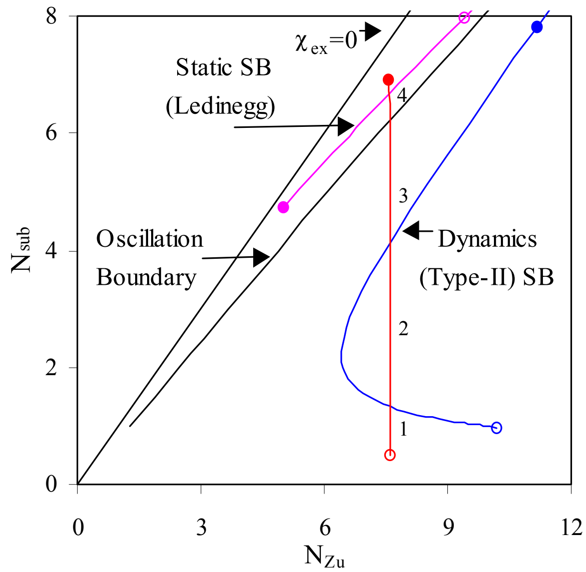



Figure 4a. Dynamic and static SB's of a forced circulation two-phase flow channel.

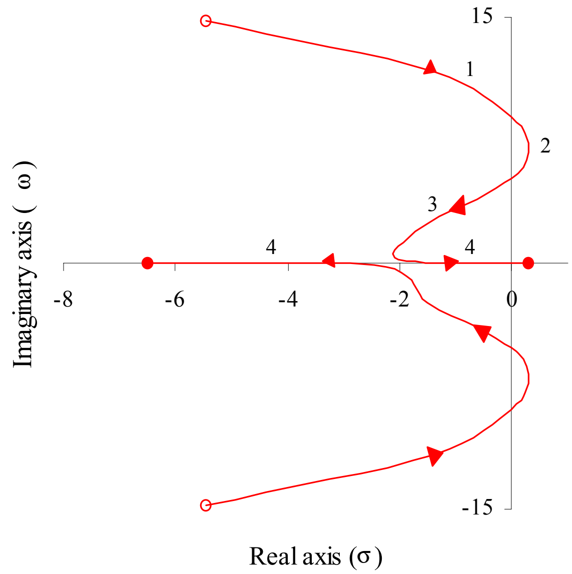



Figure 4b. The complex conjugate pole pair of the system along the red trajectory of operating points in figure 4a. The third, real, pole of the system is not depicted being irrelevant for the stability.

The dynamic SB separates the stable and the so-called Type-II unstable operating region The static or Ledinegg instability at high subcooling is inherent to forced circulation systems and caused by the interaction of the internal pressure drop characteristic of the channel and the driving external pressure drop. These results were obtained with a constant external pressure drop boundary condition. Figure 4b shows the poles of the system along a trajectory of operating points in figure 4a. Since this simple boiling channel is modeled as a third-order system it has 3 poles, 2 of which form a complex conjugate pole pair in operating points below the oscillation boundary. This complex conjugate pole pair is responsible for the Type-II instability as they shift into the right half plane of the complex domain when the operating conditions move to the Type-II unstable region. Moving further up with the operating conditions in figure 4a, at higher subcooling, the system becomes again stable and the oscillation boundary is reached. At that point, the complex pole pair becomes real and the two real poles start to move in opposite directions. As the Ledinegg SB is reached, the pole moving into the positive direction crosses the origin and the system becomes static unstable.

The characteristic equation (2) is also valid for the static (Ledinegg) instability as $s \to 0$. Figure 5 shows how the different pressure drop terms in the characteristic equation change along the static and dynamic SB. In the dynamic case (Figure 5b) these terms are complex and represented by their gain and phase.

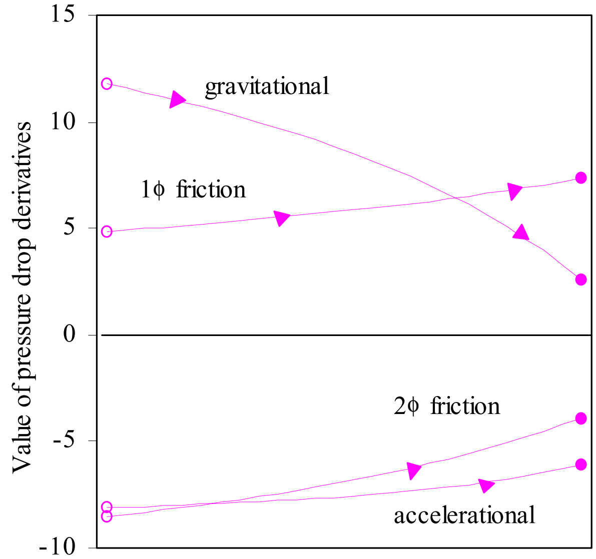



Figure 5a. The values of the different pressure drop derivatives along the static (Ledinegg) SB. Arrows indicate the direction of moving towards higher $N_{sub.}$

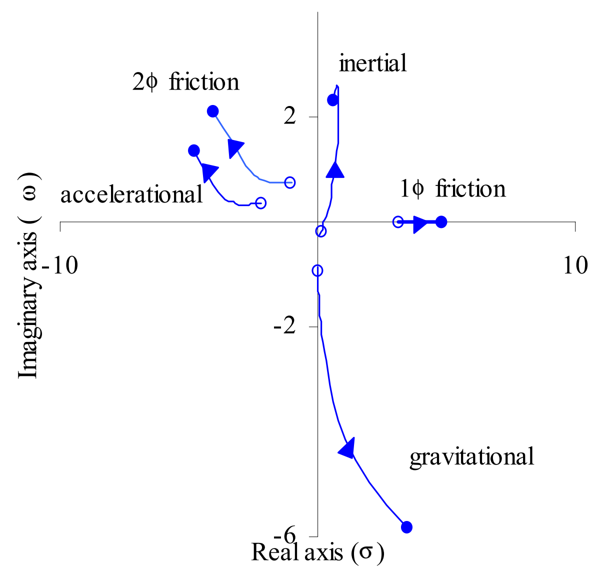


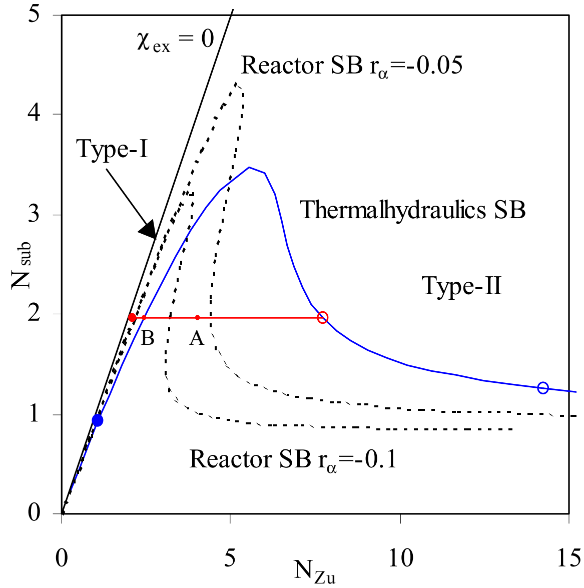

Figure 5b. The locus of the complex vectors of the different pressure drop transfer functions at operating conditions along the dynamic (Type-II) SB.

In case of the static (Ledinegg) instability, the two-phase frictional and accelerational pressure drop derivatives are negative (thus destabilizing), while the gravitational and one-phase frictional pressure drop are stabilizing, which is plausible on physical ground. The static instability occurs at high subcoolings (low exit qualities) because then the negative (destabilizing) pressure drop derivatives become too large to be compensated by the stabilizing terms. This is due to the abrupt change in the void fraction induced by fluctuations in the flow quality at low exit qualities.

In case of the dynamic (Type-II) instability, the above pressure drop terms play the same stabilizing (gravity, one-phase friction, inertia) or destabilizing (two-phase friction, acceleration) role as for the static instability. This can be seen from the phases of the different terms in figure 5b.

### 3.2.2. NATURAL CIRCULATION SYSTEM

In the following, a natural circulation thermalhydraulic system with riser placed on the top of the heated section is considered. In the analysis of a system with a riser two different approaches can be used in the model. One is a nodal approach that divides the riser into several nodes and uses spatial approximations for the flow quality and mass flux within a node. The other is an exact treatment of the propagation of density waves in the riser using frequency domain analysis.



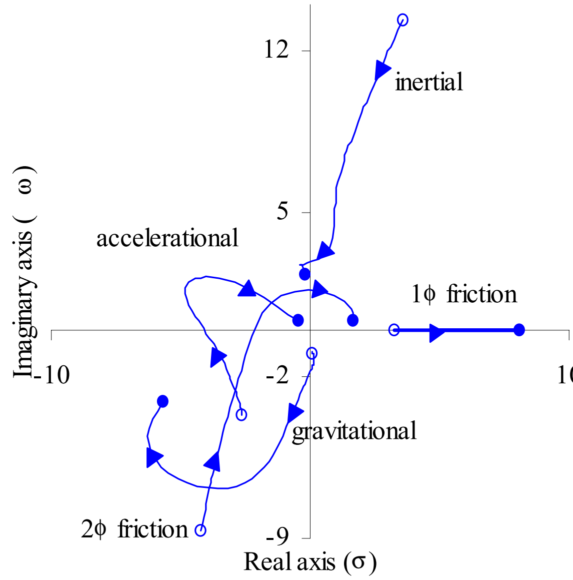




Figure 6a. The SB of the thermalhydraulic subsystem of Dodewaard (blue solid line) and SB's of coupled reactor systems with different void coefficients (dashed lines).

Figure 6b. The locus of the pressure drop transfer functions along the SB of the natural circulation thermalhydraulic subsystem. The arrows show the direction of moving from the Type-II to the Type-I SB.

For the Dodewaard reactor, using four spatial nodes is a reasonably good approximation (Van Bragt [4]). The exact frequency-domain treatment of the riser results in the appearance of complex exponential (delay) terms in the transfer functions describing the propagation and delay of void fraction and quality in the riser. It makes the model infinite order, having infinite number of complex conjugate pole pairs. Using the nodal approach with four riser nodes, the thermalhydraulic subsystem is modeled as a 7$^{th}$ order system having three complex conjugate pole pairs and one real pole. We have used this approach.

The SB, the solid line given in figure 6, shows that a new region of instability emerges close to the zero quality line: the Type-I instability region. This classification of boiling channel instabilities was given by Fukuda and Kobori [6]. The Type-I instability is caused by the large gravitational pressure drop change in the riser due to core exit quality fluctuations at low core exit qualities again owing to the abrupt change in void fraction as a function of the quality (Fukuda, Kobori [6]). Thus the physical origin of the phenomenon is similar to the Ledinegg instability. However, the latter one does not occur in natural circulation but in forced circulation systems.

Similar to figure 5b, figure 6b shows how the different pressure drop transfer functions change along the SB. One can see that as the Type-I region is approached, the phase of the gravitational pressure drop transfer function gradually becomes destabilizing and its relative importance (gain) increases. The gravitational pressure drop term is indeed responsible for the Type-I instability.

3.3 ANALYSIS OF A NATURAL CIRCULATION BOILING WATER REACTOR

We continue with the analysis of a natural circulation reactor by including and varying the strength of the void reactivity feedback in the previously examined thermalhydraulic system.

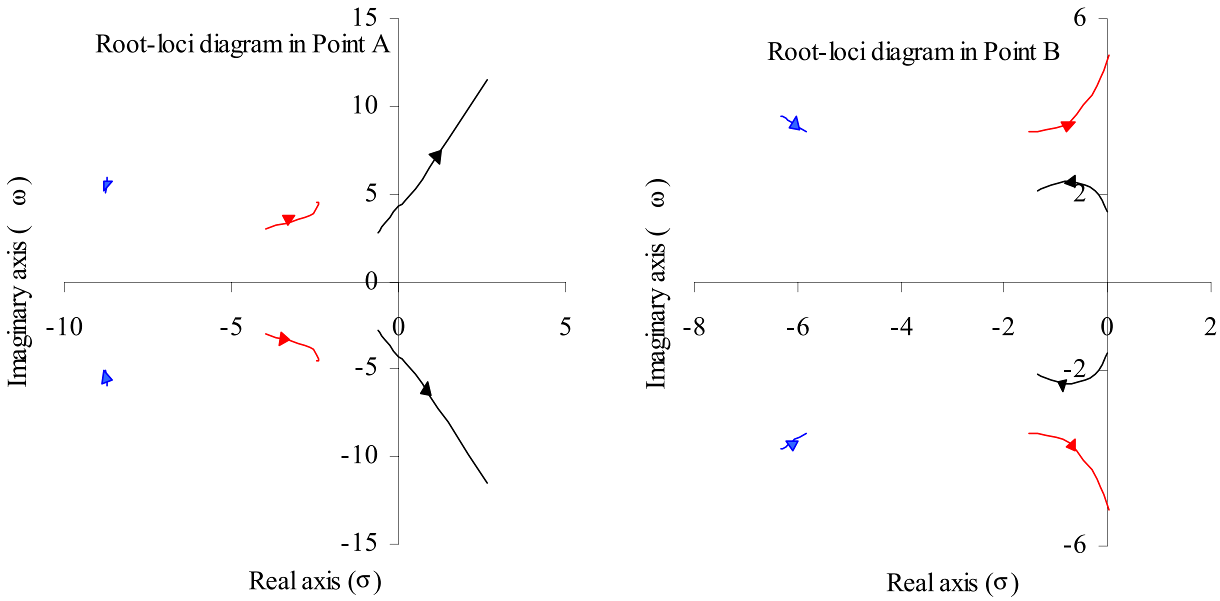



Figure 7. Root-loci diagram of a natural circulation reactor using the void coefficient as parameter. The Type-II oscillations are destabilized in point A in figure 6a (left figure), while in point B the Type-I oscillations are stabilized (right figure) by increasing void feedback. Arrows indicate the direction of increasing void coefficient (only the complex conjugate pole pairs are shown).

The approach is similar to that of March-Leuba[5] who has also used a simplified model. The value of the void reactivity coefficient is varied at two different operating points denoted by A and B in figure 6a. Parts of the root-locus curves of the system using the void reactivity coefficient ($r_\alpha$) as parameter are drawn in figure 7 for the three complex pole pairs. The locations of the reactor SB's in figure 6a already show that the Type-II unstable region increases by increasing (in absolute sense) the void coefficient, consequently point A becomes unstable at a certain value of the void coefficient. However, the Type-I region decreases and point B (which was exactly on the Type-I SB for the thermalhydraulic system) becomes stable. The same can be seen in more detail in the root-locus diagrams in figure 7. It is well known in linear stability analysis that the root-loci of the feedback (coupled) system approach the poles of the so-called open-loop transfer function if the feedback coefficient approaches zero (Hetrick [8]). The beginning of the root-locus curves in figure 7 corresponds to this ($r_\alpha$=0) situation. In our model, the open-loop transfer function is (ignoring the small contribution of the Doppler coefficient) the product of the zero power reactor transfer function, the (first-order) transfer function from fuel to coolant temperature and the transfer function of the thermalhydraulic subsystem. The first two transfer functions have only real poles while the thermalhydraulic subsystem is modeled as a $7^{th}$-order system having three complex conjugate pair and a real pole. Thus, the complex pole pairs are the poles of the thermalhydraulic subsystem at $r_\alpha=0$ and their position is gradually changed (and thus the stability of the system) as $r_\alpha$ is increased.

It is remarkable in case of point B that the Type-I thermalhydraulically unstable (low frequency) pole is stabilized by an increasing void coefficient, but the pole at a higher frequency, which was originally stable, becomes unstable, in fact, Type-II unstable. This happens at such a high value of the void coefficient that the SB is shifted over point B too, getting it into the Type-II unstable region. As Van Bragt[4] has pointed out, the Type-II oscillations are destabilized by increasing the strength of the (negative) void feedback because at the relative high frequencies of the (thermalhydraulics) Type-II oscillations a large phase lag is caused by the fuel transfer function and the thermalhydraulic subsystem. This large phase lag turns the negative feedback into positive thus destabilizing the system. The frequency of the Type-I oscillations is generally low, therefore the phase lags are much smaller and the void reactivity feedback has a stabilizing effect on the thermalhydraulic oscillations. This feature is the origin of the interchange between the two dominant poles in figure 7b. At higher frequencies, the gain of the fuel transfer function is considerably smaller. This has to be compensated by a high feedback strength (void coefficient), which is needed for the second (higher frequency) pole pair to become unstable.

Similar behavior of interchanging the dominance between two different pole pairs can be found if one follows trajectories (the red line in figure 6a) of operating points leading from the Type-II to the Type-I region for different void reactivity feedback coefficients. First, we follow this trajectory in case of the thermalhydraulic subsystem without void reactivity feedback. Figure 8a shows that the same pole pair remains the dominating pair in both types of instability regions, although we have seen that different types of physical mechanisms are responsible for the instabilities. This shows that one pole (pair) of the natural circulation thermalhydraulic system cannot be identified with one physical phenomenon. On the contrary, each pole contains a contribution from each physical instability mechanism.

Similar calculations have been done for reactor systems with two different void feedback coefficients. The results can be seen in figure 8b. For a low value of the void reactivity coefficient, a similar behavior can be observed as for the thermalhydraulic subsystem: the same pole is the dominating one in both regions. This is plausible because for weak feedback the coupled system behaves more or less in the same manner as the thermalhydraulic subsystem. However, for stronger feedback the two pole pairs change their dominance. First, the higher frequency pole dominates in the Type-II region due to the same reason as in case of figure 7b. However, in the Type-I region (in the left-most point of the red trajectory in figure 6a) the lower frequency pole dominates and moves into the right half plane. At this operating point the void fraction in the core is very low (low exit quality), the void feedback becomes quite weak, and the system behaves again almost like the thermalhydraulic subsystem shown in figure 8a. The joining of the different void coefficient loci shows this just as the fact that they approach the pole of the thermalhydraulic subsystem, which is indicated by a blue solid square in the figure.

The interchanging of the dominating pole pair has an interesting consequence. The positions of the poles show that in the vicinity of the interchange point (around σ = -0.9) the dominating higher-frequency pole has a higher DR, but a more negative real value (σ) than the lower-frequency pole.

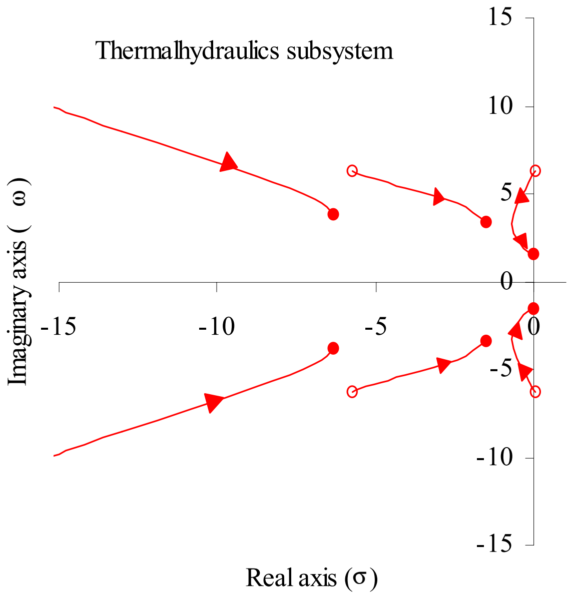



Figure 8a. Poles of the natural circulation thermalhydraulics subsystem as moving along the red trajectory in figure 6a from the Type-II to the Type-I region.

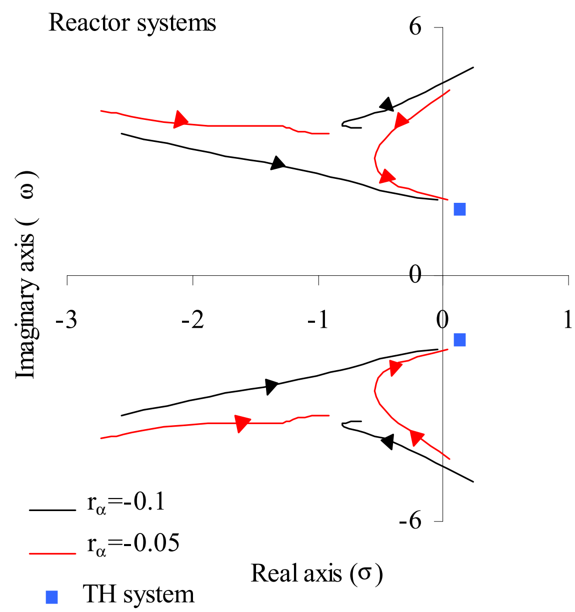



Figure 8b. Poles of the reactor systems along the same trajectory of operating points for two different void reactivity feedback coefficients.

## CONCLUSIONS

Numerical investigations using a reduced-order BWR dynamics model have been carried out to examine the relation of different stability indicators (DR, operational stability margins) in case of a natural circulation BWR. It is shown that the system might be much closer to instability than the value of the DR would indicate, thus one might draw erroneous conclusions about the operational stability margins relying solely on the DR.

The stability characteristics of BWR's, which is the root of the relation between the DR and stability margins, have been investigated by examining the role of the different poles of the reactor system using linear system analysis. It was confirmed that the origin of the instabilities lies in the thermalhydraulic subsystem. To simplify the treatment and to make it more transparent, the relation between the poles of the thermalhydraulic subsystem and the different pressure drop terms has been analyzed. It was demonstrated that the poles of the thermalhydraulic subsystem of a natural circulation BWR cannot be identified clearly with one instability mechanism. It is clear, however, which types of pressure drop terms are responsible for the different types of instabilities. For Type-II instabilities the two-phase frictional and the accelerational pressure drop are the important destabilizing terms, whereas for the Type-I instability the gravitational pressure drop is the most important destabilizing term. In case of our reference natural circulation reactor with strong void reactivity feedback an interchange of the dominating poles has been found for the transition between the two different instability types, whereas for the system with small feedback or for the thermalhydraulic system, the same pole remains the dominating one. The interchange of poles warns that one has to consider the different poles watchfully to be able to determine the correct stability characteristics (DR's, stability margins) of a natural circulation BWR using a reduced-order model.

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